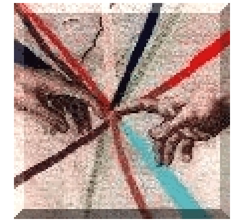
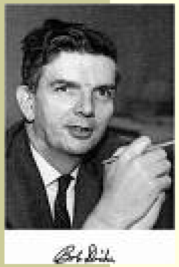


Doorway States and the Super-Radiant Mechanism

N.Auerbach, TAU and MSU



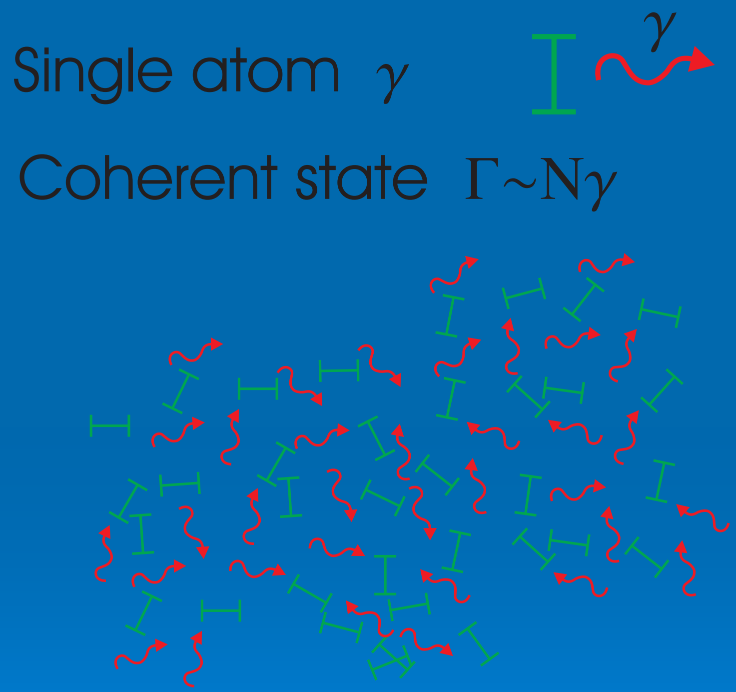
Coherence in Spontaneous Radiation Process

- R.H.Dicke, Phys.Rev. **93**, 99 (1954)
- *"In the usual treatment of spontaneous radiation by a gas, the radiation process is calculated as though the separate molecules radiate independently of each other…..*
- *It is clear that this model is incapable of describing a coherent spontaneous radiation process…This simplified picture overlooks the fact that all the molecules are interacting with a common radiation field and hence cannot be treated as independent."...*
- *"A gas that radiates strongly because of coherence will be called "super-radiant"".*

Superradiance, collectivization by decay

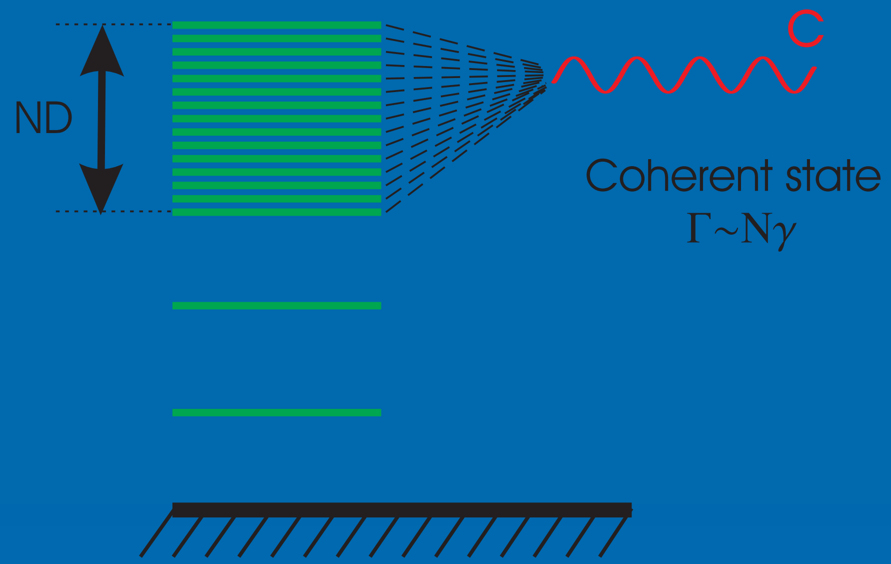
Dicke coherent state

N identical two-level atoms
coupled via common radiation



Analog in nuclei

Interaction via continuum
(Trapped states) self-organization



The Effective Hamiltonian

The total wave function of the system ,

$$|\Psi\rangle = Q|\Psi\rangle + P|\Psi\rangle$$

satisfies the Schrodinger equation

$$H|\Psi\rangle = E|\Psi\rangle$$

that can be decomposed into a set of coupled equations:

$$(E - H_{QQ})Q|\Psi\rangle = H_{QP}P|\Psi\rangle$$

and

$$(E - H_{PP})P|\Psi\rangle = H_{PQ}Q|\Psi\rangle$$

where the notation is $H_{AB} = AHB$

Effective Hamiltonian (cont'd)

Eliminating $P|\Psi\rangle$ we obtain:

$$\left(E - H_{QQ}^{ef}\right)Q|\Psi\rangle = 0$$

with the effective Hamiltonian:

$$\mathrm{H}_{\mathrm{QQ}}^{ef} = H_{QQ} + H_{QP}\frac{1}{E^{(+)} - H_{PP}}H_{PQ}$$

Here $E^{(+)} = E + i0$

The second term of the effective Hamiltonian contains a real and imaginary part of the propagator

$$\mathrm{G}^{(+)}(E) = \frac{1}{E^{(+)} - H_{PP}}$$

The Effective Hamiltonian (cont'd)

These originate from the principal value and delta function $\delta\left(E - H_{PP}\right)$.

The imaginary part, $-\left(i/2\right)W$ is given by:

$$W = 2\pi \sum_{c} H_{QP} \left|c\right\rangle \left\langle c\right| H_{PQ}$$

where c are the open channels.

The effective Hamiltonian in Q-space is non-Hermitian

$$\mathrm{H}^{\mathrm{eff}} = \bar{H} - \frac{i}{2} W$$

where $\bar{H} \equiv \bar{H}_{QQ}$ is a symmetric real matrix that includes, apart from the original Hamiltonian of the Q-space , H_{QQ} ,

the principal value contribution of the QP-coupling.

The cross section for a reaction $a \rightarrow b$ is determined by the square of the scattering amplitude:

$$T^{ba}\left(E\right) = \sum_{q,q'} \left\langle a\right| H_{QP} \left|q\right\rangle \left(\frac{1}{E^{(+)} - H^{eff}} \right)_{qq'} \left\langle q'\right| H_{QP} \left|b\right\rangle \quad ,$$

$$S = 1 - iT$$

The eigenvalues of H^{eff} , $\varepsilon = \mathrm{E} \text{-} \left(i/2\right)\Gamma$, are complex poles of the scattering matrix, corresponding to the resonances in the cross section.

Single Channel

- To demonstrate in a simple way the effect of the anti-Hermitian term we look at the case of a single channel. Then the matrix W has a completely separable form:

$$\langle q | W | q' \rangle = 2\pi A_q^c A_{q'}^{c*}\ , \ \text{where}$$

$$\mathrm{A}_{\mathrm{q}}^{\mathrm{c}} = \langle q | H_{QP} | c \rangle$$

Separable interaction

$$H = \varepsilon I - \frac{i}{2}\begin{bmatrix} A_1^2 & A_1 A_2 & A_1 A_3 \ldots\ldots \\ A_2 A_1 & A_2^2 & A_2 A_3 \ldots\ldots \\ A_3 A_1 & A_3 A_2 & A_3^2 \ldots\ldots \\ . & & \\ . & & \end{bmatrix}$$

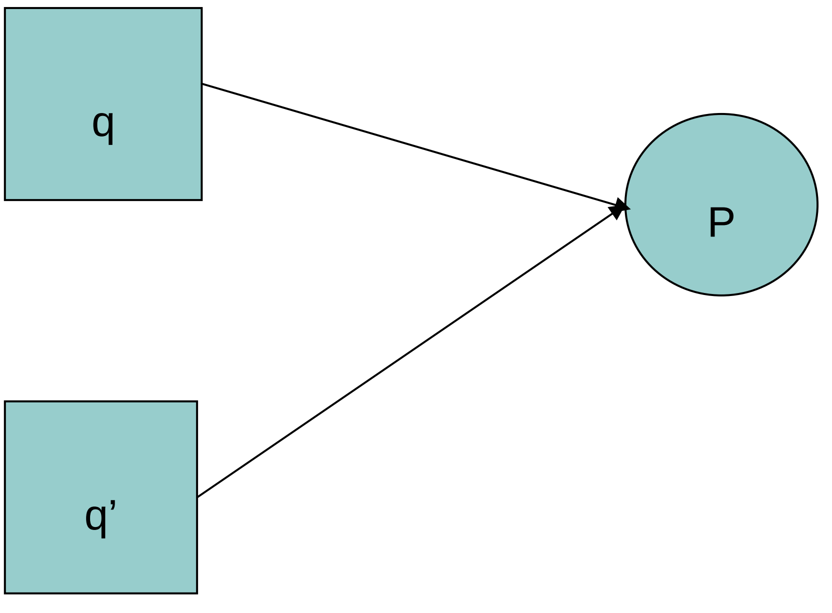
“Super-radiant” state

- The rank of the factorized matrix is 1, so that all eigenvalues of W are zero, except one that has the value equal to the trace of this matrix:

$$\Gamma_0 = \sum_q \langle q|W|q\rangle = 2\pi \sum_q \left|A_q^c\right|^2 \equiv \sum_q \Gamma_q^{\uparrow}$$

$$(\Gamma_q^{\uparrow} = 2\pi \left|A_q^c\right|^2)$$

Effective interaction in the Q-space



“Super-radiant” state

- The special unstable state is often referred to as the *“super-radiant” (SR),* in analogy to the Dicke coherent state of a set of two-level atoms coupled through a common radiation field. Here, the coherence is generated by the common decay channel. The stable states are *trapped* and decoupled from the continuum.
- *R.H.Dicke, Phys.Rev. 93, 99 (1954)*

General case

- The phenomenon of super-radiance survives in a general situation of N intrinsic states and N_c open channels provided $N_c \ll N$, if the mean level spacing D of internal states and their characteristic decay widths satisfy the conditions

$$\kappa^c = \frac{\gamma^c}{D} >> 1$$

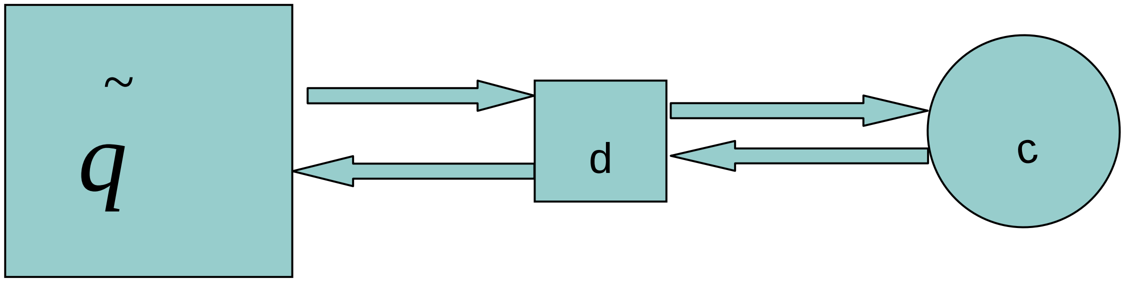
General case

- In this regime of overlapping resonances their interaction through the common continuum channels leads to restructuring of the complex energy spectrum, similarly to the formation of Dicke's coherent state. Since the rank of the factorized W matrix is N_c , it has only N_c non-zero positive eigenvalues.
- The intrinsic space Q is now divided into the SR subspace of dimension N_c and the subspace of trapped states It> of dimension $N-N_c$.

Doorways

- Frequently only a subset of intrinsic states {Q} connects directly to the {P} space of channels. The rest of states in {Q} will connect to {P} states due to the admixtures of these selected states . The special states coupled directly to the continuum are the *doorways Id>.* They form the doorway subspace {D}. The corresponding projection operator will be denoted as *D.*
- The remaining states will be denoted as $\left|\tilde{q}\right\rangle$

Doorways



Doorways

The full Hamiltonian can be decomposed in the following way:

$$\mathrm{H} = \left(H_{\tilde{Q}\tilde{Q}} + H_{DD} + H_{\tilde{Q}D} + H_{D\tilde{Q}} \right) + \left(H_{PP} + H_{DP} + H_{PD} \right)$$

Diagonalizing the first part in this expression will give back the states $\left|\mathrm{q}\right\rangle$ with the components $\left|\mathrm{d}\right\rangle$ mixed with states $\left|\tilde{\mathrm{q}}\right\rangle$

Doorways (Corridors)

Single doorway

Assume one important doorway $\left| d \right\rangle$. The matrix elements of the effective operator W in the Q-space are now given by:

$$\langle \mathrm{q} | W | q' \rangle = 2\pi \sum_{c=1}^{N_c} \langle q | H_{DP} | c \rangle \langle c | H_{PD} | q' \rangle$$

Under the doorway assumption,

$\langle q | H_{DP} | c \rangle = \langle q | d \rangle \langle d | H_{DP} | c \rangle$, where $\langle \mathrm{q} | \mathrm{d} \rangle$ is the admixture of the doorway to the state $| q \rangle$

Then,

$$\langle q | W | q' \rangle = 2\pi \langle q | d \rangle \langle d | q' \rangle \sum_c \left| \langle d | H_{DP} | c \rangle \right|^2$$

This matrix element is again separable, irrespective of the number of channels, and again one finds a single broad state with a widths:

$$\Gamma_s = 2\pi \sum_q \left| \langle q | d \rangle \right|^2 \sum_c \left| \langle d | H_{DP} | c \rangle \right|^2$$

naturally this width is simpy the decay width $\Gamma_d^{\uparrow}$ of the doorway.

When is this picture valid?

The criterion of validity is that the average spacing between levels in {Q} -space is smaller than the decay width of such a state "before" the SR is set at work. Consider the spreading width , $\Gamma_{\mathrm{d}}^{\downarrow}$, of the doorway state for the fragmentation into compound states $\left|\tilde{\mathrm{q}}\right\rangle$. If N_{q} is the number of compound states in the interval covered by the spreading width, their average energy spacing is :

$$\bar{D} \approx \frac{\Gamma_d^{\downarrow}}{N_q}$$

Before the SR mechanism is turned on, the average decay width of a $\left|\mathrm{q}\right\rangle$ is

$$\Gamma_{\mathrm{q}}^{\uparrow} = 2\pi \sum_c \left|\left\langle q \right| H_{QP} \left| c \right\rangle\right|^2$$ that can be estimated:

$$\Gamma_q^{\uparrow} = \frac{\Gamma_s}{N_q} \ , \quad \text{therefore :} \quad \frac{\Gamma_q^{\uparrow}}{D_q} \approx \frac{\Gamma_s}{\Gamma_d^{\downarrow}} \approx \frac{\Gamma_d^{\uparrow}}{\Gamma_d^{\downarrow}} \quad \text{thus,} \quad \frac{\Gamma_{\mathrm{d}}^{\uparrow}}{\Gamma_d^{\downarrow}} > 1$$

Examples

- *Single-particle resonance*

Consider a s.p. state $\left|\phi_{s.p.}\right\rangle$ that belongs to the $\{Q\}$ – space but its energy is above the decay threshold. The interaction spreads this state among the complicated compound states. These compound states couple to the continuum via the s.p. component $\left\langle\phi_{s.p.}\middle|q\right\rangle$. Thus $\left|\phi_{s.p.}\right\rangle$ serves as a doorway. This mechanism will produce a state with a large s.p. width $\Gamma_{s.p.} = 2\pi\sum\left|\left\langle\phi_{s.p.}\middle|H_{QP}\middle|c\right\rangle\right|^2$. This width should be identified with the width of s.p. resonances in the framework of the optical potential in Feshbach's theory.

The rest of the narrow states represent the neutron resonances.

Examples

- *Isobaric analog state (IAS).*

The IAS, $\left|\mathrm{A}\right\rangle$ is the result of action of the isospin lowering operator T_- on the parent state $\left|\pi\right\rangle$,

$\left|\mathrm{A}\right\rangle = const \cdot T_- \left|\pi\right\rangle$

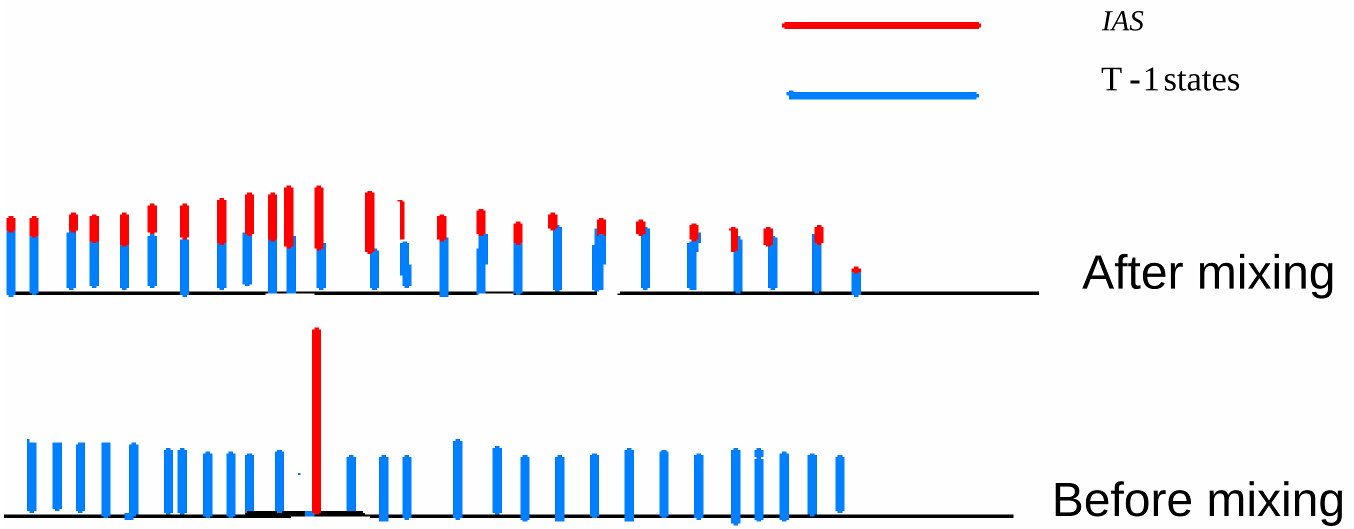
In the compound nucleus, the IAS is surrounded by many compound states $\left|q\right\rangle$ of lower isospin $\mathrm{T}_< = T - 1$. The Coulomb interaction does not conserve isospin fragmenting the strength of the IAS over many states $\left|q\right\rangle$ that results in the spreading width $\Gamma_A^{\downarrow}$ of the IAS.

If located above the threshold, the IAS can also decay into several continuum channels that gives rise to the decay width $\Gamma_A^{\uparrow}$. In heavy nuclei $\Gamma_A^{\uparrow} > \Gamma_A^{\downarrow}$.

The SR mechanism is relevant to this case, providing an explanation why the IAS appears as a single resonance with a decay width given by that of $\left|A\right\rangle$,

$$\Gamma_{IAS}^{\uparrow} = 2\pi \left|\left\langle A\right| H_{QP} \left|P\right\rangle\right|^2$$

Mixing of the IAS with T-1 states

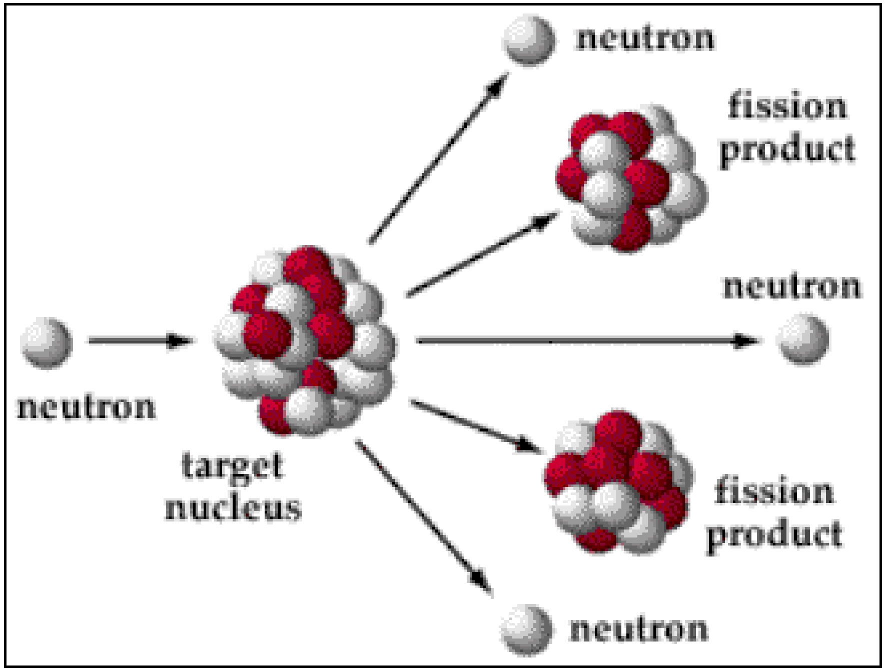


Example ^{208}Pb

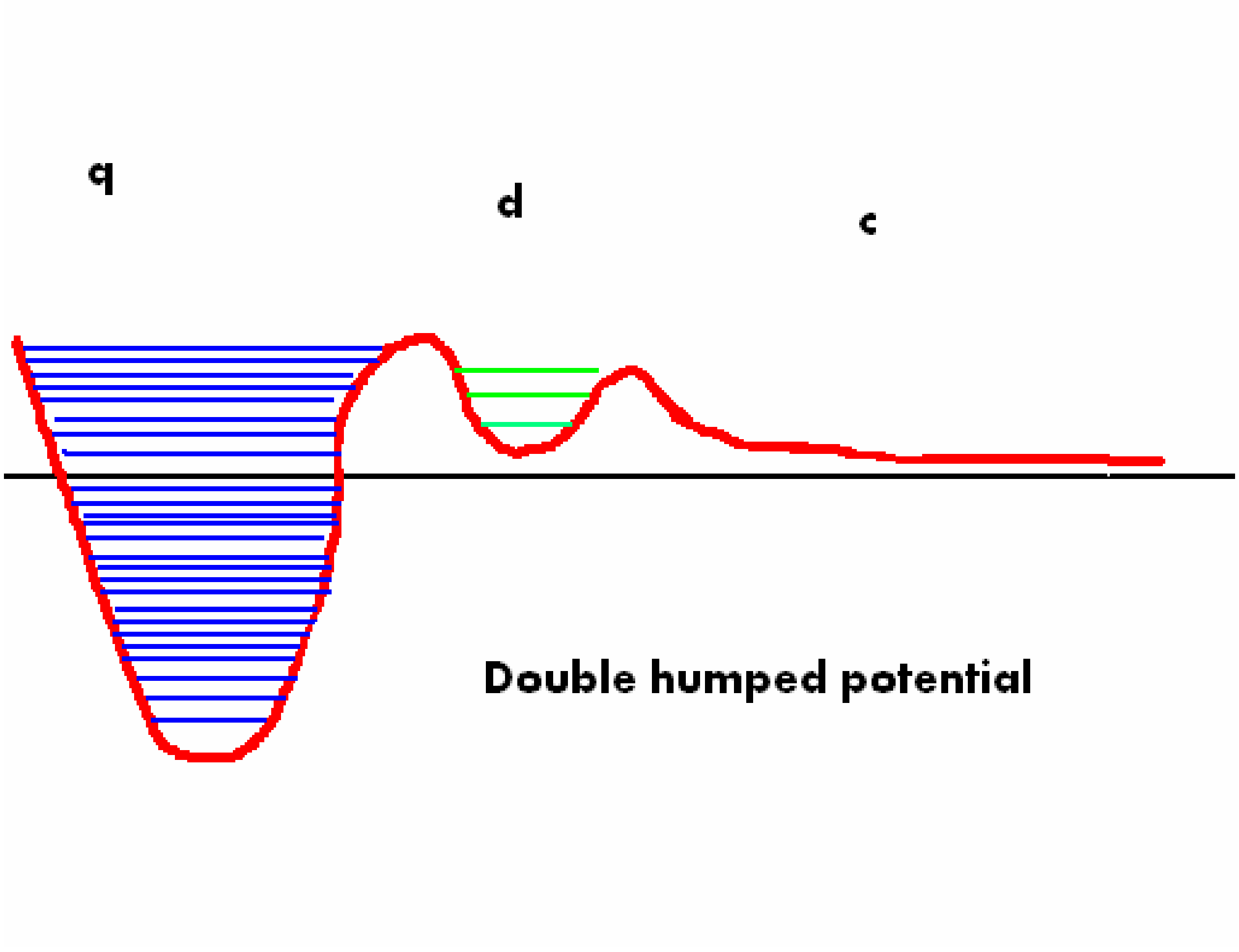
In ^{208}Bi the total width of the IAR is about $\Gamma_{\mathrm{A}} = 250keV,$ of which $\Gamma_{\mathrm{A}}^{\uparrow} = 170keV$ and $\Gamma_{A}^{\downarrow} = 80keV.$

Certainly $\frac{\Gamma_{\mathrm{A}}^{\uparrow}}{\Gamma_{A}^{\downarrow}} > 1$

Neutron Induced Fission



Fission



Double humped potential in fission

- An outstanding example are the fission isomers in several heavy, nuclei such as Pu.
- The fission cross section in the neutron induced reaction shows structures with larger widths spaced several hundred eV apart , in addition to the usual compound resonances which are spaced a few eV. This phenomenon is interpreted as a direct consequence of a double humped potential in the fission process of the compound nucleus. The energy spacing between the few excited (usually collective-rotational or vibrational) states in the second shallower well are larger than between the compound states in the deeper well. When the compound states in the deeper well are in the vicinity of a state in the second well, they couple forming mixed states which, couple to the fission channel via the admixture of the state from the second well.

Double humped potential in fission (intermediate structure)

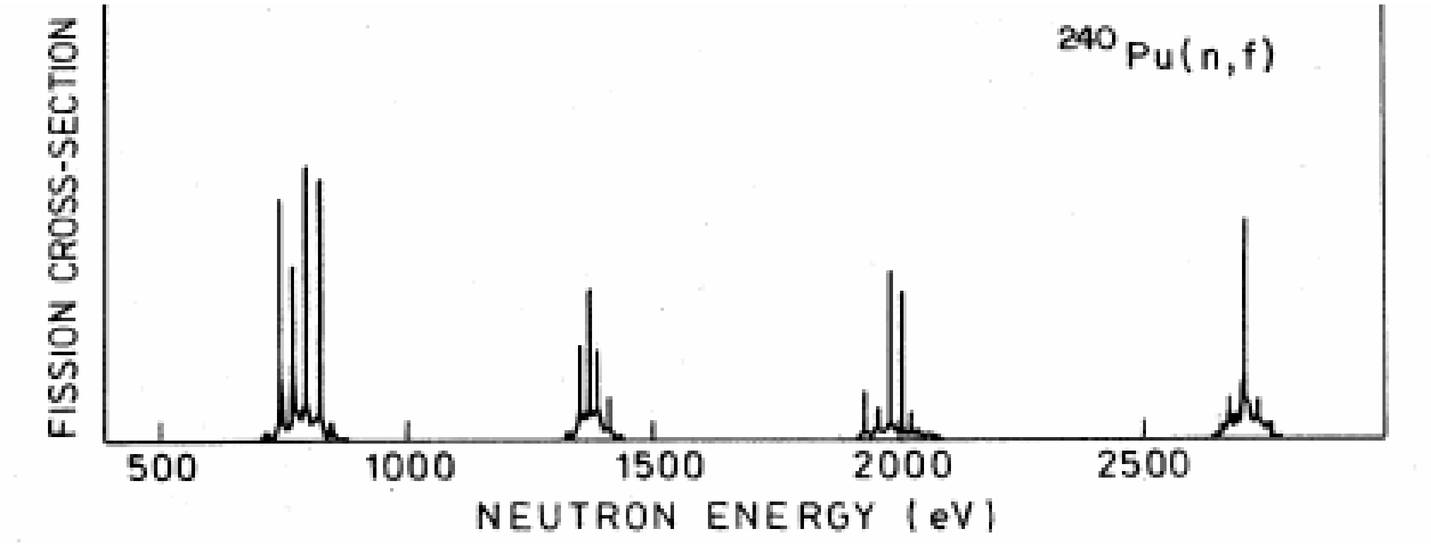
In the present formalism the states in the second minimum are doorways $|d\rangle$ for the decay of a compound state into the open fission channels $|f\rangle$. The matrix W for the compound states $|q\rangle$ in the vicinity of the doorway $|d\rangle$ can be written as :

$$\langle q|W|q'\rangle = \langle q|d\rangle\langle d|q'\rangle\sum_f\left|\langle d|V|f\rangle\right|^2$$

After diagonalization, one will find a considerably wider decay widths than that of a typical compound state but smaller than a single-particle decay width. This situation repeats itself for each of the states of the second well. As a result, one will observe intermediate structure in the fission cross section.

G. 13.9. The sub threshold fission cross section ^{241}Pu (n,f) [taken from Migneco and eobold (68)].

Intermediate Structure in Fission



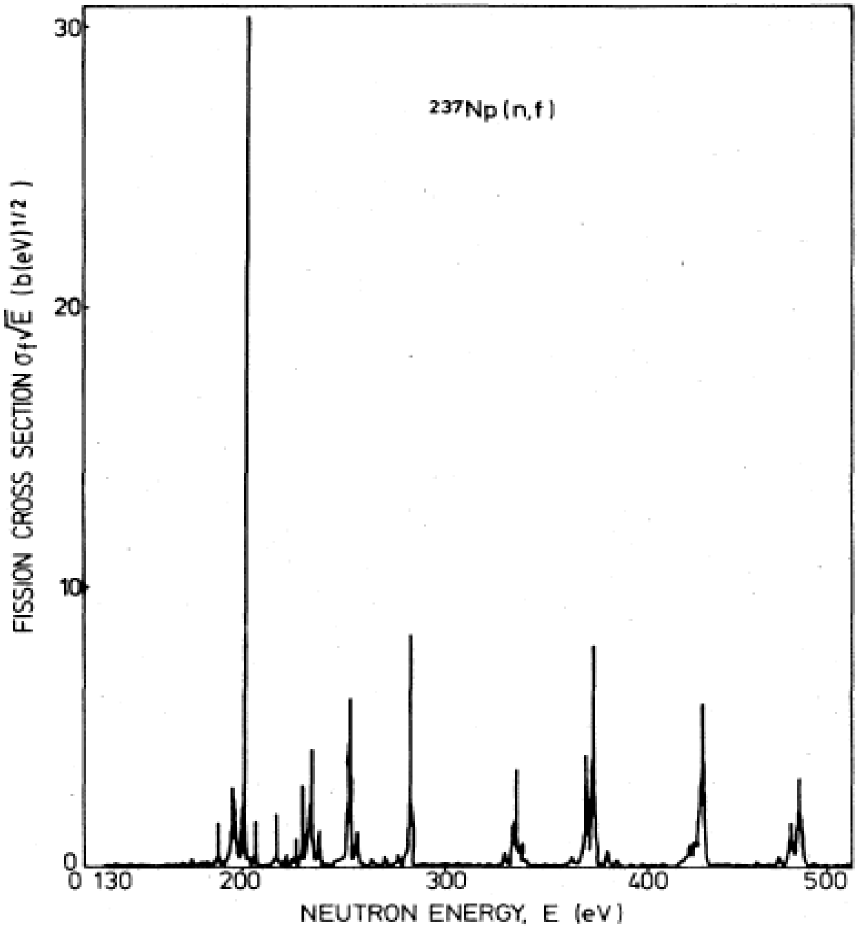


FIG. 119. Neutron fission cross section of ^{237}Np in region 100–500 eV [from Plattard *et al.* (1976)].

Example; *Giant Resonances (GR)*

- One describes the giant resonances in nuclei in terms of 1p-1h configurations. The residual interaction forms collective states out of these configurations. However, usually the GRs are located in the particle continuum. The 1p-1h are surrounded by a vast spectrum of 2p-2h excitations which will mix with the GR.

Denote these states as $|\mathrm{b}\rangle$, and the giant resonance by $|\mathrm{G}\rangle$. The 2p-2h states will decay into the continuum via the admixture $\langle \mathrm{G}|\mathrm{b}\rangle$ (of the GR into the 2p-2h states).

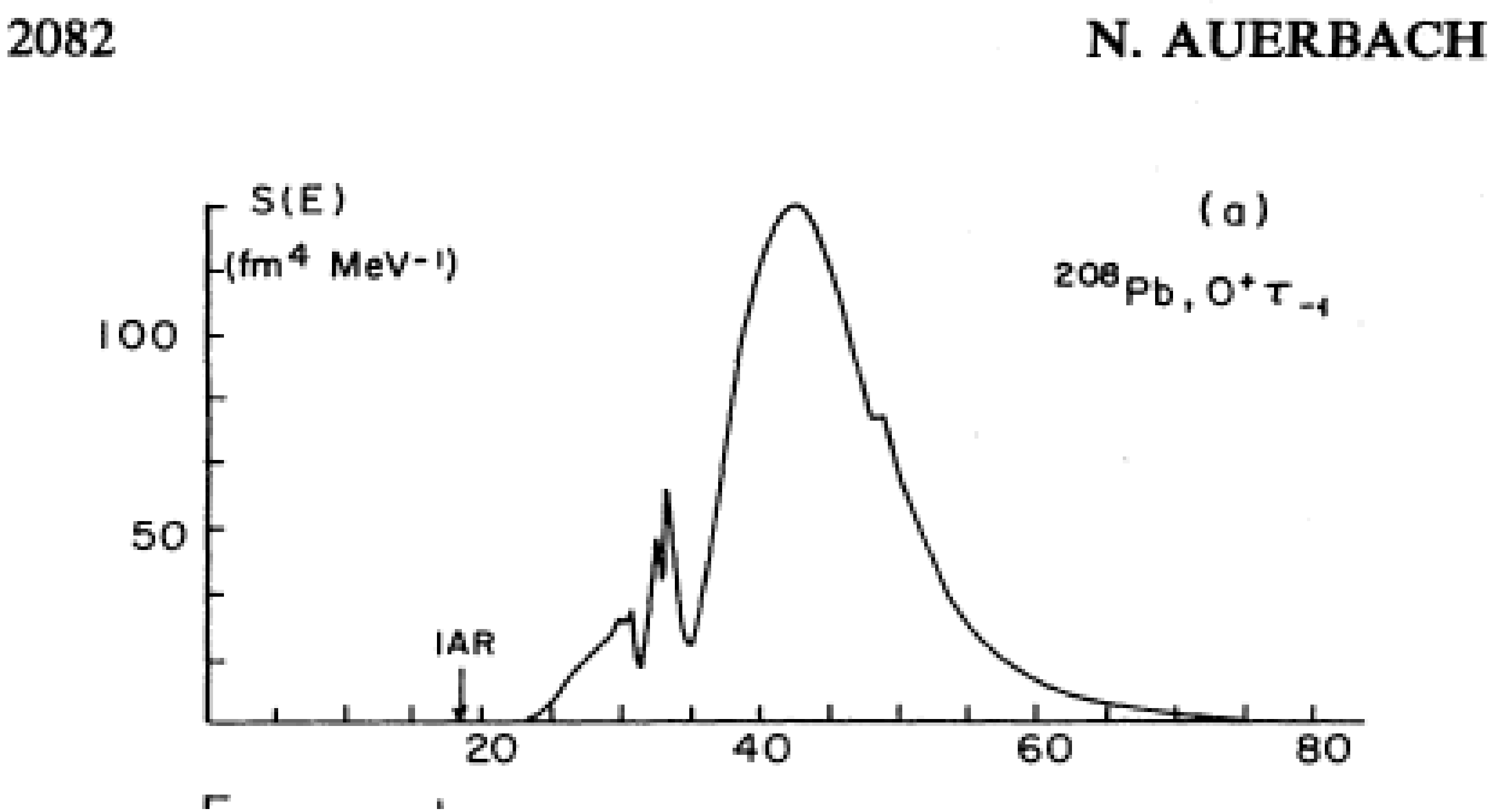
The GR serves as a doorway. The W matrix elements are given by:

$$\langle b|W|b'\rangle = \langle b|G\rangle\langle G|b'\rangle \sum_c \langle G|V|c\rangle\langle c|V|G\rangle$$

Again the W matrix is of rank one and the SR state will have the decay width:

$$\Gamma_G^{\uparrow} = 2\pi \sum_c \left|\langle G|V|c\rangle\right|^2 \text{, under the condition that } \frac{\Gamma_G^{\uparrow}}{\Gamma_G^{\downarrow}} > 1$$

Giant Resonances (CRPA)



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NUCLEAR PHYSICS A

Doorway states in nuclear reactions as a manifestation of the "super-radiant" mechanism

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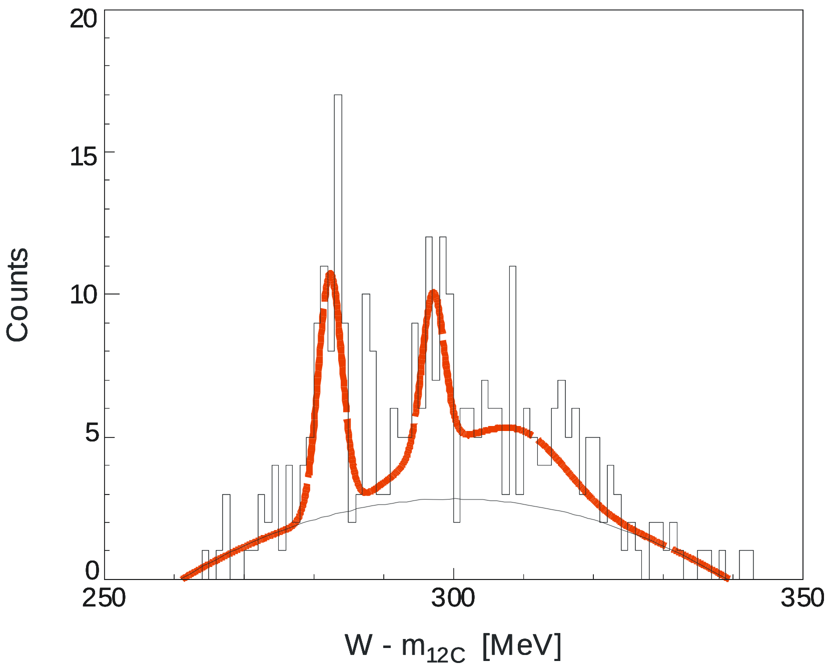
Multi-quark states

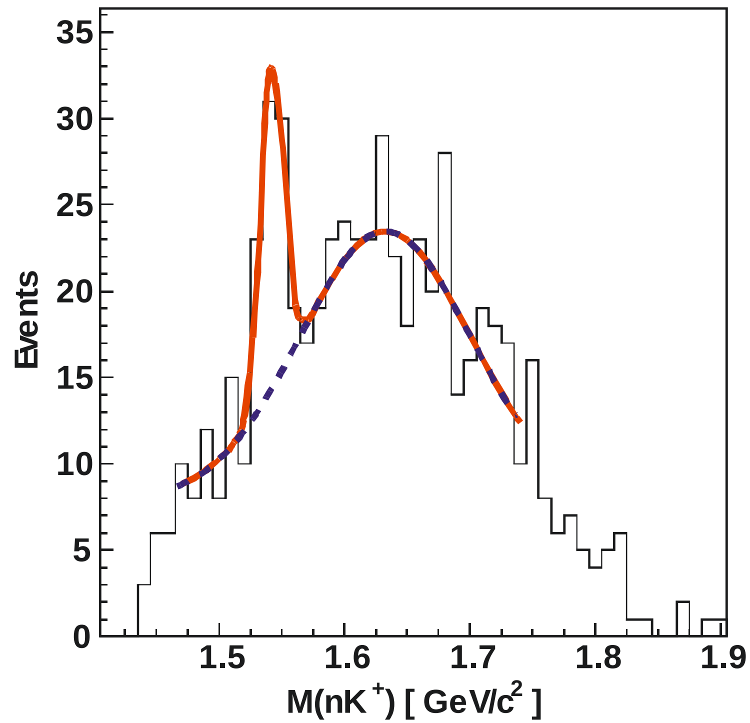
- The SR mechanism is universal and can take place in very distinctively different systems.
  It is possible that in the sector of quark physics there are situations in which preconditions exist for the appearance SR states followed by very narrow trapped resonances. Some examples are taken from the multi- qark systems. In some cases it is claimed that uncharacteristically narrow resonances are observed. For example the 1545 MeV pentaquark, the X(3872) tetraquark and other tetraquarks.

Superradiance in resonant spectra



Narrow resonances and broad superradiant state in ^{12}C 👎

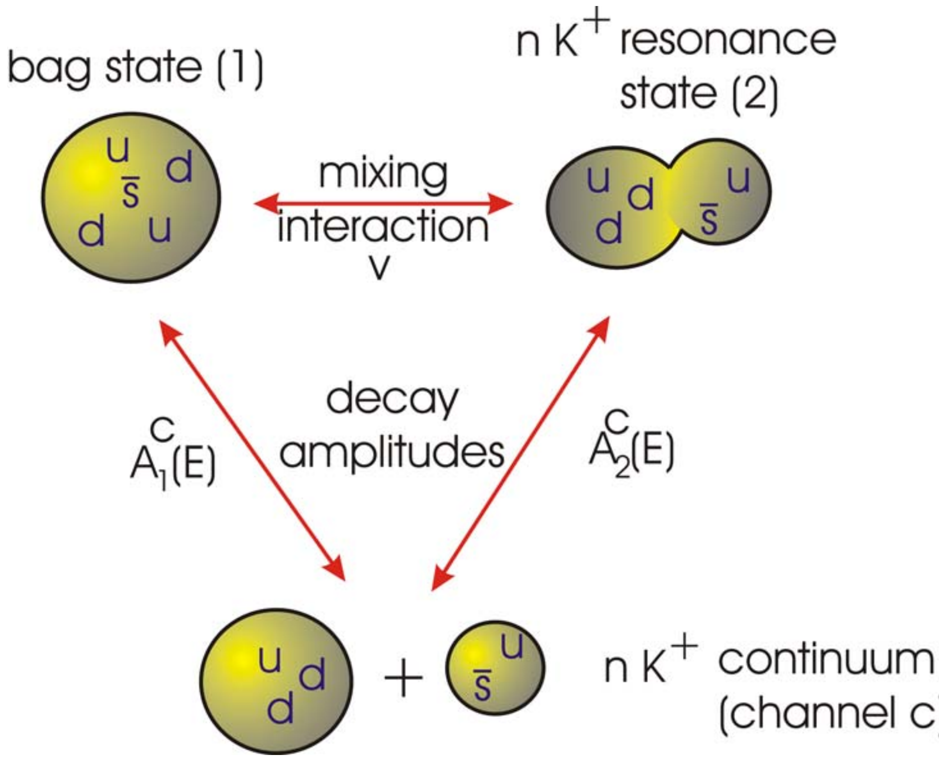
Bartsch *et.al.* Eur. Phys. J. A **4**, 209 (1999)



Pentaquark as a possible candidate for superradiance

Stepanyan *et.al.* hep-ex/0307018

✈$^+$ pentaquark as a two-state interference

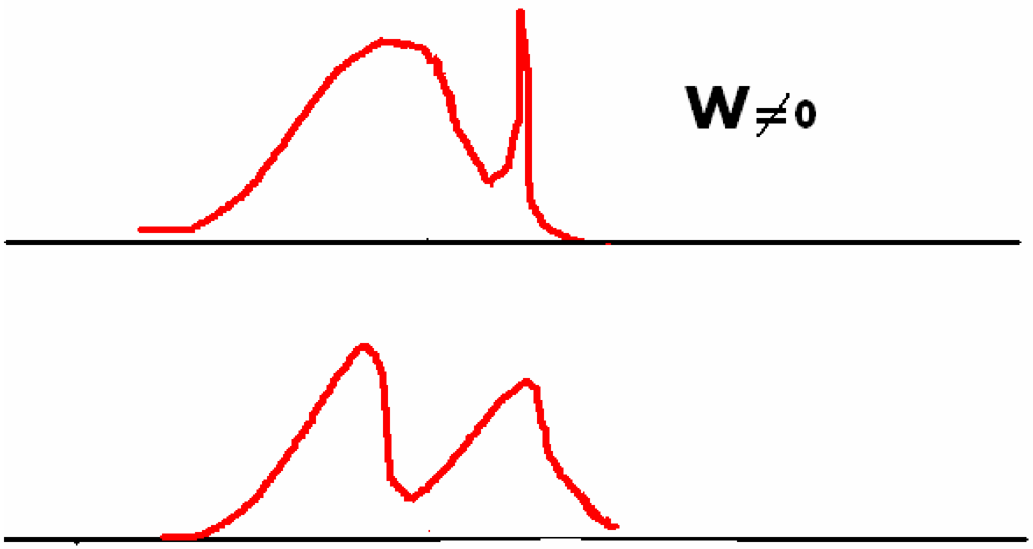


$$\gamma_i = A_i^2 \, , i = 1{,}2$$

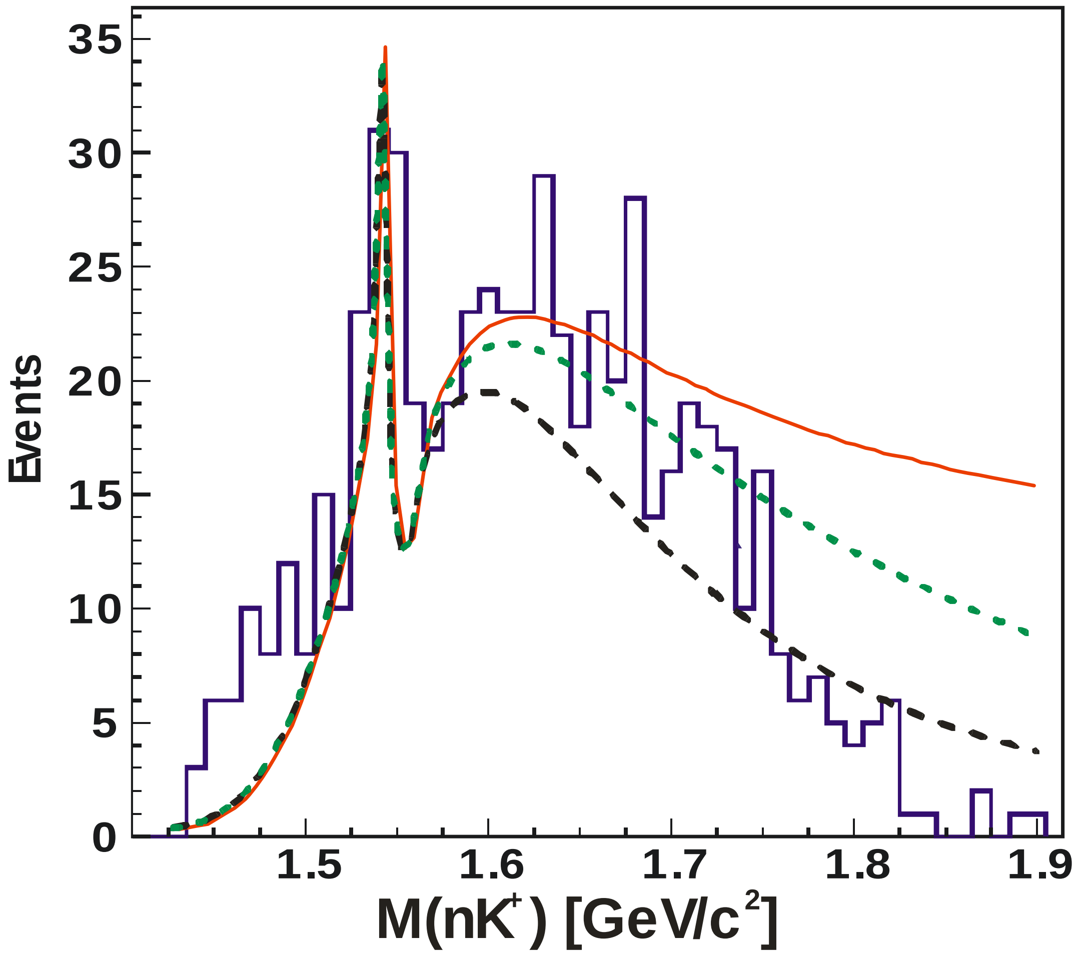
Effective Hamiltonian

$$\mathcal{H}(E) = \begin{pmatrix} \epsilon_1 - \frac{i}{2}\gamma_1 & v - \frac{i}{2}A_1A_2 \\ v - \frac{i}{2}A_1A_2 & \epsilon_2 - \frac{i}{2}\gamma_2 \end{pmatrix}$$

Two-level interaction via the continuum



Kn scattering crossection



Sensible parameters
under requirement
•Resonant energy E_r=1540 MeV
•Kn threshold energy
•Width of broad peak

♏📂 🖬1535 MeV
♑$_1$(E_r) =120 MeV
♏$_2$ =1560 MeV
♑$_2$(E_r) =60 MeV
v=1 MeV
☹🖬🗏🗀🗀 (green), 500 (red)
MeV

Other examples

One could envisage other situations in the field of intermediate energy when the SR mechanism might produce narrow states in addition to a very broad state. Narrow resonances in deeply bound hadronic atoms (pionic, anti-nucleonic), in deeply bound anti-kaons, in sigma hypernuclei, etc