CCM08, Dresden, 5-8.03.2008

SCALAR-FIELD MODELS:

BIFURCATION, DEFORMATION AND NETWORK OF TOPOLOGICAL DEFECTS

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Outlook:

1. 1 & 2 scalar fields
2. Bifurcation
3. Deformation
4. Network of defects

1: Single field

Relativistic systems, 1+1 dimensions

$$\mathcal{L} = \frac{1}{2}\left(\frac{\partial\phi}{\partial t}\right)^2 - \frac{1}{2}\left(\frac{\partial\phi}{\partial x}\right)^2 - V(\phi)$$

kinetic … gradient … potential

non-negative

$$V = \frac{1}{2}W_\phi^2 = \frac{1}{2}\left(\frac{dW}{d\phi}\right)^2$$

$$\phi = \phi(x,t);\ EoM:$$

$$\frac{\partial^2\phi}{\partial t^2} - \frac{\partial^2\phi}{\partial x^2} + \frac{dV}{d\phi} = 0$$

Static solutions …

$$\frac{d^2\phi}{dx^2} = W_\phi W_{\phi\phi} \quad \longleftrightarrow \quad \frac{d\phi}{dx} = W_\phi$$

1: Single field

$$\mathcal{L} = \frac{1}{2}\left(\frac{\partial\phi}{\partial t}\right)^2 - \frac{1}{2}\left(\frac{\partial\phi}{\partial x}\right)^2 - V(\phi)$$

kinetic … gradient … potential

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$$V = \frac{1}{2}W_\phi^2 = \frac{1}{2}\left(\frac{dW}{d\phi}\right)^2$$

Bogomol'nyi bound:

$$\epsilon(x) = \frac{1}{2}\left(\frac{d\phi}{dx}\right)^2 + \frac{1}{2}W_\phi^2 = \frac{1}{2}\left(\frac{d\phi}{dx} - W_\phi\right)^2 + \frac{dW}{dx}$$

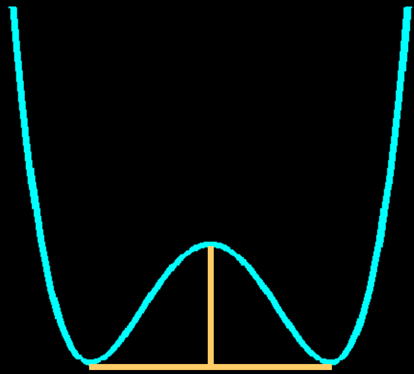
Energy minimized to: $E = \Delta W$

1: Single field

Standard example

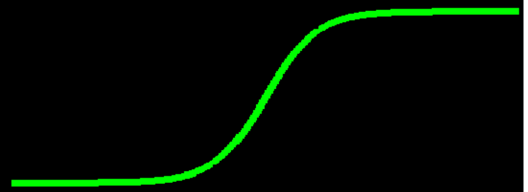
$$\phi^4 \; : W(\phi) = \phi - \frac{1}{3}\phi^3$$

potential

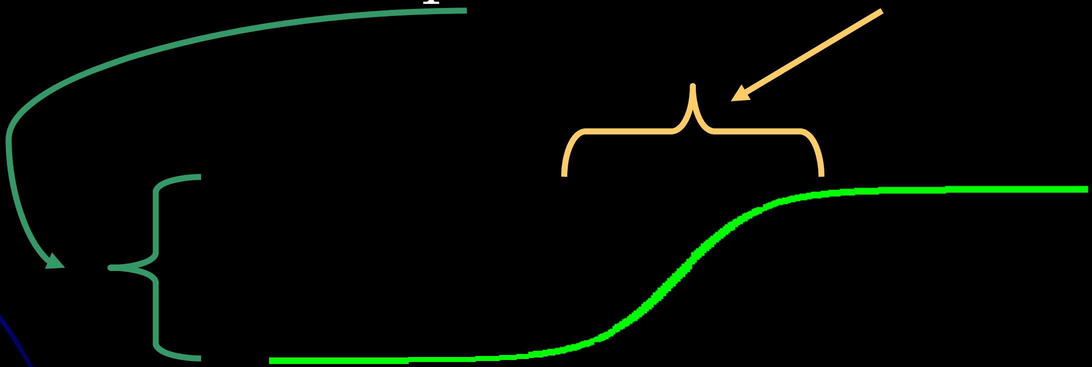


solution

$\tanh(x)$



Two parameters : amplitude and width

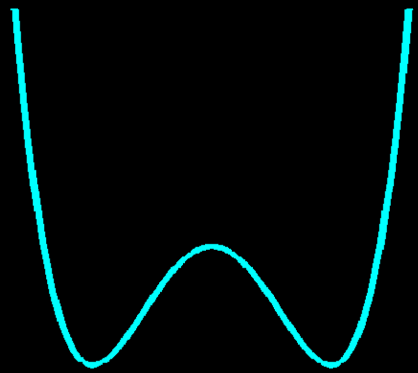


1: Single field

Standard example

$$\phi^4 \;:\; W(\phi) = \phi - \frac{1}{3}\phi^3$$

potential



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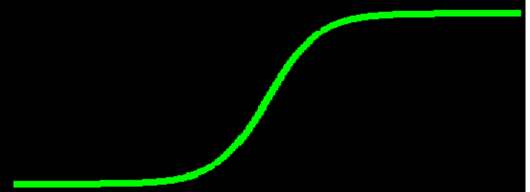
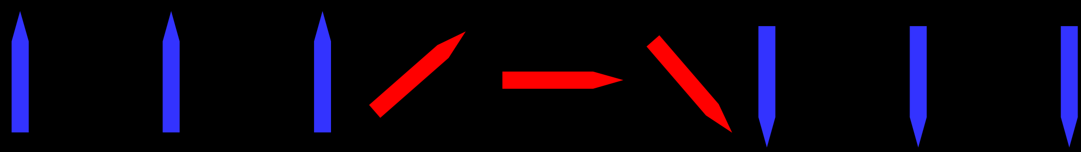


It may be used to describe this:



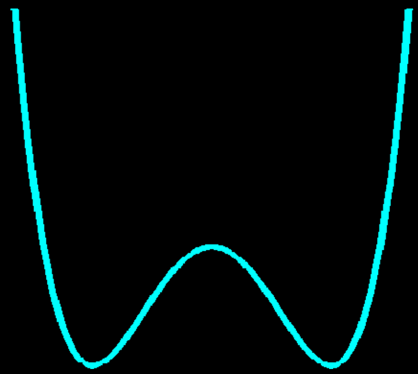
or this:

1: Single field

Standard example

$$\phi^4 \;:\; W(\phi) = \phi - \frac{1}{3}\phi^3$$

potential



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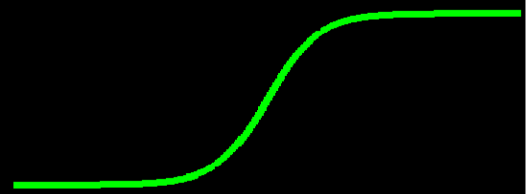

Applications:
Magnetic systems…
Pattern formation…
Also:
Cosmology: DB,Gomes, Losano, RMenezes PLB(2006)
Braneworld: Afonso, DB, Losano PLB (2006)
Brane/cosmology: DB, Brito, Costa PLB(2008)

1: Two or more fields

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - V(\phi,\chi)$$

We choose :

$$V(\phi,\chi) = \frac{1}{2}W_\phi^2 + \frac{1}{2}W_\chi^2$$

non-negative

Minima in (ϕ,χ) plane

junction of defects

$$\epsilon(x) = \frac{1}{2}\left(\frac{d\phi}{dx}\right)^2 + \frac{1}{2}W_\phi^2 + \frac{1}{2}\left(\frac{d\chi}{dx}\right)^2 + \frac{1}{2}W_\chi^2$$

$$= \frac{1}{2}\left(\frac{d\phi}{dx} - W_\phi\right)^2 + \frac{1}{2}\left(\frac{d\chi}{dx} - W_\chi\right)^2 + \frac{dW}{dx}$$

1: Two or more fields

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non-negative

$\text{first} - \text{order equations}$

Door to DS

$$\frac{d\phi}{dx} = W_\phi \quad \& \quad \frac{d\chi}{dx} = W_\chi$$

$$E = \Delta W$$

1: Two or more fields

Good way to describe junctions

$$\mathcal{L} = \partial_\mu \bar{\varphi} \partial^\mu \varphi - V(|\varphi|) \qquad V = |W'(\varphi)|^2$$

$$\varphi = \phi + i\chi$$

First-order framework:
Abraham, Townsend NPB(1991)
DB, JMenezes, Santos PLB(2001); NPB(2002)

1: Two or more fields

Good way to describe junctions

$$\mathcal{L} = \partial_\mu \bar{\varphi} \partial^\mu \varphi - V(|\varphi|) \qquad V = |W'(\varphi)|^2$$

$$\text{Example}: \begin{cases} W(\varphi) = \varphi - \dfrac{1}{N+1}\varphi^{N+1} \\ \text{minima}: \varphi^N = 1;\ \text{symmetry}: Z_N \end{cases}$$

roots of unit

nice discrete symmetry appears very naturally…

1: Two or more fields

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Example :
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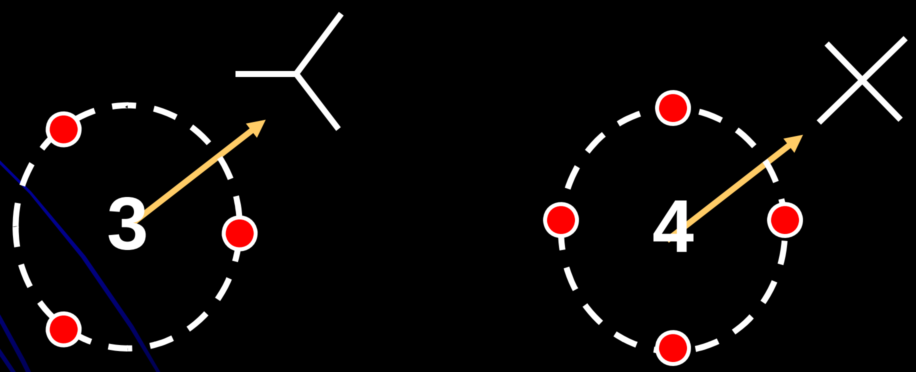
Illustration :



pattern formation :

Gibbons, Townsend, PRL99
DB, Brito, PRL2000

2: Bifurcation

In the plane :

interaction

$$V(\phi,\chi)=V(\phi)+V(\chi)+\lambda v(\phi,\chi)$$

$$V(\phi)=\frac{1}{2}W_\phi^2;\quad V(\chi)=\frac{1}{2}W_\chi^2$$

$$W(u)=\sqrt{\frac{3}{2}}u-\frac{1}{3}\sqrt{\frac{2}{3}}u^3;\ u=\phi,\chi$$

Elphick, Hagberg, Meron, PRL (1998)
phase front instability in a GL model

We consider the case

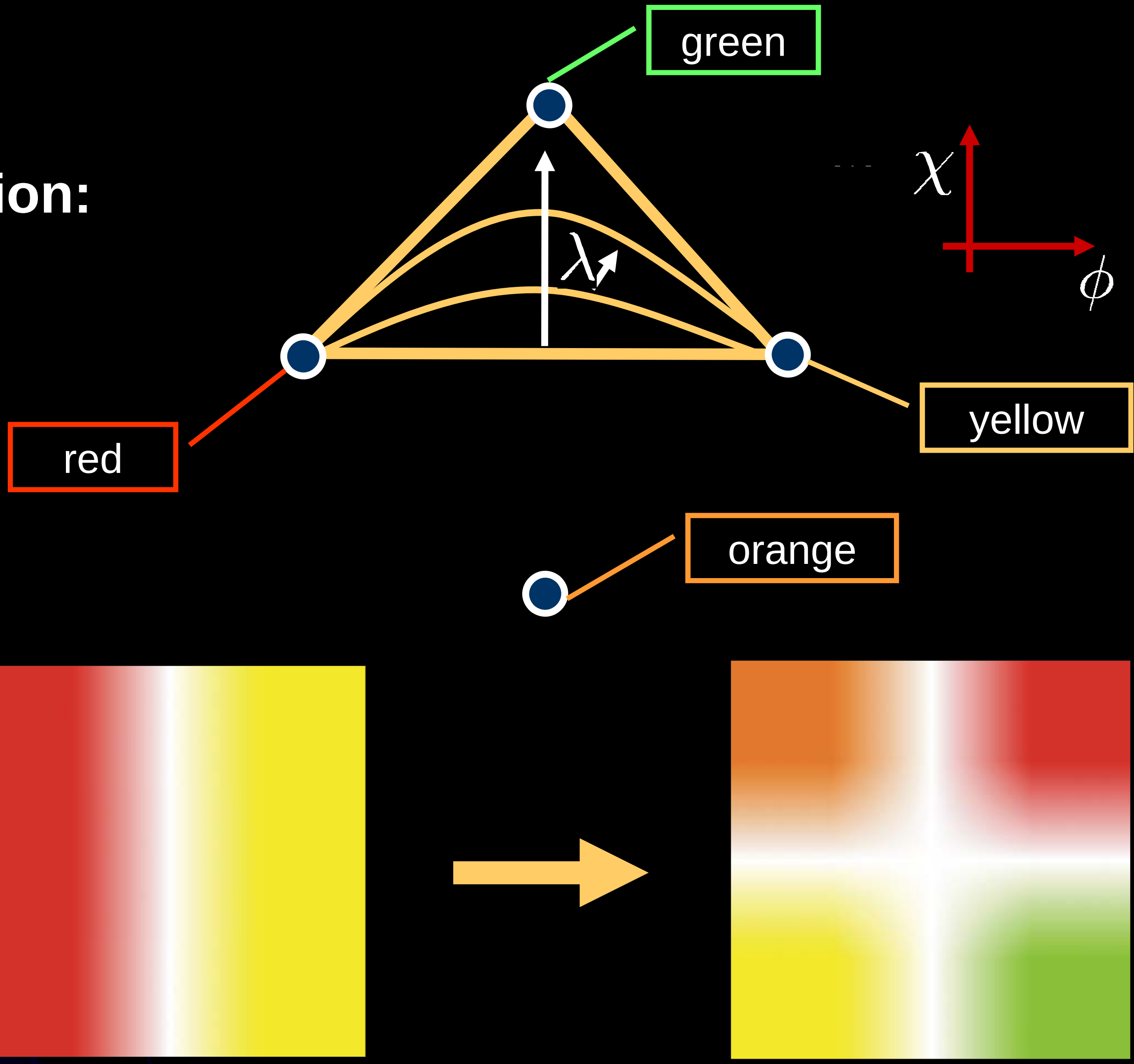
$$v=\frac{1}{2}\phi^4+\frac{1}{2}\chi^4-3\phi^2\chi^3-\frac{9}{2}$$

DB, Brito, Losano, EPL, 2006

…but it breaks first-order structure

2: Bifurcation

Illustration:



3: Deformation

$$\text{Deformation procedure}: \begin{cases} \mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi); \quad V = \frac{1}{2}W_\phi^2 \\ \mathcal{L}_D = \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - U(\chi) \quad U = \frac{1}{2}\mathcal{W}_\chi^2 \end{cases}$$

$$\text{Take } f = f(\chi) \text{ such that}: \quad \mathcal{W}_\chi = \frac{W_\phi(\phi \to f(\chi))}{f'(\chi)}$$

DB, Losano, Malbouisson, PRD(R), 2002

Theorem:

if $\phi(x)$ is static solution for model $\mathcal{L}$, then

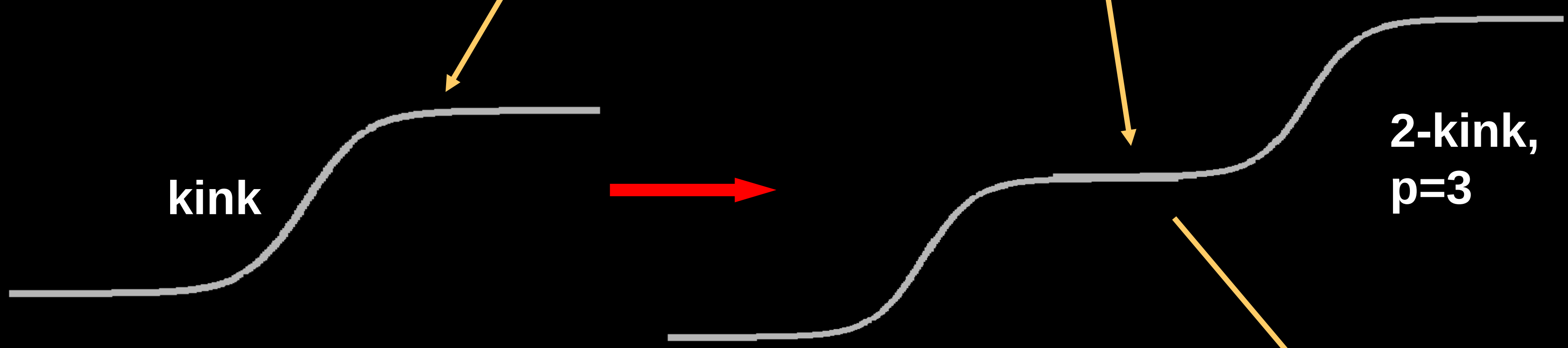
$$\chi(x) = f^{-1}(\phi(x))$$

is solution for new model

3: Deformation

***p* odd integer**

$$\text{Example}:\ \phi^4 \quad \& \quad f(\chi)=\chi^{1/p}$$



DB, JMenezes, RMenezes, PRL(2003)

Direction: 3-kink, 4-kink,…, to build a stair… In preparation

Another direction: deformation with 2 fields, PRD, 2007;

Network of defects, PLB(2008)

3: Deformation

RAPID COMMUNICATIONS

PHYSICAL REVIEW B **69**, 220410(R) (2004)

Magnetic domain walls in constrained geometries

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(Received 23 April 2004; published 22 June 2004)

Magnetic domain walls have been studied in micrometer-sized $Fe_{20}Ni_{80}$ elements containing geometrical constrictions by spin-polarized scanning electron microscopy and numerical simulations. By controlling the constriction dimensions, the wall width can be tailored and the wall type modified. In particular, the width of a 180° Néel wall can be strongly reduced or increased by the constriction geometry compared with the wall in unconstrained systems.

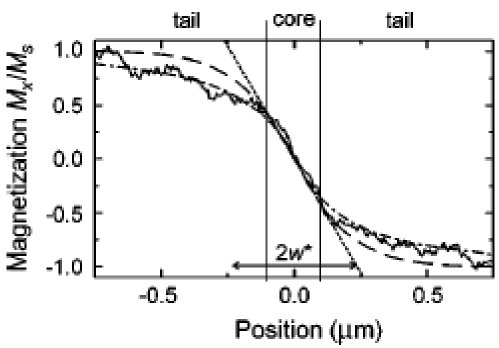
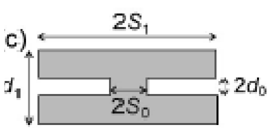


FIG. 2. Domain-wall profile measured for a 10 μm $\times$ 4 μm $\times$ 7.5 nm $Fe_{20}Ni_{80}$ element with a constriction size of d_0=225 nm and S_0=500 nm. The experimental profile is compared with the calculation (dashed-dotted line) and an hyperbolic tangent function (dashed line). The profile is averaged over 80% of the constriction length $2S_0$.



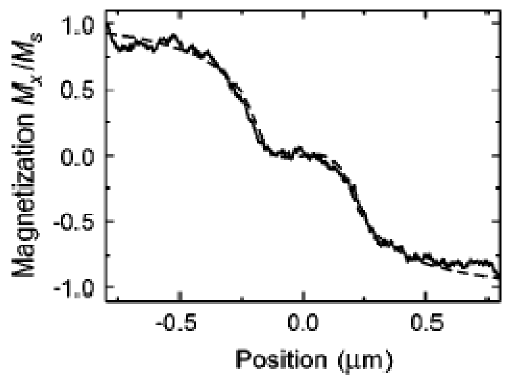


FIG. 5. Wall profile determined in a wide constriction with S_0 =250 nm and d_0=250 nm showing the splitting of the wall into two individual 90° walls. Experimental data are shown as a solid line, the simulation as a dashed line; element dimensions: 10 μm $\times$ 4 μm $\times$ 7.5 nm.

4: Network

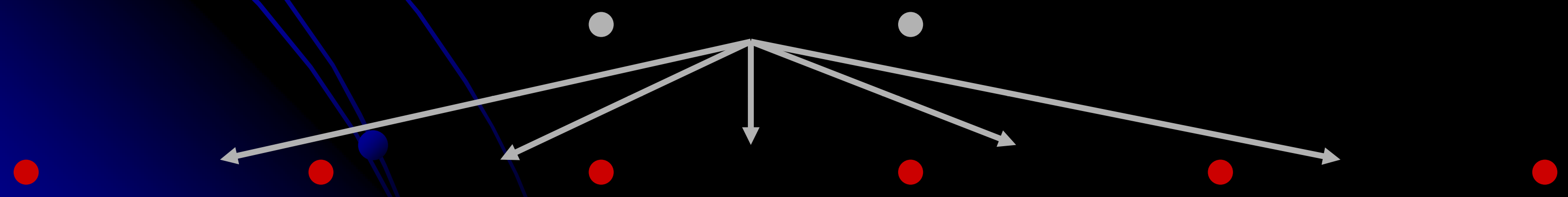
One field, in the line…

Consider the model: $$V(\chi) = (1/2)(1-\chi^2)^2$$

and the deformation: $$\chi \to f(\phi) = \sin(\phi)$$

Then, we get: $$V(\phi) = \frac{1}{2}\cos^2(\phi)$$

Sine-Gordon

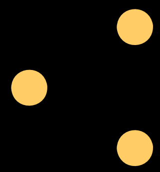


4: Network

Complex field, in the plane

Afonso, DB,GonzalezLeon, Losano,MateosGuilarte, PRD(2007); PLB(2008)

The Z_3 case:



$$V(\chi) = \frac{1}{2}(1-\chi^3)(1-\bar{\chi}^3)$$

freedom to choose deformation function

$$\text{the choice}: \ f(\phi) = \mathcal{W}(\phi)$$

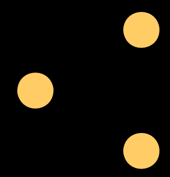
$$\text{leads to the ODE}: f'^2 = f^3(\phi) - 1$$

$$\text{Weierstrass equation}: P'^2(z) = 4P^3(z) - g_1 P(z) - g_2$$

0 **1**

4: Network

The Z_3 case:



$$\text{Weierstrass equation}: P'^2(z) = 4P^3(z) - g_1 P(z) - g_2$$

0 **1**

$$P_{01} : \ \text{periodic,}$$

with periodic

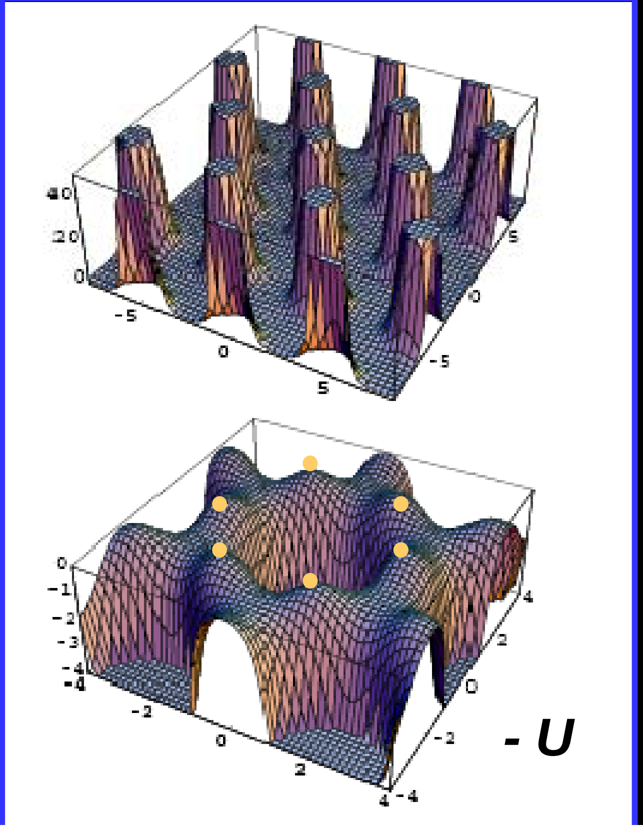
divergence!!!

4: Network

The deformed potential is:

$$U = \frac{1}{2} P'_{01}(4^{-\frac{1}{3}}\phi)\bar{P}'_{01}(4^{-\frac{1}{3}}\phi)$$

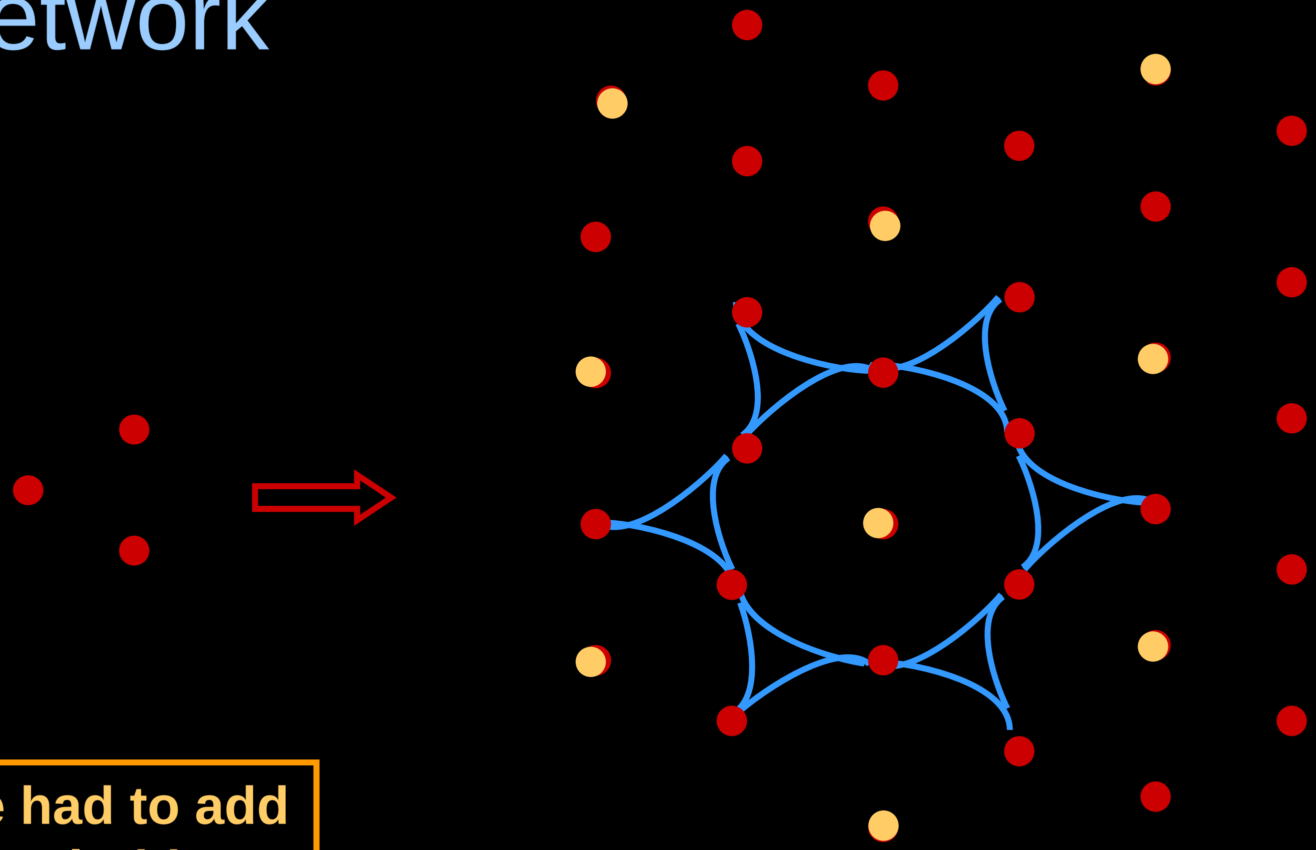
It is depicted as:



Hexagonal structure
of minima around singularity

4: Network

Thus:



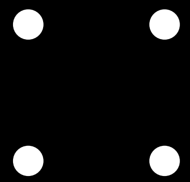
We had to add singularities:

But nice orbits and network:

4: Network

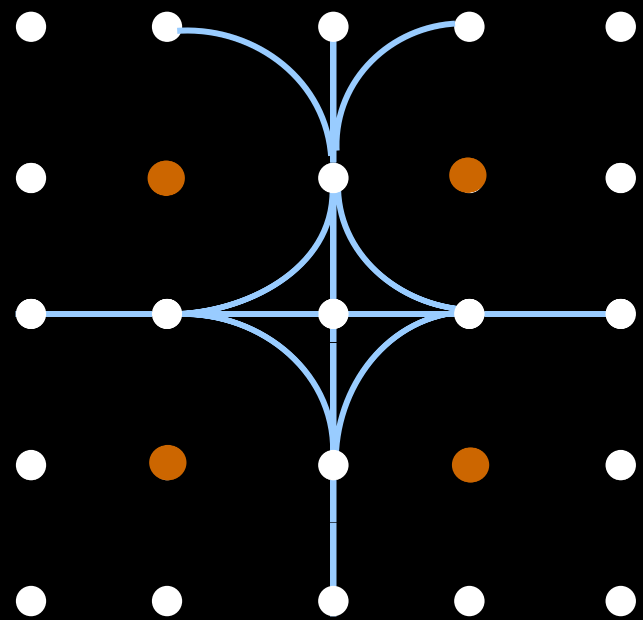
Z_3: Weierstrass → Z_4: elliptic Jacobi

Z_4 is similar:



singularities

and orbits…



We can also make asymmetric patterns: in preparation…

Thanks to:

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Miguel Angel Gonzalez Leon, Salamanca U

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