Tunneling of Composite Objects

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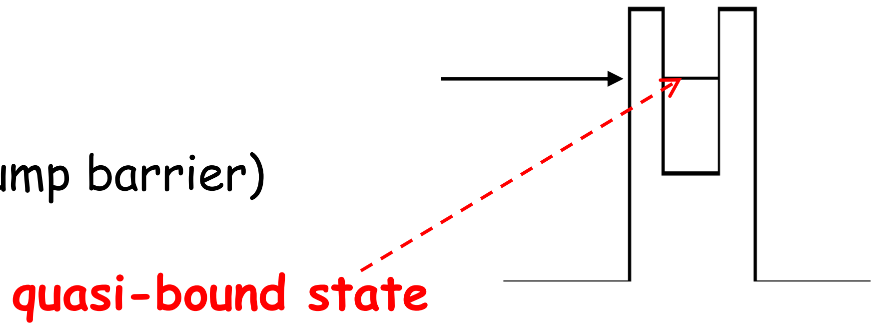
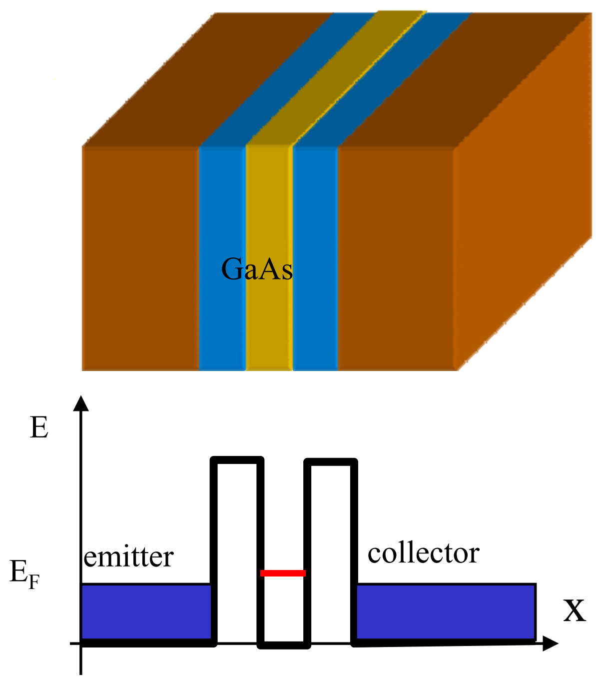
Resonant tunneling

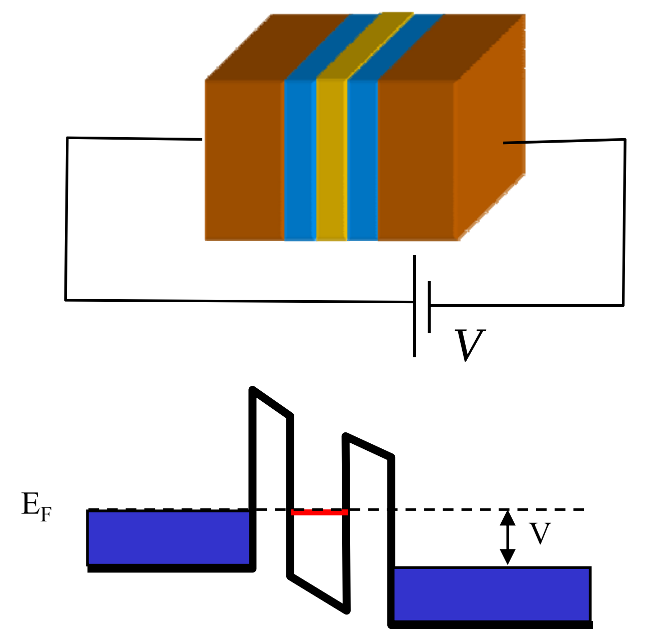
(point particle + double-hump barrier)



Well-known. Used in device technology.

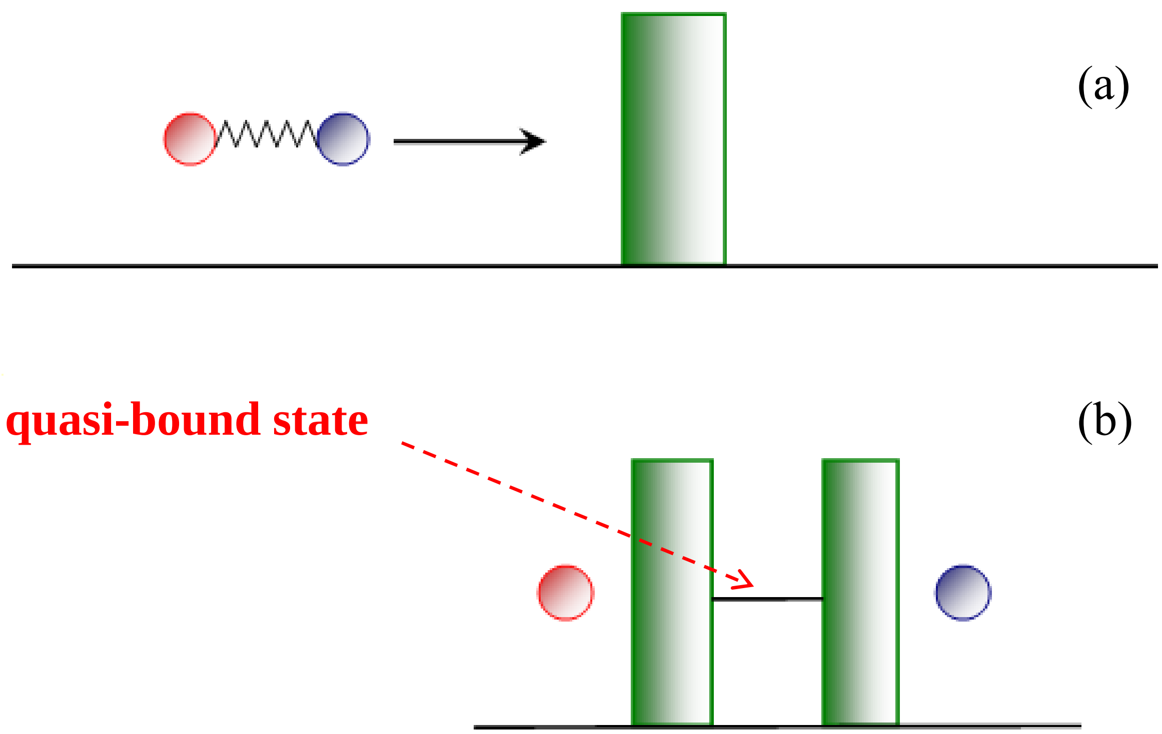
E.g. Resonant Diode Tunneling device.





Resonant tunneling

(composite particle + single-step barrier)



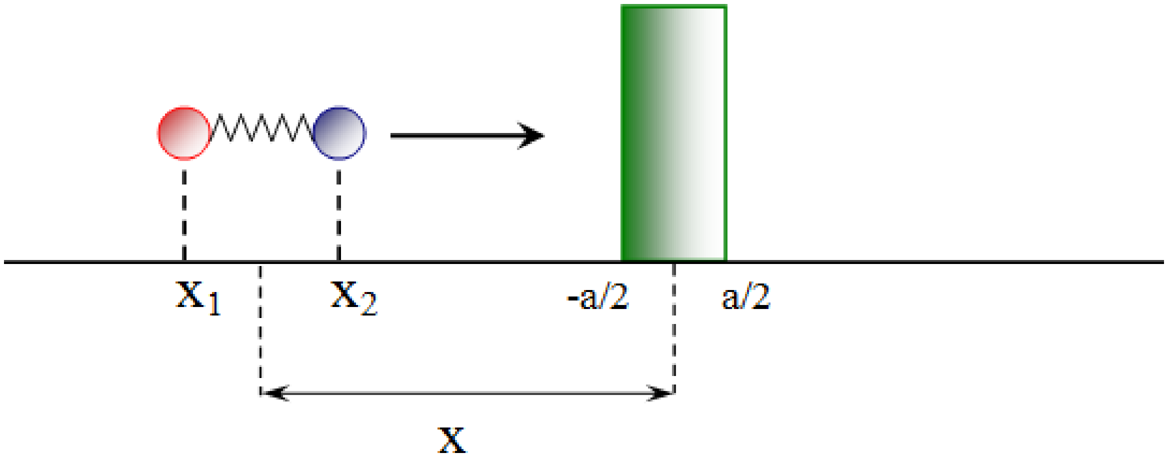
Poorly known. Occurs in atomic and nuclear systems.

E.g. fusion of loosely-bound nuclei.

Step-barrier V + square-well U

barrier

interaction between particles



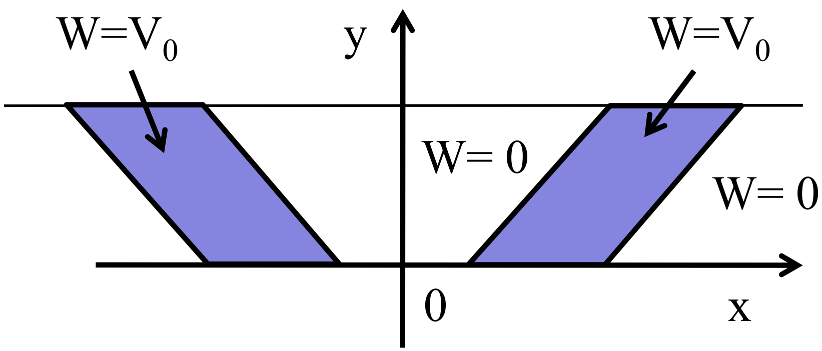
$$H = -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2}\right) + V(x_1) + V(x_2) + U\left(\left|x_1 - x_2\right|\right)$$

$$V(x) = \begin{cases} V_0, & -a/2 \le x \le a/2 \\ 0, & \text{otherwise} \end{cases}$$

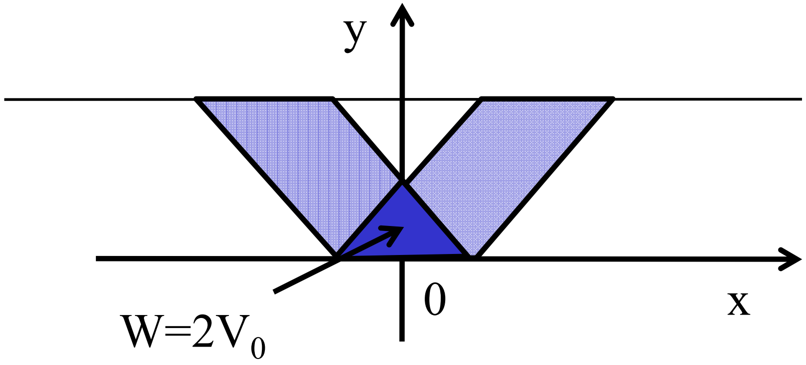
$$U(x) = \begin{cases} 0, & -d/2 \le x \le d/2 \\ \infty, & \text{otherwise} \end{cases}$$

$$x = \left(\frac{x_1 + x_2}{2} \right), \qquad y = x_1 - x_2 + \frac{d}{2}$$

$$\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} - \frac{\hbar^2}{2m}\frac{\partial^2}{\partial y^2} + U(y - \frac{d}{2}) + W(x,y) \right] \Psi(x,y) = E\Psi(x,y)$$

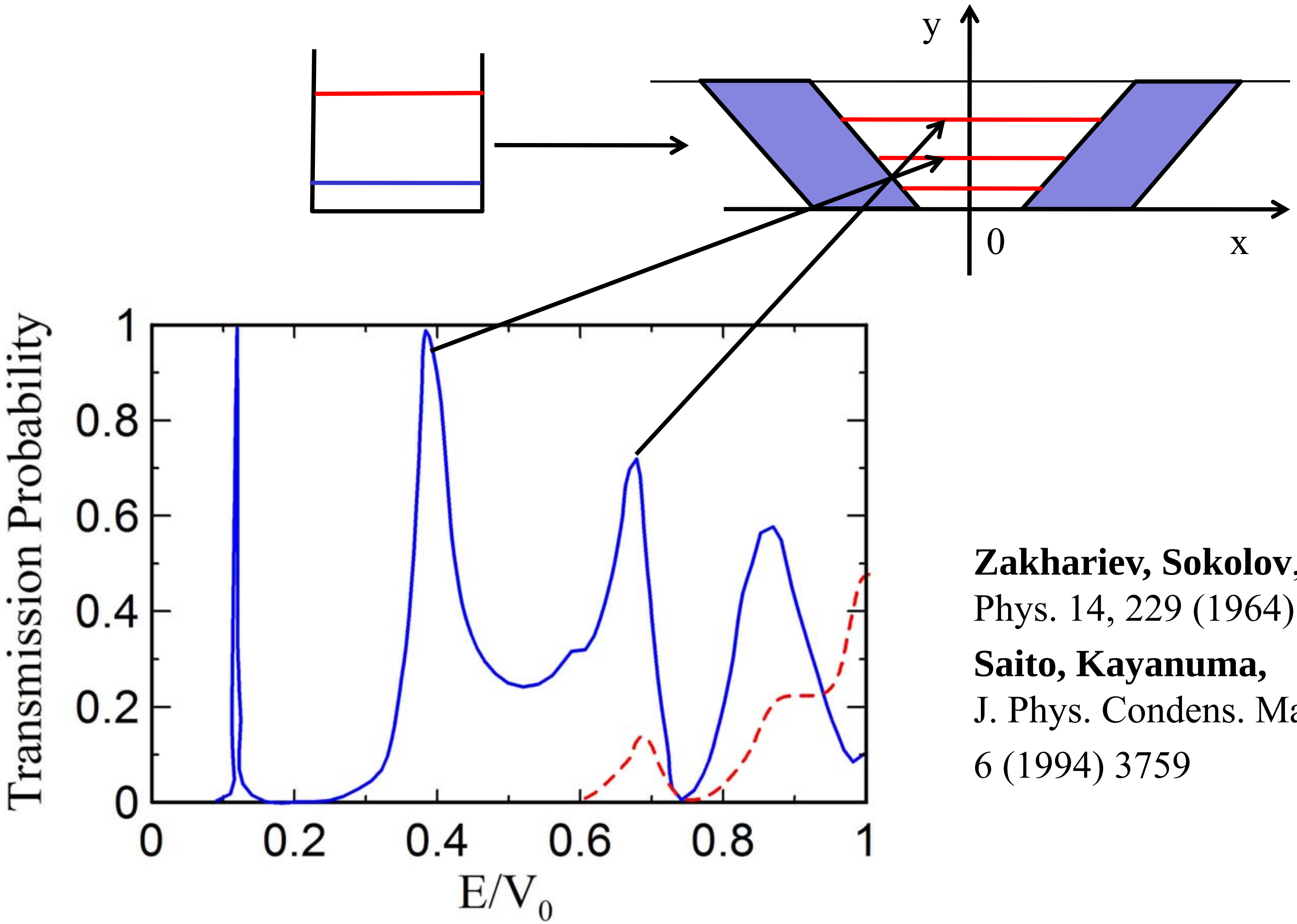


(a) d/2 > a



(b) d/2 < a

Problem equivalent to a single particle (c.m. coordinate x) tunneling through 2 barriers.

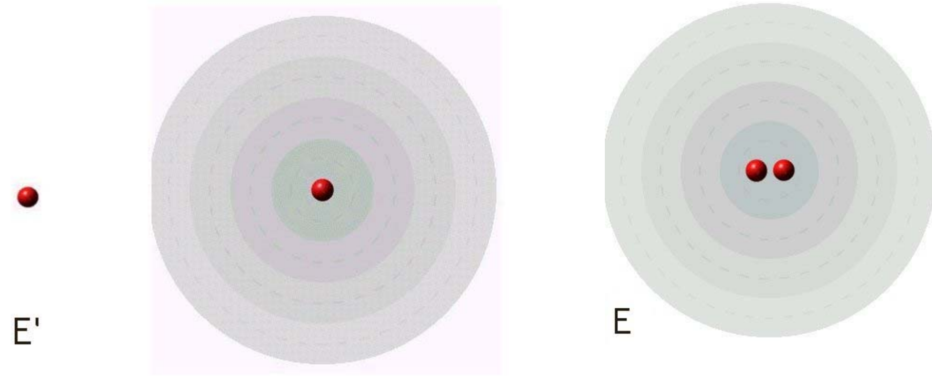


Zakhariev, Sokolov, Ann. d. Phys. 14, 229 (1964)

Saito, Kayanuma,
J. Phys. Condens. Matter
6 (1994) 3759

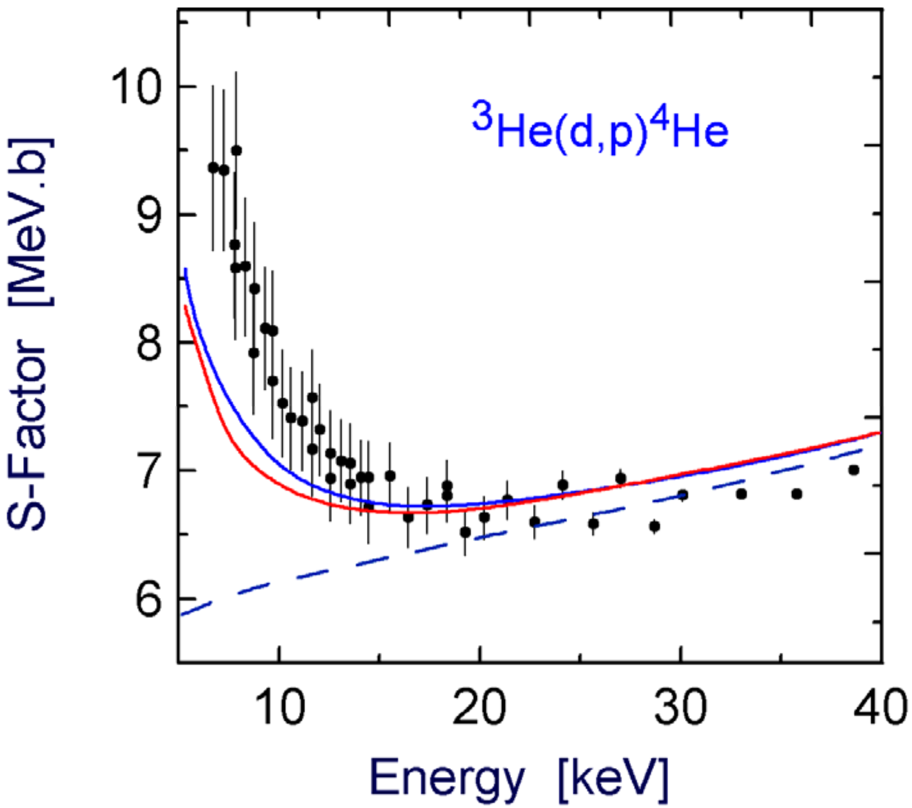
Example 1

Nuclear fusion with electron screening (in the laboratory)



Adiabatic model: $\Delta E = E' - E$

$$\sigma_{lab}^{fusion} \sim \sigma_{bare}(E+\Delta E)$$
$$\sim \exp\left[\pi\,\eta(E)\frac{\Delta E}{E}\right]\sigma_{bare}(E)$$



- - - - - S_{bare}

——— Dynamic

——— Adiabatic

Rolfs, 1995 Reaction	ΔE [eV] experiment	ΔE [eV] adiabatic limit
$d(^3He, p)^4He$	180 ± 30	119
$^6Li(p, \alpha)^3He$	470 ± 150	186
$^6Li(d, \alpha)^4He$	380 ± 250	186
$^7Li(p, \alpha)^4He$	300 ± 280	186
$^{11}B(p, \alpha)2^4He$	620 ± 65	348

Amplification of small effects

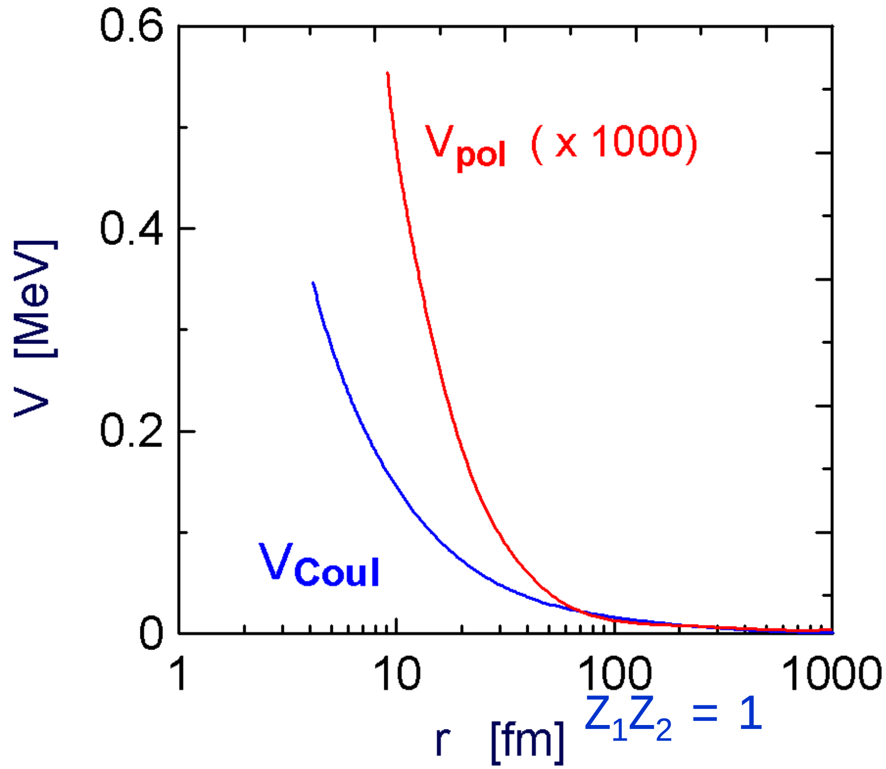
- Thermal motion, lattice vibrations, beam energy spread
- Nuclear breakup channels (in weakly-bound nuclei)
- Dynamics of tunneling

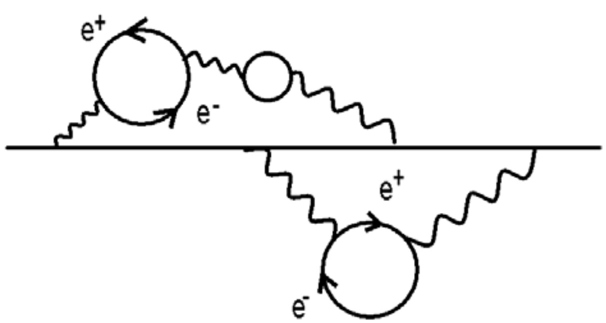
Balantekin, Hussein, CB, NPA 1997

Corrections	
Vaccuum Polarization	$\sim 1\%$
Relativity	10^{-3}
Bremsstrahlung	10^{-3}
Atomic porarization	10^{-5}
Nuclear polarization	$< 10^{-10}$

all ≤ 1%

Not a solution! (we need ~ 100%)

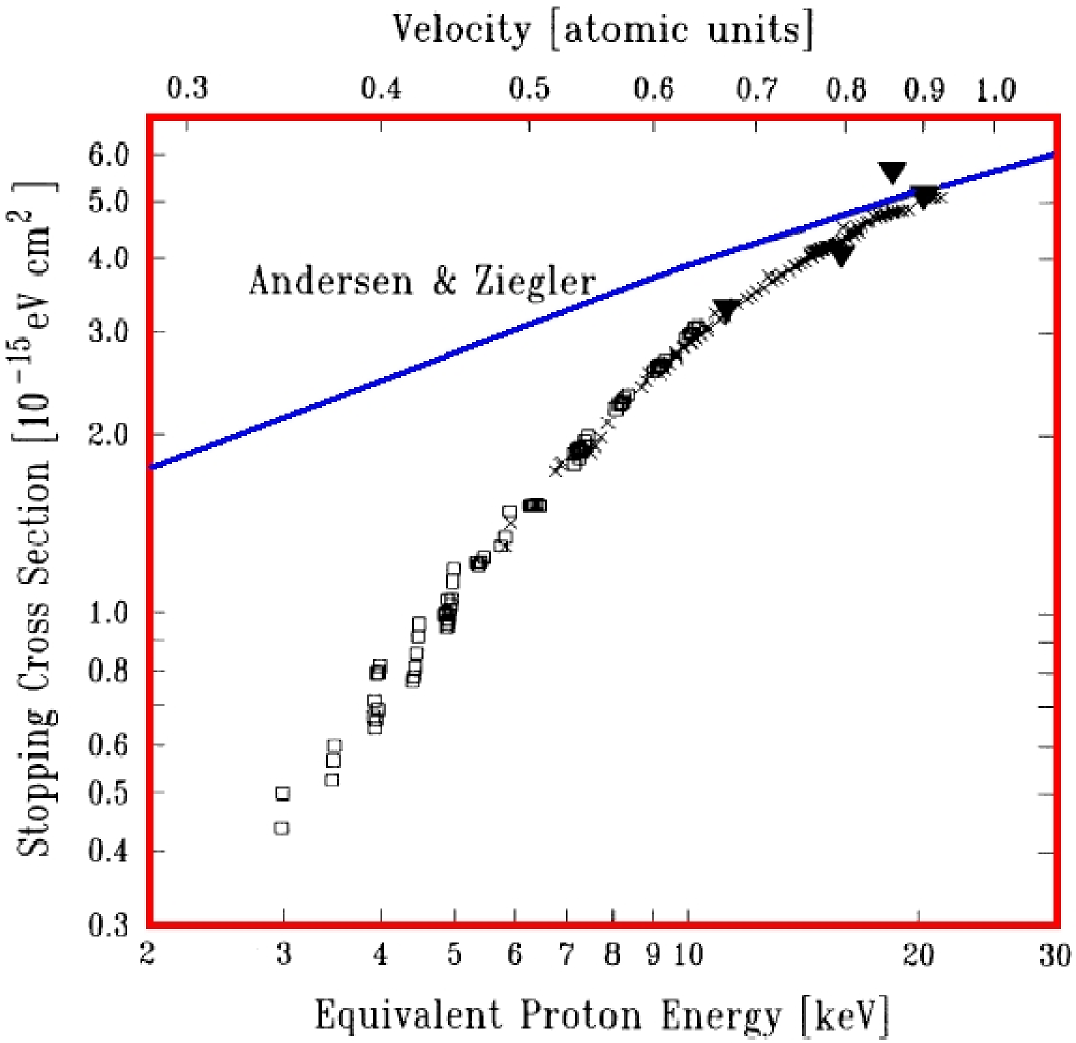


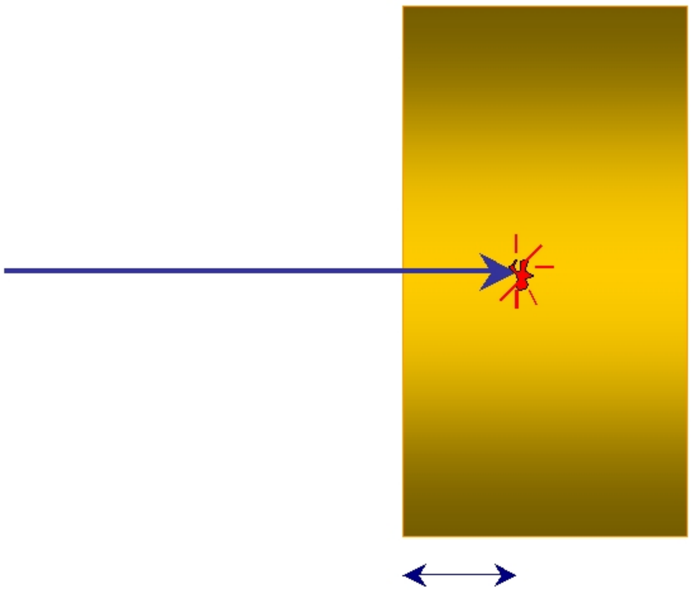


Vacuum polarization

Wrong extrapolation of stopping power?

Bang, PRC 1996; Langanke, PLB 1996





$$S_p = -\frac{dE}{dx}$$

$E' = E - S_p.\Delta x$

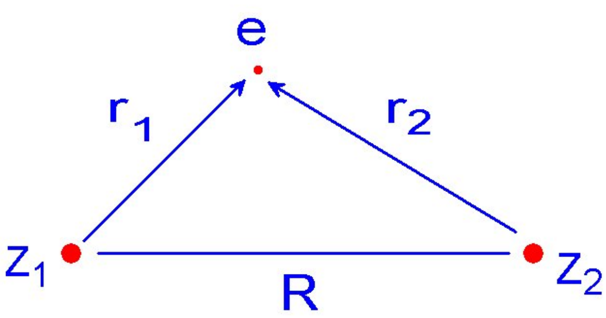
H + He

Golser and Semrad, PRL 1991

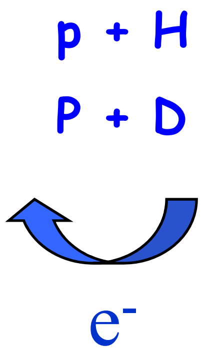
Mainly charge-exchange

Test with the simplest system

CB, de Paula, PRC 2000, PLB 2004



Elliptic coordinates



Charge exchange (pickup)

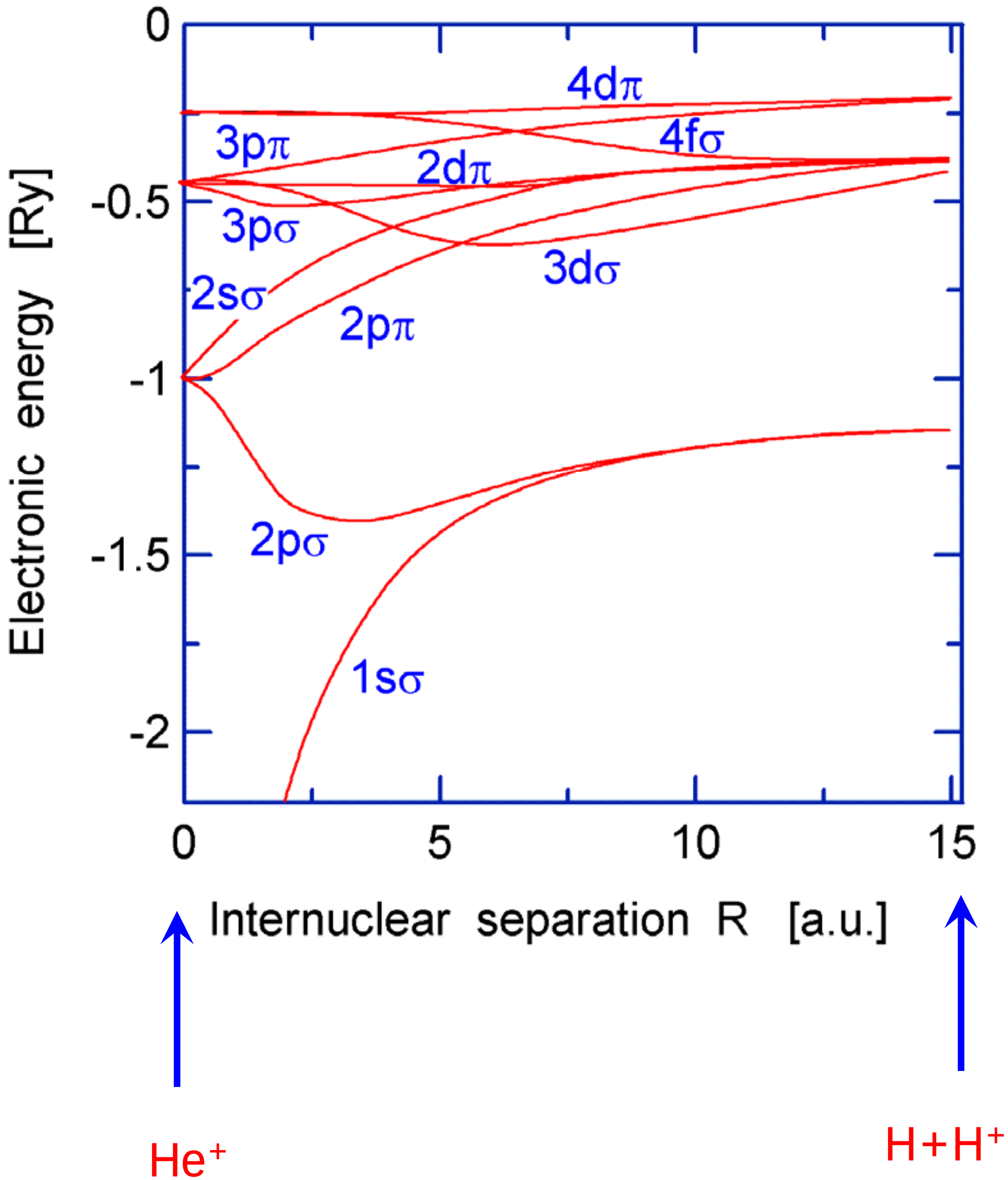
$$\xi = \frac{r_1 + r_2}{R}; \qquad \eta = \frac{r_1 - r_2}{R}; \qquad \phi$$

$$\Psi = F(\xi)\,G(\eta)\,e^{im\phi}$$

$$\frac{d}{d\xi}\left[\left(\xi^2 - 1\right)\frac{dF}{d\xi}\right] + \left[\frac{R^2\xi^2}{2}E + 2R\xi - \frac{m^2}{\xi^2 - 1}\right]F(\xi) = 0$$

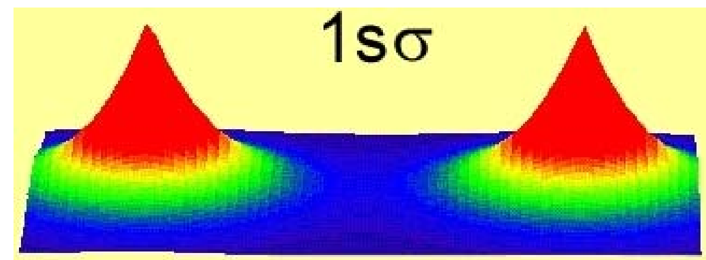
$$\frac{d}{d\eta}\left[\left(1 - \eta^2\right)\frac{dG}{d\eta}\right] - \left[\frac{R^2\xi^2}{2}E + 2R\xi + \frac{m^2}{\eta^2 - 1}\right]G(\eta) = 0$$

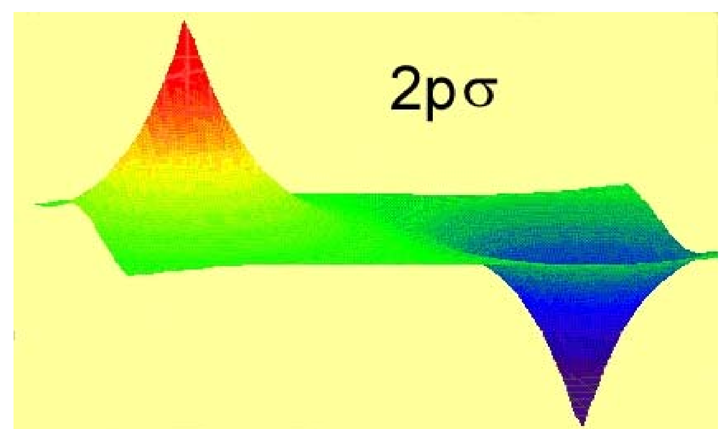
Expansion basis: molecular orbitals for p+H



$$l_z \Phi_s = \pm \lambda \Phi_s$$

Value of λ	0	1	2	3
Code letter	σ	π	δ	$\phi, \cdots$



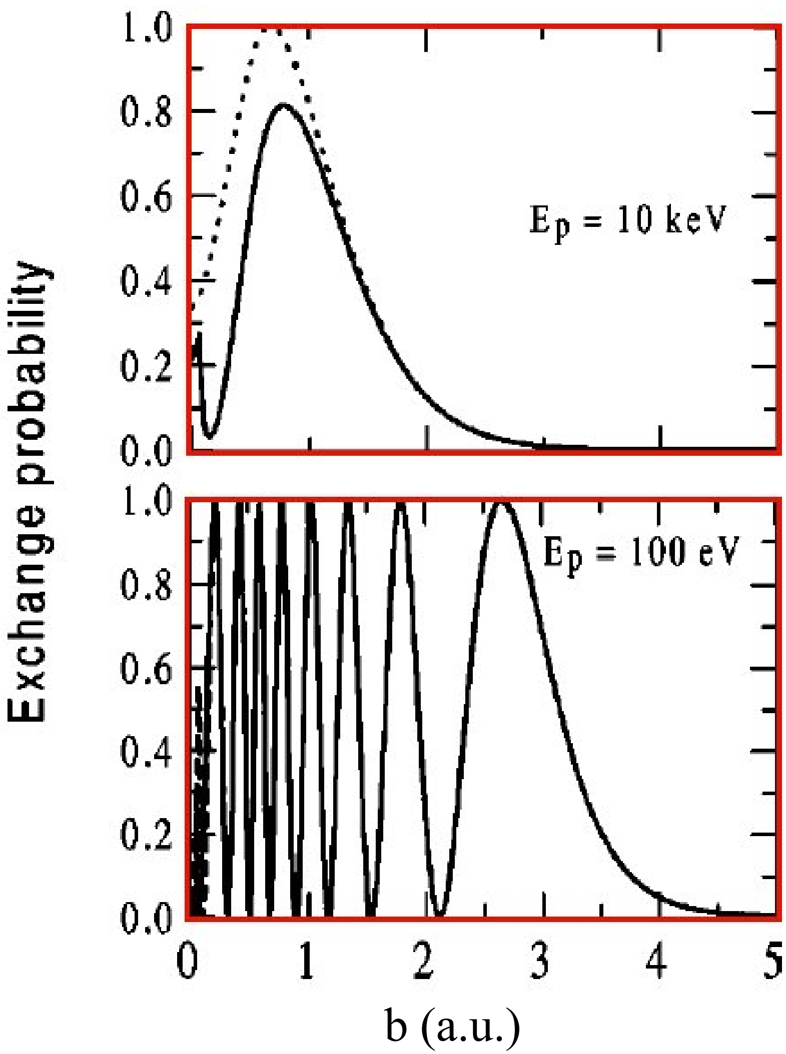


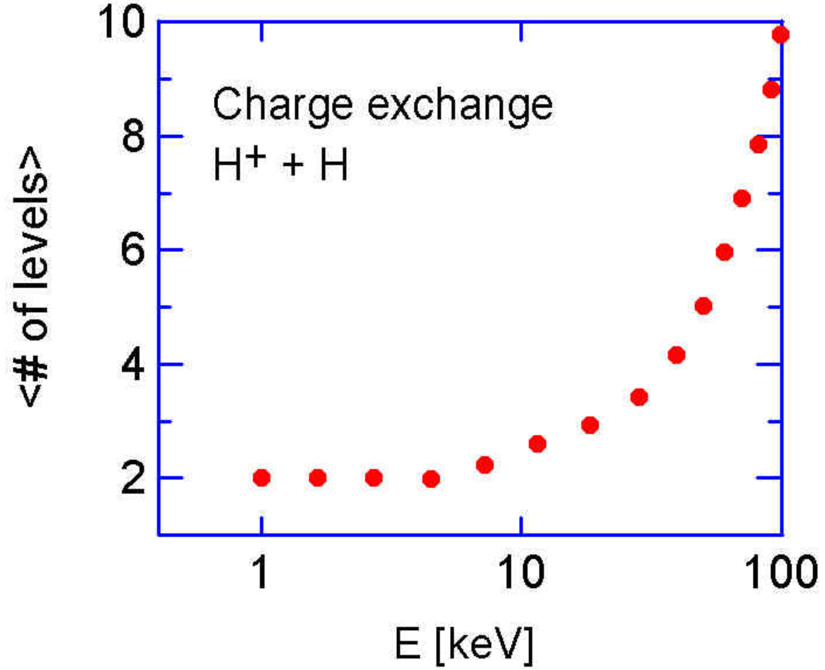
t.d. calculations

$$i\hbar \frac{d}{dt} a_m(t) = E_m(t) a_m(t) - i\hbar \sum_n a_n(t) \left\langle m \left| \frac{d}{dt} \right| n \right\rangle$$

$$\left\langle m \left| \frac{d}{dt} \right| n \right\rangle = \frac{\langle m | dV_p / dt | n \rangle}{E_n(t) - E_m(t)},$$

(Hellman, Feynmann relation)





For E_p < 10 keV, only 1sσ and 2pσ

2-level problem - resonant exchange

$$P_{exch} \approx \frac{1}{2} + \frac{1}{2} \cos\left\{ \frac{1}{\hbar} \int_{-\infty}^{\infty} \left[E_{2p}(t) - E_{1\sigma}(t) \right] dt \right\}$$

Stopping in H⁺ + He collisions

Slater-type orbitals

$$\phi = N\, r^{n-1}\, e^{-\xi r}\, Y_{lm}(\theta,\phi)$$

Two-center basis for two-electrons Hartree-Fock equations

$$\Phi_i = \sum_{i=1}^{n}\left[c_{ji}^{A}\phi_i^{A} + c_{ji}^{B}\phi_i^{B}\right]$$

→ **F . C = S . C . E**

$$\mathrm{F}_{\mu\nu} = H_{\mu\nu} + \sum_{\lambda\rho} P_{\lambda\rho}\left[(\mu\nu\,|\,\lambda\rho) - \frac{1}{2}(\mu\rho\,|\,\lambda\nu)\right]$$

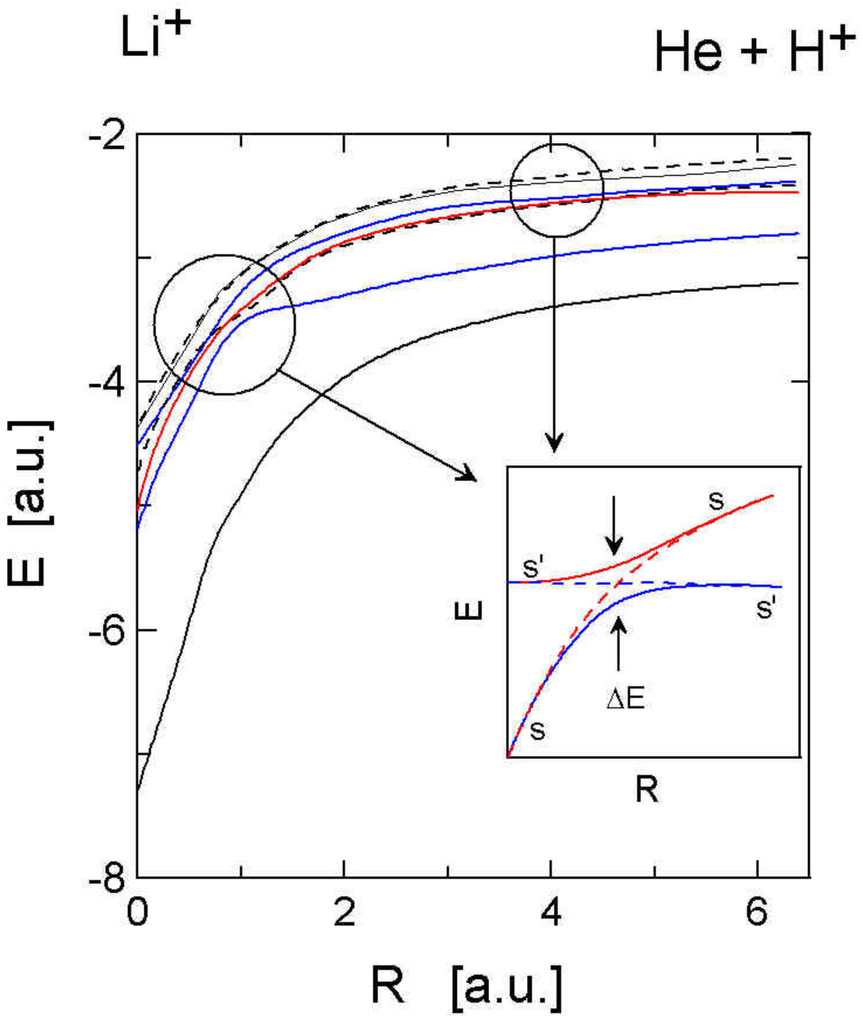
$$H_{\mu\nu} = \iint \phi_{\mu}^{*}(1)\left[-\frac{1}{2}\nabla_1^2 - \sum_{A}\frac{1}{r_{1A}}\right]\phi_{\nu}^{*}(1)\, d\tau_1\,, \qquad P_{\lambda\rho} = 2\sum_{i=1}^{occ} c_{\lambda i} c_{\rho i}$$

$$(\mu\nu\,|\,\lambda\rho) = \iint \phi_{\mu}(1)\,\phi_{\nu}(1)\frac{1}{r_{12}}\phi_{\lambda}(2)\,\phi_{\rho}(2)\, d\tau_1 d\tau_2\,, \qquad S_{\mu\nu} = \int \phi_{\mu}(1)\,\phi_{\nu}(1)\, d\tau_1$$

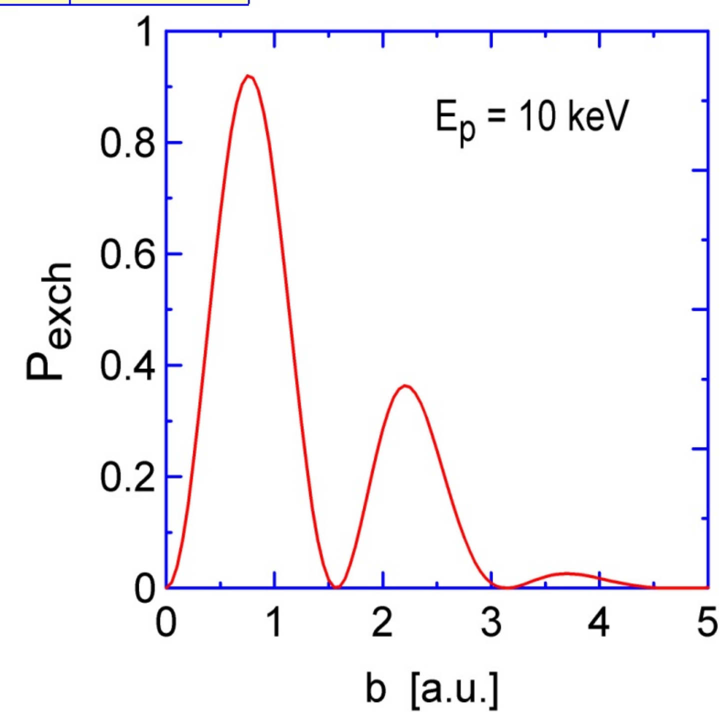
→ t. d. coupled-channels equations

Damping of resonant exchange

H(1s) ⇔ He(1s2s)



Separated atom	United atom
$H^+ + He(1s^2)$	0Σ
$H(1s) + He^+(1s)$	1Σ
$H^+(1s) + He(1s2s)$	2Σ
$H(n=2) + He^+(1s)$	1Π
$H(n=2) + He^+(1s)$	3Σ
$H(n=2) + He^+(1s)$	4Σ
$H^+ + He(1s1p)$	5Σ
$H^+ + He(1s1p)$	2Π



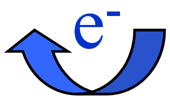
Landau-Zener

$$P = e^{-\Gamma\,\Delta t_{coll}} \cos^2\left[\frac{H_{12}\,a}{2v}\right]$$

Stopping power at very low energies

p + H

P + D



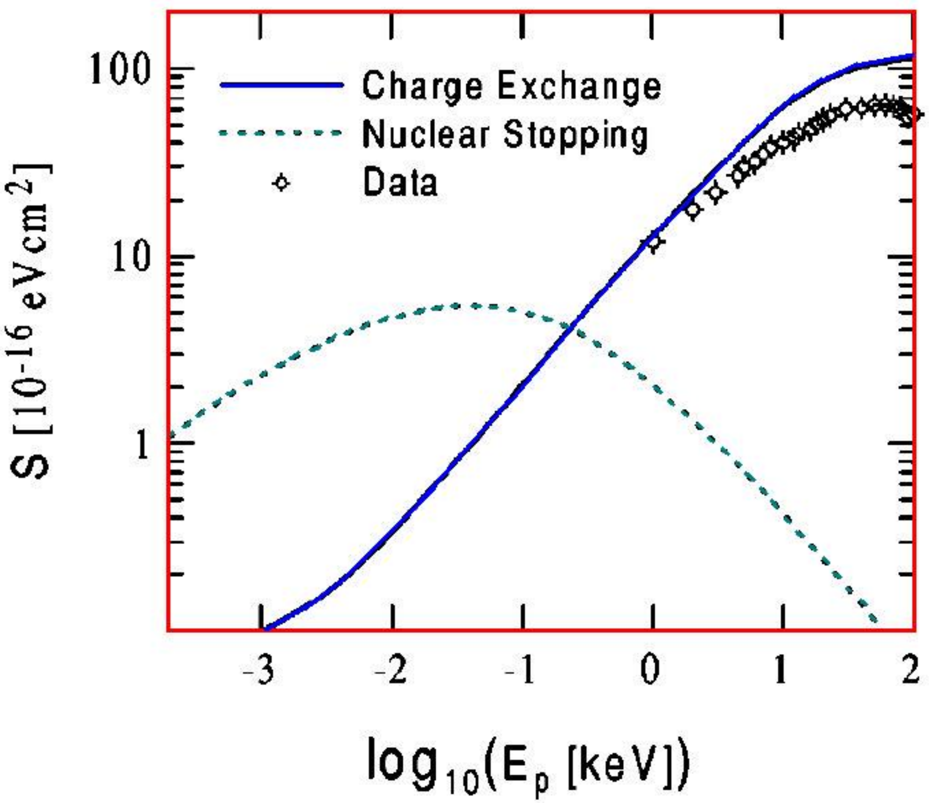
CB, PLB 2004

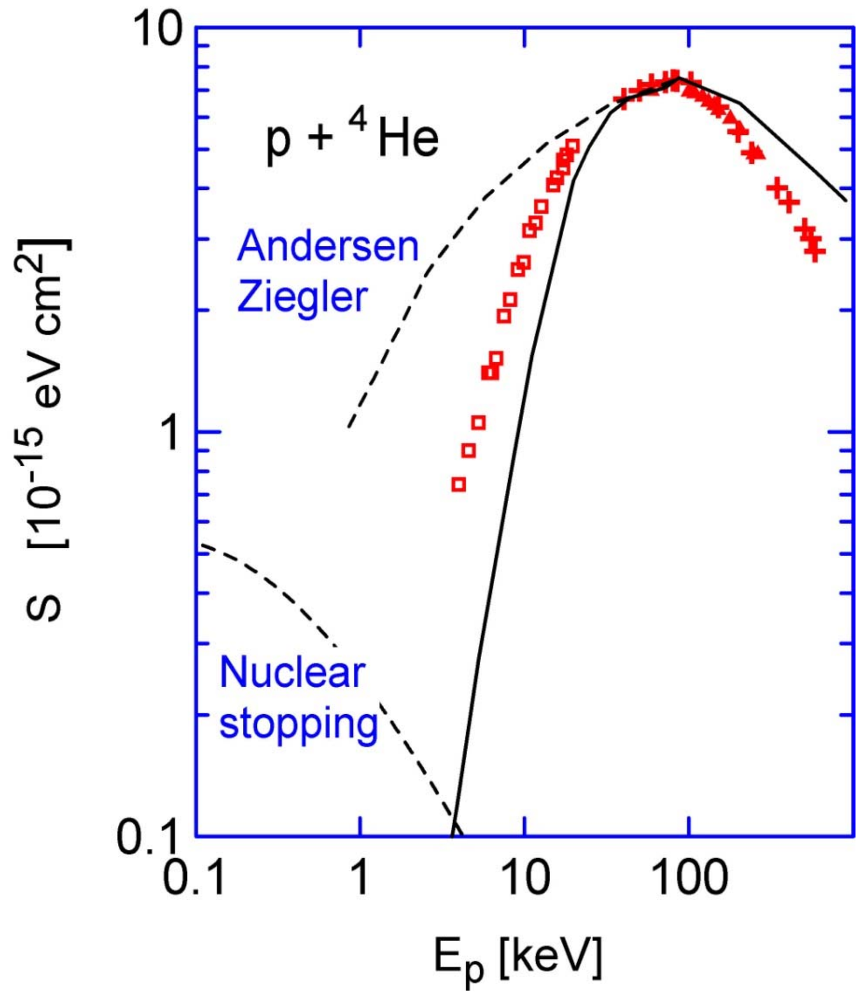
Threshold effect

$$E_P \geq \frac{\mu^2}{4M_P m_e} \Delta E \geq 8 \quad \text{keV}$$

He: $1s^2 \rightarrow 1s2s$: 19.8 eV

CB, de Paula, PRC 2000



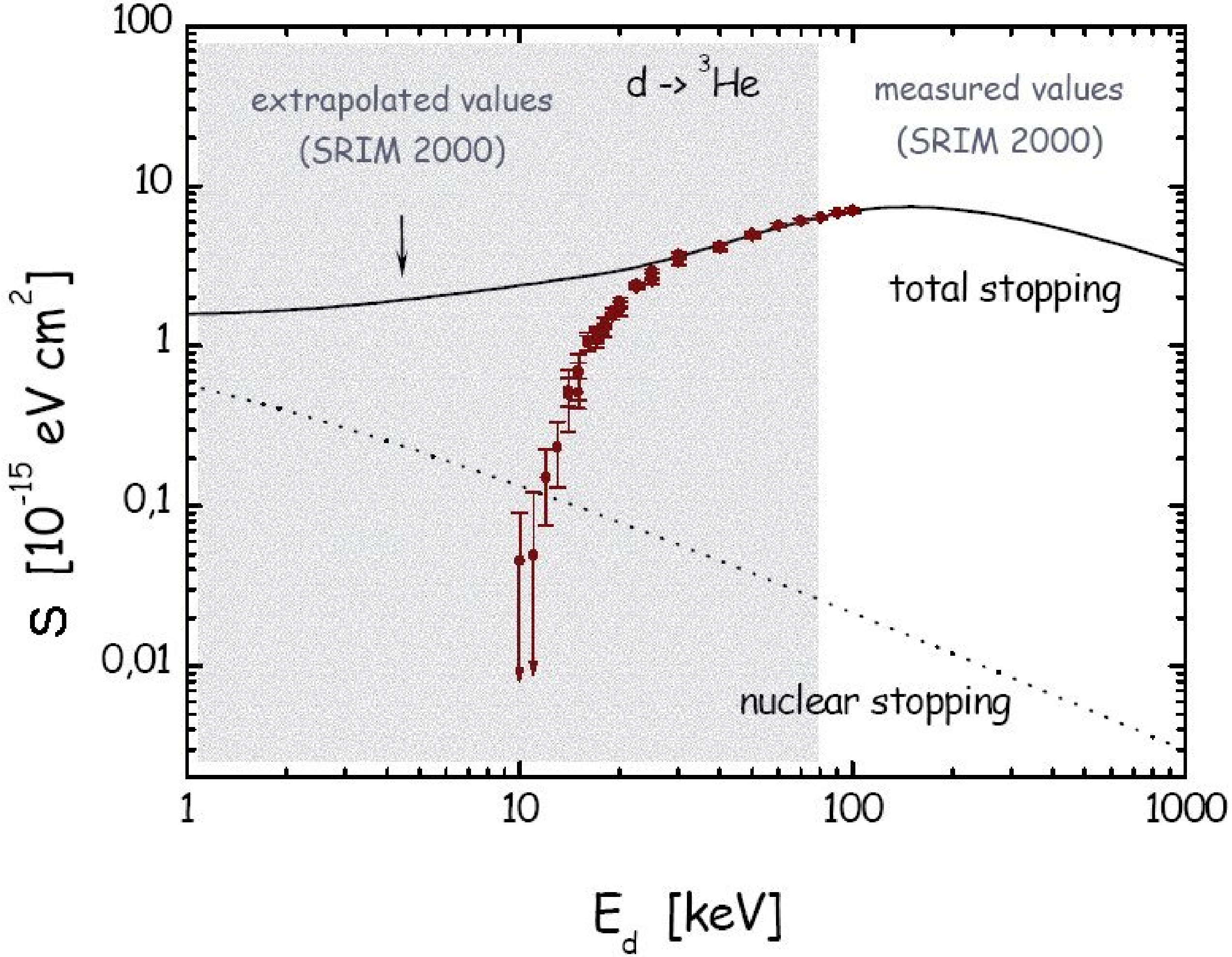


Experimental data

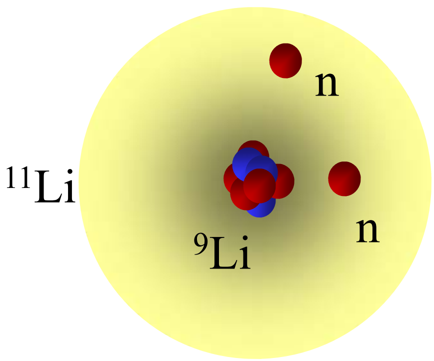
Formicola et al, Eur. Phys. J. A 2000

Raiola et al, EPJ 2002

Rolfs, PTP 2004



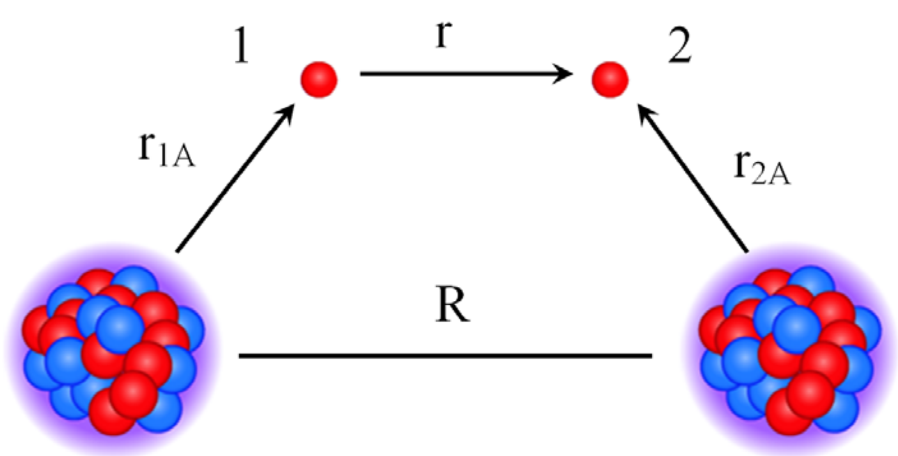
Example 2 Fusion of halo nuclei



$$\mathcal{H} = -\frac{\hbar^2}{2m}\nabla_1^2 - \frac{\hbar^2}{2m}\nabla_2^2 + V_A(r_{1A}) + V_B(r_{1B}) + V_A(r_{2A}) + V_B(r_{2B})$$

$$\Psi_\pm \cong \Psi_A(r_{1A}) \pm \Psi_B(r_{1B})$$

$$E(R) = \frac{\langle \Psi | \mathcal{H} | \Psi \rangle}{\langle \Psi | \Psi \rangle} = \frac{S_{2n}(1+\mathcal{O}^2) + 2\mathcal{O}\mathcal{J} + 2\mathcal{I}}{1+\mathcal{O}^2}$$

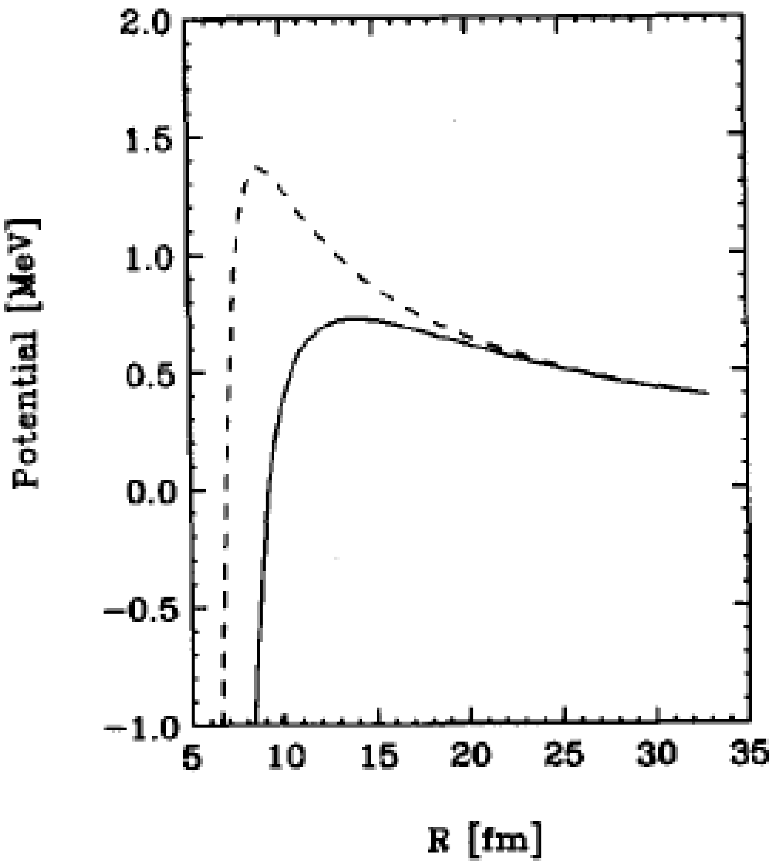


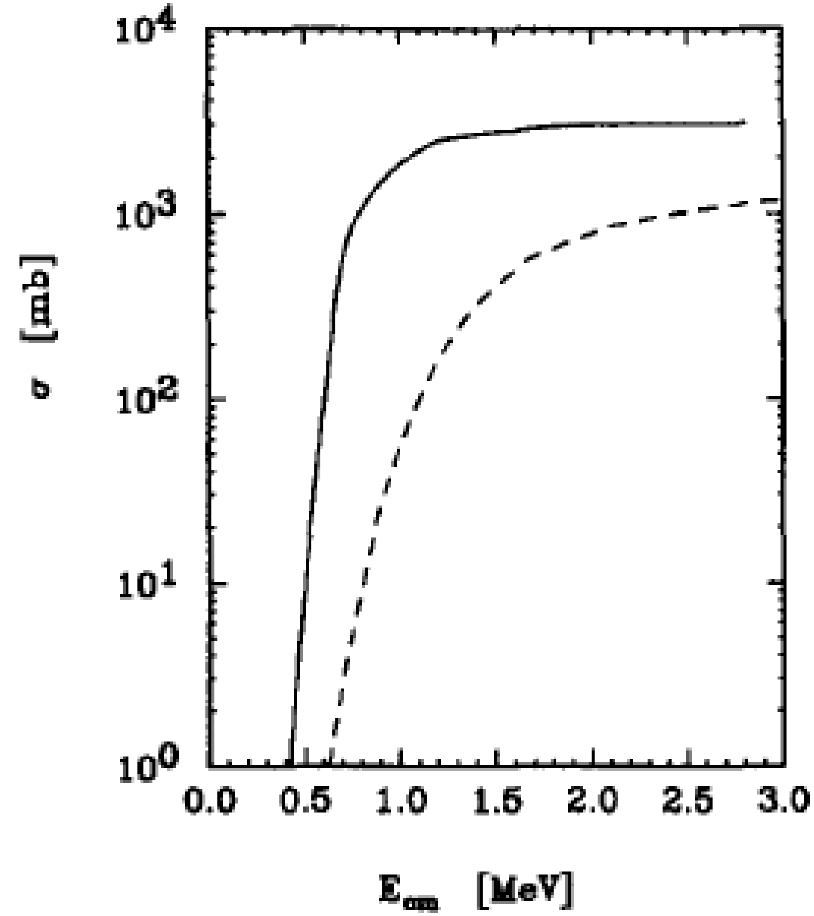
$$\mathcal{I} = \langle \Psi_A \| V_B(r_{1B}) \| \Psi_A \rangle$$

$$\mathcal{L} = \langle \Psi_A \| V_B(r_{1B}) \| \Psi_B \rangle$$

$$\mathcal{O} = \langle \Psi_A | \Psi_B \rangle$$

$^{9}Li+^{11}Li$



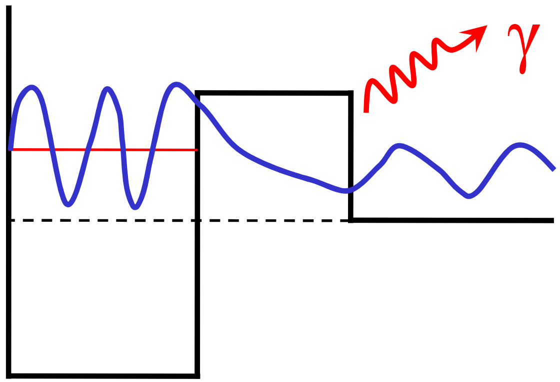


$$V(R) = E(R) - S_{2n}$$

$$\sigma(E_{cm}) = \frac{\pi}{k^2}\sum_l (2l+1) T_l(E_{cm})$$

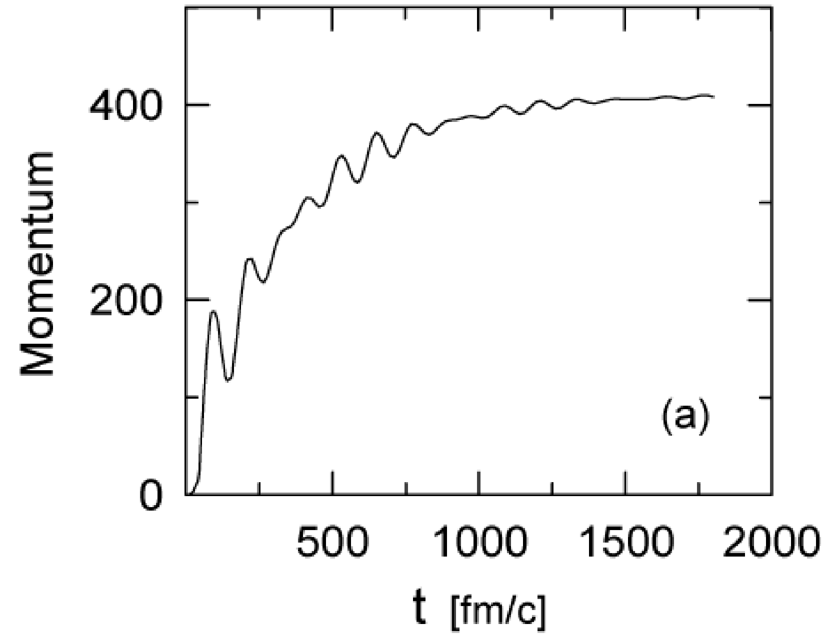
CB, Balantekin, PLB 314, 275 (1993)

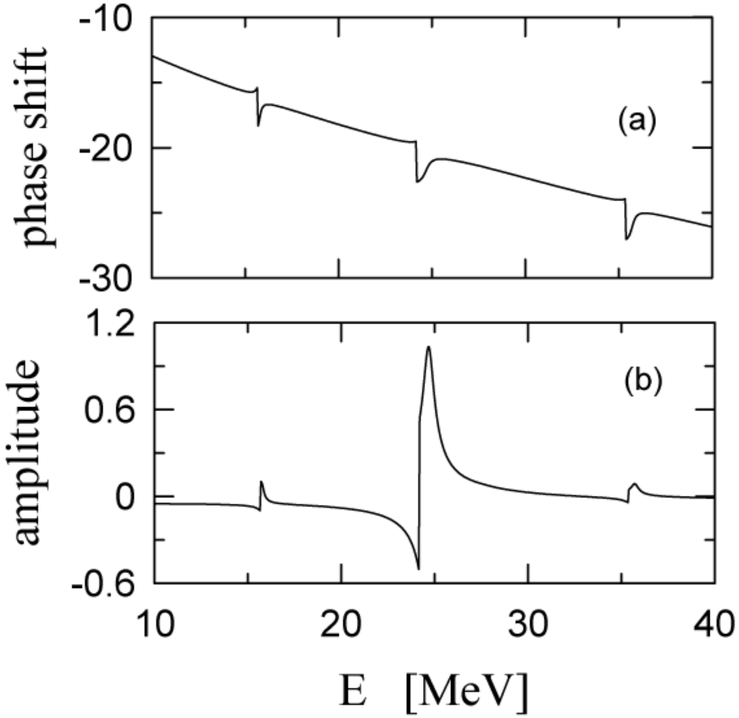
Example 3 Bremsstrahlung in α-decay

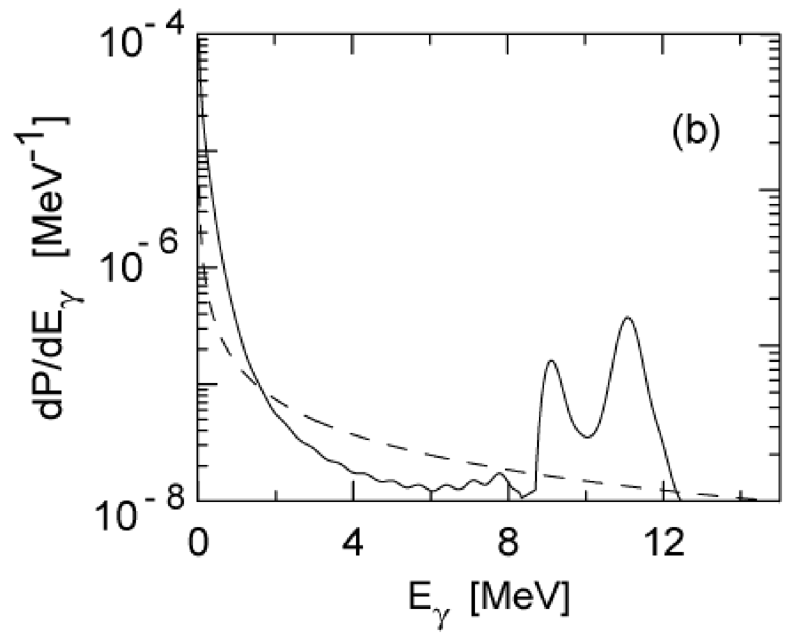


$$\frac{dP}{dE_{\gamma}} = \frac{1}{\mathrm{E}_{\gamma}} \frac{\mathrm{dE}(\omega)}{\mathrm{dE}_{\gamma}}$$

$$dE(\omega) = \frac{8\pi\omega^2}{3m^2c^3} Z^2 e^2 \left| \langle \Psi | \hat{\mathbf{p}} | \Psi \rangle \right|^2$$

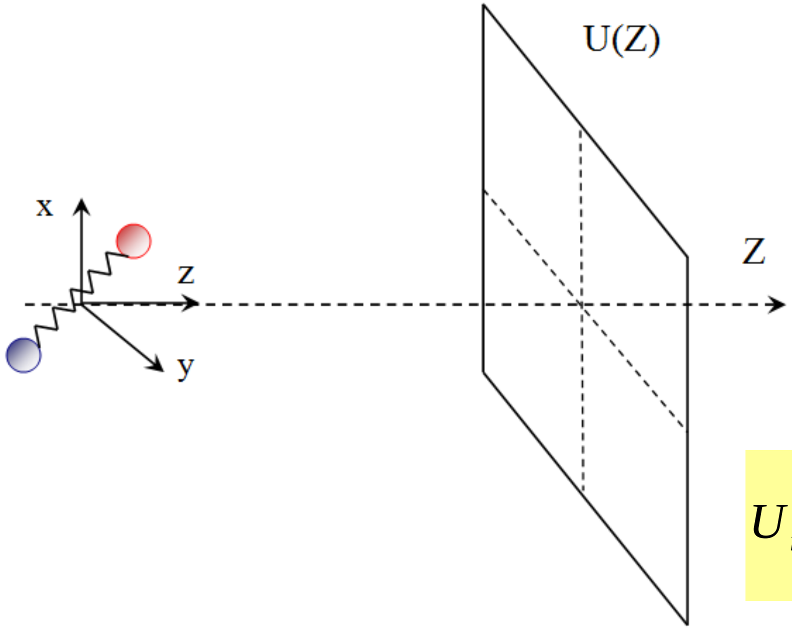






CB, de Paula, Zelevinsky,
PRC 60, 031602(R) (1999)

Composite particle tunneling through a barrier



$$\mathcal{H} = \frac{\mathbf{P}^2}{4m} + \frac{\mathbf{p}^2}{m} + U\left(Z - \frac{z}{2}\right) + U\left(Z + \frac{z}{2}\right) + V(r)$$

$$\left[\frac{\mathbf{p}^2}{m} + V(r)\right]\phi_n(\mathbf{r}) = \varepsilon_n \phi_n(\mathbf{r})$$

$$\Psi(Z,\mathbf{r}) = \sum_{n=0}^{\infty} \psi(Z)\phi_n(\mathbf{r})$$

$$U_{nm}(Z) = \frac{4m}{\hbar^2}\int \left[U\left(Z + \frac{z}{2}\right) + U\left(Z - \frac{z}{2}\right)\right]\phi_n^*(\mathbf{r})\phi_m(\mathbf{r})d\mathbf{r}$$

$$\left(\frac{d^2}{dZ^2} + k_n^2\right)\psi_n(Z) - \sum_m U_{nm}(Z)\psi_m(Z) = 0$$

$$k_n^2 = \frac{4m}{\hbar^2}(E - \varepsilon_n)$$

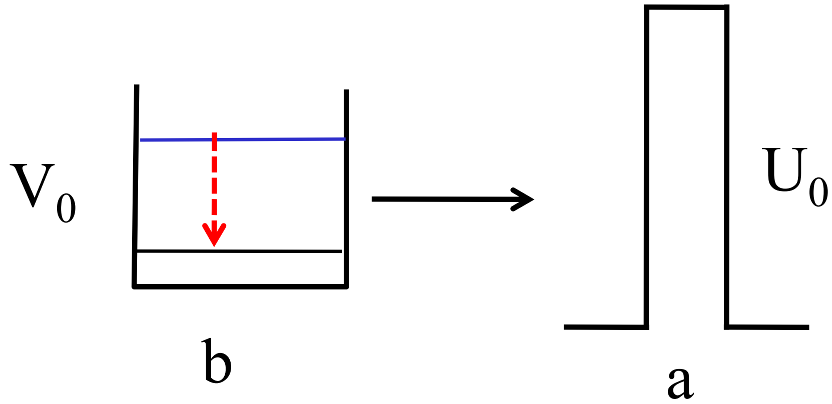
$$\psi_{nl}(Z) = e^{ik_n Z}\delta_{nl} + \frac{1}{2ik_n}\sum_{m=0}^{\infty}\int_{-\infty}^{\infty} e^{ik_n(Z-Z')}U_{nm}(Z')\psi_{ml}(Z')dZ'$$

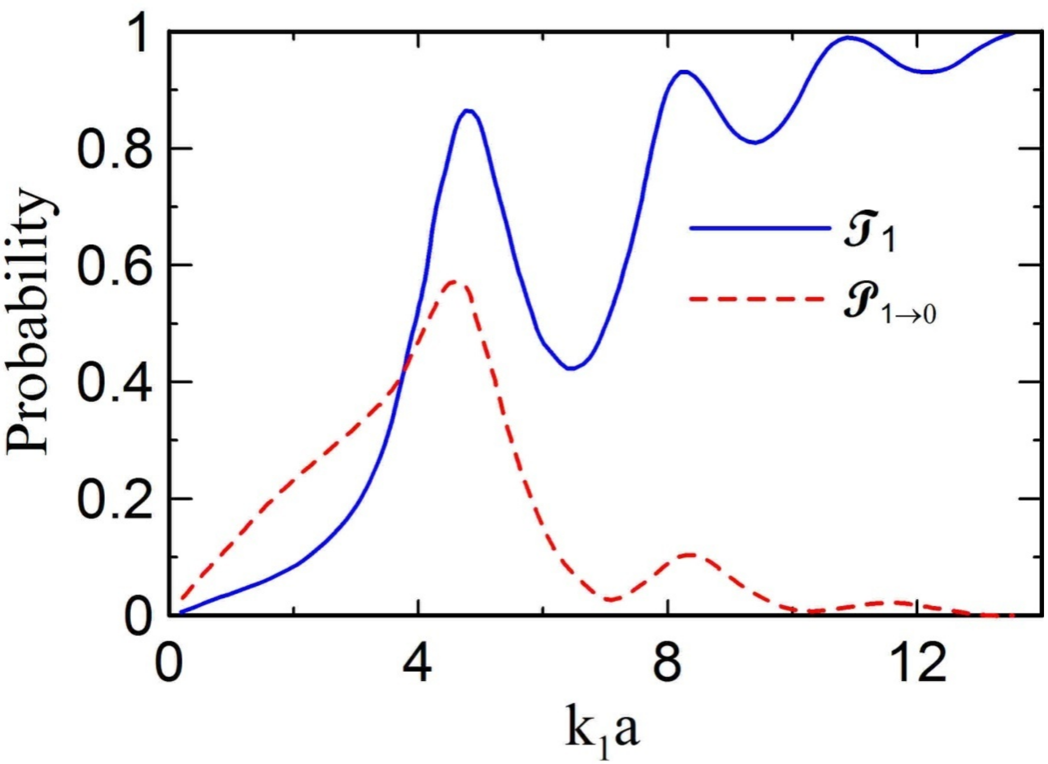
$$R_{nl}(Z) = \frac{1}{2ik_n}\sum_{m=0}^{\infty}\int_{-\infty}^{\infty} e^{ik_n Z'}U_{nm}(Z')\psi_{ml}(Z')dZ'$$

$$T_{nl}(Z) = \delta_{nl} + \frac{1}{2ik_n}\sum_{m=0}^{\infty}\int_{-\infty}^{\infty} e^{-ik_n Z'}U_{nm}(Z')\psi_{ml}(Z')dZ'$$

$$\mathcal{R}_l = \sum_{n=0}^{\infty} \frac{k_n}{k_l} |R_{nl}|^2, \qquad \mathcal{T}_l = \sum_{n=0}^{\infty} \frac{k_n}{k_l} |T_{nl}|^2$$

$$\mathcal{P}_{n\to l} = \frac{k_n}{k_l} \left(|R_{nl}|^2 + |T_{nl}|^2 \right)$$





Step barrier

$$\frac{U_0 a}{\hbar c} = 10, \qquad \sqrt{\frac{mU_0}{\hbar^2}}a = 6$$

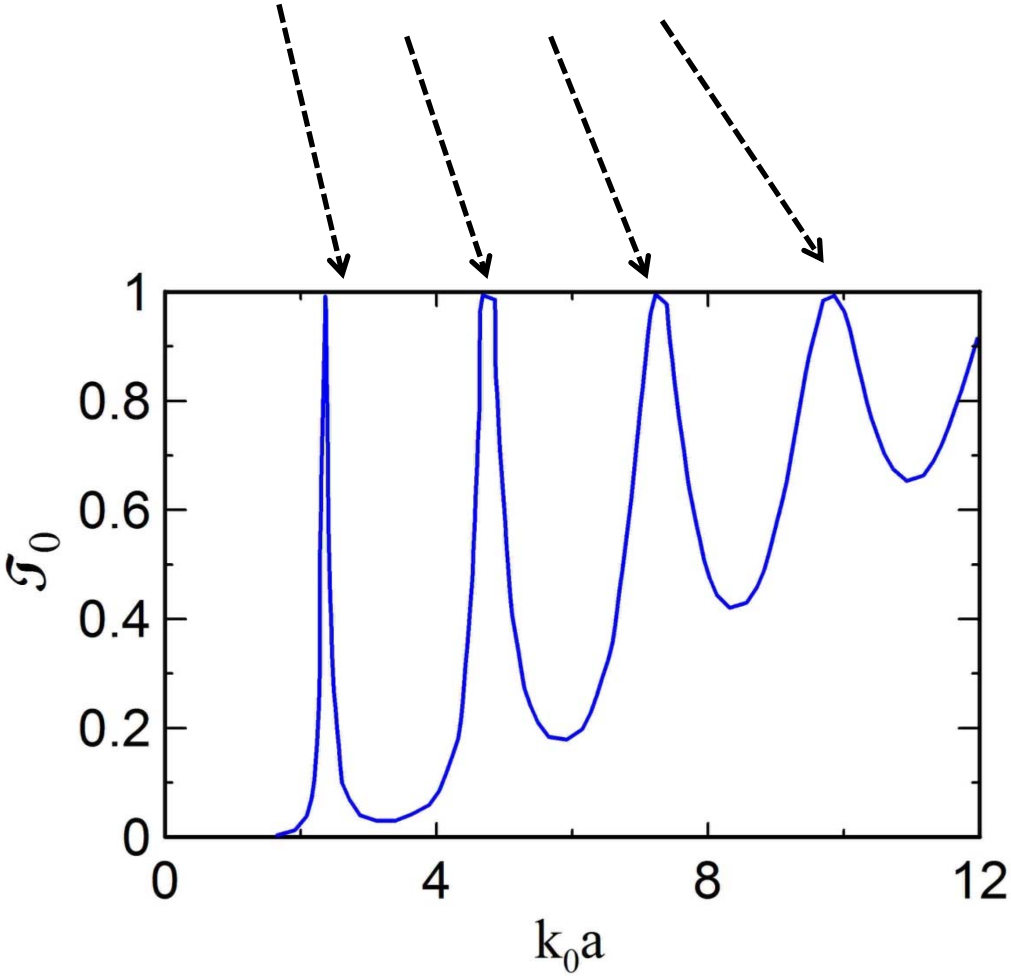
Square-well particle-particle interaction

$$\frac{V_0 b}{\hbar c} = 2, \qquad \sqrt{\frac{mV_0}{\hbar^2}}b = 4$$

Deuteron tunneling through a barrier

$$k_0 + \frac{2mU_0 a}{\hbar^2} \tan\left(k_0 \sqrt{\langle r^2 \rangle} \right) = 0$$

valid for $U_0 a \rightarrow \delta(Z)$



For d + d → ^{4}He

With a and U_0 simulating Coulomb barrier

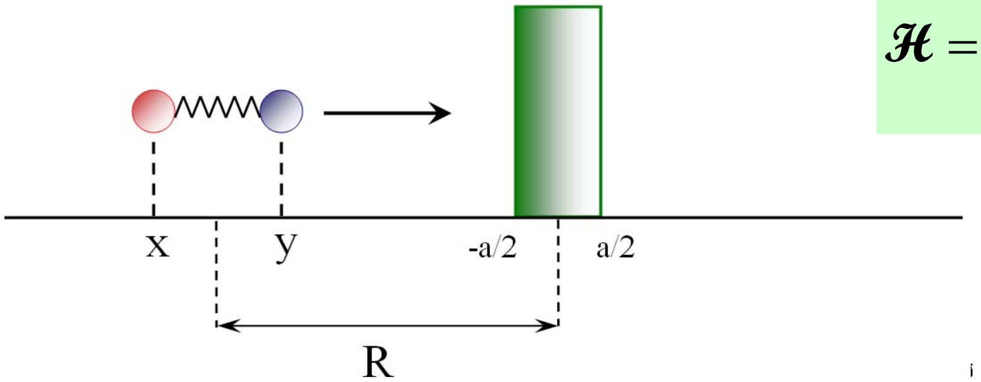
→No tunneling resonance

For large Z's, A's

→ Many resonances possible, if $\langle r^2 \rangle$ large (loosely-bound)

Effective potential and fusion enhancement

CB, Flambaum, Zelevinsky, JPG 34, 1 (2007)



$$\mathcal{H} = -\frac{p_x^2}{2m_x} + \frac{p_y^2}{2m_y} + V(x-y) + U(x) + U(y)$$

$$x, y \to r, R \qquad \Psi(r,R) = \psi(R)\phi(r,R)$$

$$\alpha(R) = \left\langle \phi \middle| \frac{\partial \phi}{\partial R} \right\rangle$$

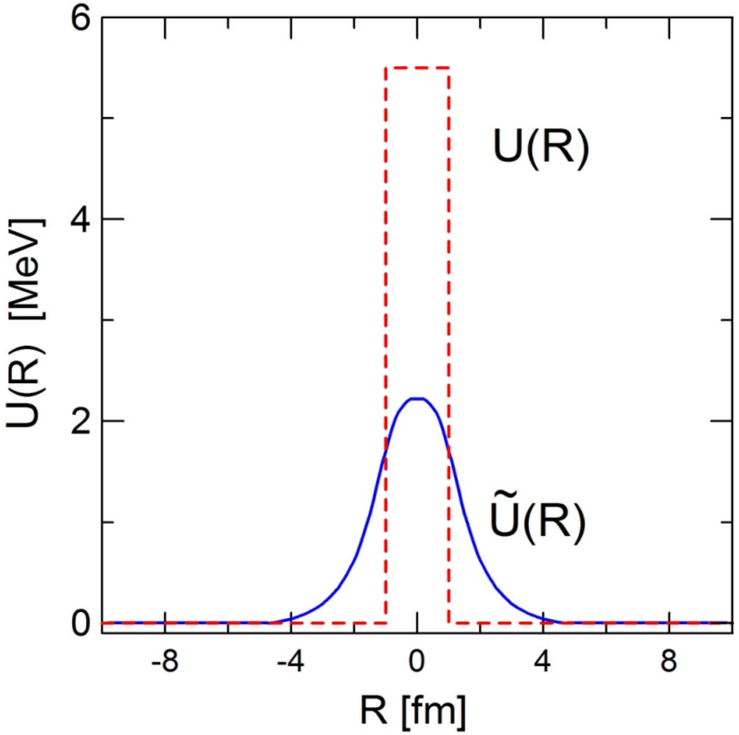
ϕ normalized

$$\psi(R) = u(R)\exp\left[-\int^R \alpha(R')dR'\right]$$

$$u''(R) + \frac{2M}{\hbar^2}\left[E - \widetilde{U}(R)\right]u(R) = 0$$

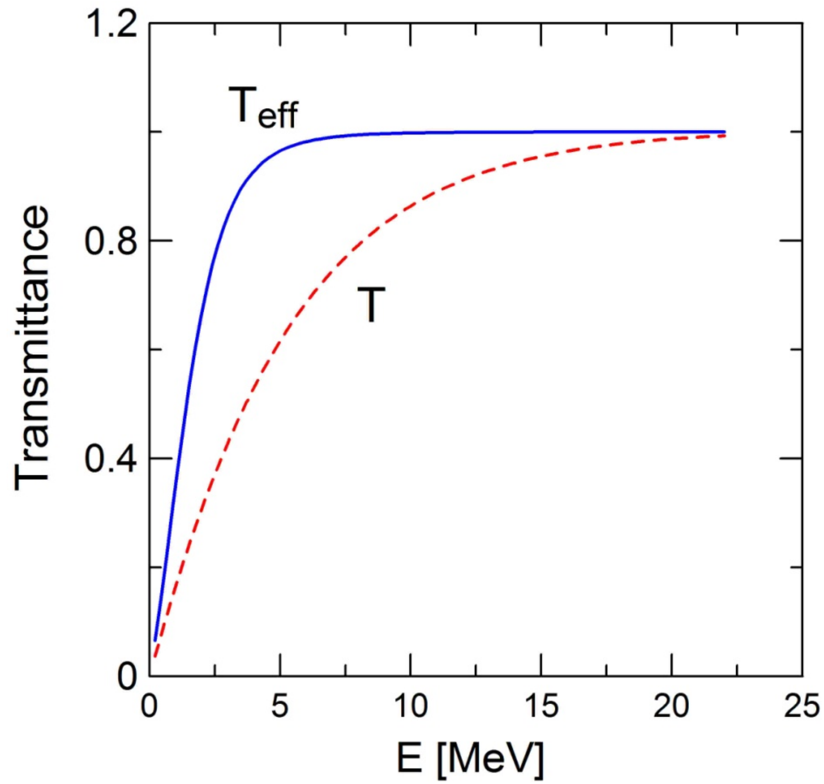
$$\widetilde{U}(R) = \varepsilon(R) - E_0 + \frac{\hbar^2}{2M}\left[\alpha^2(R) + \alpha'(R) + \beta(R)\right]$$

$$\beta(R) = \left\langle \phi \middle| \frac{\partial^2 \phi}{\partial R^2} \right\rangle$$



$$E_0 = -2.225 \text{ MeV}$$
$$r_0 = 2 \text{ fm}$$

CB, Flambaum, Zelevinsky,
JPG 34, 1 (2007).



Conclusions & Perspectives:

- Tunneling of molecules and atoms

  Lauhon, Ho, PRL 85, 4566 (2000) (H-atom tunnels a copper surface)
  Yazdani, Nature 409, 471 (2001)

- Tunneling of Cooper pairs

  Zelevinsky, Flambaum, JPG 34, 355 (2005)

- Tunneling of excitons

  Saito, Kayanuma, Phys. Rev. B 51, 5453 (1995)
  Jin et al., Acta Phys. Sin. 53 3211 (2004)

- Fusion of loosely-bound nuclear systems

  C.B., Zelevinsky, in progress