Incorporating wave effects in ray billiards:

Challenging ray-wave correspondence in open systems

Martina Hentschel

MPIPKS Dresden

mpipks Mesoscopic Systems

Emmy Noether-Programm
Deutsche Forschungsgemeinschaft DFG

and DFG Research Group 760 "Scattering systems with complex dynamics"

The Mesoscopic Regime

system size L $\leq$ phase coherence length l_φ

l_φ

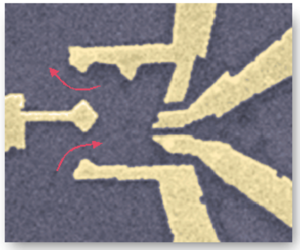
Interference

Microscopic world	**Mesoscopic regime**	**Macroscopic world**
(quantum mechanical)	(semiclassical)	(classical)
≤ 1 nm	~ nm … μm	≥ 0.1 mm

Decoherence

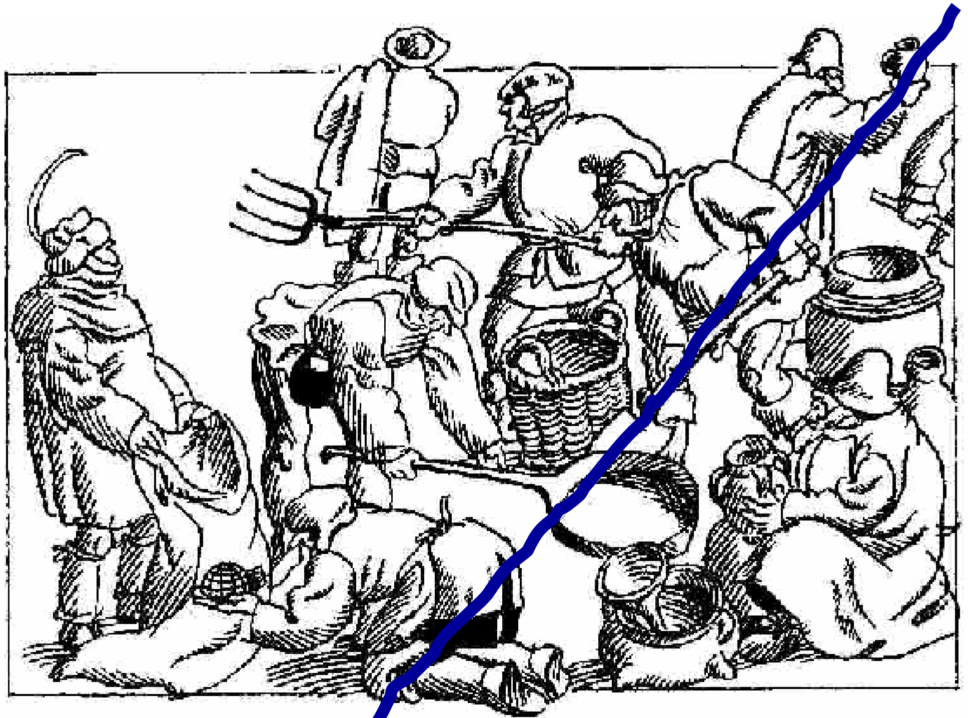
Examples

- **Electrons** in semiconductor heterostructures, quantum dots, and nanoparticles (l_φ ~ 1 μm)
- **Light** in optical microcavities made from glass fibers, polymers or semiconductor heterostructures (l_φ ~ 100 μm)



1μm
Marcus Lab (Harvard)

Have to catch light



Die Schildbürger bauen ein Rathaus

Sie traten ein. Drinnen war es so dunkel, dass sie einander nicht sehen konnten. Sie bekamen einen großen Schreck und überlegten, warum es im Rathaus so dunkel ist. Sie gingen nach draußen. Alle drei Mauern waren fest und gerade, das Dach war gut gedeckt und überall draußen war es hell.

Wieder gingen die Schildbürger in das Rathaus, aber drinnen war es dunkel wie zuerst. Sie überlegten und überlegten, aber sie erkannten nicht, dass sie die Fenster in ihrem Rathaus vergessen hatten. Schließ-

Johann Friedrich von Schönberg (?) (1598)

use

total internal reflection

at thinner medium (air)

i.e. cavity refractive index $n > 1$

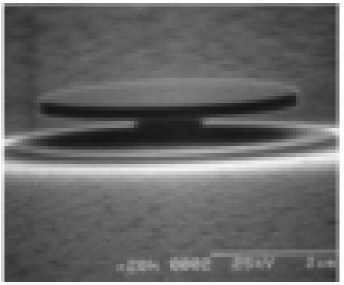
incident angle ≥ critical angle

$$\chi \geq \arcsin(1/n)$$

Optical Microcavities

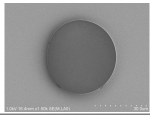
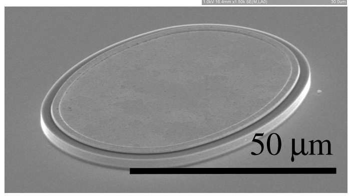
Examples

- Semiconductor Heterostructures

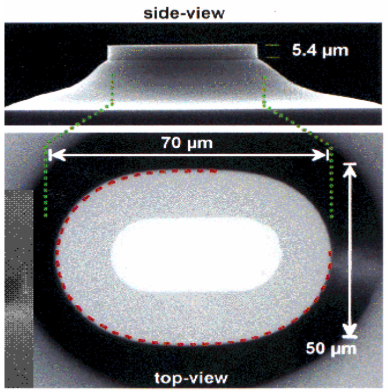


Bell Labs

n = 3.3
n = 2.6

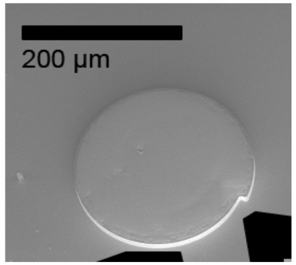



Harayama group 2005



Gmachl et al. 1992

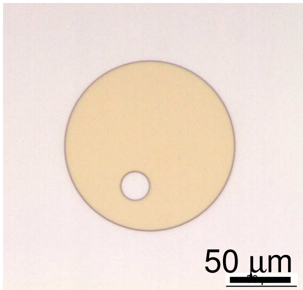
Kneissl et al. 2004



Capasso group 2007

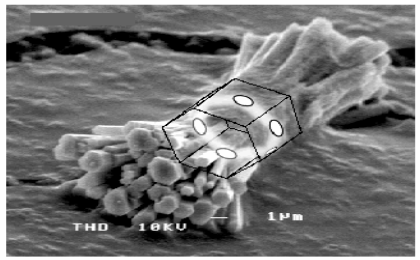
- Polymer structures
  n = 1.5

- Glass fibers
  n = 1.4 .. 1.8



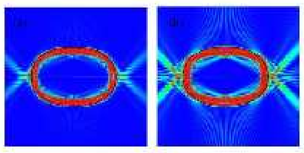
Lebenthal et al. 2006

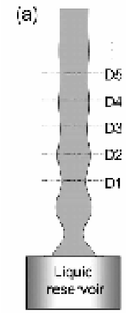
- Minerals
  Zeolithe
  n = 1.47

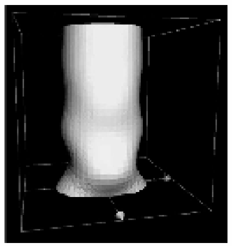


Braun et al. 2000

- Jets and Droplets
  Ethanol, n = 1.36

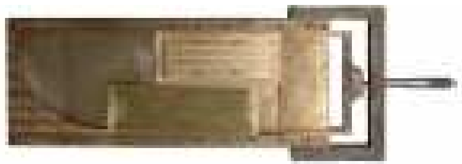






Kim et al., 2006

- Microwave resonators
  Stöckmann and Richter groups



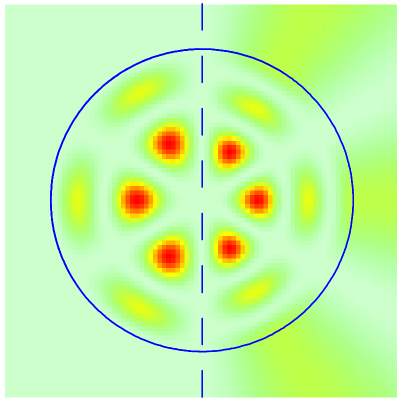
Why is it interesting?

I. Optical microcavities as model systems for quantum chaos

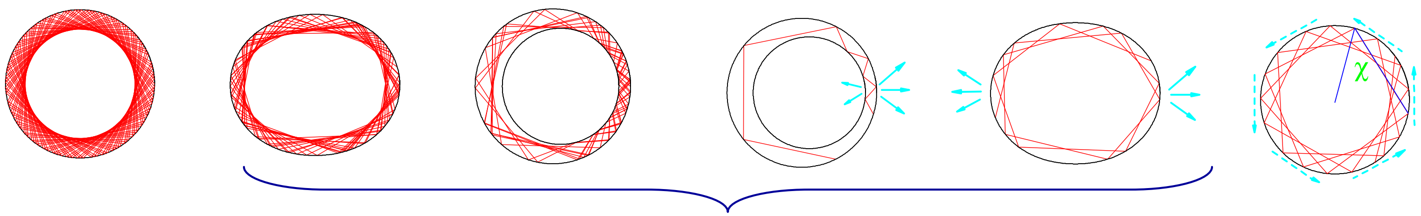
analogy between quantum mechanical particles and light in 2 dimensions:

Schrödinger equation ~ Helmholtz equation

closed systems **open systems** (refractive escape)



II. Directional emission from microlasers



break rotational symmetry

holy grail: unidirectional emission

Contents

I. Optical microcavities as model systems for quantum chaos

classical-quantum correspondence as paradigm of quantum chaos

But: **deviations from ray-wave correspondence seen in optical microcavities**

types of deviations observed

introducing an *adjusted reflection law* due to openness of optical microcavities can explain many of those

leads to *non-Hamiltonian dynamics,*
for example, the formation of attractors and repellers

II. Directional emission from microlasers

… while keeping a very high Q-factor

the Limaçon-shape can do it

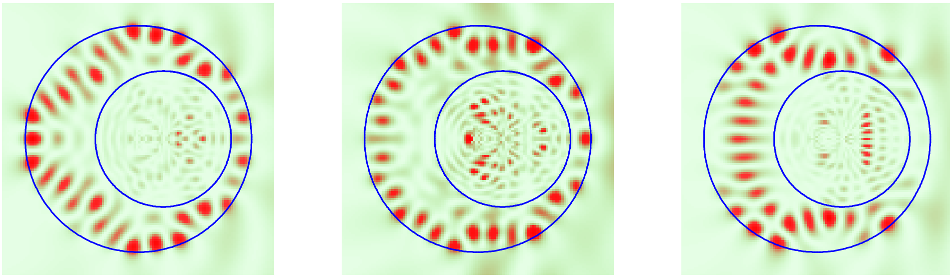
latest from spiral cavities

I. Optical microcavities as model systems for quantum chaos

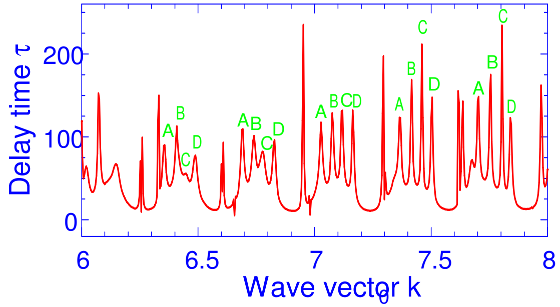
Ray-wave correspondence

in real space:

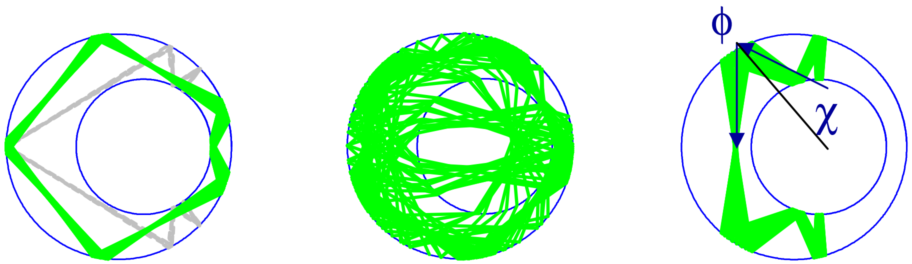
• Resonances (Example: Annular Billiard)



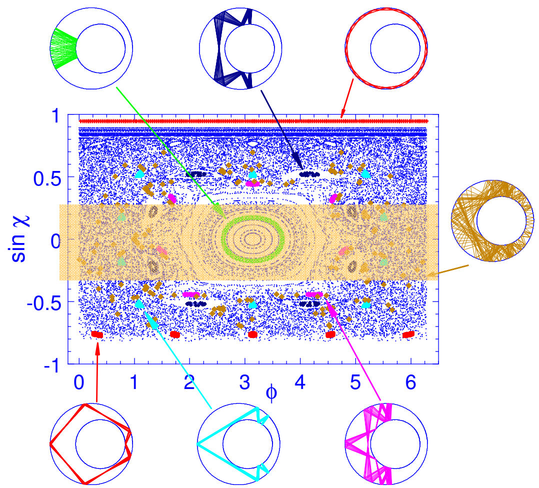
S-matrix method



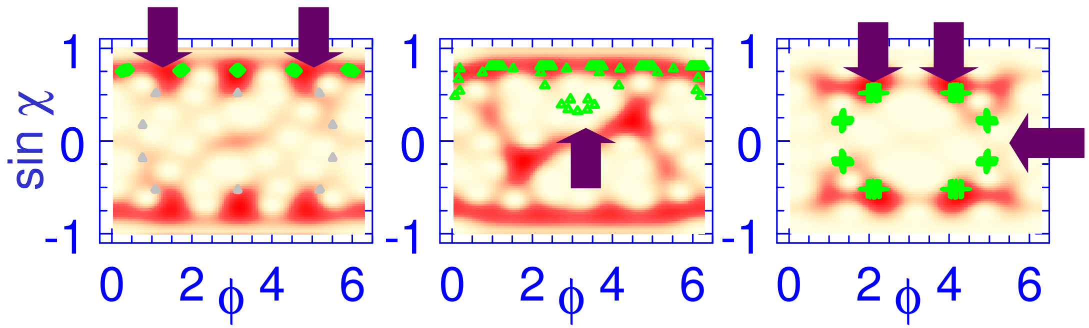
• stable Orbits



Poincaré surface of section



in phase space: Poincaré SOS and Husimi function



M.H. and K. Richter, PRE 2002

Bracket [**Husimi functions at dielectric interfaces**

M.H., H. Schomerus, R. Schubert, Europhys. Lett. 2003

Husimi-function = "measure wavefunction with a
coherent state centered around (ϕ, sin χ) "

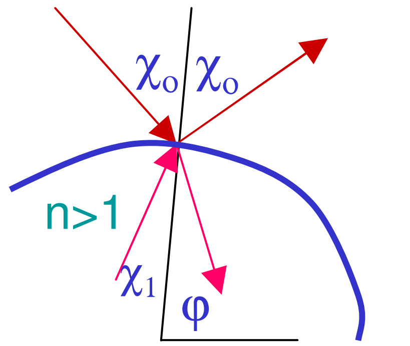
Hard wall billiards: Dirichlet boundary conditions, $\Psi|_{bound} = 0$, $\Psi'|_{bound} \neq 0$

Optical systems: mixed boundary conditions: $\Psi|_{bound} \neq 0$, $\Psi'|_{bound} \neq 0$

have to generalize definition of Husimi-function

$\exists$ four rays

four corresponding Husimi functions desirable



(only) **simultaneously solvable**

$$H(\varphi, \sin\chi) = \frac{k}{2\pi}\left| \pm F\, h_{\Psi}(\varphi, \sin\chi) \pm \frac{i}{k\,F} h'_{\Psi'}(\varphi, \sin\chi)\right|^2 \quad \text{with} \quad F_{0,1} = \sqrt{n\cos\chi_{0,1}}$$

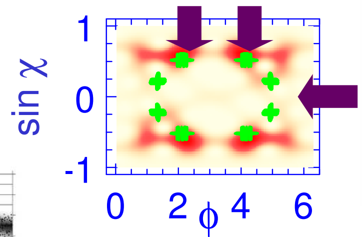
inside/ outside (first ±)

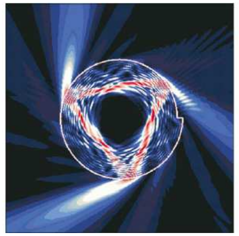
incident/ outgoing (second ±)

] Bracket

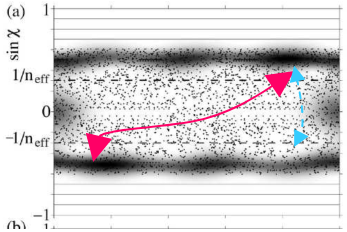
Deviations from the ray picture

- ray-wave correspondence only qualitatively, e.g. quasi-scars in spirals (Lee et al., PRL 2004)



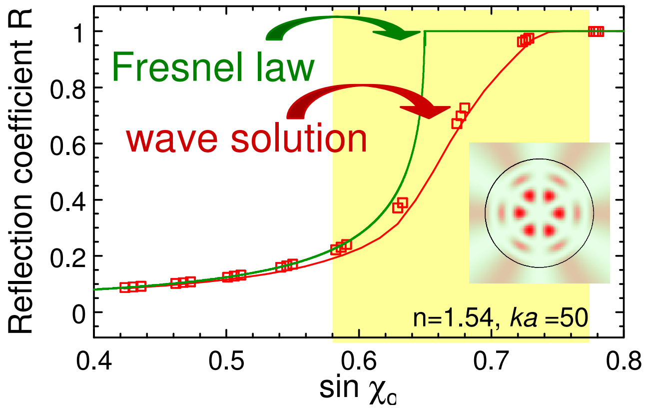


- Husimi functions lack the time reversal symmetry χ $-\chi$ (despite axial symmetry of the billiard)

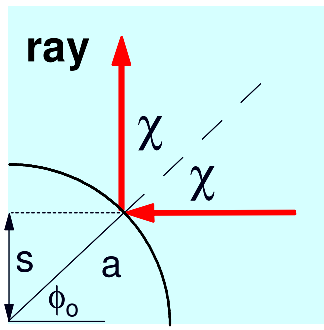


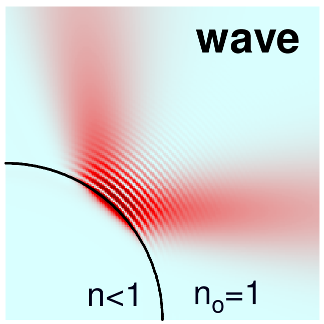
e.g.
M.H. and K. Richter, PRE 2002
J. Wiersig and M.H., PRA 2006

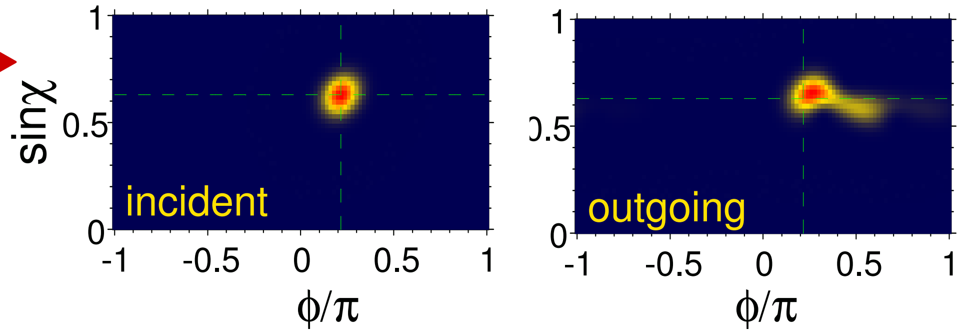
- reflection coefficients deviate from Fresnel's law near critical incidence (M.H. and H. Schomerus, PRE 2002)

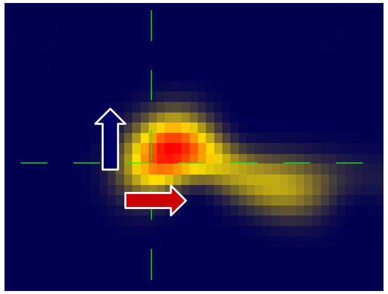


- reflection of a light beam at curved interface: reflection law violated (H. Schomerus and M.H., PRL 2006)



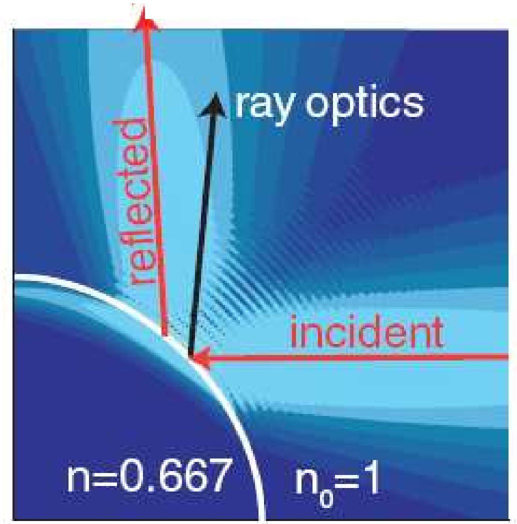


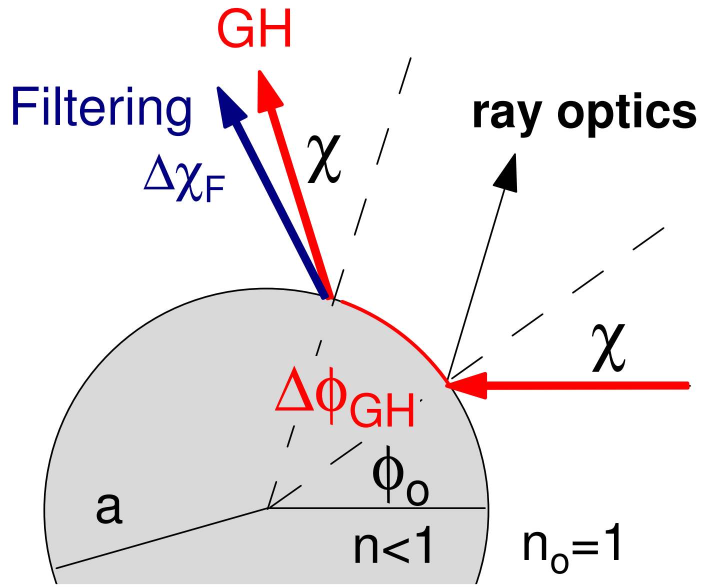




Deviation in two independent directions in phase space:

Goos-Hänchen shift and Fresnel Filtering





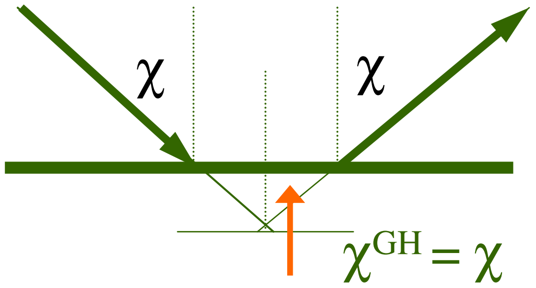
Correction $\Delta\phi_{GH}$ in ϕ – direction Goos-Hänchen shift

$\Delta\chi_F$ in χ – direction Fresnel Filtering

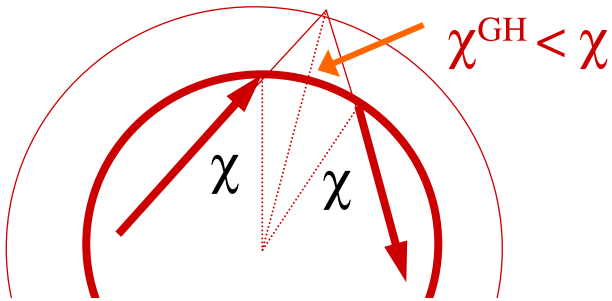
Origin of corrections:

- Fresnel filtering (Ff): beam = built from rays with a distribution of angles χ
  Transition regime to total internal reflection
- Goos-Hänchen shift (GHs): semiclassical interference of those rays
  lateral shift of beam upon total reflection

- GHs: planar interface



- GHs: curved interface



analytical formula

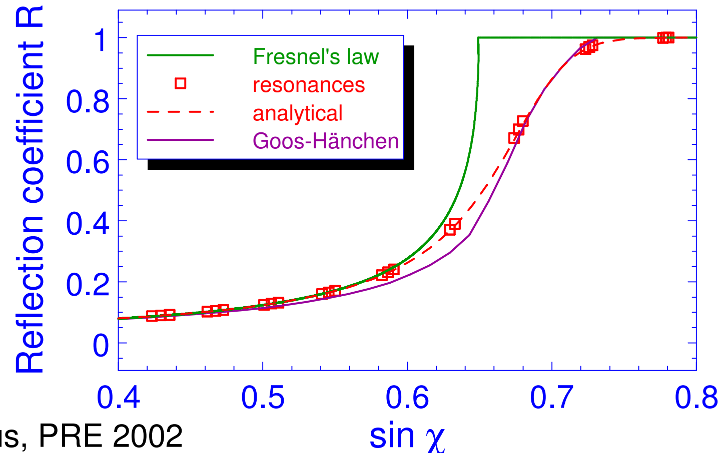
$$R = \left| \frac{\cos\chi + iF}{\cos\chi - iF} \right|^2$$

$$F^{TE} = in\cos\eta \left[1 + \frac{1}{\sin^2\eta} \left(\frac{K_{2/3}(z)}{K_{1/3}(z)} - 1 \right) \right]$$

$$F^{TM} = \frac{F^{TE}}{n^2}$$

$$z = -i \,\mathrm{Re}\, k r_c \frac{\cos^3\eta}{3\sin^2\eta}$$

semiclassical correction to Snell's and Fresnel's law



M.H. and H. Schomerus, PRE 2002

Introducing an adjusted reflection law into billiard dynamics

$$\begin{pmatrix}\phi_r\\ \chi_r\end{pmatrix}=\begin{pmatrix}\phi_i\\ \chi_i\end{pmatrix} \quad\mapsto\quad \begin{pmatrix}\phi_r\\ \chi_r\end{pmatrix}=\begin{pmatrix}\phi_i+\Delta\phi\,(\phi_i,\chi_i)\\ \chi_i+\Delta\chi\,(\phi_i,\chi_i)\end{pmatrix}$$

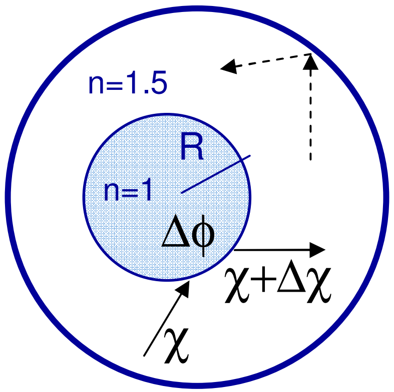
GHs

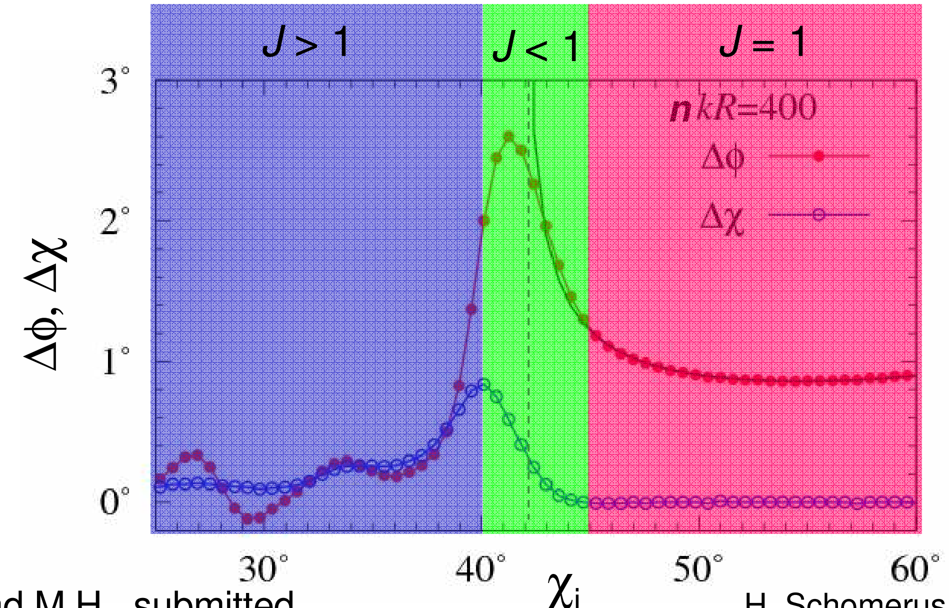
Ff

Example: Annular billiard with air inclusion, but hard outer walls

(with E. Altmann and G. DelMagno, subm. to PRL)

Jacobian: $J=\dfrac{\cos(\chi_i+\Delta\chi)}{\cos\chi_i}\left(1+\dfrac{\partial\,\Delta\chi}{\partial\,\chi_i}\right)$

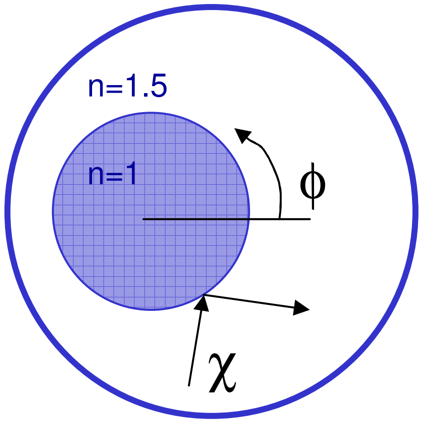


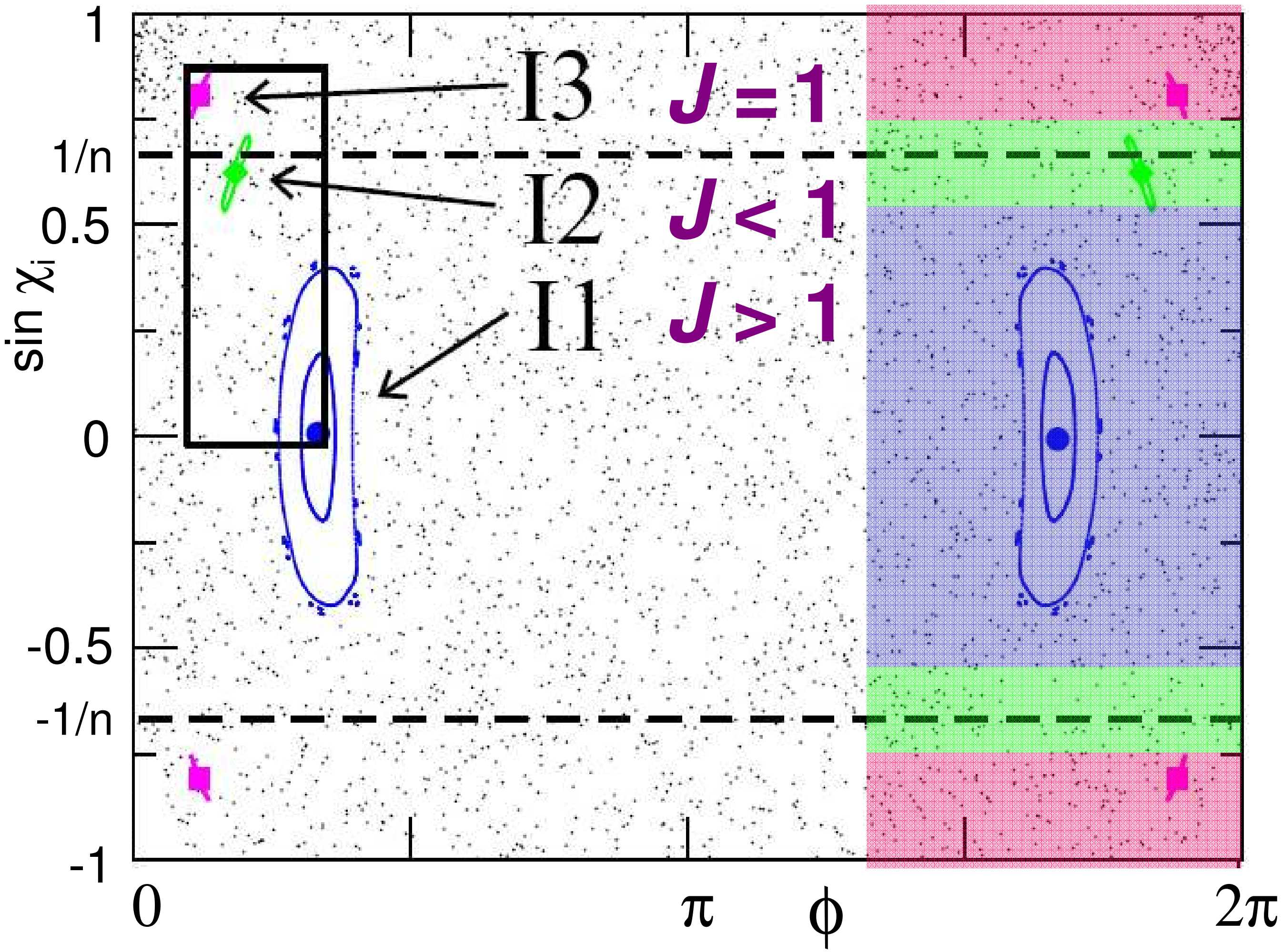


E. G. Altmann, G. Del Magno, and M.H., submitted

H. Schomerus and M.H., PRL 2006

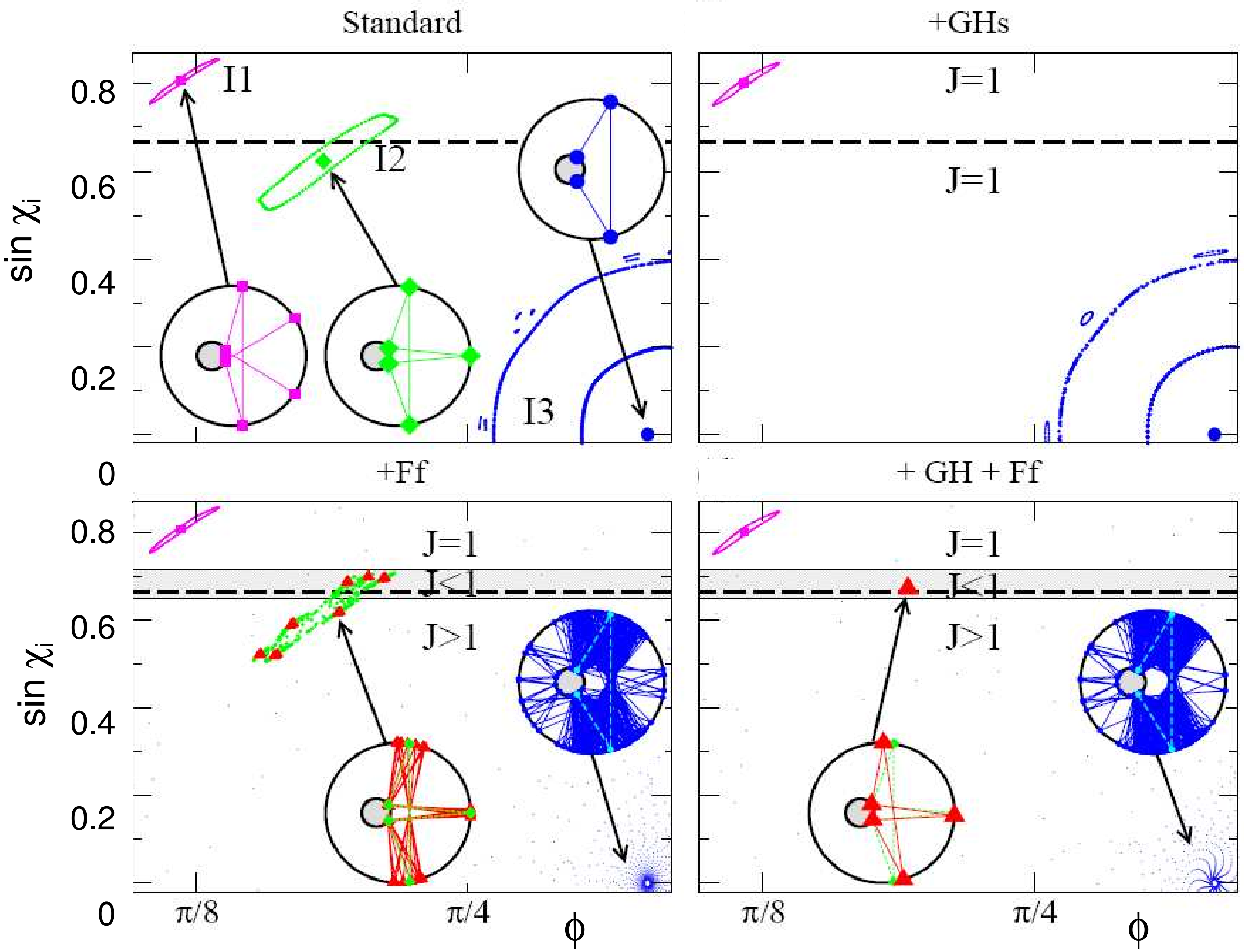
Phase space structure with conventional reflection law





E. G. Altmann, G. Del Magno, and M.H., submitted

Phase space structure with adjusted reflection law: Role of Goos-Hänchen shift and Fresnel Filtering



E. G. Altmann, G. Del Magno, and M.H., submitted

Formation of attractors and repellers depending on the Jacobian:

Non-Hamiltonian dynamics in quantum-chaotic systems

- GHs and Ff can dramatically change the phase-space structure

- if J>1, J<1, or J=1 depends on the (non-trivial) dependence of $\Delta\phi$ and $\Delta\chi$ on ϕ_i and χ_i

- strongest effects around the critical angle

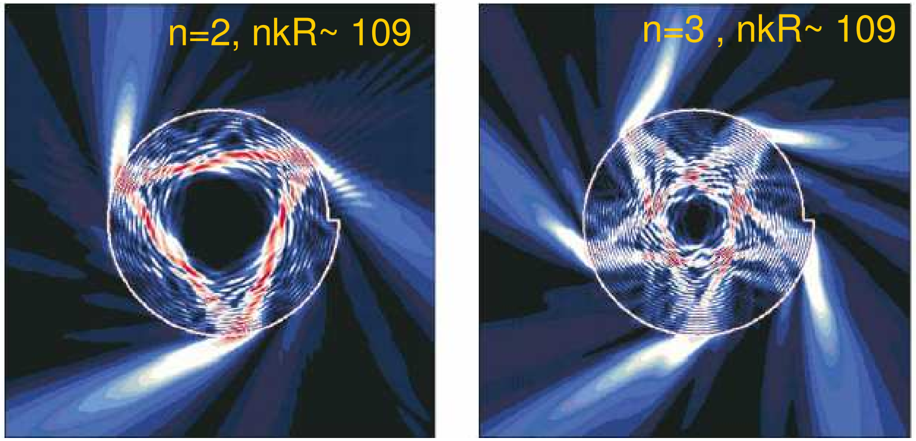
- for constantly curved interface: Fresnel filtering induces the non-Hamiltonian features

- origin: openness of the system
  (this implies Fresnel filtering as one correction to the conventional reflection law)

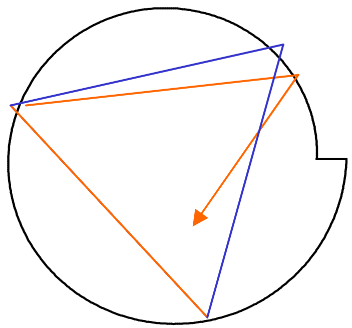
- Note: contracting (J<1, attractors) *as well as* expanding (J>1, repellers) phase-space
  volume possible, Hamiltonian dynamics persists well above the critical angle

- Fresnel filtering violates time-reversal symmetry
  asymmetries in the Husimis
  principle of ray-path reversibility is broken
  (distortion of path depends on the sign of χ due to GHs and Ff)

Quasi-scars in the spiral are now understood as "normal" periodic orbits



Lee et al., PRL 2004:
Resonances follow regular orbits despite the chaotic dynamics of the spiral – "quasi-scars"



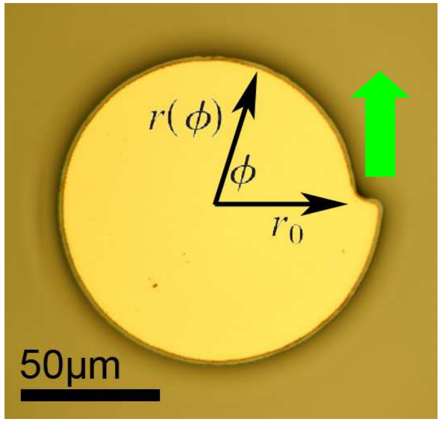
Idea:

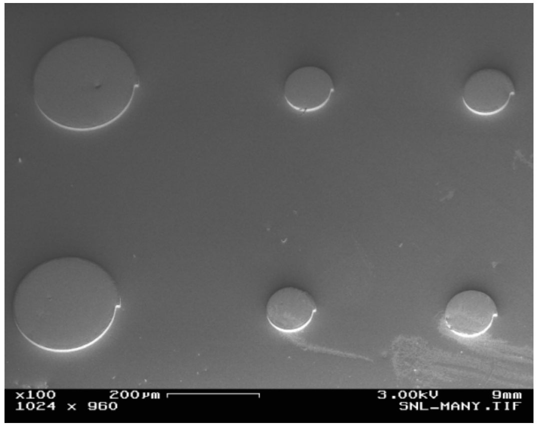
- Conservation of angular momentum **violated**
  - but **re-established** through Fresnel filtering
    ($\Delta\chi$ =1.737^{o} consistent with nkR ~ 100)
- resulting periodic orbit is unstable
  "real" scars were observed

Note: for almost all ϕ exists $\Delta\chi$ such that a periodic orbit exists, and these appear slightly rotated in space

II. Directional emission from microlasers

Spiral-shaped microlasers
and the quest for directional emission





Federico Capasso,
Michael Belkin
Harvard University

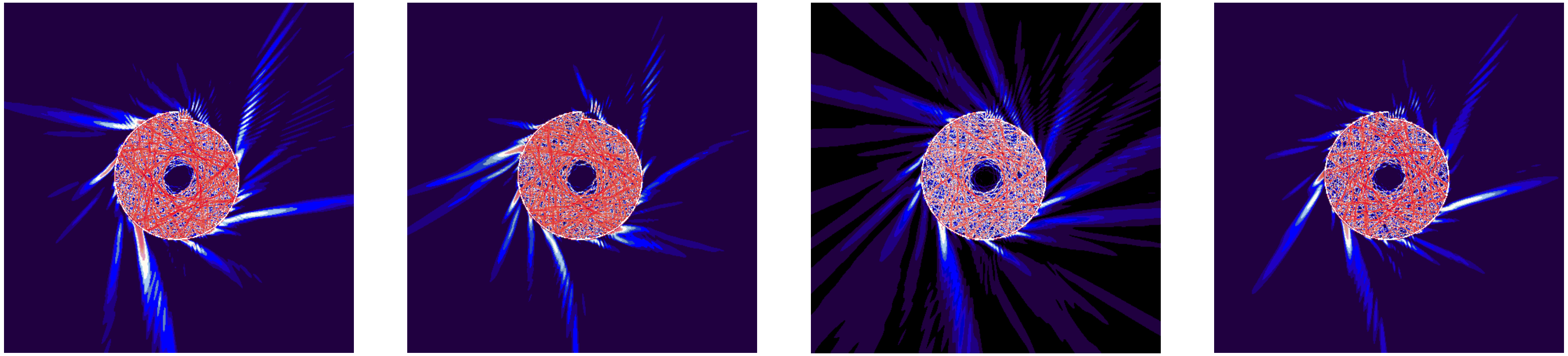
n=3.2

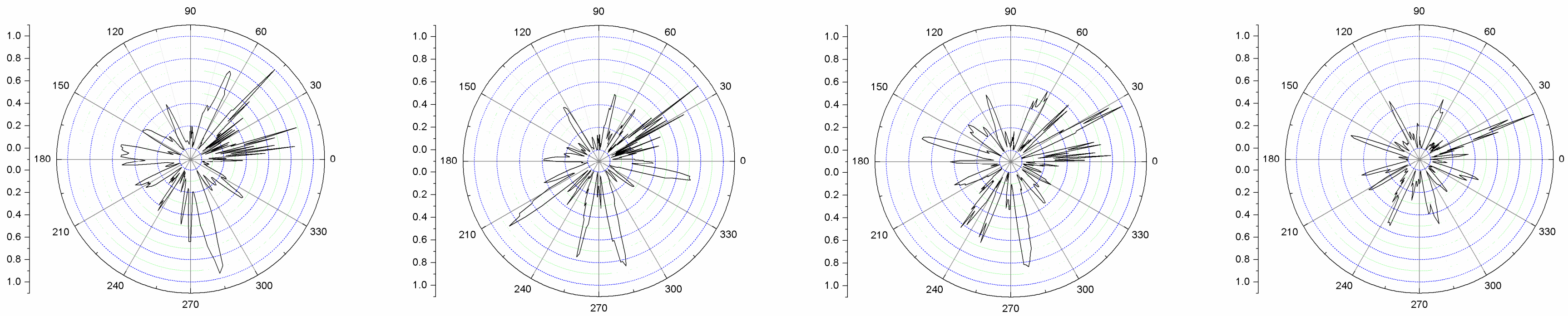
directional emission from "notch" was highly expected

Far-field patterns:
Expectations from wave simulations

(with Dr. Tae-Yoon Kwon, MPIPKS)

r_0 = 80 μm, Δr = 10 μm, nkR ~250

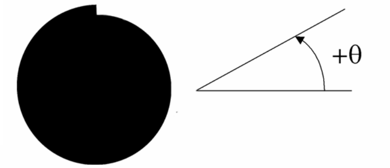




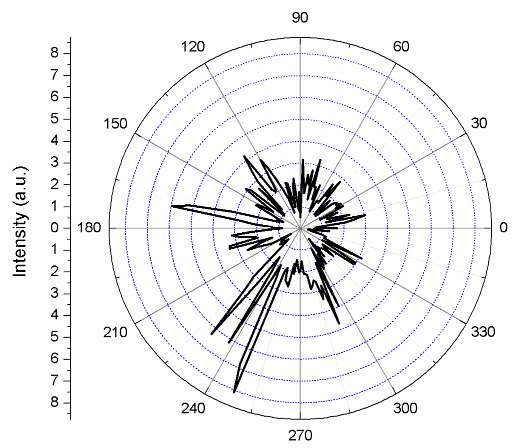
⇒ no directional emission from spiral quantum-cascade laser devices expected

T.-Y. Kwon, M.H., F. Capasso et al., to be submitted

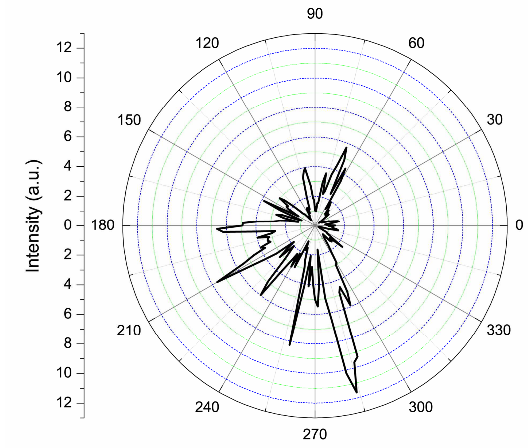
Far-field patterns: Measurements



- **Experiments** no directional emission as well

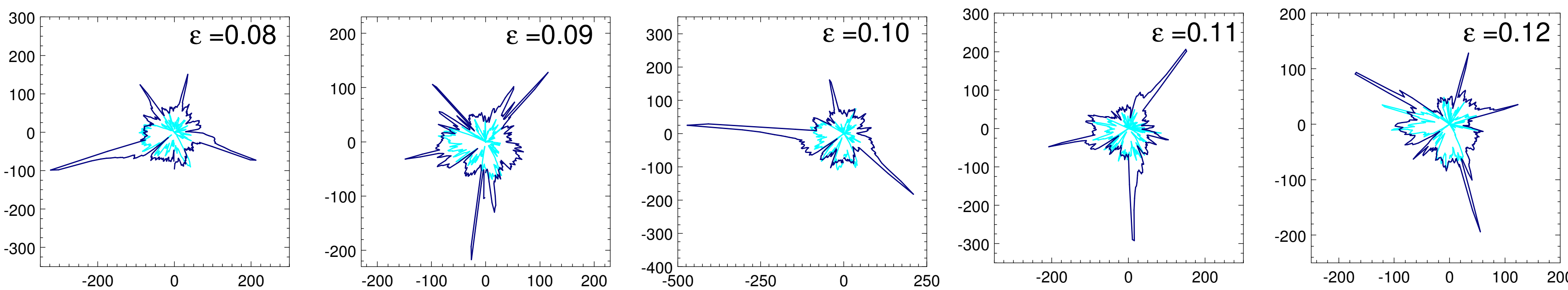


$r_0 = 80\ \mu m$, $\Delta r = 10\ \mu m$



$r_0 = 110\ \mu m$, $\Delta r = 10 \mu m$

- **Ray simulations** reveal extreme sensitivity on notch size



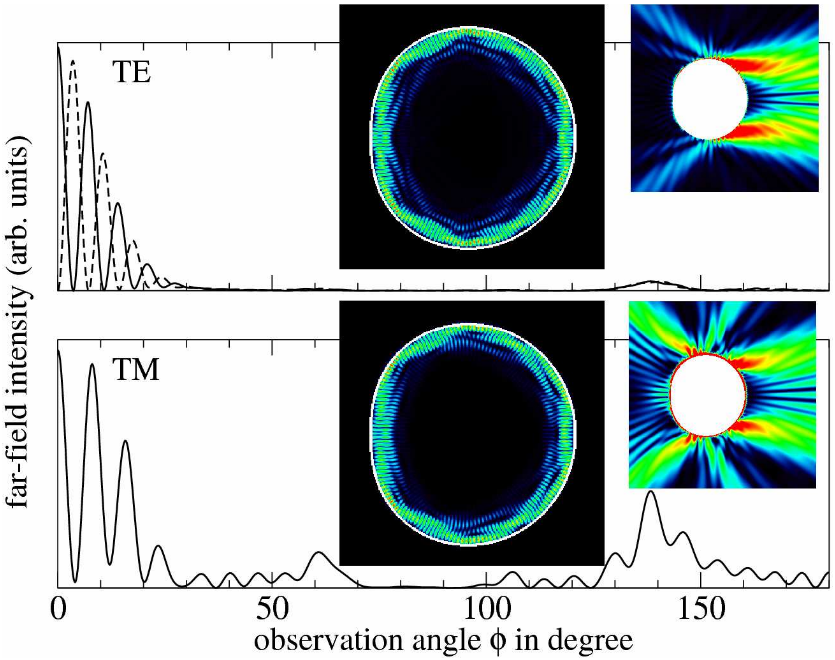
T.-Y. Kwon, M.H., F. Capasso et al., to be submitted

Note: directional emission from spiral possible when pumped near outer boundary

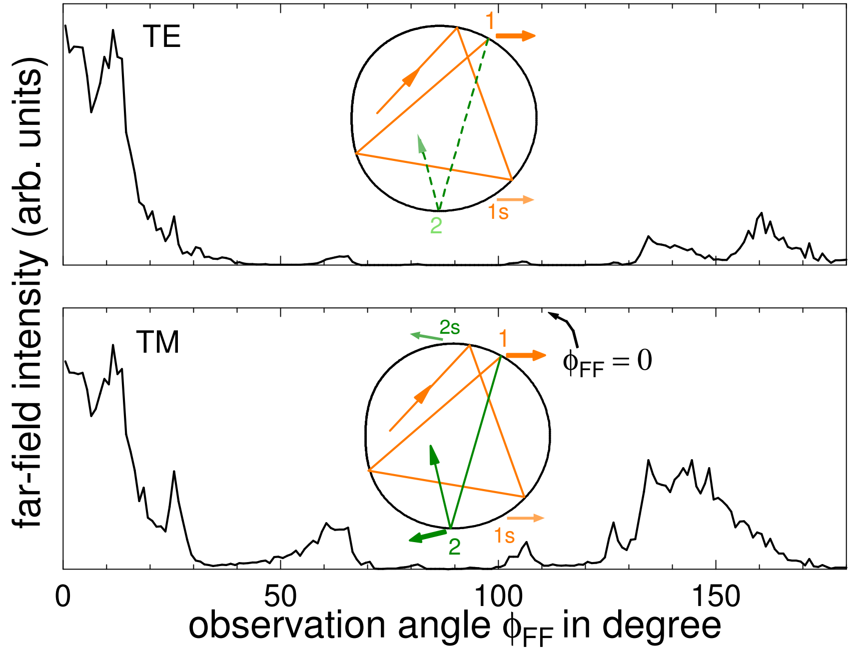
Directional emission from the Limaçon cavity

J. Wiersig and M.H., PRL 2008

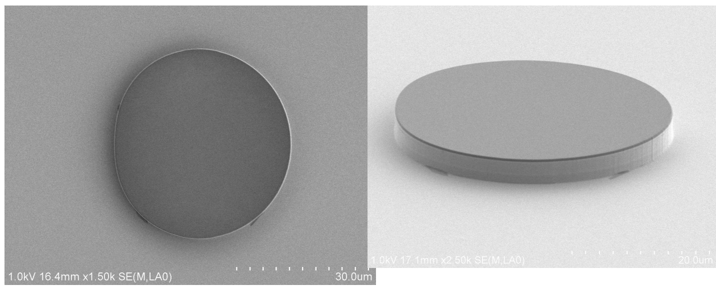
wave calculation

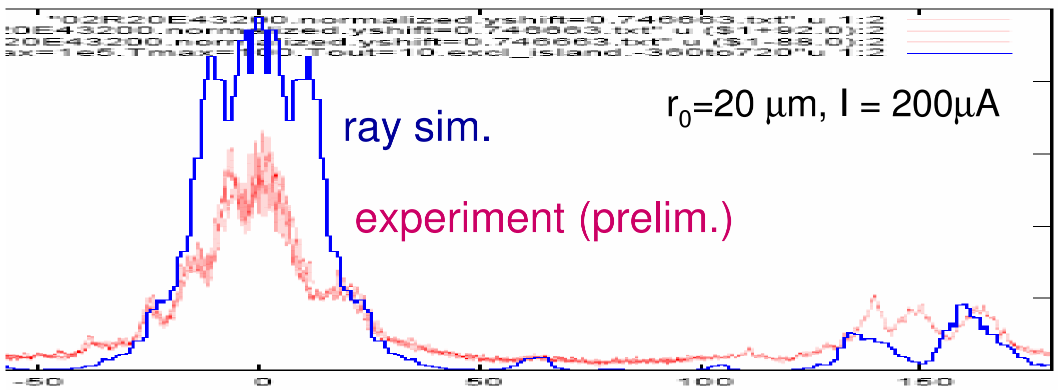


ray simulation



Confirmed in experiments:
Harayama group (ATR Kyoto)

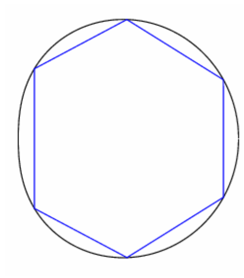




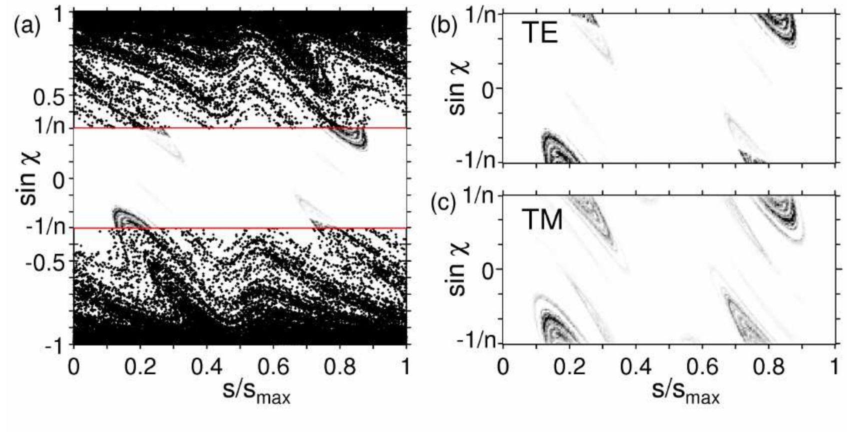
Find convincing ray-wave correspondence!

Why?

- far-field pattern determined by **unstable manifold** of hexagonal, scarred orbit
  (well in the critical regime with J=1, very high Q-factor *and* directional emission)

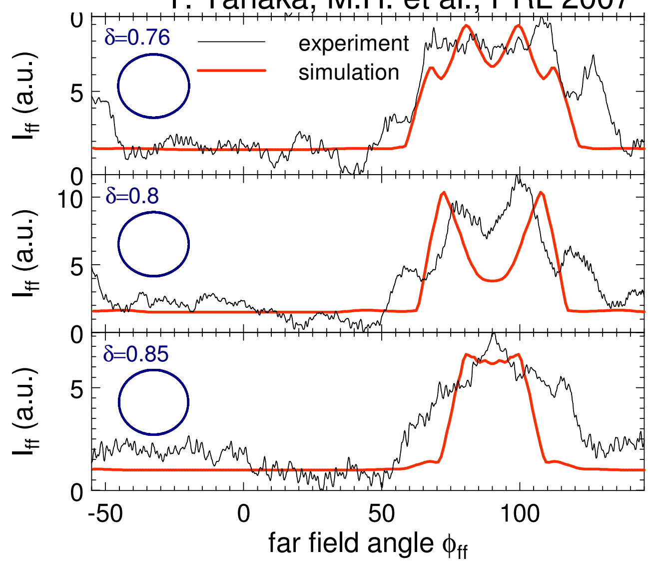


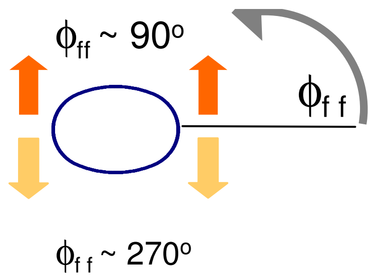
J. Wiersig and M.H., PRL 2008



- **ray-wave correspondence holds whenever unstable manifold matters**
  (cf. also works in groups of Ch.-M. Kim, S.-W. Kim, T. Harayama, A.D. Stone)

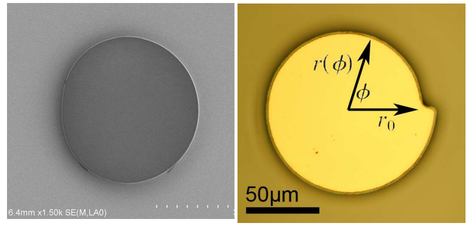
T. Tanaka, M.H. et al., PRL 2007





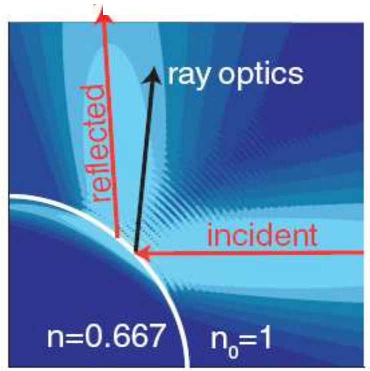
Conclusions

• optical microcavities as generalized model systems for quantum chaos

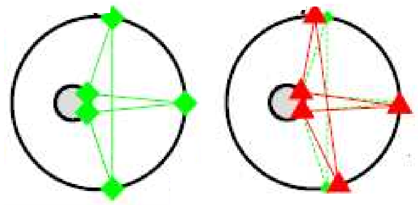


• application: microlasers, quest for directional emission
study and 'optimize' unstable manifold

• ray-wave correspondence may be violated due to intrinsic
openness of optical boundaries **adjusted reflection law (Goos-Hänchen shift and Fresnel filtering)**



• **non-Hamiltonian dynamics**
(formation of attractors and repellers,
breaking of time-reversal symmetry)



• adjusted reflection laws apply also to acoustic,
seismic, or chemical waves; analogy to soft-wall billiards

Outlook: dramatic change in phase-space structure as *one* reason for
the many regular orbits ["scars"] observed in optical systems

Conference "New Frontiers of Quantum Chaos in Mesoscopic systems"
May 19-30, 2008, MPIPKS Dresden – deadline March 31, www.mpipks-dresden.mpg.de/~nfqc08