Chaotic Scattering of Microwaves in Billiards: Induced Time-Reversal Symmetry Breaking and Fluctuations in GOE and GUE Systems

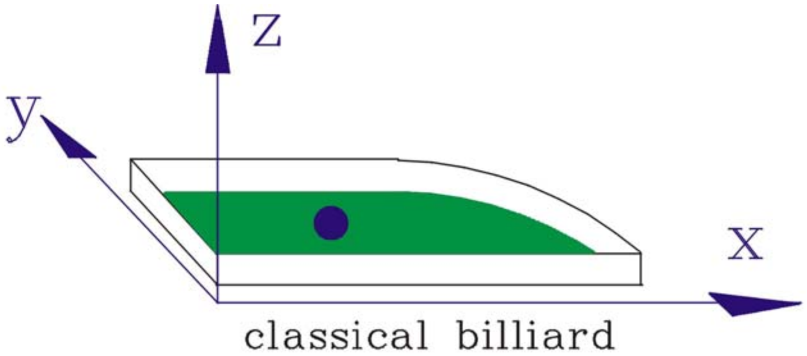
2008

- Quantum billiards and microwave resonators as a model of the compound nucleus
- Induced time-reversal symmetry breaking in billiards - isolated resonances
- Fluctuation properties of S-matrix elements - overlapping resonances
- Test of models based on RMT for GOE and GUE systems

Supported by DFG within SFB 634

S. Bittner, B. Dietz, T. Friedrich, M. Miski-Oglu, P. Oria-Iriarte, A. R., F. Schäfer
H.L. Harney, J.J.M. Verbaarschot, H.A. Weidenmüller

The Quantum Billiard and its Simulation



Schrödinger ↔ Helmholtz

quantum billiard

$$\left(\Delta + k^2\right)\Psi = 0$$

$$k = \sqrt{\frac{2mE}{\hbar^2}}$$

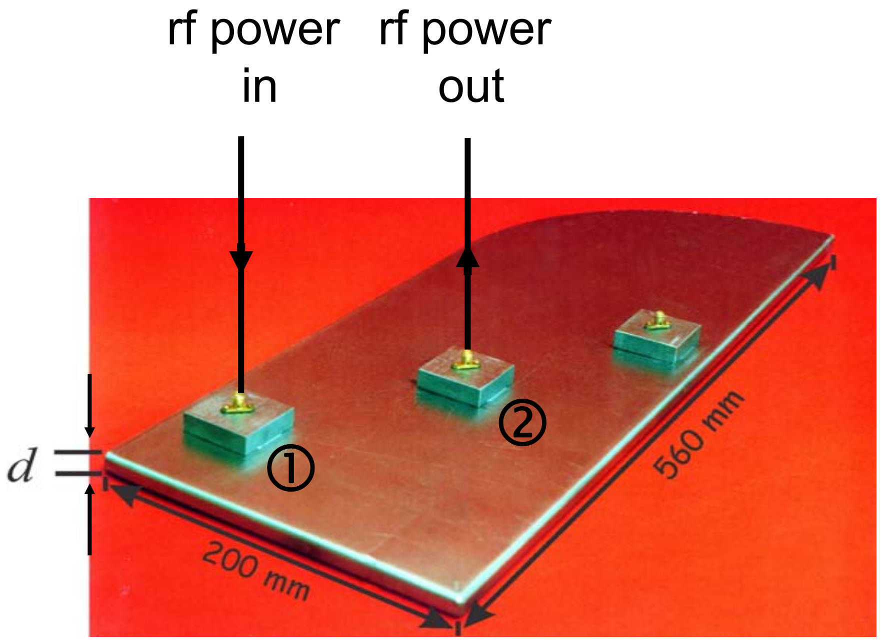
2D microwave cavity: $h_z < \lambda_{min}/2$

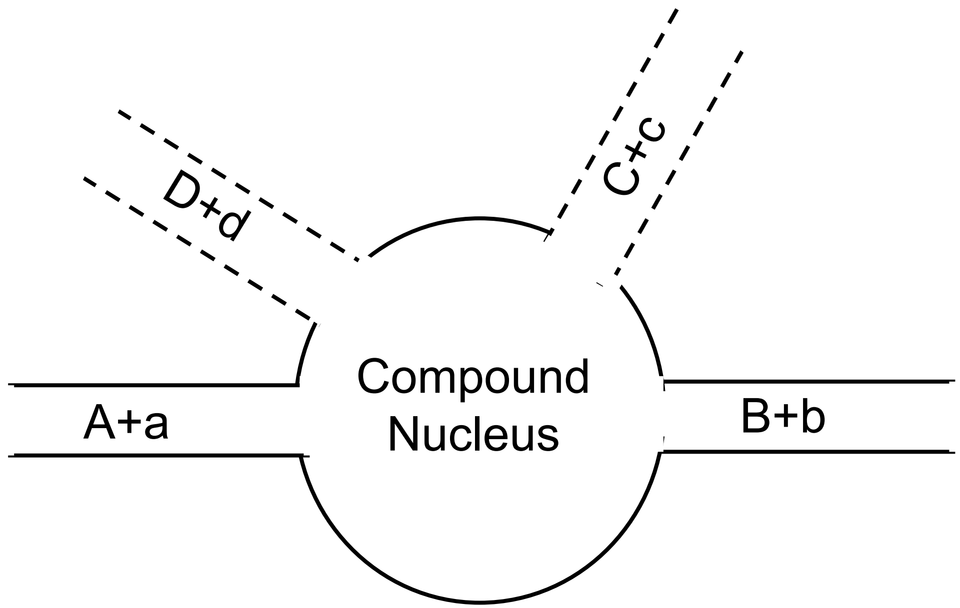
$$\left(\Delta + k^2\right)E_z = 0$$

$$k = \frac{2\pi f}{c}$$

Helmholtz equation and Schrödinger equation are equivalent in 2D. The motion of the quantum particle in its potential can be simulated by electromagnetic waves inside a two-dimensional microwave resonator.

Microwave Resonator as a Model for the Compound Nucleus





- Microwave power is **emitted** into the resonator by antenna ①
  and the output signal is **received** by antenna ②
  → **Open scattering system**
- The antennas act as **single scattering channels**
- **Absorption into the walls is modelled by additive channels**

Scattering Matrix Description

- Scattering matrix for both scattering processes

$$\hat{S}(E) = \mathbb{1} - 2\pi i\, \hat{W}^T (E\mathbb{1} - \hat{H} + i\pi\, \hat{W}\hat{W}^T)^{-1}\, \hat{W}$$

Compound-nucleus reactions		**Microwave billiard**
nuclear Hamiltonian	$\leftarrow \hat{H} \rightarrow$	resonator Hamiltonian
coupling of quasi-bound states to channel states	$\leftarrow \hat{W} \rightarrow$	coupling of resonator states to antenna states and to the walls

- **RMT description**: replace $\hat{H}$ by a GOE / GUE matrix for T-inv / T-noninv systems

Excitation Spectra

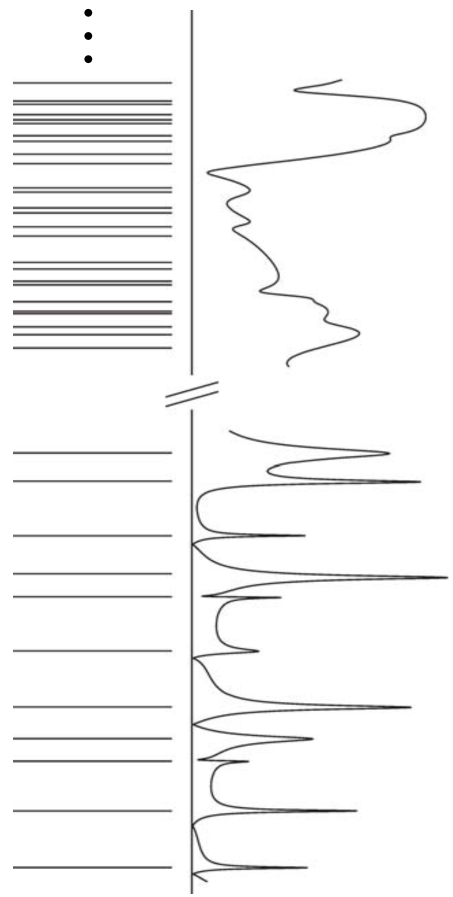
atomic nucleus

$\rho \sim \exp(E^{1/2})$

overlapping resonances
for $\Gamma/D>1$
Ericson fluctuations

isolated resonances
for $\Gamma/D<<1$

microwave cavity



$\rho \sim f$

Search for Time-Reversal Symmetry Breaking in Nuclei

VOLUME 19, NUMBER 9 PHYSICAL REVIEW LETTERS 28 AUGUST 1967

UPPER LIMIT OF T NONCONSERVATION IN THE REACTIONS $^{24}Mg + \alpha \rightleftarrows {}^{27}Al + p$

W. von Witsch, A. Richter, and P. von Brentano*
Max Planck Institut für Kernphysik, Heidelberg, Germany
(Received 28 June 1967)

Time-reversal invariance has been tested via detailed balance in the compound-nuclear reactions $^{24}Mg+\alpha \rightleftarrows {}^{27}Al+p$. The relative differential cross sections agree within the experimental uncertainties, leading to an estimated upper limit for the ratio of the T-nonconserving to the T-conserving reaction amplitudes of $(2\text{-}4)\times10^{-3}$. The same upper limit is found for the nuclear matrix elements which are odd with respect to time reversal.

VOLUME 51, NUMBER 5 PHYSICAL REVIEW LETTERS 1 AUGUST 1983

Improved Experimental Test of Detailed Balance and Time Reversibility in the Reactions $^{27}Al+p \leftrightarrows {}^{24}Mg+\alpha$

E. Blanke,[a] H. Driller,[b] and W. Glöckle
Abteilung für Physik und Astronomie, Ruhr Universität Bochum, D-4630 Bochum, Germany

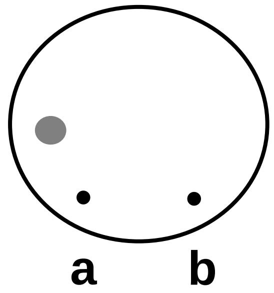
and

H. Genz, A. Richter, and G. Schrieder
Institut für Kernphysik, Technische Hochschule Darmstadt, D-6100 Darmstadt, Germany
(Received 25 April 1983)

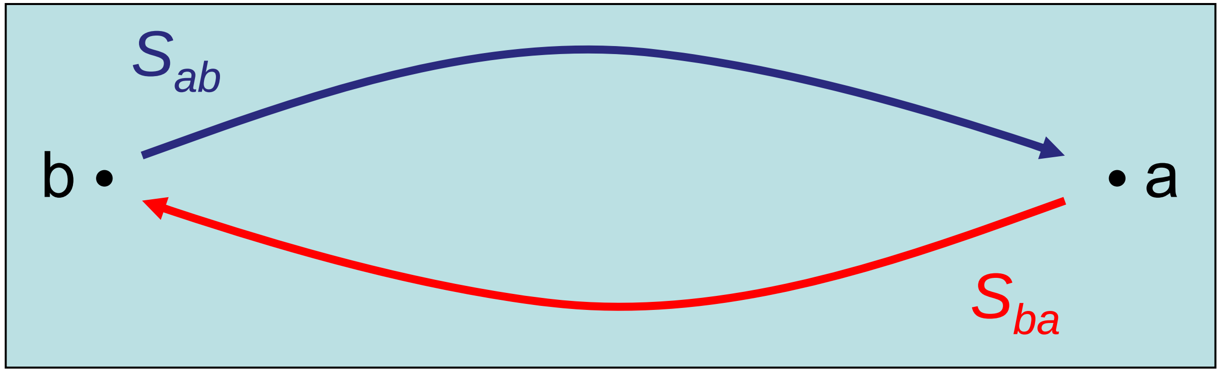
A new test of the principle of detailed balance in the nuclear reactions $^{27}Al(p,\alpha_0)^{24}Mg$ and $^{24}Mg(\alpha,p_0)^{27}Al$ at bombarding energies 7.3 MeV $\le E_p \le$ 7.7 MeV and 10.1 MeV $\le E_\alpha \le$ 10.5 MeV, respectively, is reported. Measured relative differential cross sections agree within the experimental uncertainty $\Delta = \pm 0.51\%$ and hence are consistent with time-reversal invariance. From this result an upper limit $\xi \le 5\times10^{-4}$ (80% confidence) is derived for a possible time-reversal–noninvariant amplitude in the reaction.

Induced Time-Reversal Symmetry Breaking (TRSB) in Billiards

- T-symmetry breaking caused by a magnetized ferrite

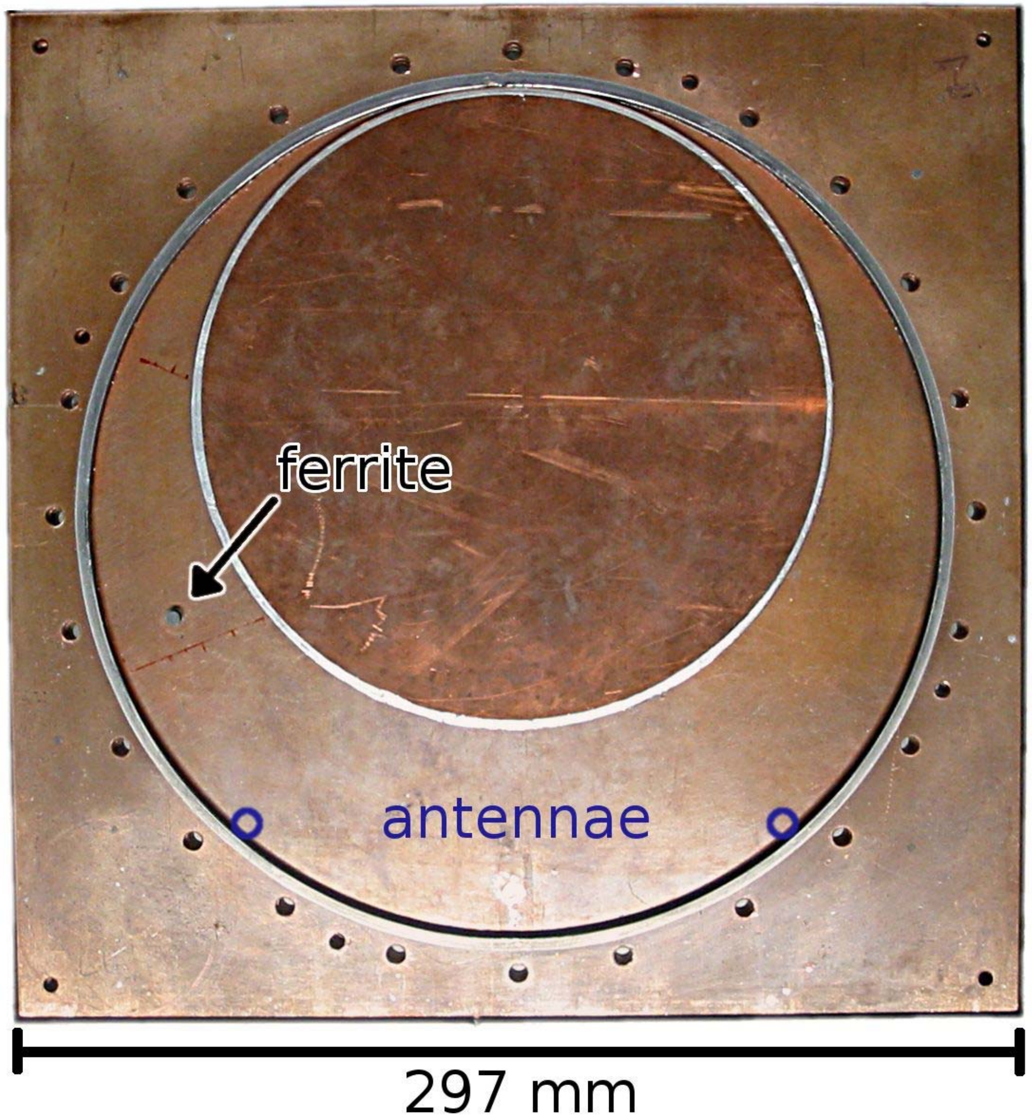


- Coupling of microwaves to the ferrite depends on the direction a ⇆ b

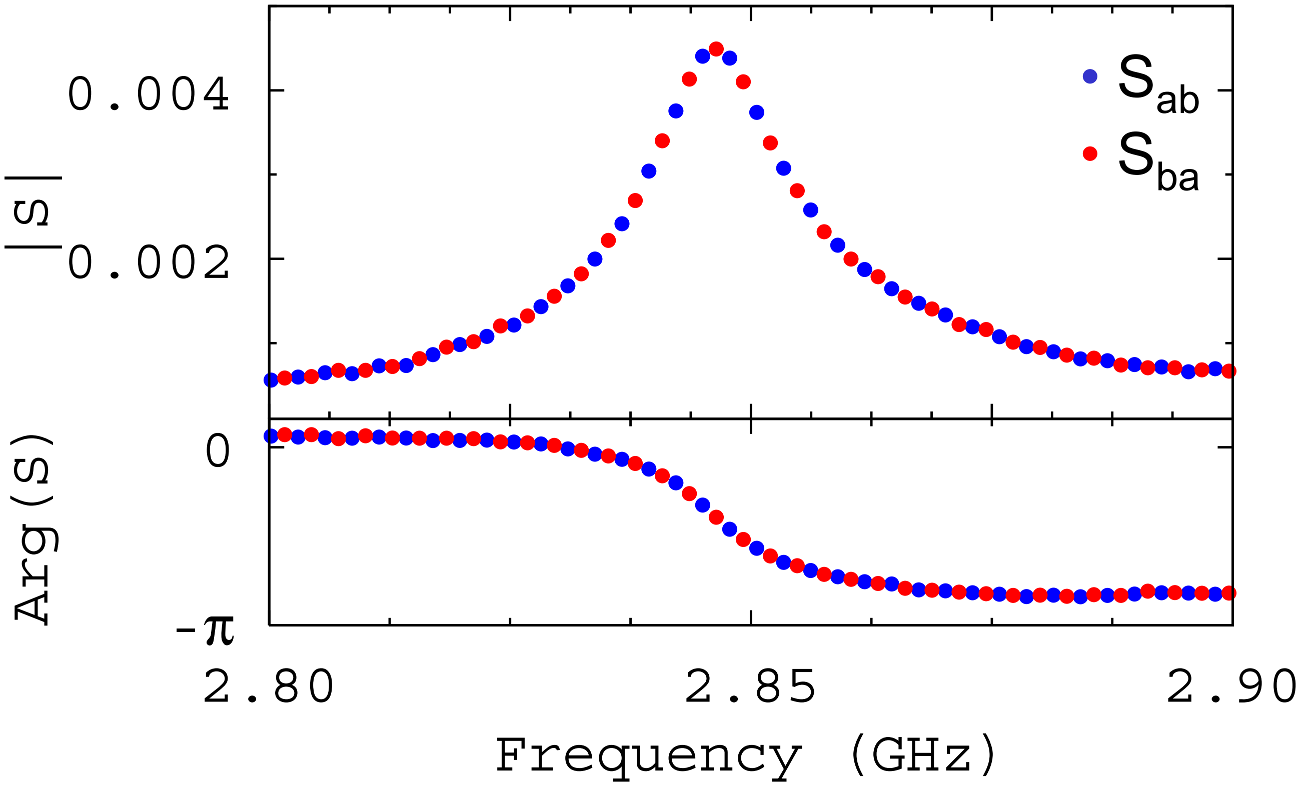


- Principle of detailed balance: $|S_{ab}|^2 = |S_{ba}|^2$
- Principle of reciprocity: $S_{ab} = S_{ba}$

Isolated Resonances - Setup

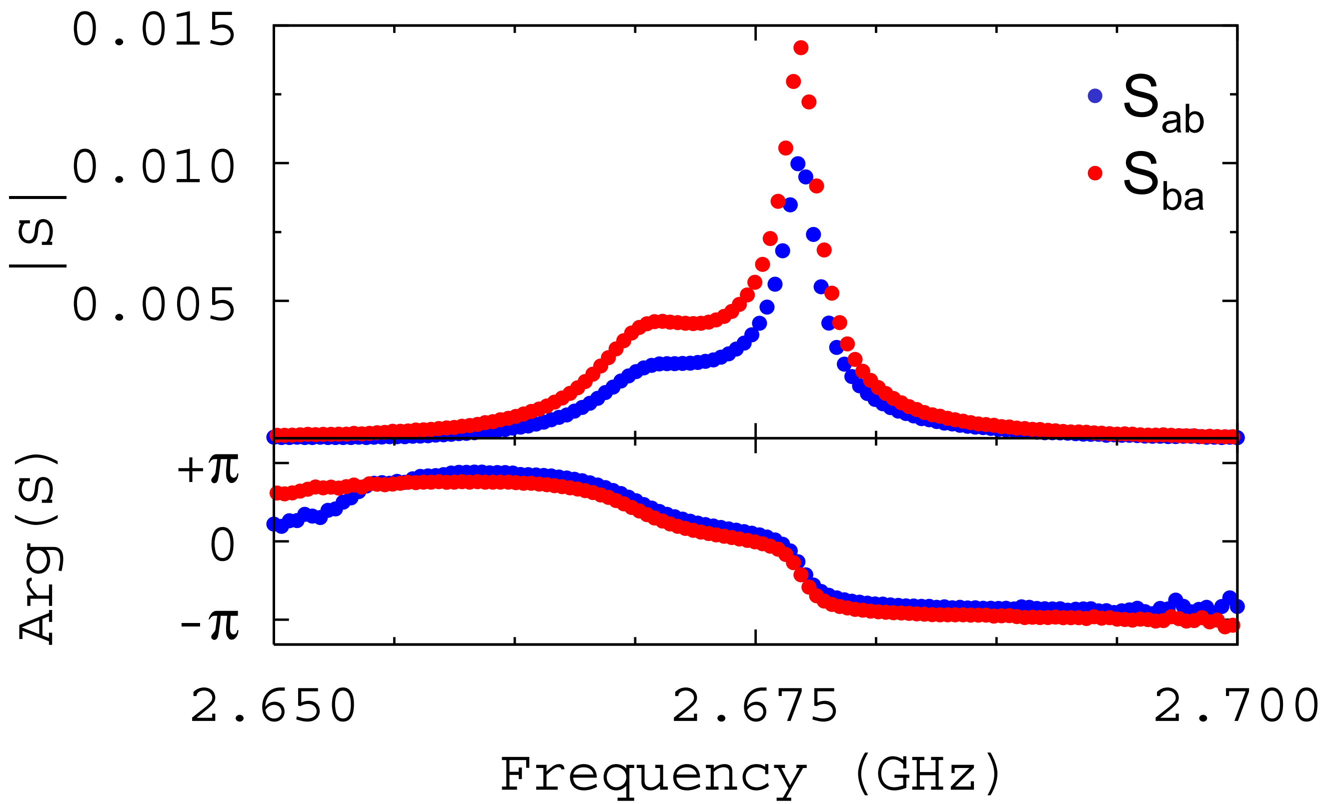


Isolated Resonances - Singlets



- Reciprocity holds → TRSB cannot be detected this way

Isolated Doublets of Resonances



- Violation of reciprocity due to interference of two resonances

Scattering Matrix and TRSB

- Scattering matrix element

$$S_{ab}(\omega) = \delta_{ab} - 2\pi i \left\langle a \middle| \hat{W}^{+} (\omega - \hat{H}^{eff})^{-1} \hat{W} \middle| b \right\rangle$$

- Decomposition of effective Hamiltonian

$$\hat{H}^{eff} = \hat{H}^{a} + \hat{H}^{s}$$

$$\downarrow$$

$$\begin{pmatrix} 0 & H_{12}^{a} \\ -H_{12}^{a} & 0 \end{pmatrix}$$

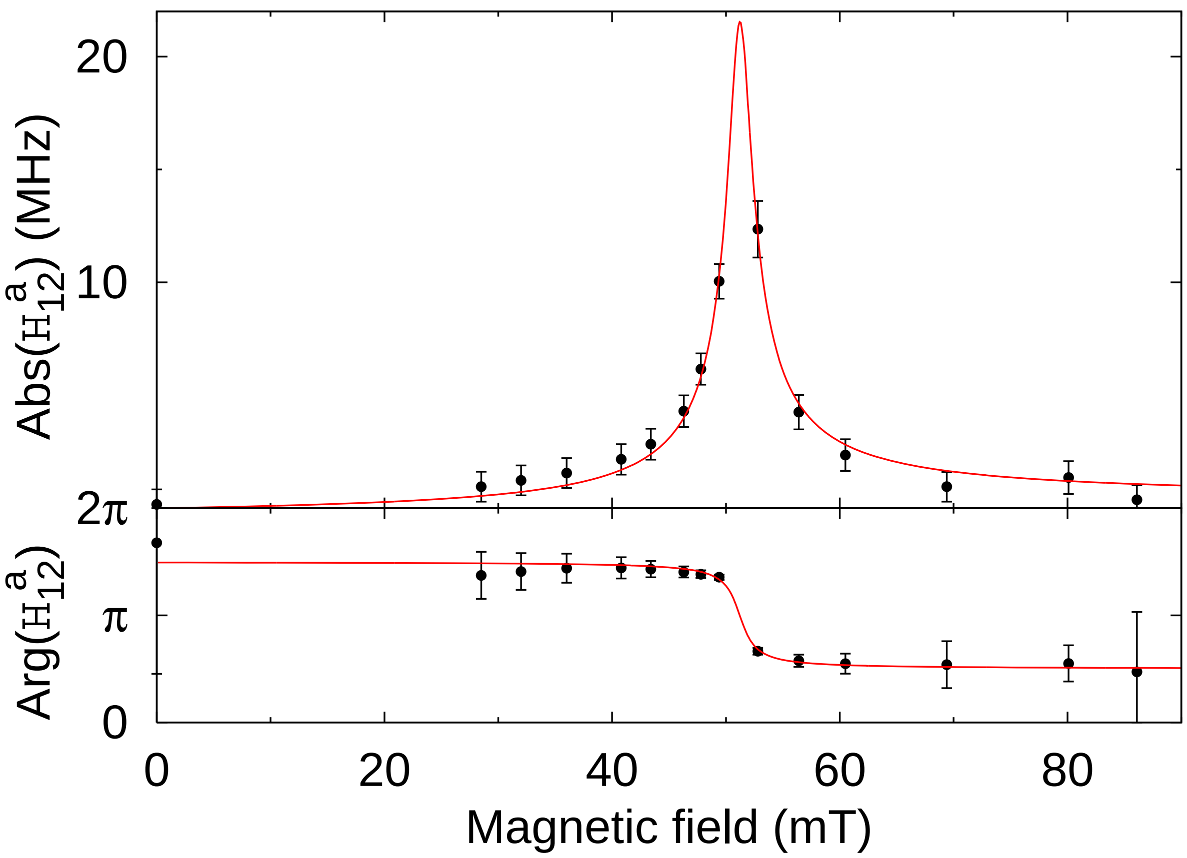
- Ansatz for TRSB incorporating the FMR and its selective coupling to the microwaves

TRSB Matrix Element

- $H_{12}^{a}(B) = \dfrac{i\pi}{2}$

- Fit parameters: λ and $\overline{\omega}$

T-Violating Matrix Element

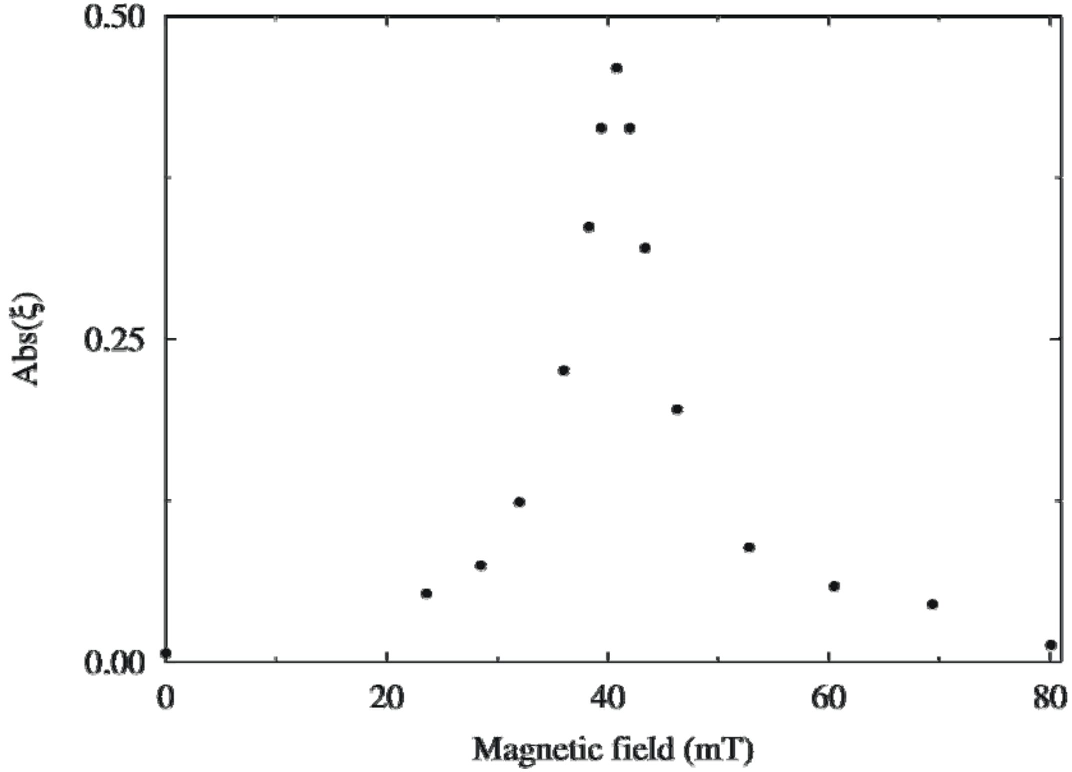


- T-violating matrix element shows resonance like structure
- Successful description of dependence on magnetic field

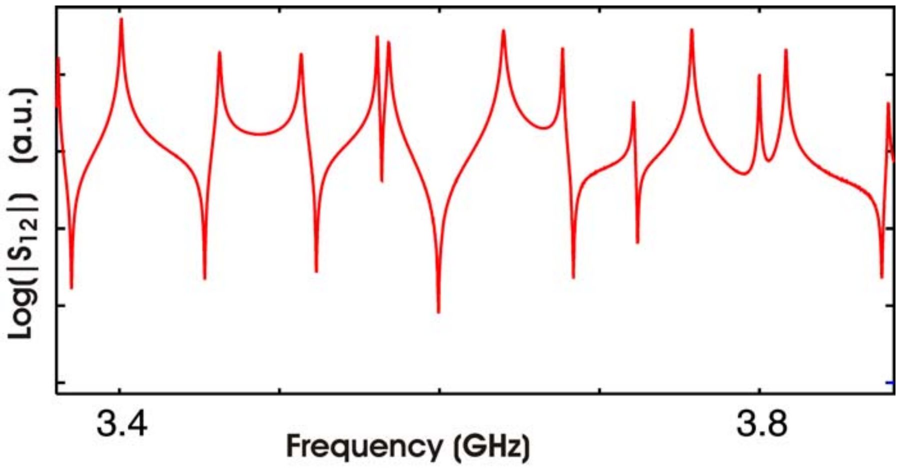
Relative Strength of T-Violation

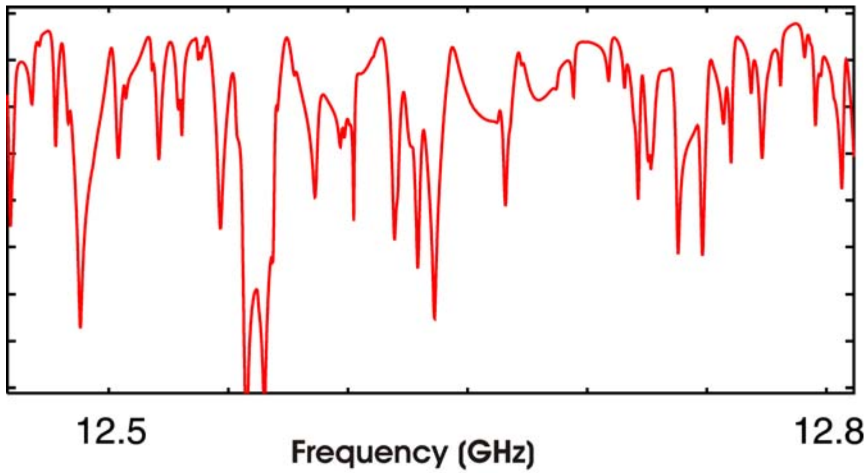
- Compare: TRSB matrix element H_{12}^{a} to the energy difference of two eigenvalues of the T-invariant system

$$\xi = \left| \frac{2H_{12}^{a}}{E_1^s - E_2^s} \right|$$



Spectra and Autocorrelation Function



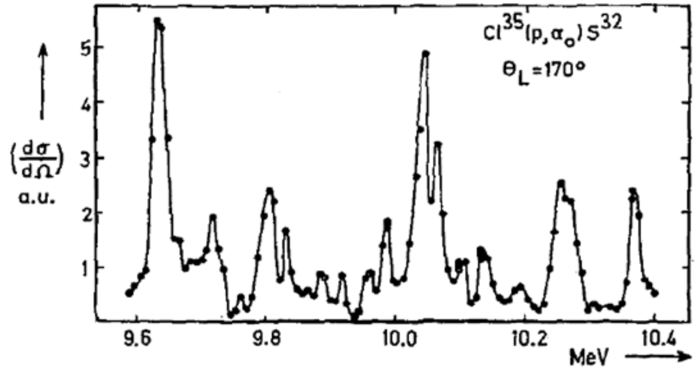


- Regime of isolated resonances
- Γ/D small
- Resonances: eigenvalues

- Overlapping resonances
- $\Gamma/D \sim 1$
- Fluctuations: Γ_{coh}

Correlation function: $C(\varepsilon) = \left\langle S(f)S^*(f+\varepsilon)\right\rangle - \left\langle S(f)\right\rangle\left\langle S^*(f+\varepsilon)\right\rangle$

Ericson's Prediction



- Ericson fluctuations (1960):

$$|C(\varepsilon)|^2 \propto \frac{\Gamma_{coh}^2}{\Gamma_{coh}^2 + \varepsilon^2}$$

- Correlation function is Lorentzian
- Measured 1964 for overlapping compound nuclear resonances

P. v. Brentano et al., Phys. Lett. 9, **48** (1964)

- Now observed in lots of different systems: molecules, quantum dots, laser cavities…
- Applicable for Γ/D >> 1 and for many open channels only

Exact RMT Result for GOE systems

- Verbaarschot, Weidenmüller and Zirnbauer (VWZ) 1984 for arbitrary Γ/D :

- VWZ-integral: $C = C(T_i, D\ ;\ \varepsilon)$

$$
\begin{aligned}
C_{ab}(\epsilon) \;=\; & \frac{1}{8}\int_0^\infty d\lambda_1 \int_0^\infty d\lambda_2 \int_0^1 d\lambda\, \mu(\lambda,\lambda_1,\lambda_2) \\
& \times \exp\left(-i\pi\epsilon(\lambda_1+\lambda_2+2\lambda)/D\right) \\
& \times J_{ab}(\lambda,\lambda_1,\lambda_2) \\
& \times \prod_e \frac{(1-T_e\lambda)}{((1+T_e\lambda_1)(1+T_e\lambda_2))^{1/2}}
\end{aligned}
$$

;

$$
\mu(\lambda,\lambda_1,\lambda_2) = \frac{\lambda(1-\lambda)|\lambda_1-\lambda_2|}{(\lambda+\lambda_1)^2(\lambda+\lambda_2)^2(\lambda_1\lambda_2(1+\lambda_1)(1+\lambda_2))^{1/2}}
$$

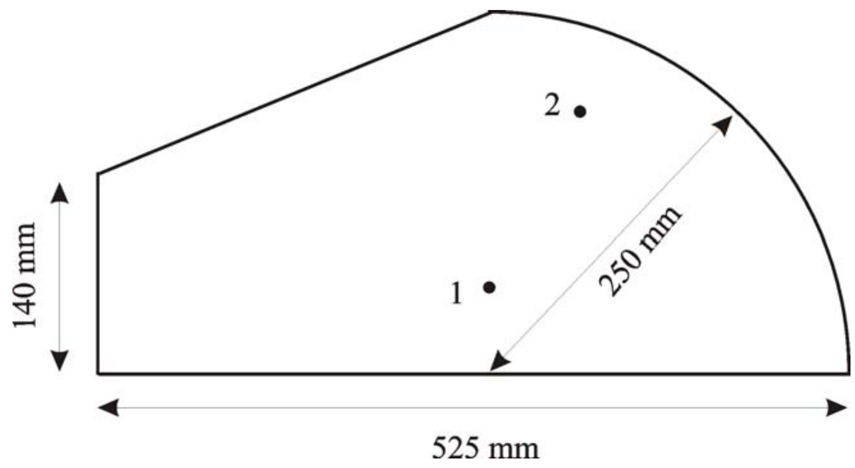
$$
\begin{aligned}
J_{ab}(\lambda,\lambda_1,\lambda_2) \;=\; & \delta_{ab}\, T_a^2\,(1-T_a) \\
& \times \left(\frac{\lambda_1}{1+T_a\lambda_1} + \frac{\lambda_2}{1+T_a\lambda_2} + \frac{2\lambda}{1-T_a\lambda}\right) \\
& + (1+\delta_{ab})\, T_a\, T_b \\
& + \left(\frac{\lambda_1(1+\lambda_1)}{(1+T_a\lambda_1)(1+T_b\lambda_1)} + \frac{\lambda_2(1+\lambda_2)}{(1+T_a\lambda_2)(1+T_b\lambda_2)} \right. \\
& \left. + \frac{2\lambda(1-\lambda)}{(1-T_a\lambda)(1-T_b\lambda)}\right)
\end{aligned}
$$

- Rigorous test of VWZ: isolate(

- Our goal: test VWZ in the intermediate regime, i.e. $\Gamma/D \approx 1$

Experimental Realisation in a Fully Chaotic Cavity

- Tilted stadium (Primack + Smilansky, 1994)



- Height of cavity 15 mm
- Becomes 3D at 10.1 GHz

- GOE behaviour checked
- Measure full complex S-matrix for two antennas: S_{11}, S_{22}, S_{12}

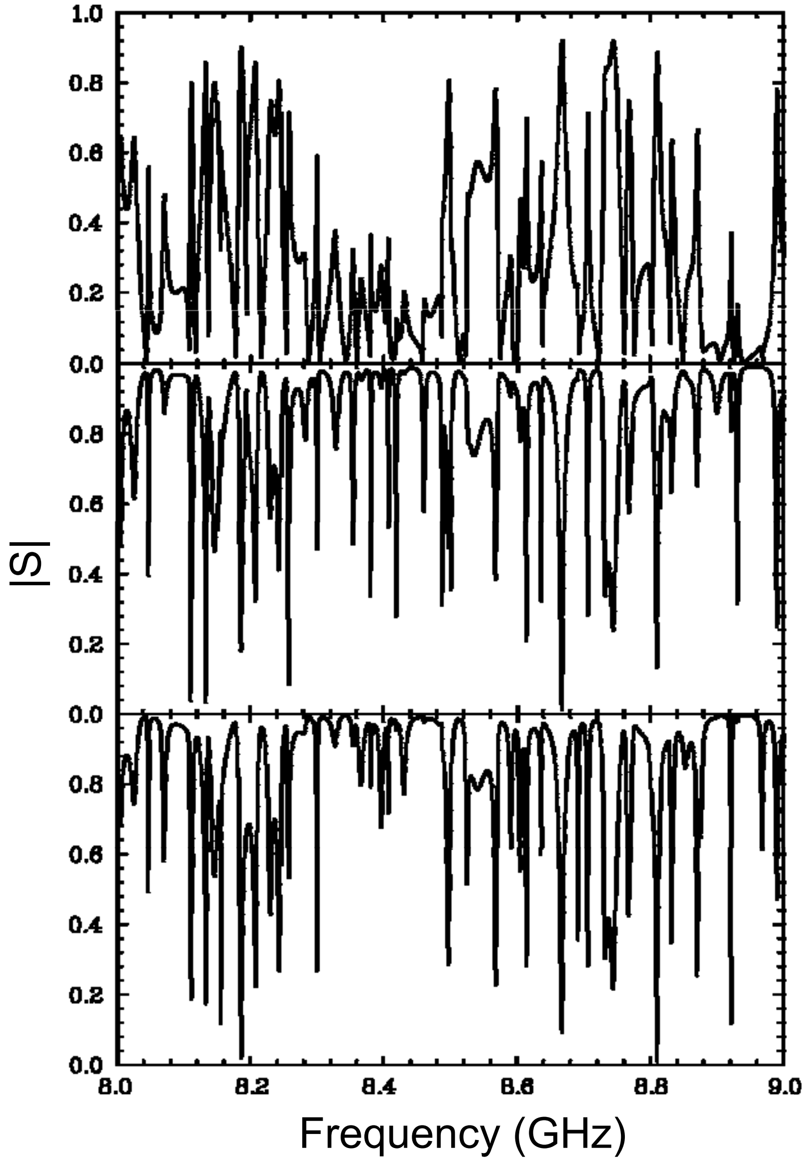
Excitation Functions of S-Matrix Elements

Example: 8-9 GHz

S_{12} →

S_{11} →

S_{22} →



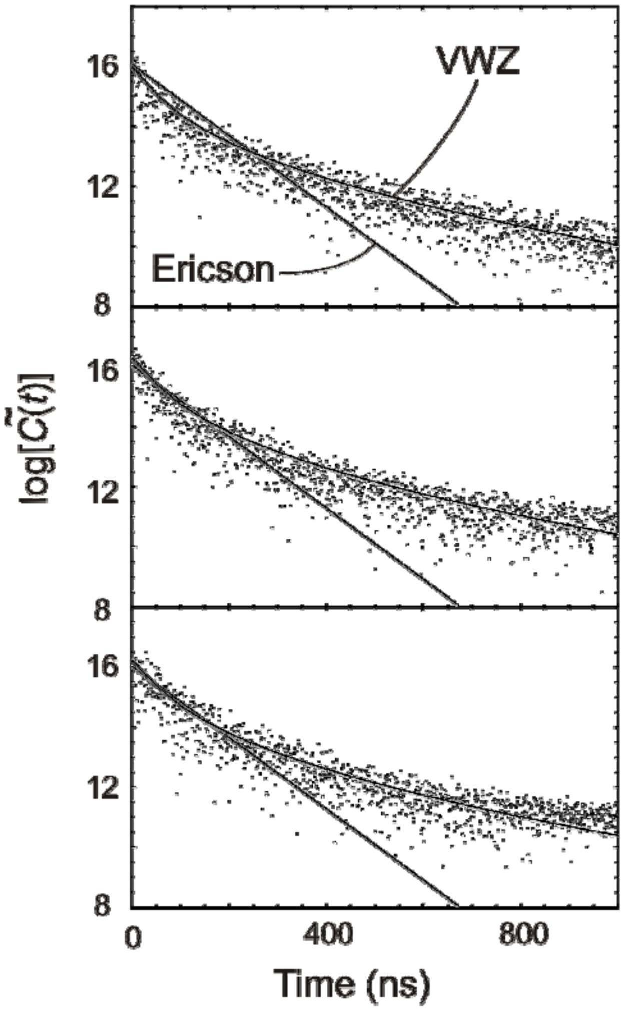
Road to Analysis

- Problem: adjacent points in $C(\varepsilon)$ are correlated
- Solution: FT of $C(\varepsilon)$ → uncorrelated Fourier coefficients $\tilde{C}(t)$
  Ericson (1965)
- Development: Non Gaussian fit and test procedure

Comparison: Experiment - VWZ

Time domain

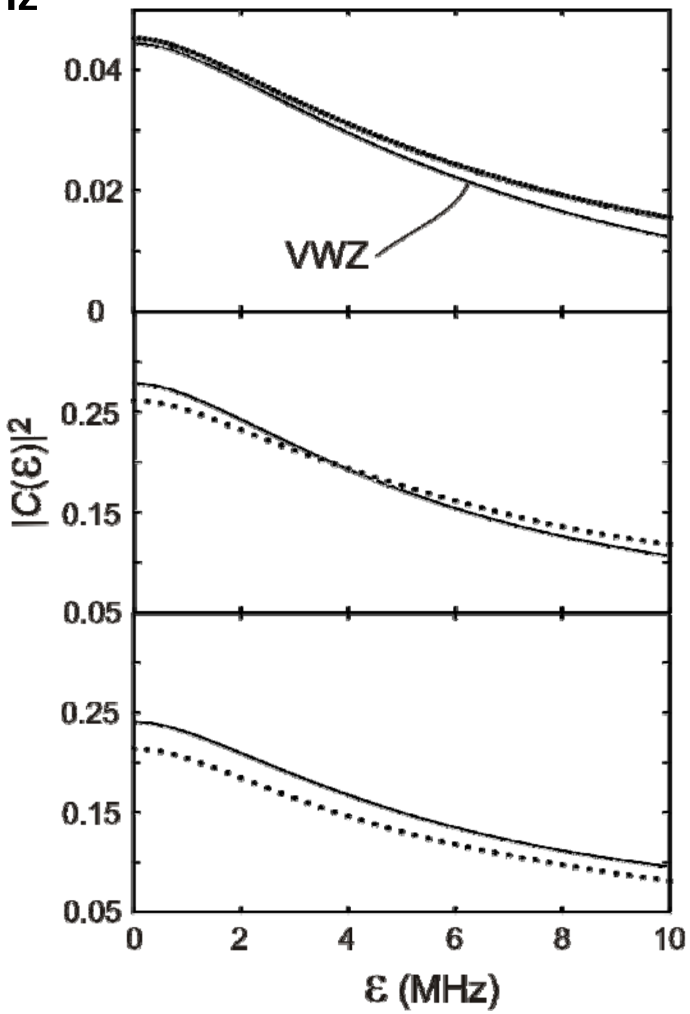
Example 8-9 GHz



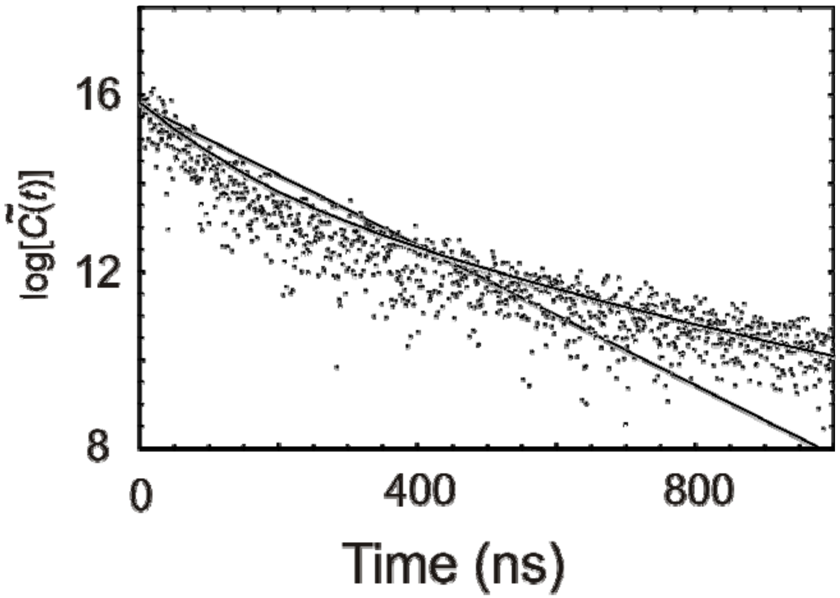
← S_{12}

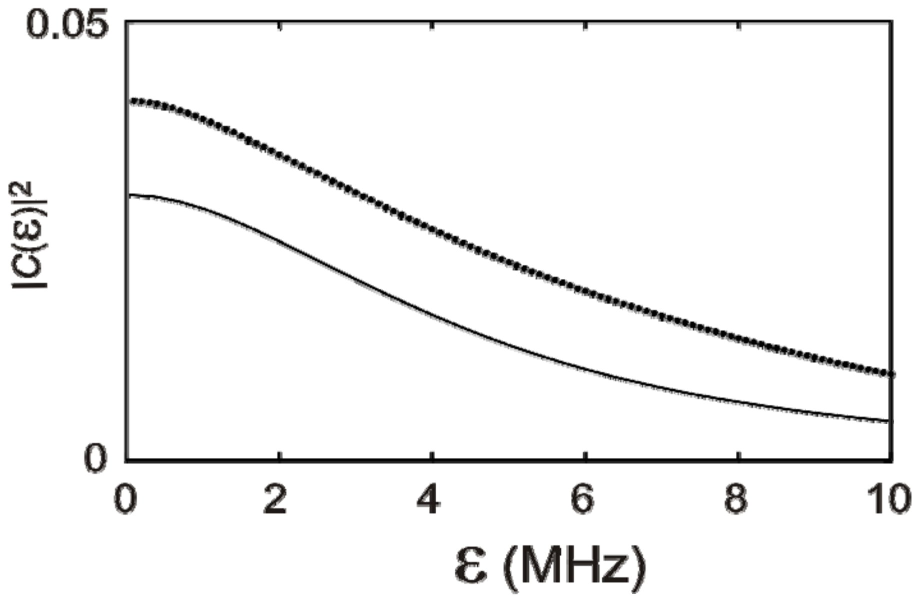
← S_{11}

← S_{22}



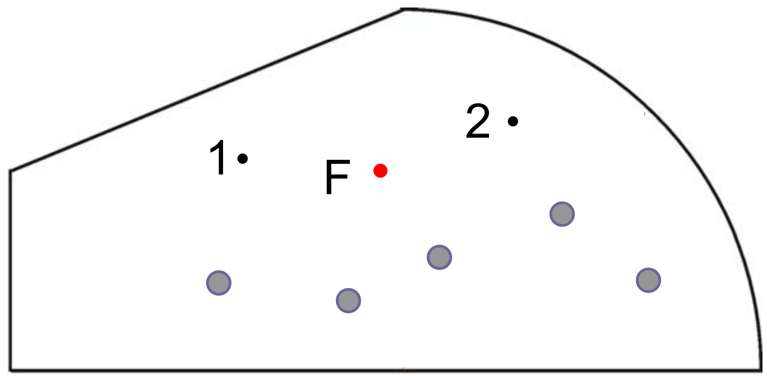
What Happens in the Region of 3D Modes?





- VWZ curve in $\tilde{C}(t)$ progresses through the cloud of points but it passes too high → GOF test rejects VWZ
- This behaviour is clearly visible in C(ε)
- Behaviour can be modelled through $\hat{H} = \begin{pmatrix} H_1^{GOE} & 0 \\ 0 & H_2^{GOE} \end{pmatrix}$

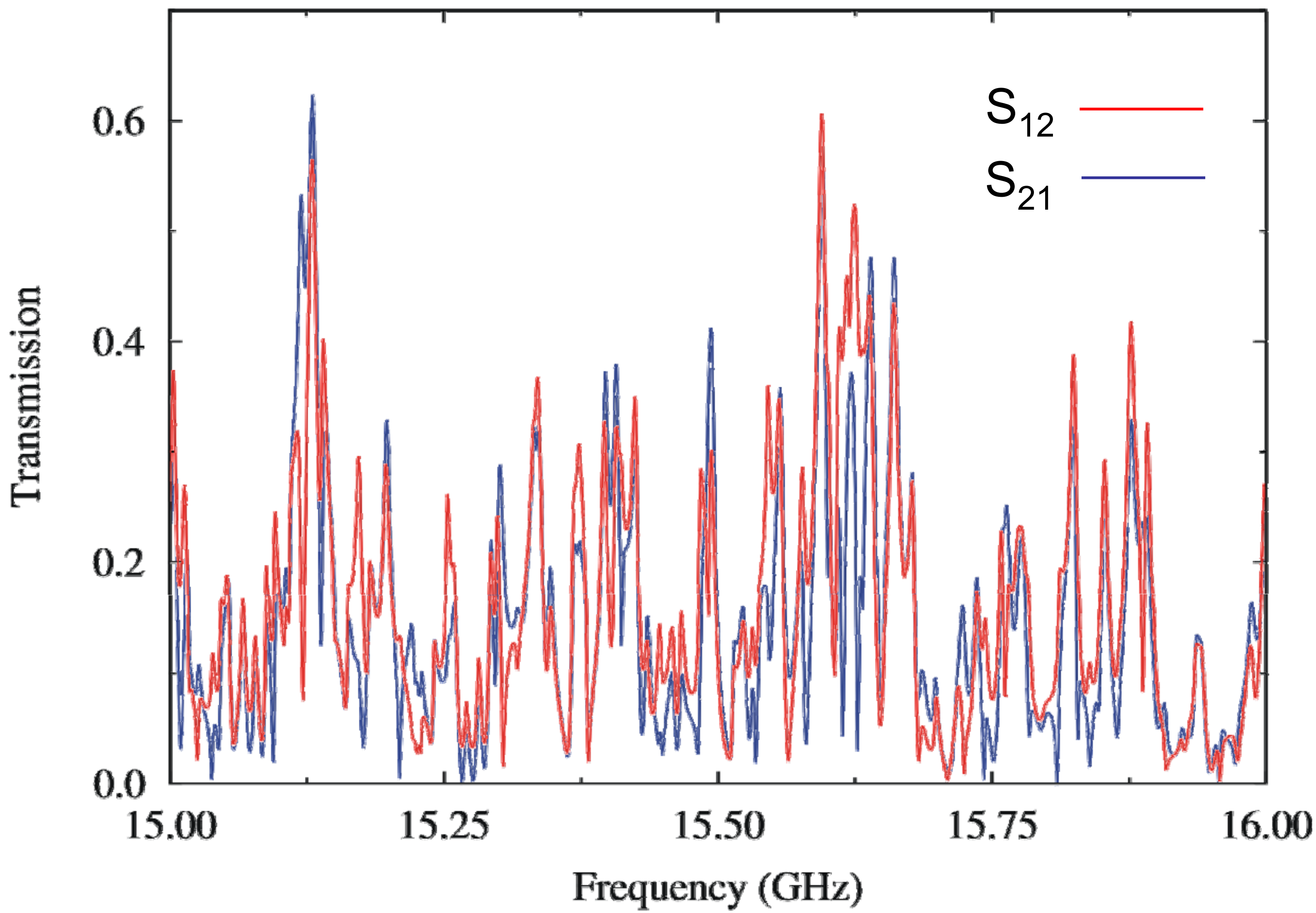
TRSB in the Region of Overlapping Resonances



- Antenna 1 and 2
- Place a magnetized ferrite F into tilted stadium billiard
- Place an additional Fe - scatterer into the stadium and move it into different positions in order to improve the statistical significance of the data sample

→ distinction between GOE and GUE becomes possible

Violation of Detailed Balance for Overlapping Resonances



Quantification of Reciprocity Violation

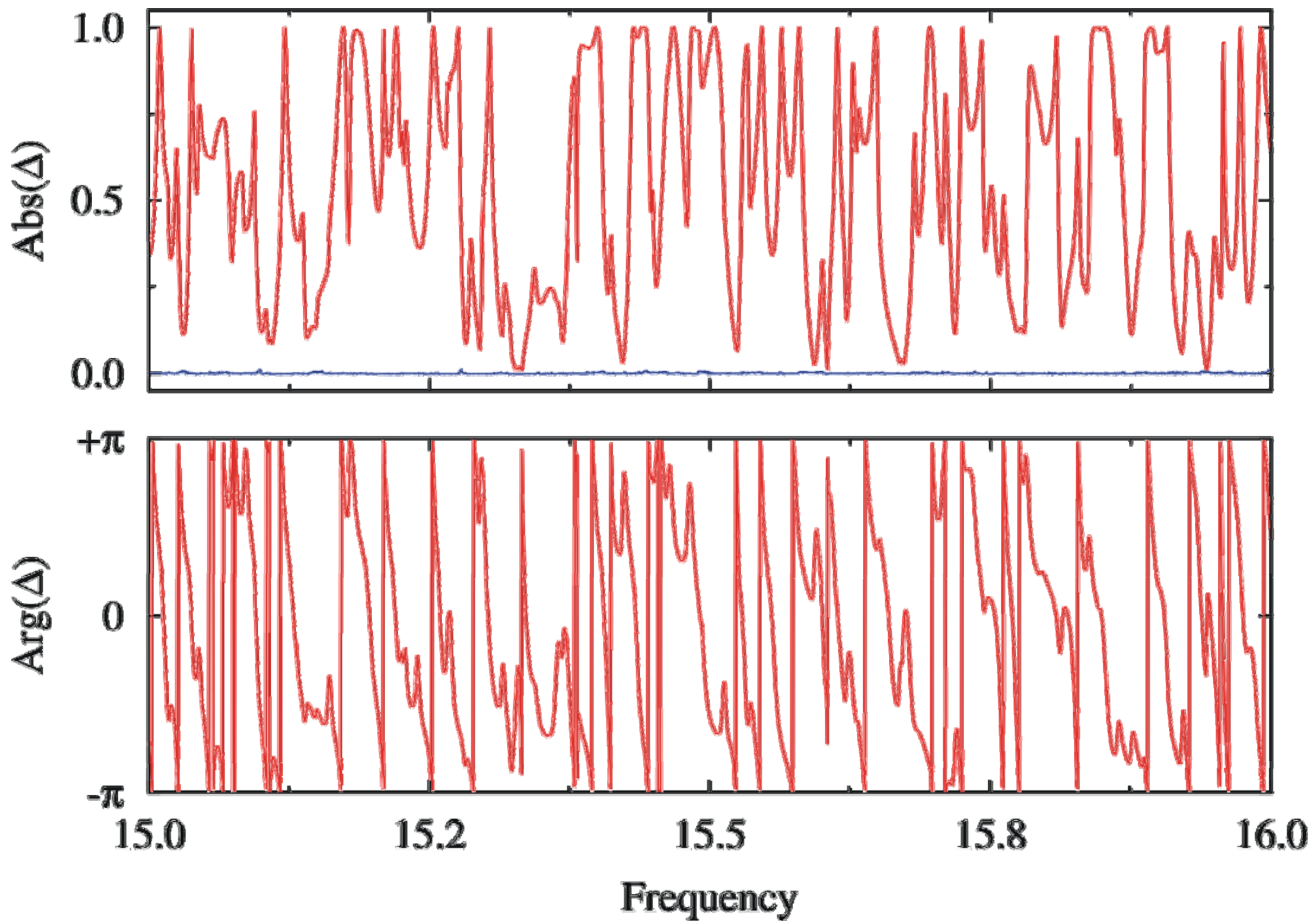
- The violation of reciprocity reflects degree of TRSB

- Definition of a contrast function

$$\Delta = \frac{S_{ab} - S_{ba}}{|S_{ab}| + |S_{ba}|}$$

- Quantification of reciprocity violation via Δ

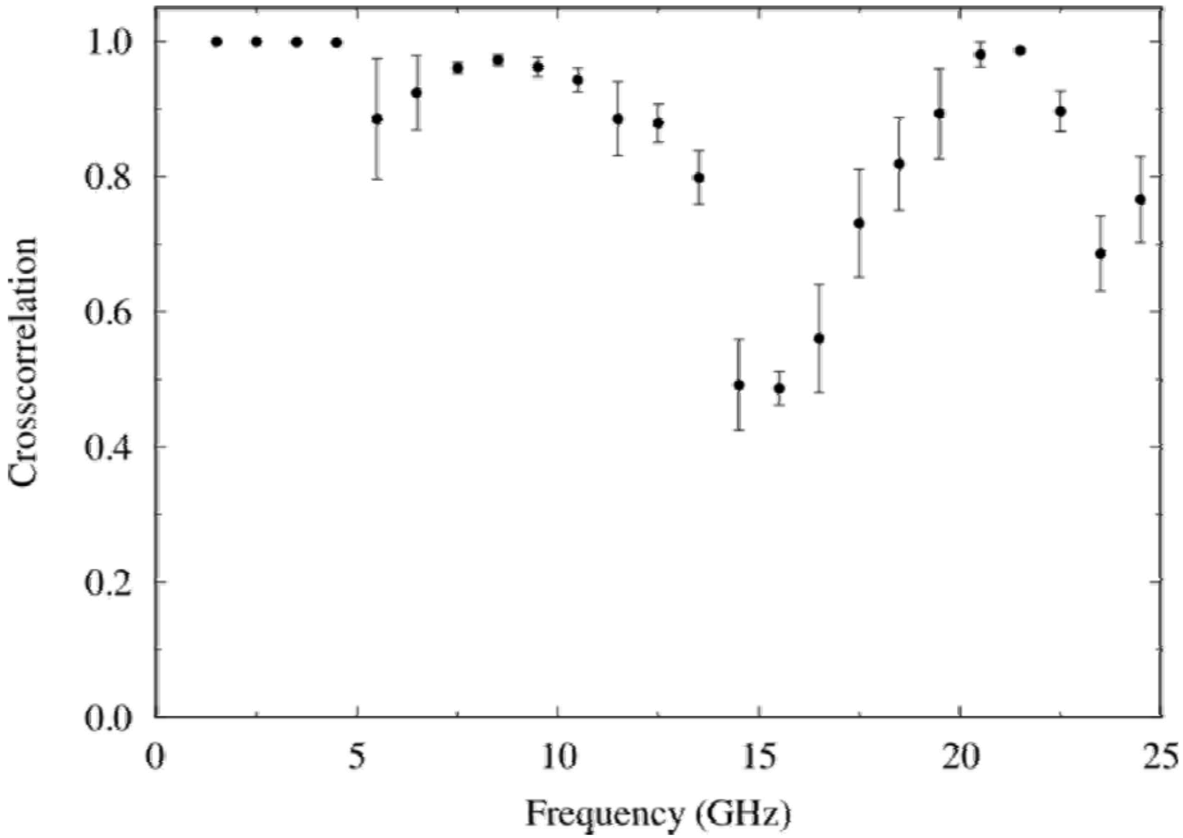
Magnitude and Phase of Δ Fluctuate



← B 🕓 200 mT

← B 🕓 0 mT:
no TRSB

Crosscorrelation between S_{12} and S_{21} at $\varepsilon = 0$



- $C(S_{12}, S_{21}^*) = \begin{cases} 1 \text{ for GOE} \\ 0 \text{ for GUE} \end{cases}$
- Data: TRSB is incomplete

S-Matrix Fluctuations and RMT

- Pure GOE → VWZ description 1984
- Pure GUE → V description 2007

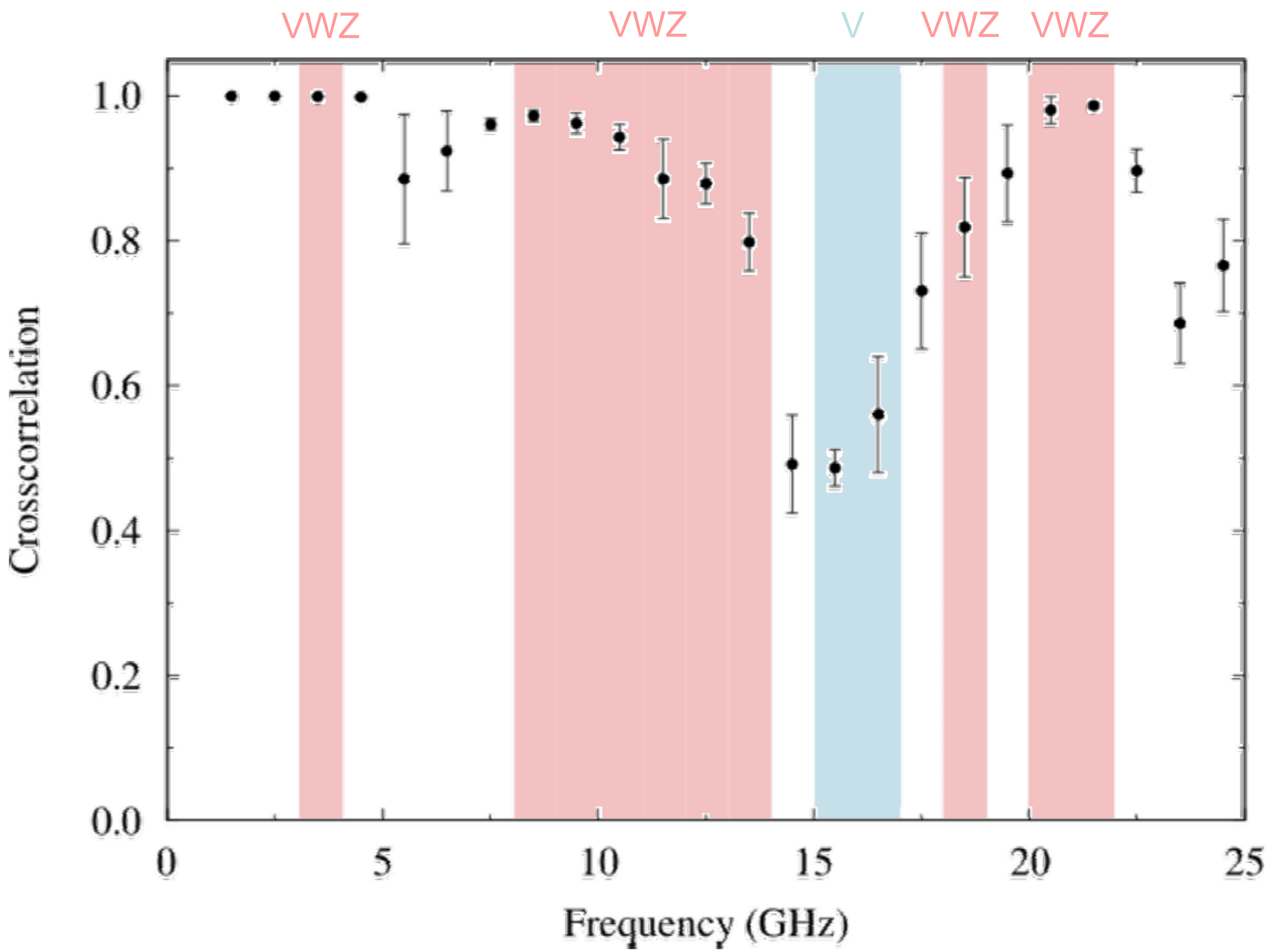
- Partial TRSB → no analytical model

- RMT → $\hat{H} = \hat{H}^{s} + i\alpha\hat{H}^{a}$

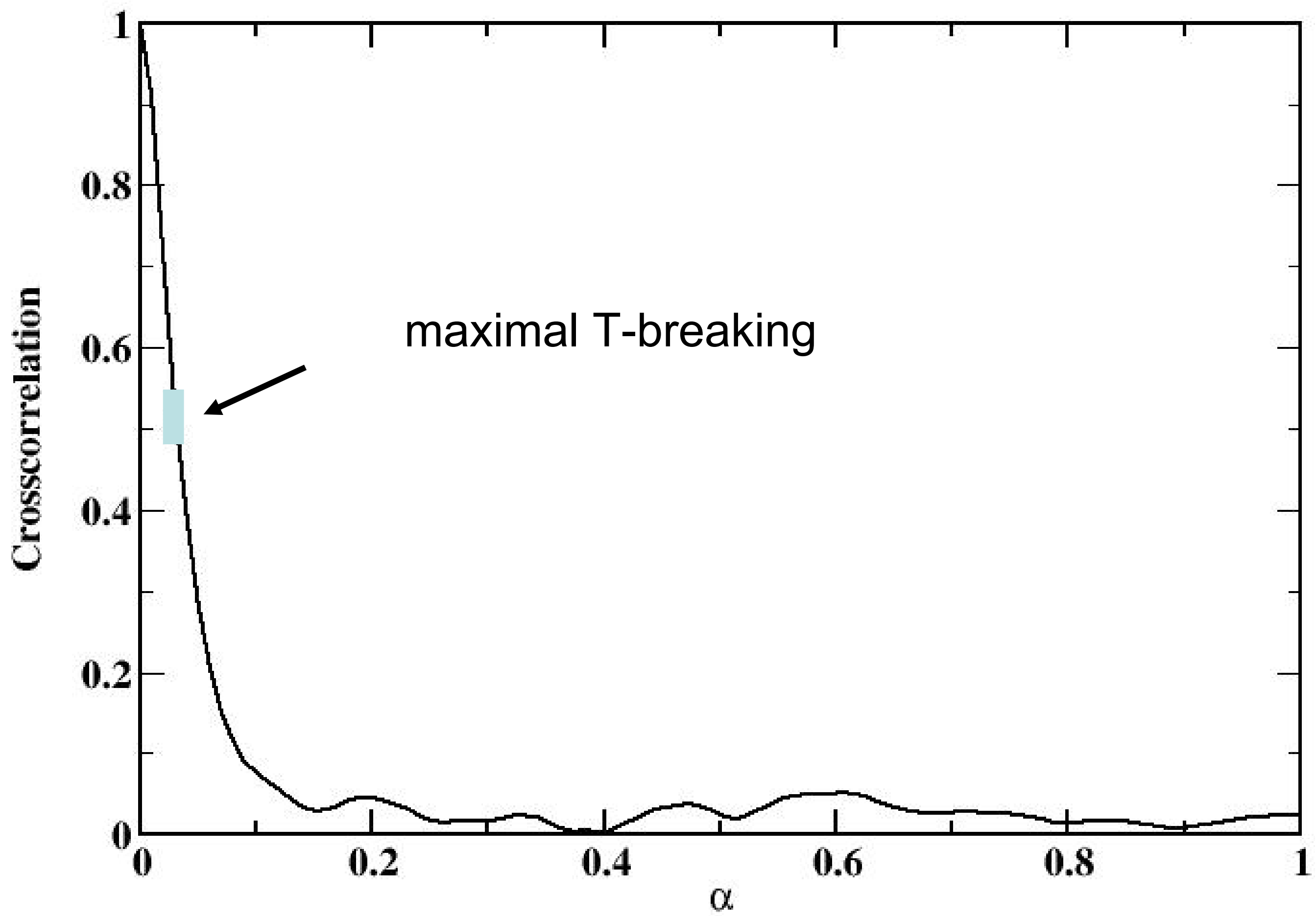
$\alpha = 0$ GOE

$\alpha = 1$ GUE

Test of VWZ and V Models



First approach towards RMT-description of experimental results



RMT → $\hat{H} = \hat{H}^s + i\alpha\hat{H}^a$

$\alpha = 0$ GOE

$\alpha = 1$ GUE

Summary

- Open microwave resonators are excellent model systems to test fluctuation properties of the compound nucleus
- RMT based models (VWZ, V) for GOE and GUE can be tested with high precision

	Γ/D << 1	Γ/D 🕓 1	Γ/D > 2
T inv	non exp decay	non exp decay	exp decay