Network inference from time-series measurements

Nicolás Rubido[1,2], Arturo C. Martí[2], Ezequiel Bianco-Martínez[1], Celso Grebogi[1], Murilo S. Baptista[1], and Cristina Masoller[3]

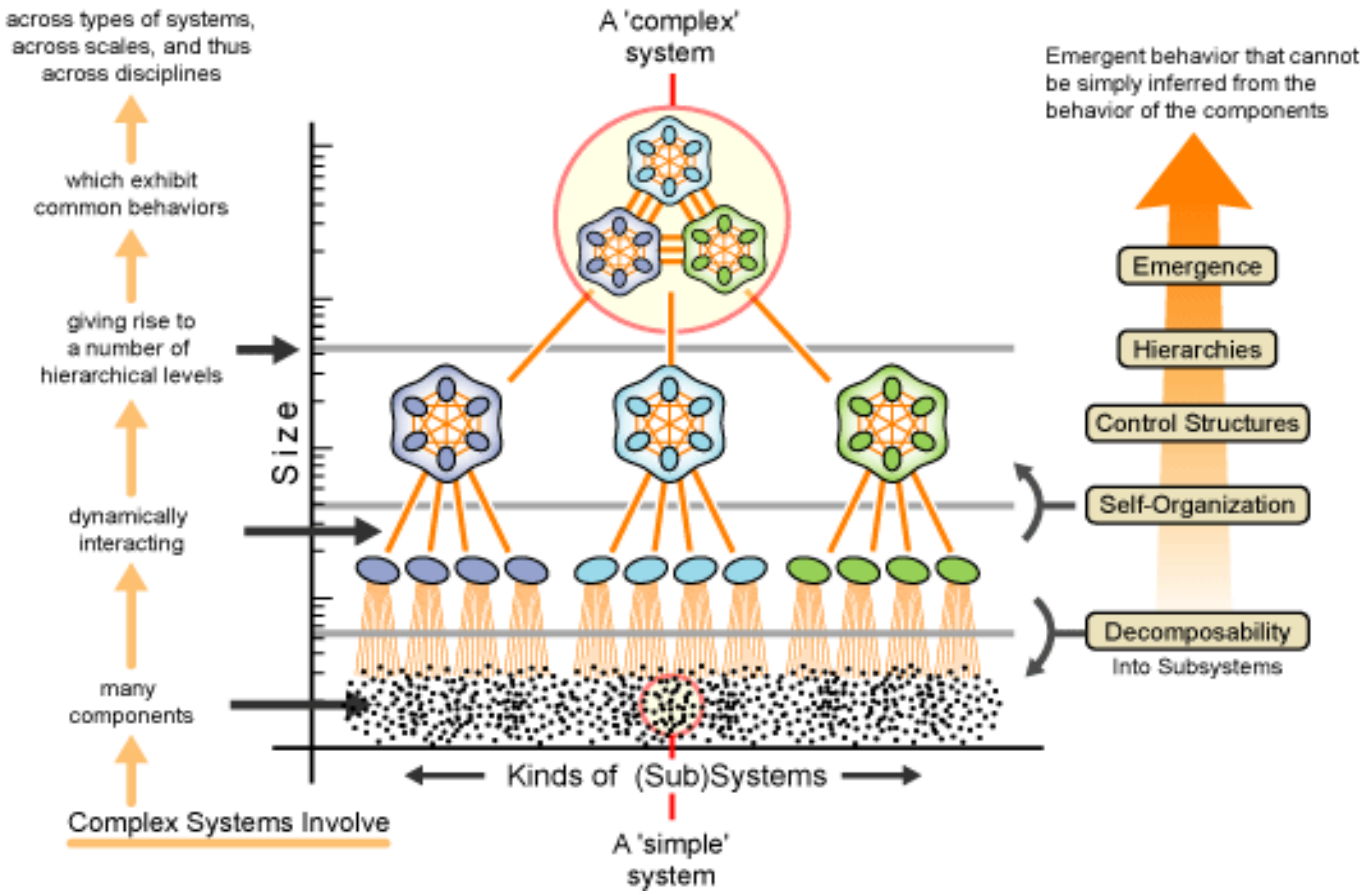
1 – Institute for Complex Systems and Mathematical Biology, University of Aberdeen, UK.
2 – Instituto de Física, Facultad de Ciencias, Universidad de la República, Uruguay
3 – Departament de Física i Enginyeria Nuclear, Universitat Politécnica de Catalunya, Colom 11, E-08222 Terrassa, Barcelona, Spain.

References:

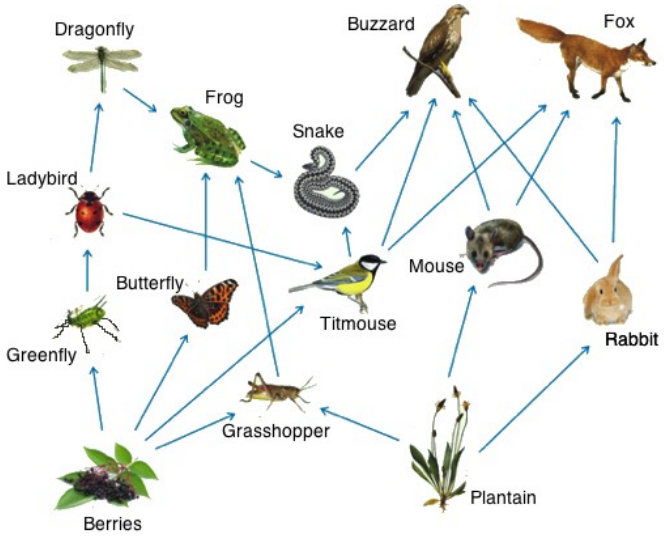
[1] A.A. Tsonis, K.L. Swanson, P.J. Roebber, *Bull. Amer. Meteor. Soc.* **87**, 585-595 (2006).
[2] M. Timme, *Phys. Rev. Lett.* **98**, 224101 (2007).
[3] A.A. Tsonis, K.L. Swanson, *Phys. Rev. Lett.* **100**, 228502 (2008).
[4] M. Valencia, J. Martinerie, S. Dupont, and M. Chavez, *Phys. Rev. E* **77**, 050905 (2008).
[5] K. Yamasaki, A. Gozolchiani, and S. Havlin, *Phys. Rev. Lett.* **100**, 228501 (2008).
[6] J.F. Donges, Y. Zou, N. Marwan, and J. Kurths, *Europhys. Lett.* **87**(4), 48007 (2009).
[7] B. Schelter, J. Timmer, and M. Eichler, *J. Neuro. Methods* **179**, 121-130 (2009).
[8] S. Bialonski, M.T. Horstmann, and K. Lehnertz, *Chaos* **20**, 013134 (2010).
[9] J. Ren, W.-X. Wang, B. Li, and Y.-C. Lai, *Phys. Rev. Lett.* **104**, 058701 (2010).
[10] J. Nawrath, M.C. Romano, M. Thiel, I.Z. Kiss, M. Wickramasinghe, J. Timmer, J. Kurths, and
[11] B. Schelter, *Phys. Rev. Lett.* **104**, 038701 (2010).
[12] S. V. Buldyrev, R. Parshani, G. Paul, H.E. Stanley, and S. Havlin, *Nat.* **464**, 1025-1028 (2010).
[13] C. Tominski, J.F. Donges, and T. Nocke, *IEEE 15th Int. Conf. Inf. Vis.* **4**, 298-305 (2011).
[14] M. Barreiro, A.C. Mart´ and C. Masoller, *Chaos* **21**, 013101 (2011).
[15] D. Hartman, J. Hlinka, M. Palus, D. Mantini, and M. Corbetta, *Chaos* **21**, 013119 (2011).
[16] S. G. Shandilya and M. Timme , *New J. Phys.* **13**, 013004 (2011).
[17] A. Gozolchiani, S. Havlin, and K. Yamasaki, *Phys. Rev. Lett.* **107**, 148501 (2011).
[18] Y.-Y. Liu, J.-J. Slotine and A.-L. Barabási, *Nat.* **473**, 167-173 (2011).
[19] N. Malik, B. Bookhagen, N. Marwan, and J. Kurths, *Clim. Dyn.* **39**, 971-987 (2012).
[20] J. Runge, J. Heitzig, N. Marwan, and J. Kurths, *Phys. Rev. E* **86**, 061121 (2012).
[21] E. Bullmore and O. Sporns, *Nat. Rev. Neuro.* **13**, 336-349 (2012).
[22] J.I. Deza, M. Barreiro, and C. Masoller, *Eur. Phys. J. Special Topics* **222**(2), 511-523 (2013).
[23] R.L. Buckner, F.M. Krienen, and B.T. Thomas Yeo, *Nat. Rev. Neuro.* **16**, 832-837 (2013).
[24] A.E. Motter, S.A. Myers, M. Anghel and T. Nishikawa, *Nat. Phys.* **9**, 191-197 (2013).
[25] F.J. Romero-Campero, *et al.*, *Front. Plant. Sci.* **4**, 291-308 (2013).
[26] K. Rehfeld and J. Kurths, *Clim. Past* **10**, 107-122 (2014).
[27] N. Molkenthin, K. Rehfeld, N. Marwan, and J. Kurths, *Sci. Rep.* **4**, 4119 (2014).

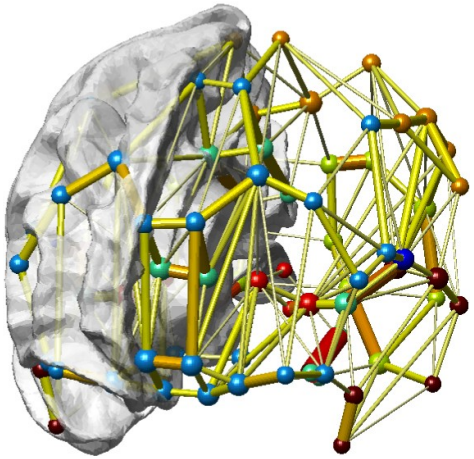
…

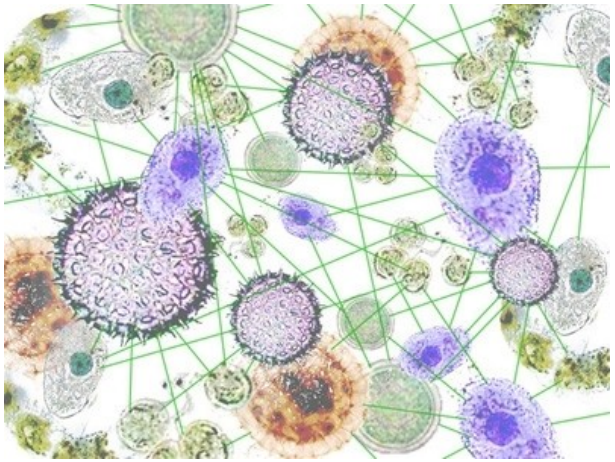
Complex Systems



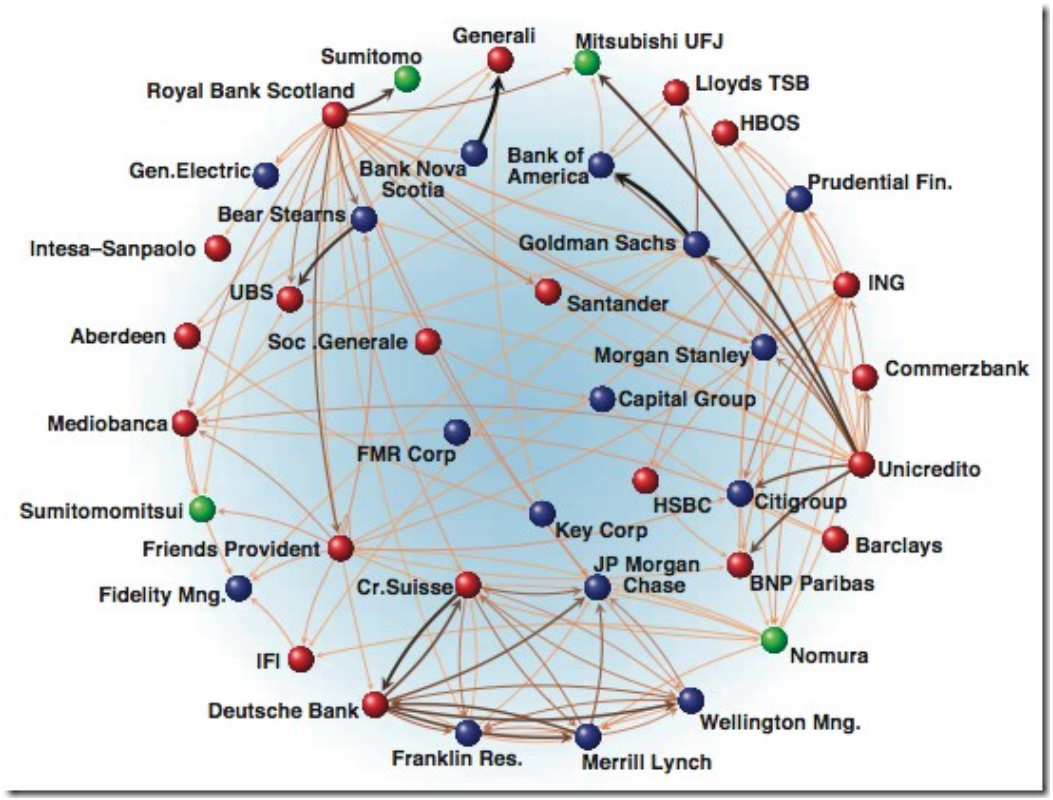
Networks

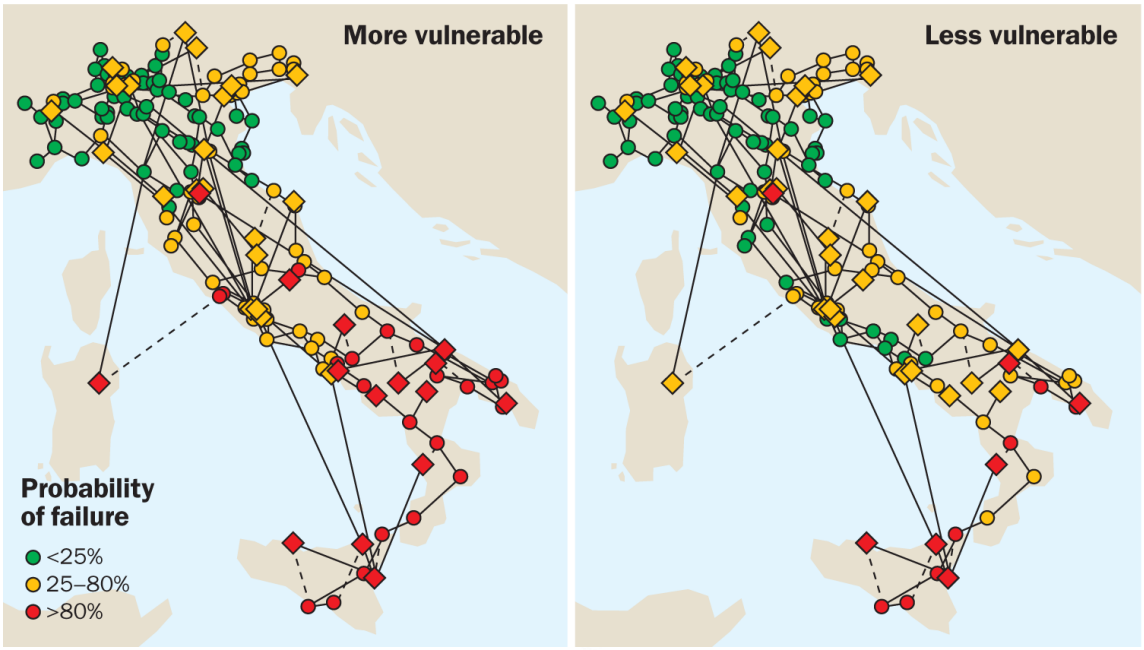

Dragonfly
Buzzard
Fox
Frog
Snake
Ladybird
Mouse
Butterfly
Titmouse
Rabbit
Greenfly
Grasshopper
Plantain
Berries



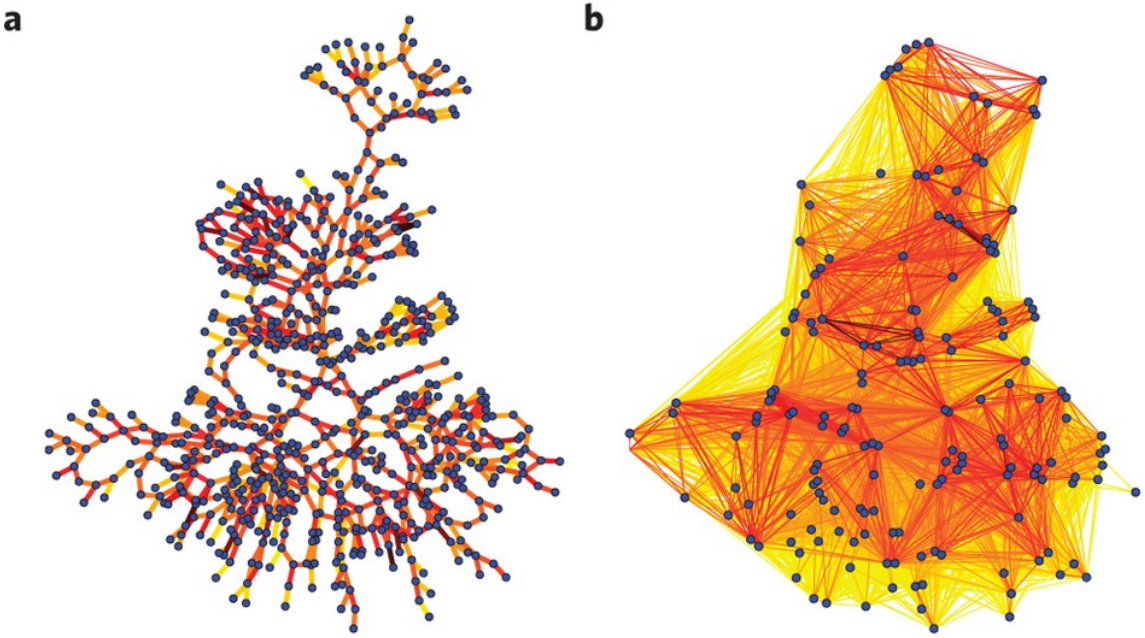



facebook
Linked in
digg
lost.fm
YouTube
SOCIAL MEDIA
In Business


Sumitomo
Generali
Mitsubishi UFJ
Royal Bank Scotland
Lloyds TSB
HBOS
Gen.Electric
Bank Nova Scotia
Bank of America
Prudential Fin.
Bear Stearns
Intesa–Sanpaolo
Goldman Sachs
ING
UBS
Santander
Aberdeen
Soc Generale
Morgan Stanley
Commerzbank
Capital Group
Mediobanca
FMR Corp
Unicredito
HSBC
Citigroup
Sumitomomitsui
Key Corp
Barclays
Friends Provident
JP Morgan Chase
BNP Paribas
Fidelity Mng.
Cr.Suisse
Nomura
IFI
Deutsche Bank
Wellington Mng.
Franklin Res.
Merrill Lynch

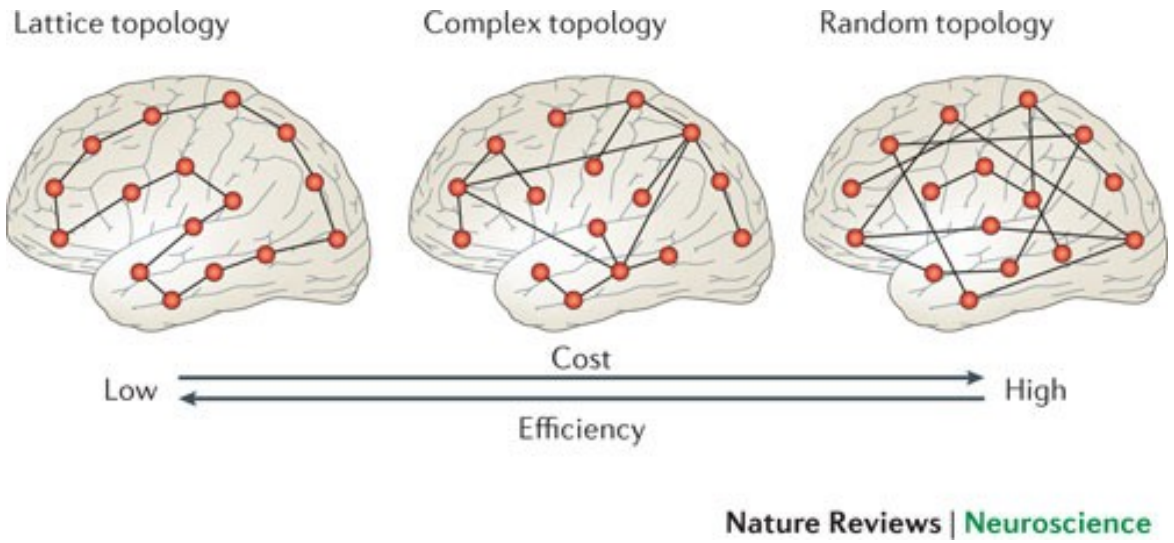


S.V. Buldyrev, R. Parshani, G. Paul, H.E. Stanley, and S. Havlin, "*Catastrophic cascade of failures in interdependent networks*", Nat. **464**, 1025-1028 (2010).

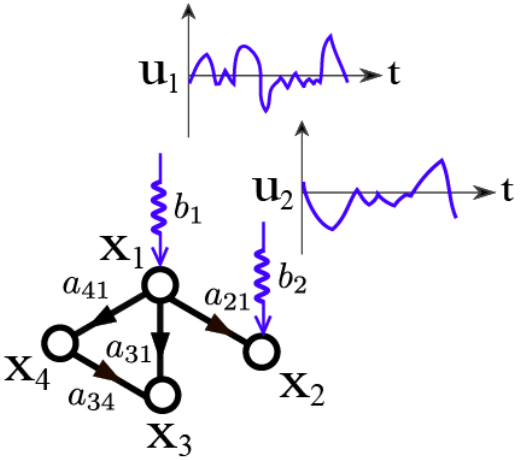


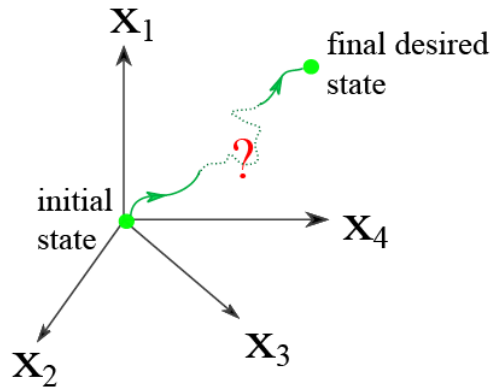
A.E. Motter, S.A. Myers, M. Anghel and T. Nishikawa, "*Spontaneous synchrony in power-grid networks*", Nat. Phys. **9**, 191-197 (2013).

E. Bullmore and O. Sporns, "*The economy of brain network organization*", Nat. Rev. Neuro. **13**, 336-349 (2012).



Y.-Y. Liu, J.-J. Slotine and A.-L. Barabási, "*Controllability of complex networks*", Nat. **473**, 167-173 (2011).

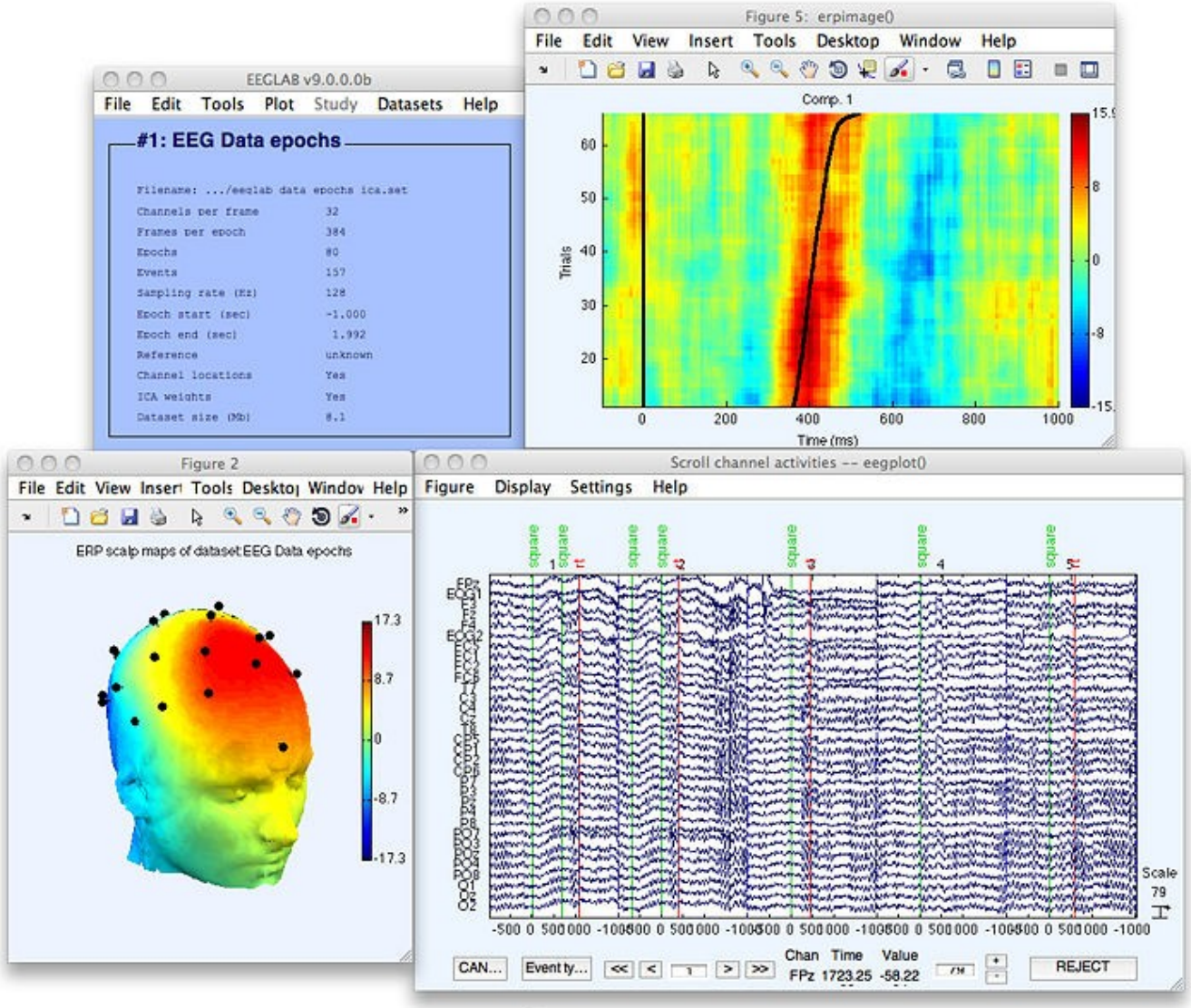


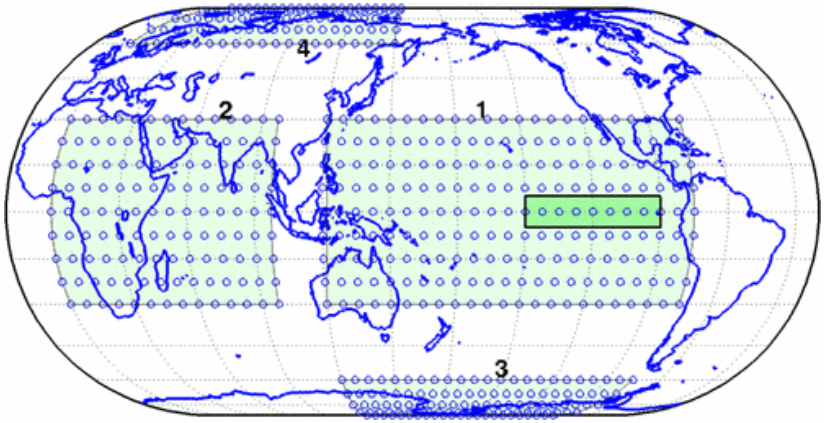


$$\mathbf{A} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ a_{21} & 0 & 0 & 0 \\ a_{31} & 0 & 0 & a_{34} \\ a_{41} & 0 & 0 & 0 \end{pmatrix}; \quad \mathbf{B} = \begin{pmatrix} b_1 & 0 \\ 0 & b_2 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}; \quad \mathbf{C} = \begin{pmatrix} b_1 & 0 & 0 & 0 & 0 & 0\,0\,0 \\ 0 & b_2 & a_{21}b_1 & 0 & 0 & 0\,0\,0 \\ 0 & 0 & a_{31}b_1 & 0 & a_{34}a_{41}b_1 & 0\,0\,0 \\ 0 & 0 & a_{41}b_1 & 0 & 0 & 0\,0\,0 \end{pmatrix}$$

$$N = 4, \; M = 2, \; \mathrm{rank}(\mathbf{C}) = 4 = N$$

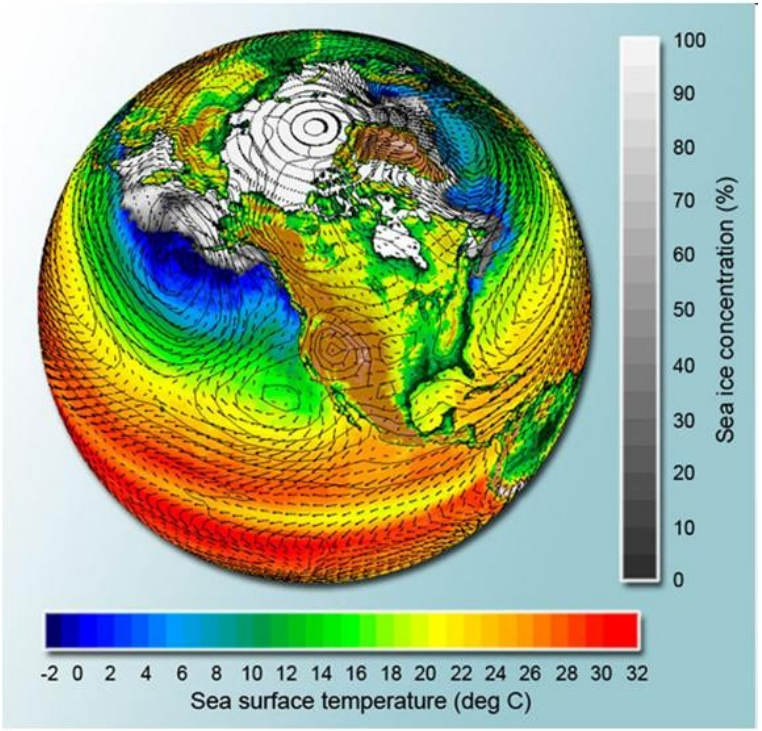
Time-series measurements

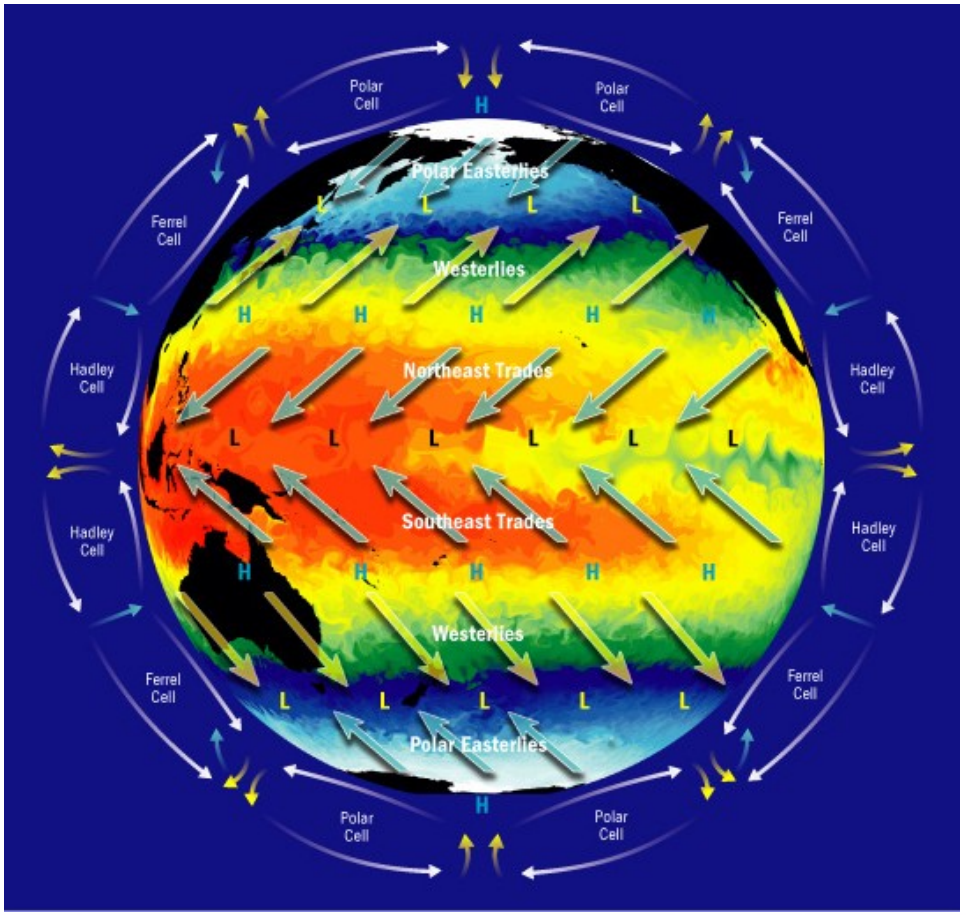




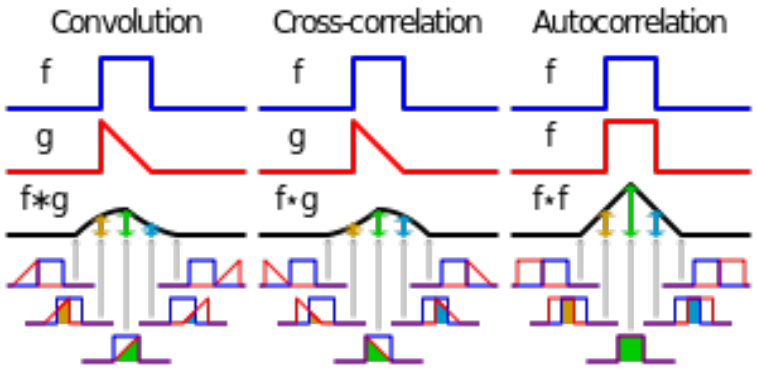
J.F. Donges, Y. Zou, N. Marwan, and J. Kurths, "*The backbone of the climate network*", Europhys. Lett. **87**(4), 48007 (2009).

C. Tominski, J.F. Donges, and T. Nocke, "*Information Visualization in Climate Research*", IEEE 15th Int. Conf. Inf. Vis. **4**, 298-305 (2011).



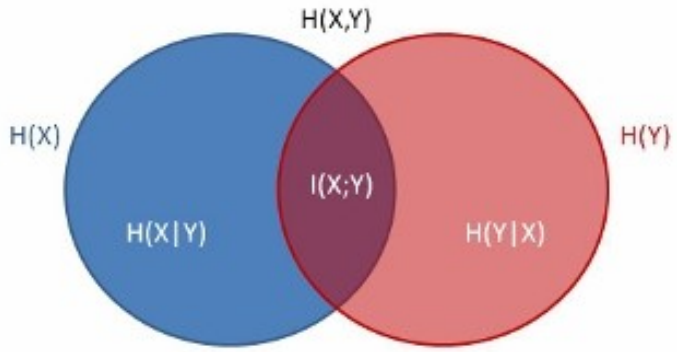


Similarity measures



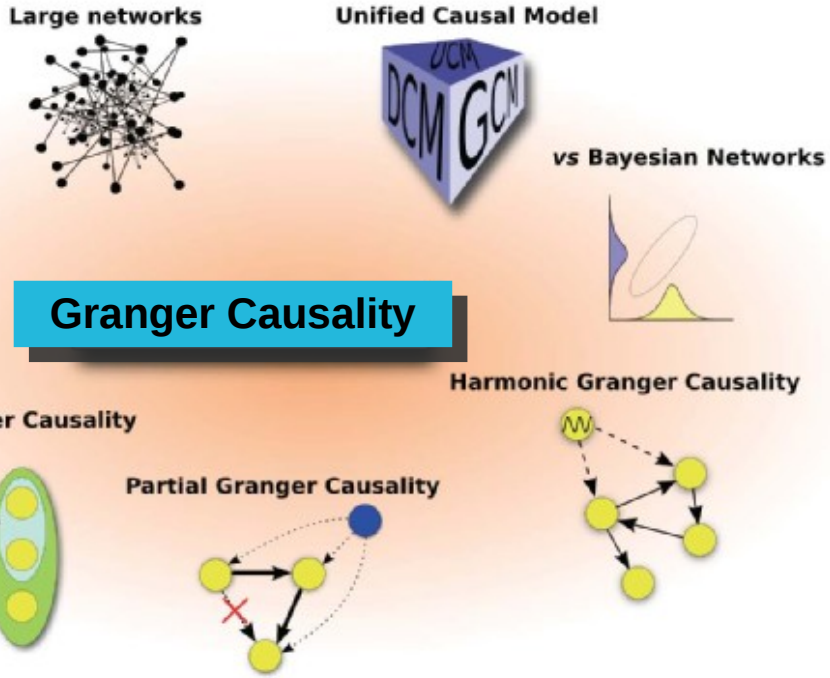
Cross-Correlation

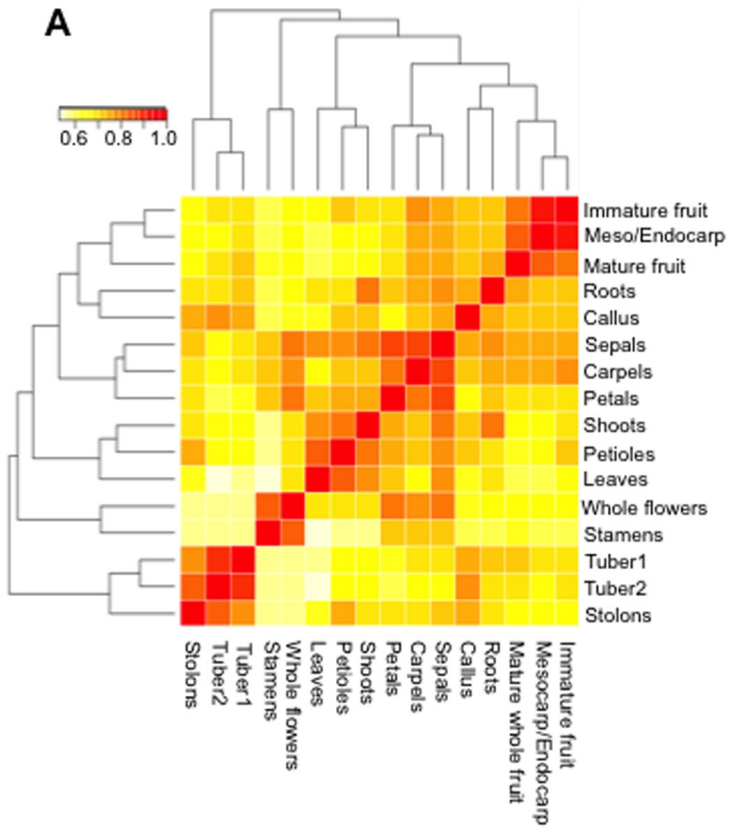
Mutual Information



$$I(X;Y) = H(X \cap Y)$$
$$H(X;Y) = H(X) + H(Y) - H(X \cap Y)$$

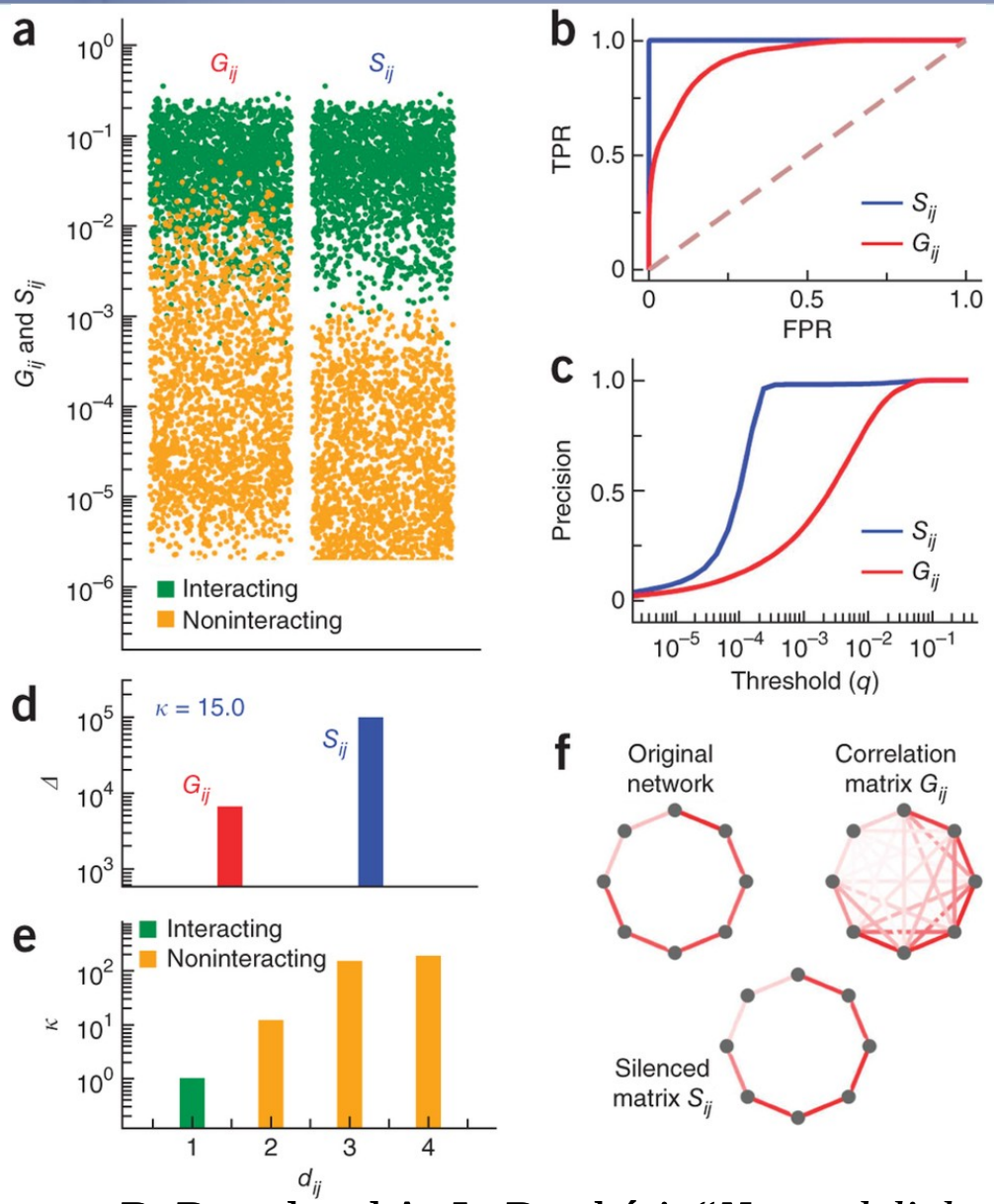
Granger Causality



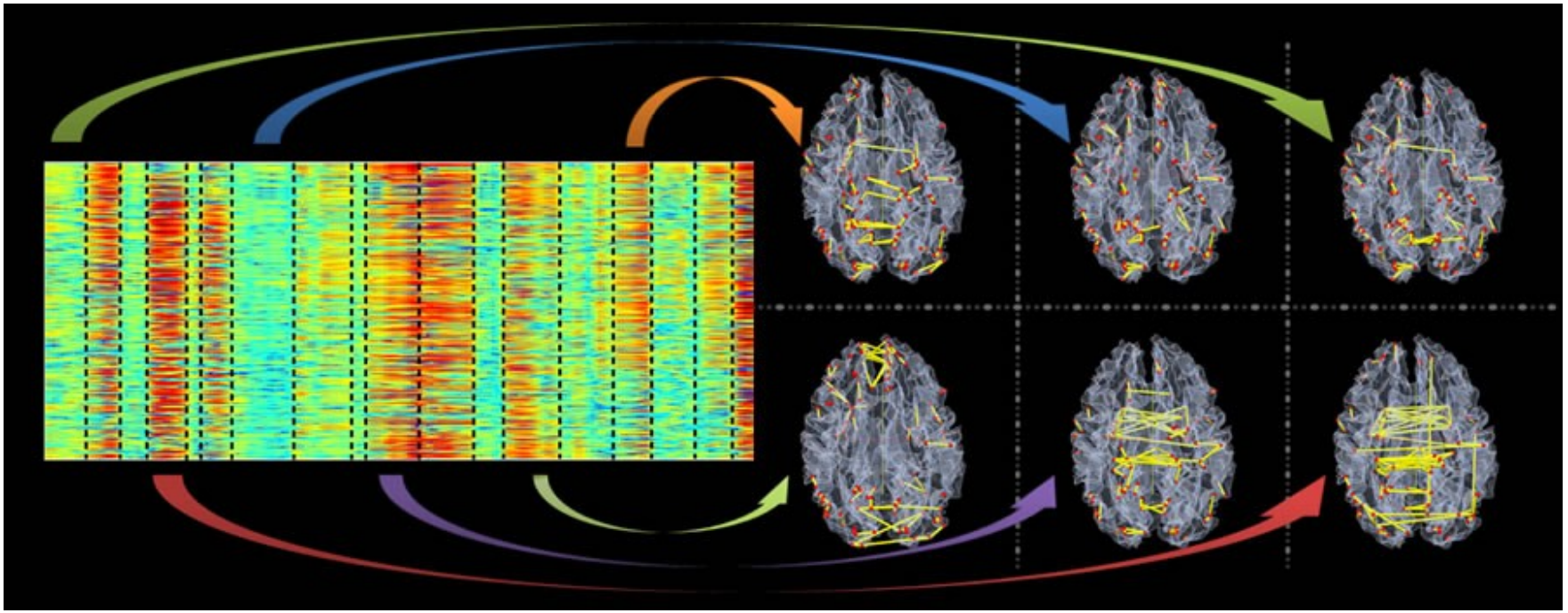


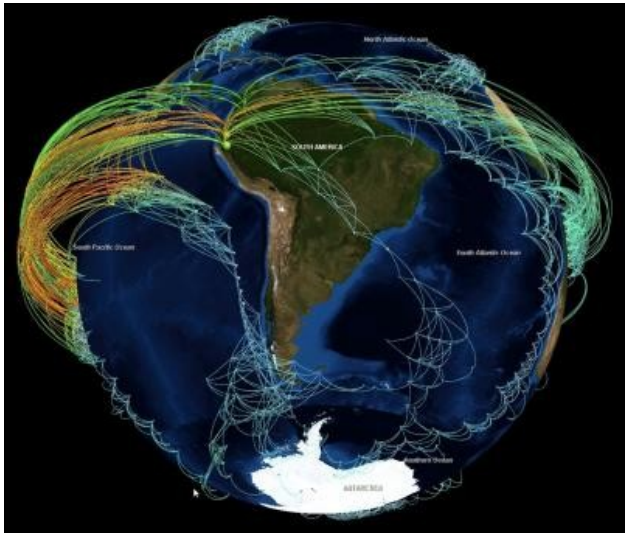
Cluster heat map of gene expression data

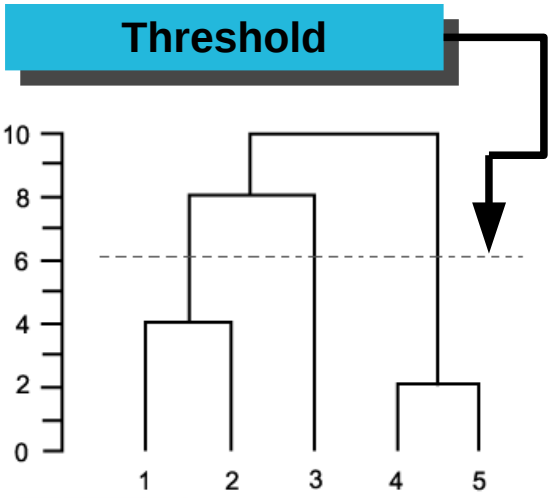
A.N. Massa, K.L. Childs, H. Lin, G.J. Bryan, G. Giuliano, and C.R. Buell, "*The Transcriptome of the Reference Potato Genome Solanum tuberosum Group Phureja Clone DM1-3 516R44*", PloS ONE **6**(10), e26801 (2011).

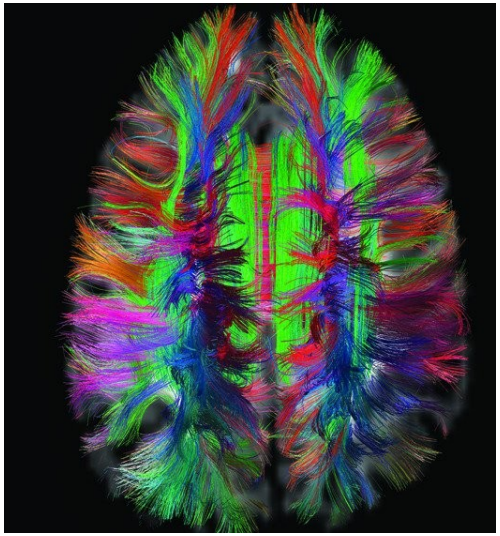


B. Barzel and A.-L. Barabási, "*Network link prediction by global silencing of indirect correlations*", Nat. Biotech. **31**, 720-725 (2013).



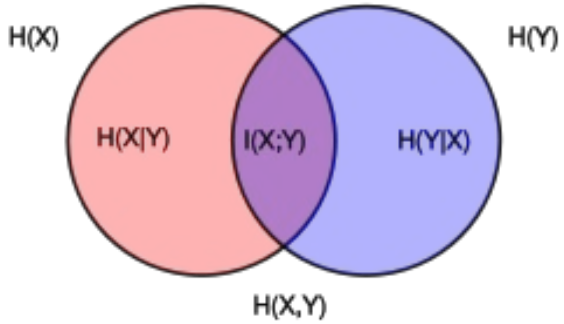



Threshold
10
8
6
4
2
0
1
2
3
4
5

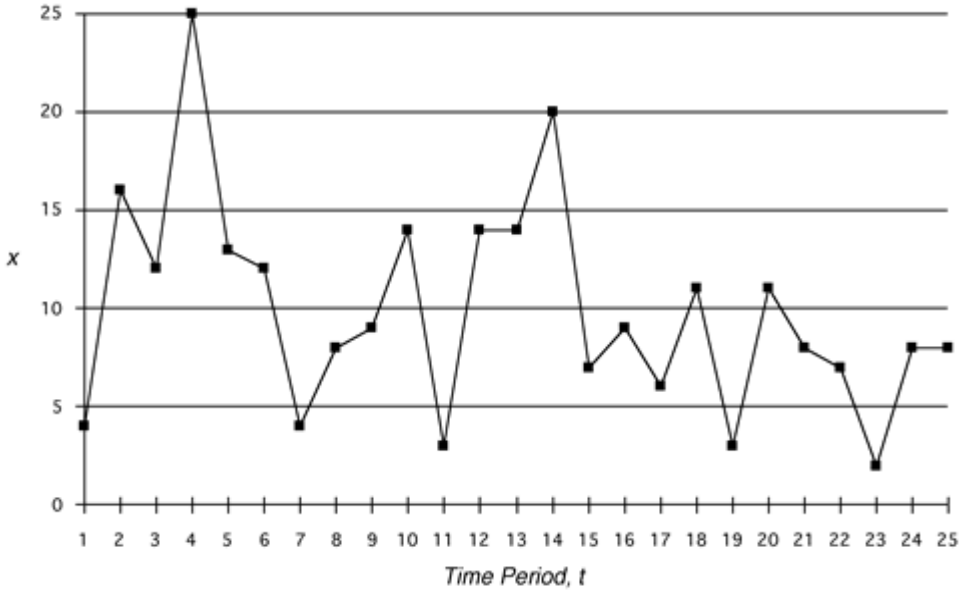


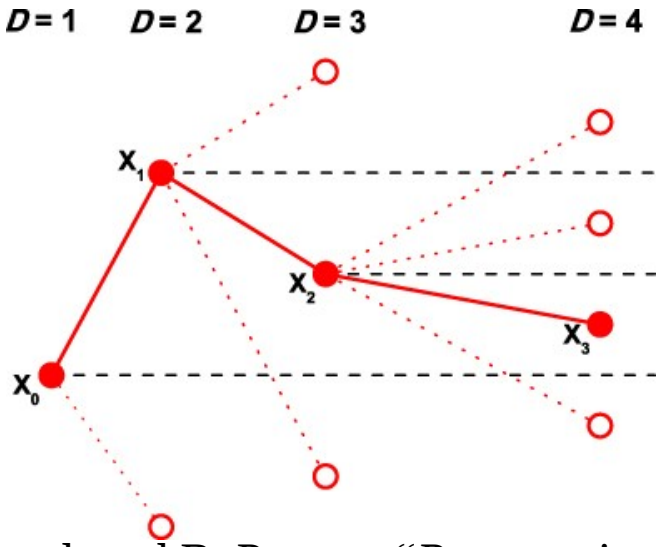
Network Inference Problems

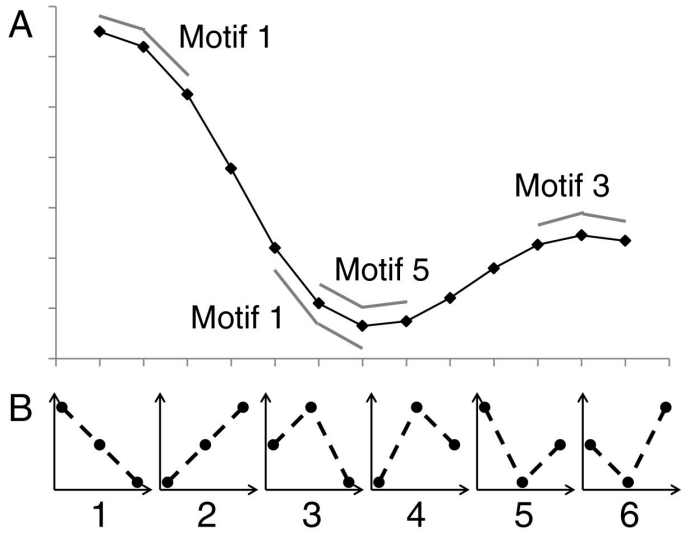
- Which similarity measure to use
- How to choose a threshold
- How much data is available
- How to avoid the (usual) noise in the data
- How to recover coupling strengths
- Which are the directions in the interactions
- How many “units” observed
- …



$$I(X;Y) = H(X) - H(X|Y)$$
$$= H(X) + H(Y) - H(X,Y),$$
$$I(X;Y) = \sum_x \sum_y p(x,y) \log \frac{p(x,y)}{p(x)p(y)},$$

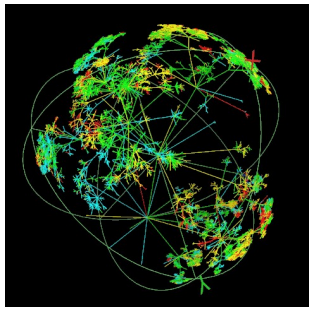




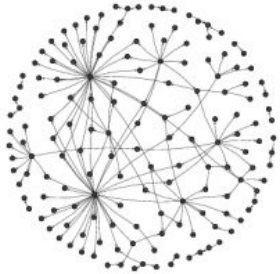


C. Bandt and B. Pompe, "*Permutation Entropy: A Natural Complexity Measure for Time Series*", Phys. Rev. Lett. **88**(17), 174102(4) (2002).

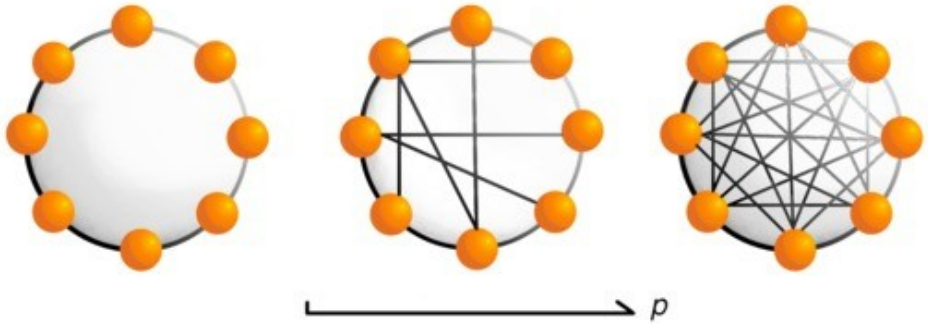
Inferred



Comparison

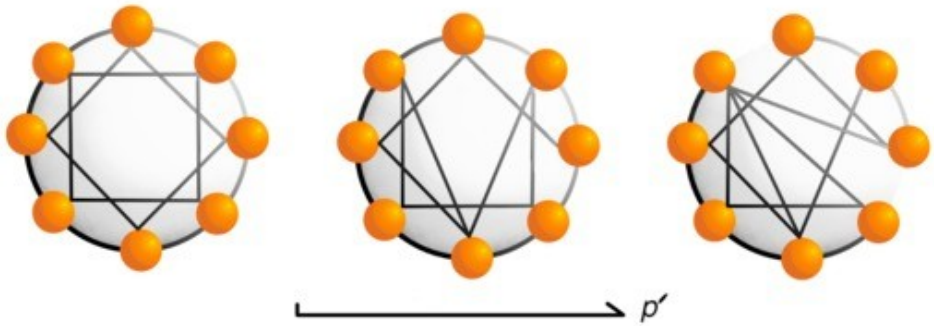


Underlying



$$E[M_N] = p\frac{N(N-3)}{2} + N$$

Expected number of edges



$$E[M_N] = \frac{N\ k}{2} = \frac{N\ (N/4)}{2}$$

Expected number of edges

Poster: N. Rubido, et al., "*Exact detection of direct links in networks of interacting dynamical units*".

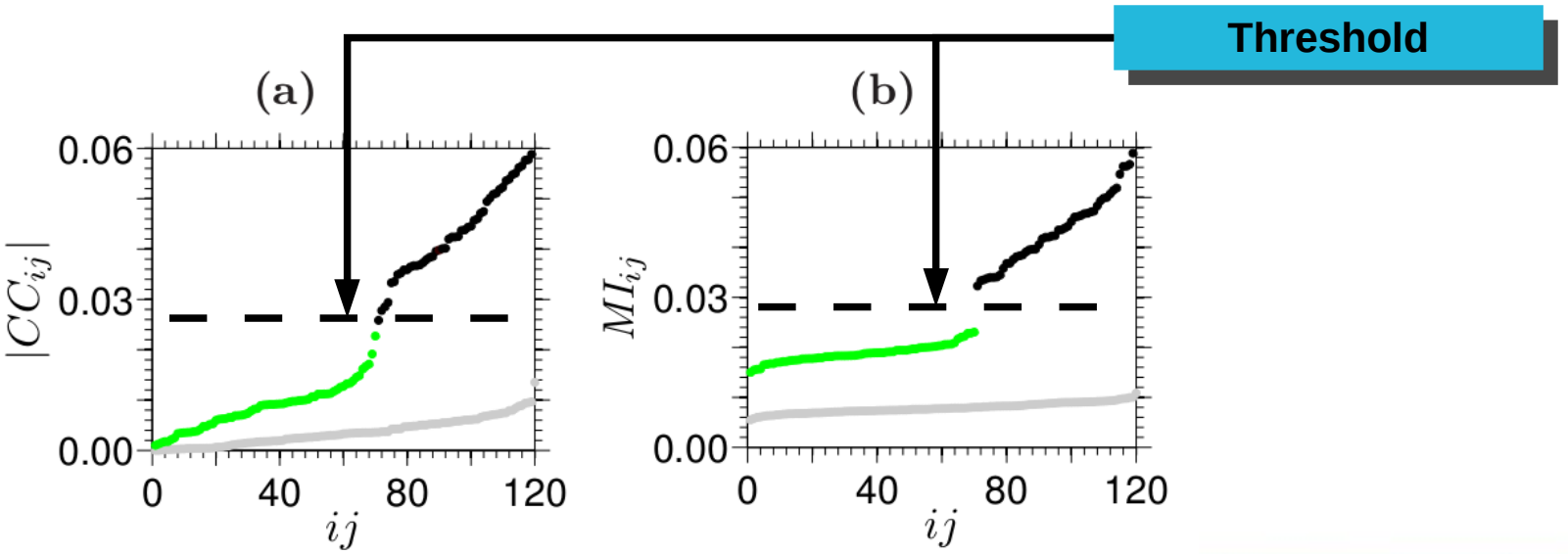
Results

$$x_{n+1}^{(i)} = (1-\epsilon)\, f_i\left(x_n^{(i)}\right) + \frac{\epsilon}{d_i}\sum_{j=1}^{N} W_{ij}\, f_j\left(x_n^{(j)}\right)$$

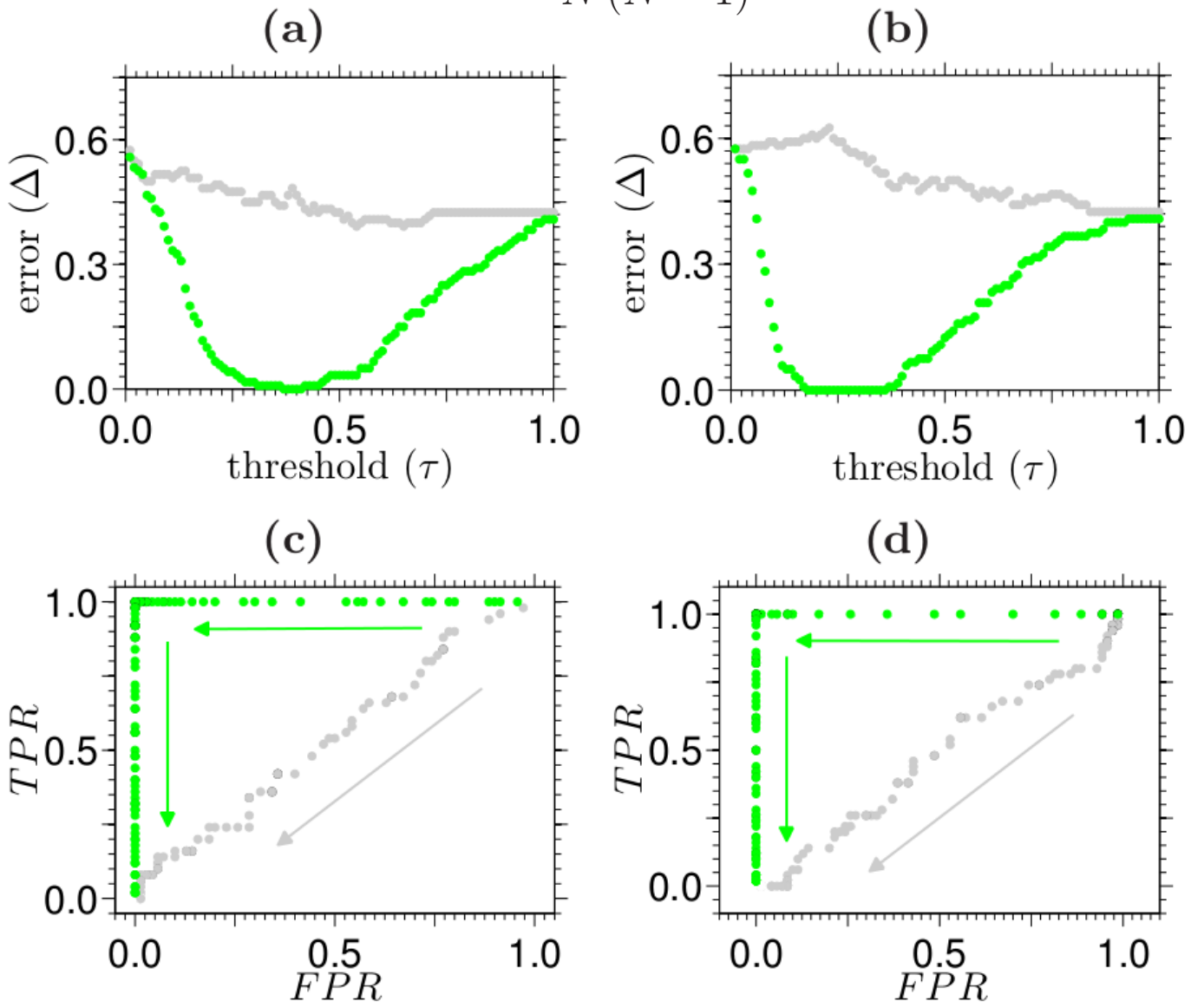
$$W_{ij} = A_{ij}\,(1 + g\,\xi_{ij})$$

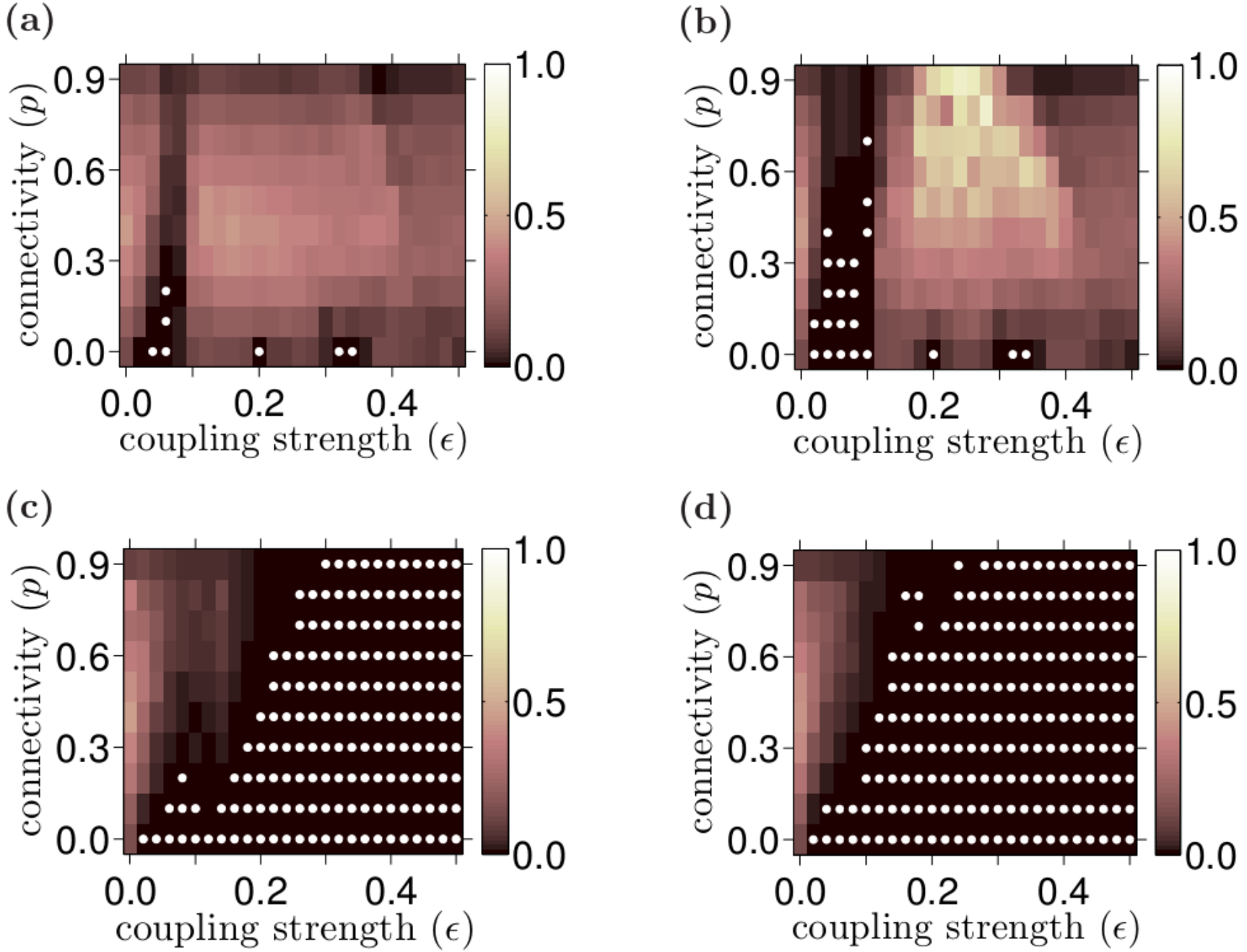
Kunihiko Kaneko, "*Overview of coupled map lattices*", Chaos **2**(3), 279 (1992).

- Logistic maps
- Circle maps
- Optical maps
- Tent maps



$$\Delta = \frac{\sum_{i,\,j=1}^{N} |A_{ij} - A_{\tau,ij}|}{N\,(N-1)}$$




(a)
(b)
(c)
(d)
connectivity (p)
coupling strength (ϵ)
0.0
0.3
0.6
0.9
0.2
0.4
0.5
1.0

Articles:

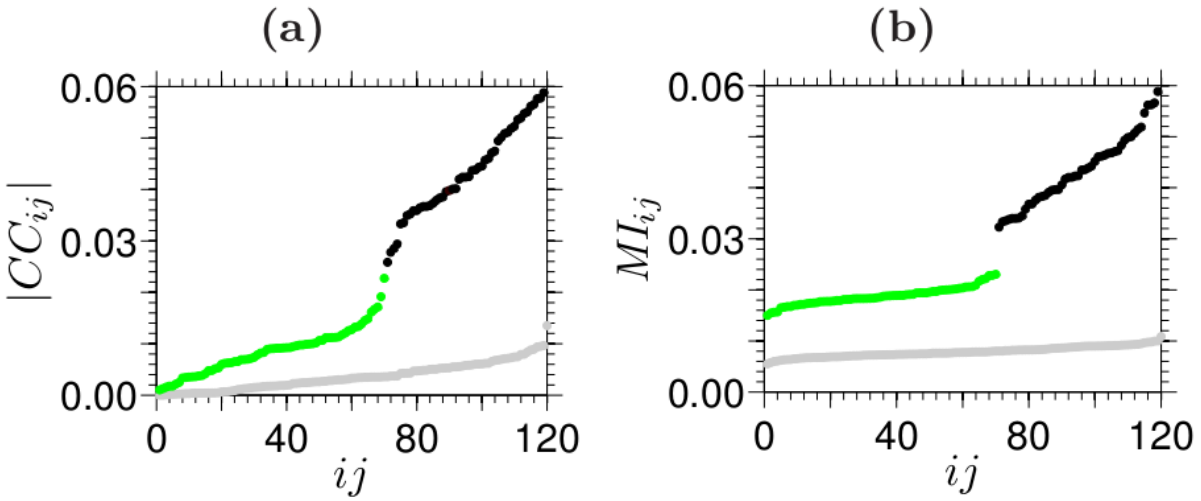
N. Rubido, A.C. Martí, E. Bianco-Martínez, C. Grebogi, M.S. Baptista, and C. Masoller, "*Exact detection of direct links in networks of interacting dynamical units*", submitted (2014) [available at: http://arxiv.org/abs/1403.4839].

CONCLUSIONS (take home messages**):**

CC and MI allow to infer the underlying networks of coupled dynamical systems, without errors, from finite-size time-series measurements.

The correct detection of links depends on the existence of a gap in the ordered values of the similarity measures between pairs of nodes.

E. Bianco-Martínez, N. Rubido, C.G. Antonopoulos, and M.S. Baptista, "*Network Inference by Mutual Information Rates from Complex Time-series*", in preparation (2014).



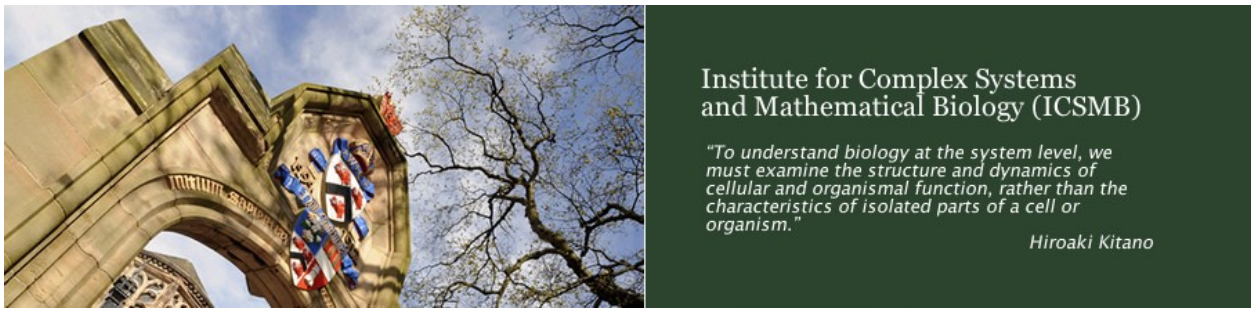
mpipks

THANK YOU

Ongoing projects:

A. L'Her, P. Amil, R. García, F. Abellá, M. S. Baptista, A. C. Martí, C. Cabeza, and N. Rubido, "*Electronic circuit implementation of a network of Logistic maps*".
Universidad de la República (UdelaR), Montevideo, Uruguay.

N. Rubido and A.J. Pons, "*Neural circuits and transfer functions*".
Universidad Politécnica de Barcelona (UPC), Terrassa, Spain.



SUPA®