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Linda Sommerlade

Assessing the strength of directed influences among neural signals:

An approach to noisy data

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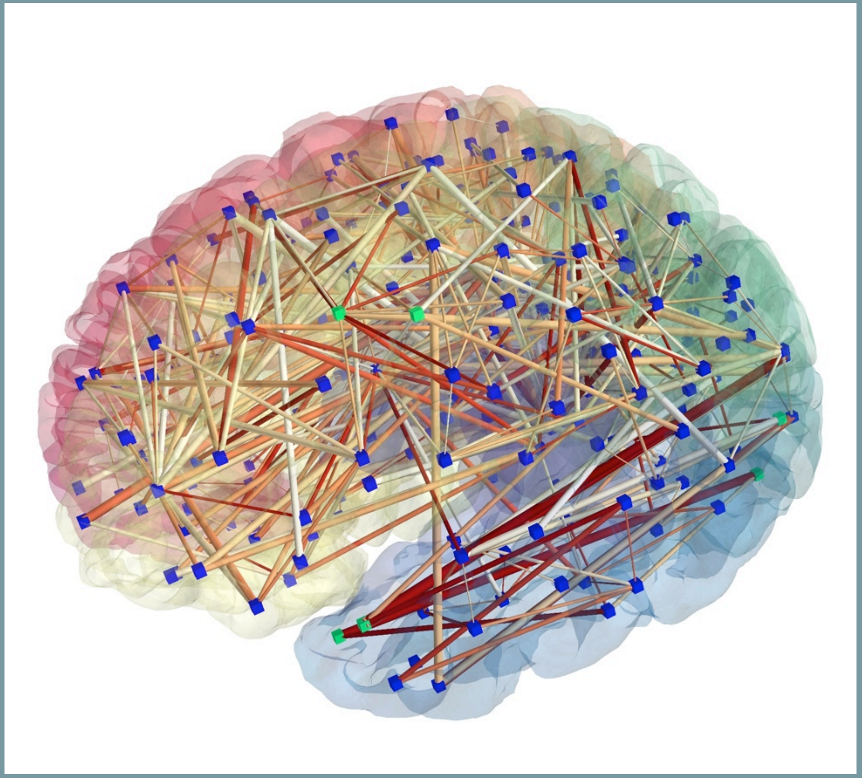
Overview

- Motivation
- Understanding
- Advanced Method
- Application
- Conclusion

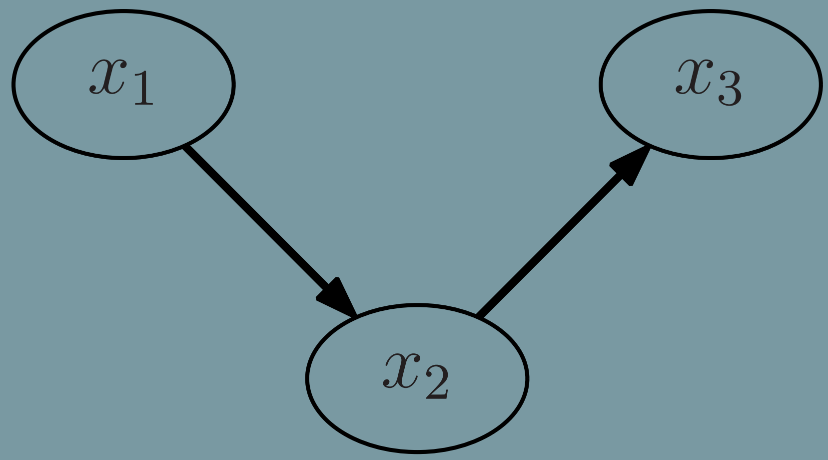
Networks

- Inference of networks from data
- Observations are afflicted with noise
- Standard measures do not consider observational noise



Granger Causality

- Causes precede effects in time
- Cause contains information on effect
- Autoregressive processes



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Simulated System

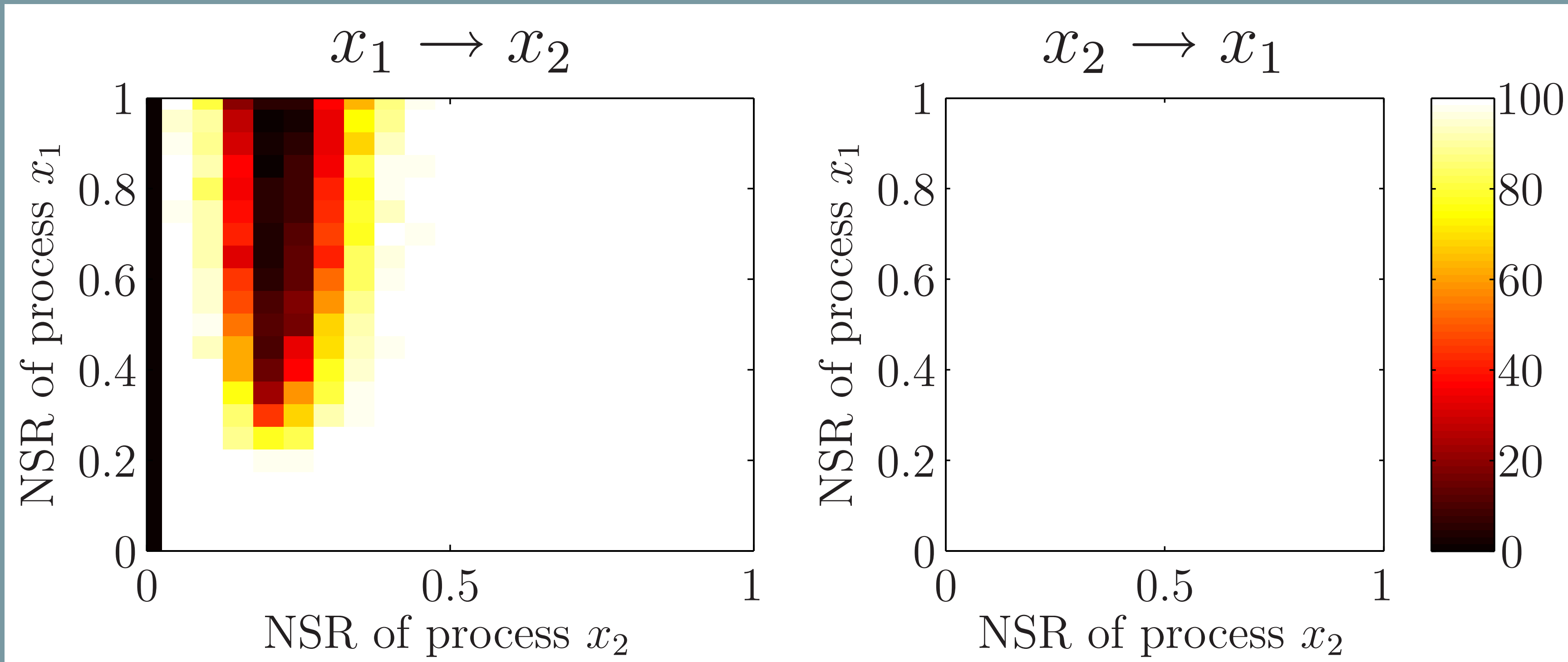
$$\vec{x}(t) = \sum_{r=1}^{2} \mathbf{a}_r \vec{x}(t-r) + \boldsymbol{\varepsilon}_x(t)$$

$$y_i(t) = x_i(t) + \sigma_i \eta_i(t) \quad i = 1, 2$$

$$\mathbf{a}_1 = \begin{pmatrix} 1.3 & c \\ 0 & 1.7 \end{pmatrix}, \ \mathbf{a}_2 = \begin{pmatrix} -0.8 & 0 \\ 0 & -0.8 \end{pmatrix}$$

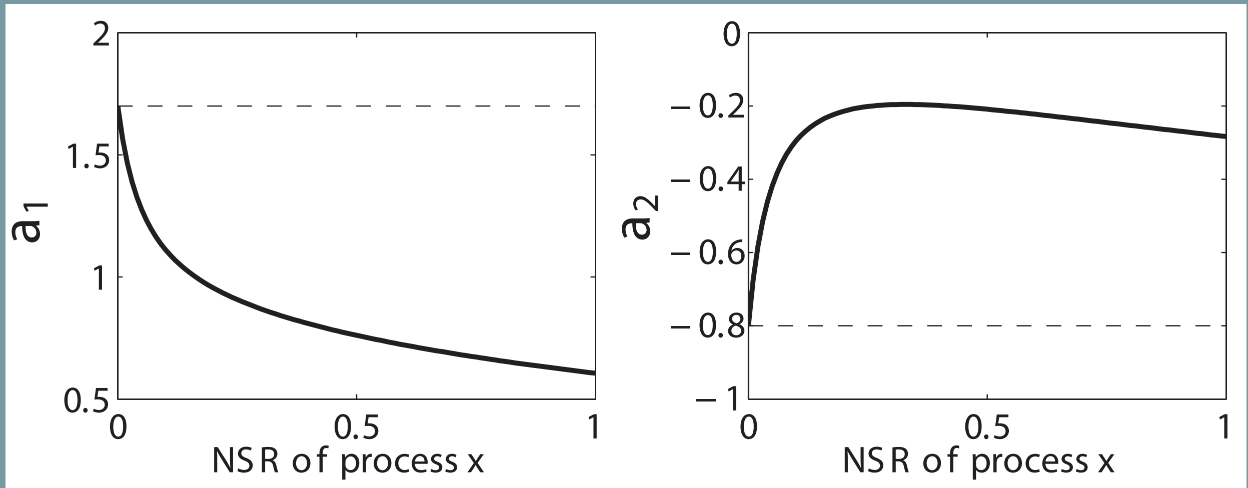
Simulation Results

Unidirectional influence



Theory (1-dimensional)

Absolute values underestimated

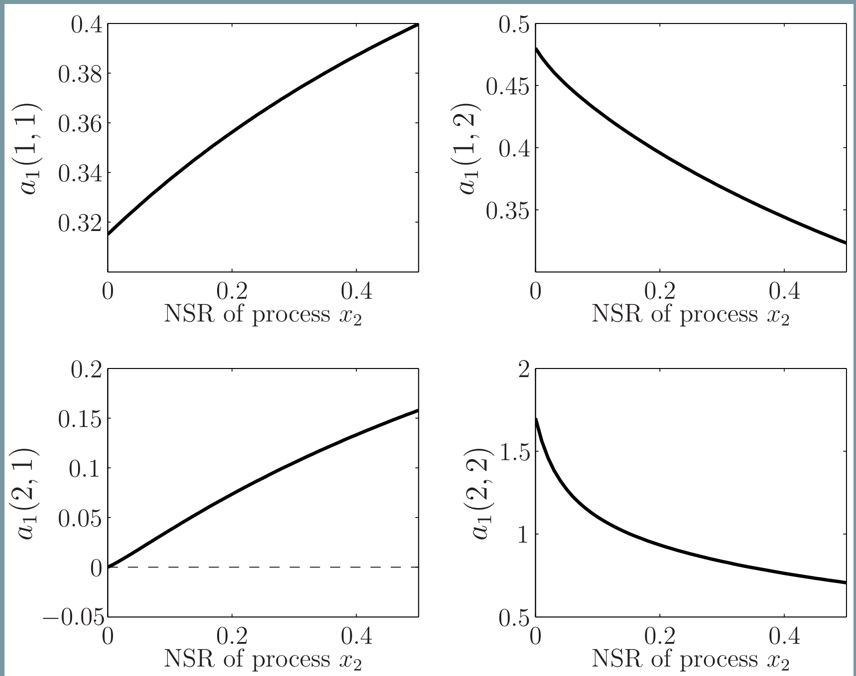


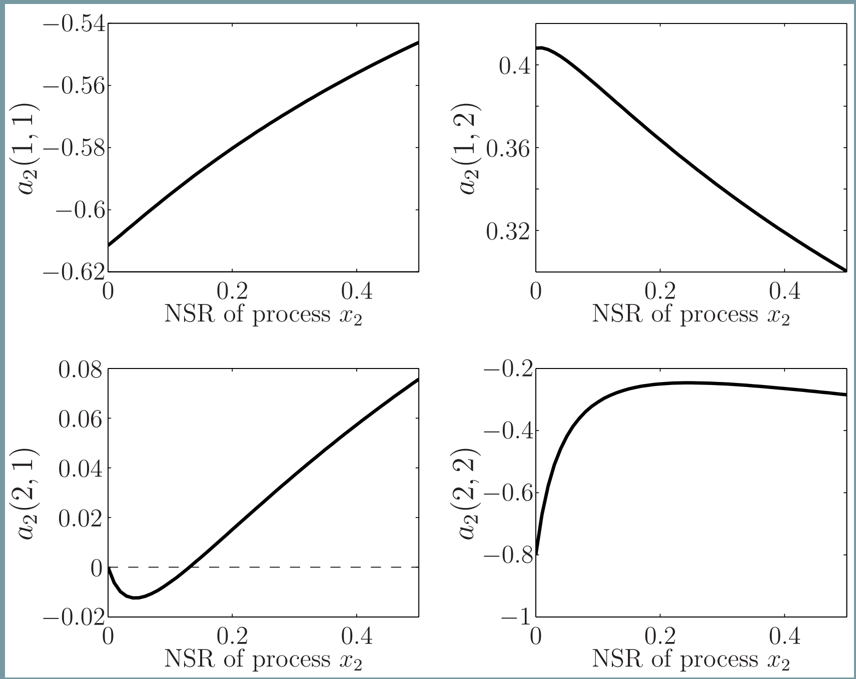
$$\hat{a}_1 = a_1 \frac{(1+\mathrm{NSR})(1-a_2) - (a_1^2 + a_2 - a_2^2)}{(1+\mathrm{NSR})^2(1-a_2)^2 - a_1^2}$$

$$\hat{a}_2 = \frac{(1+\mathrm{NSR})(a_1^2 + a_2 - a_2^2)(1-a_2) - a_1^2}{(1+\mathrm{NSR})^2(1-a_2)^2 - a_1^2}$$

Theory (2-dimensional)

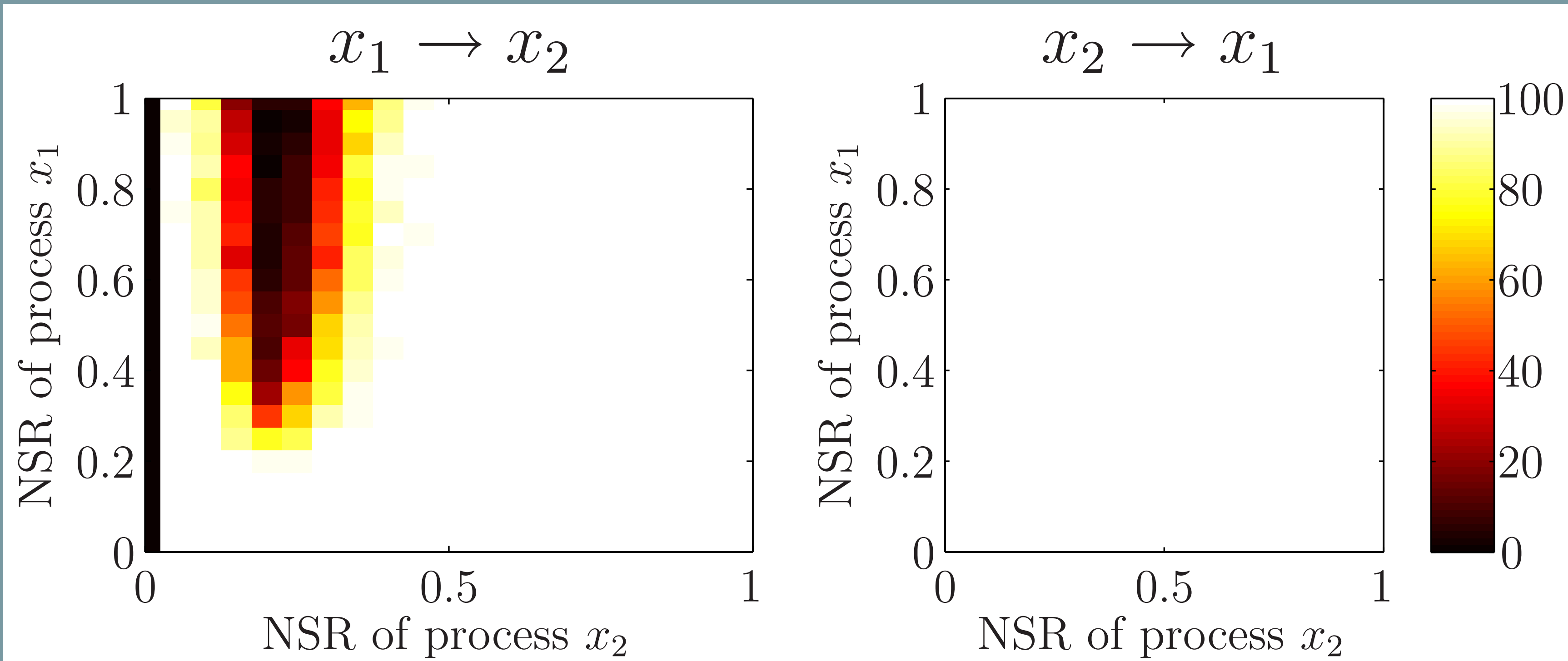
Off-diagonal entries differ from zero





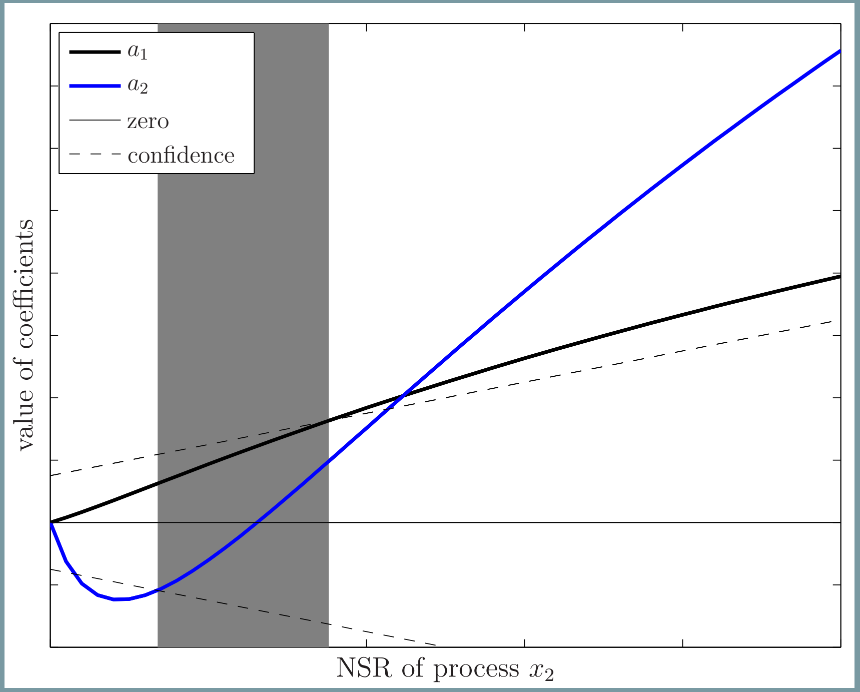
Simulation Results

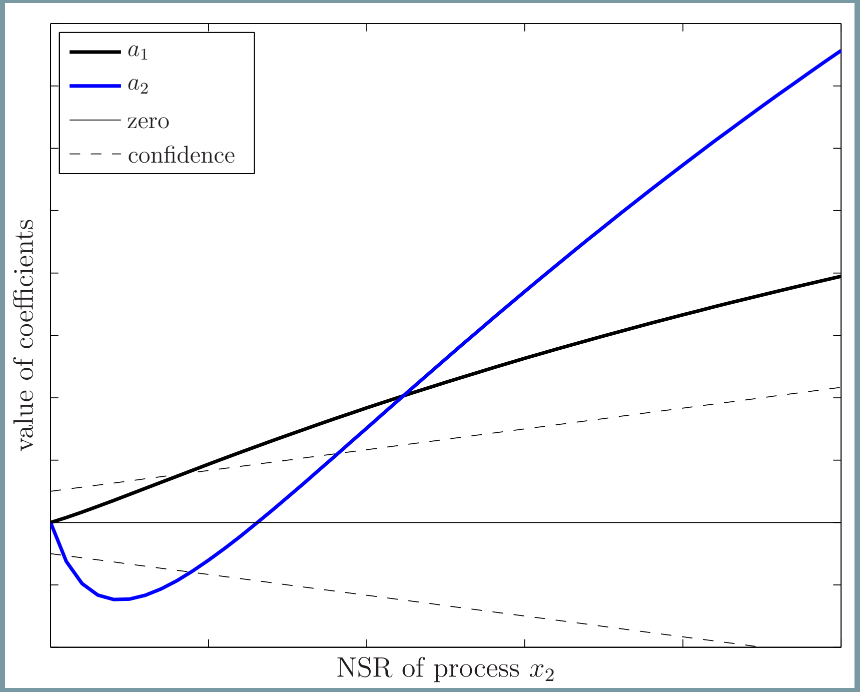
Different from theory



Schematic

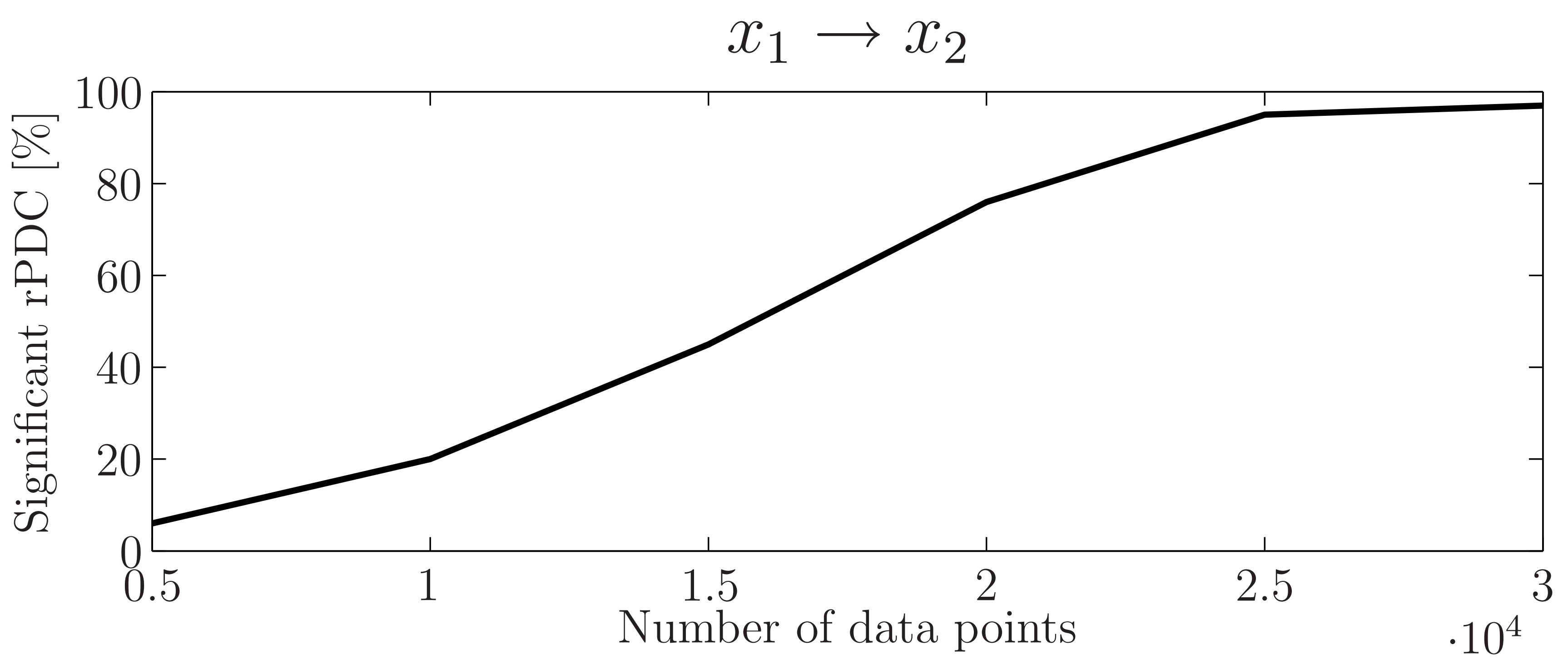
Coefficients compatible with zero





Simulation Results

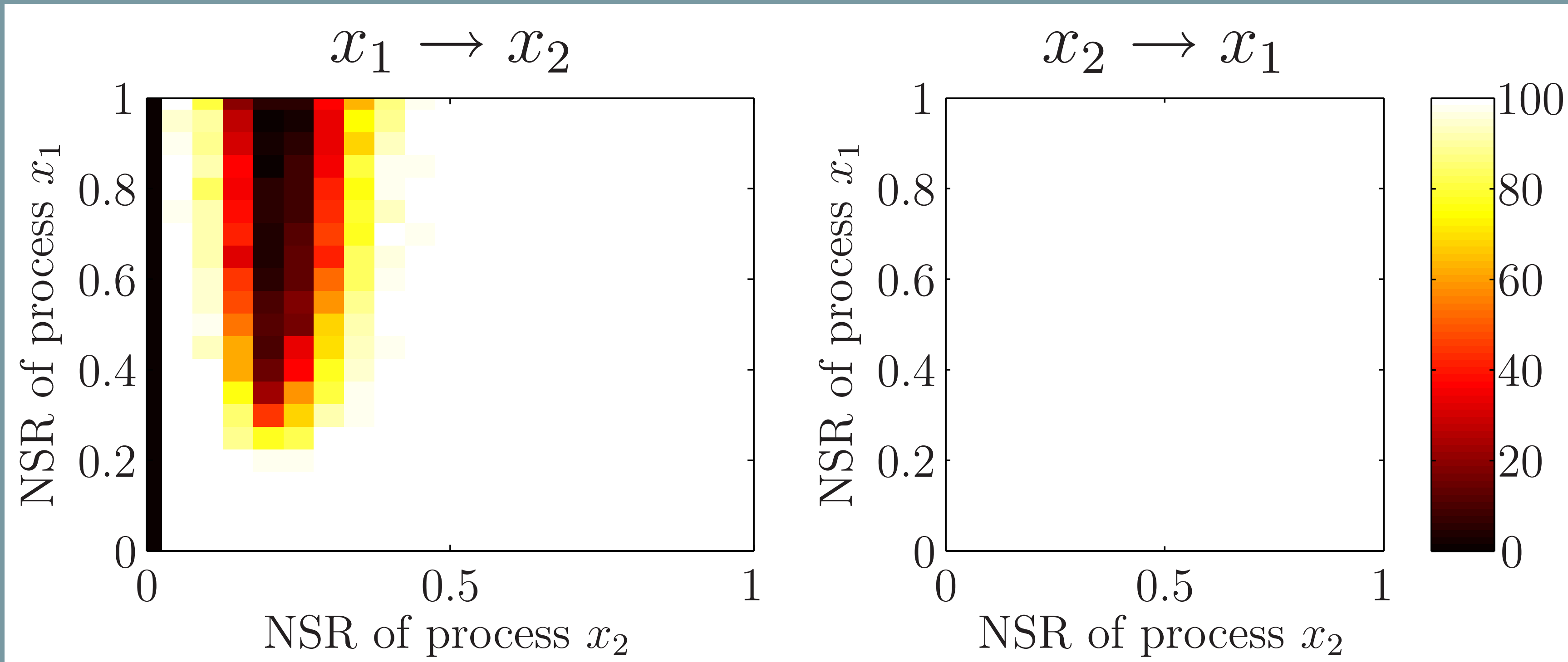
Increasing the number of data points



Simulation Results

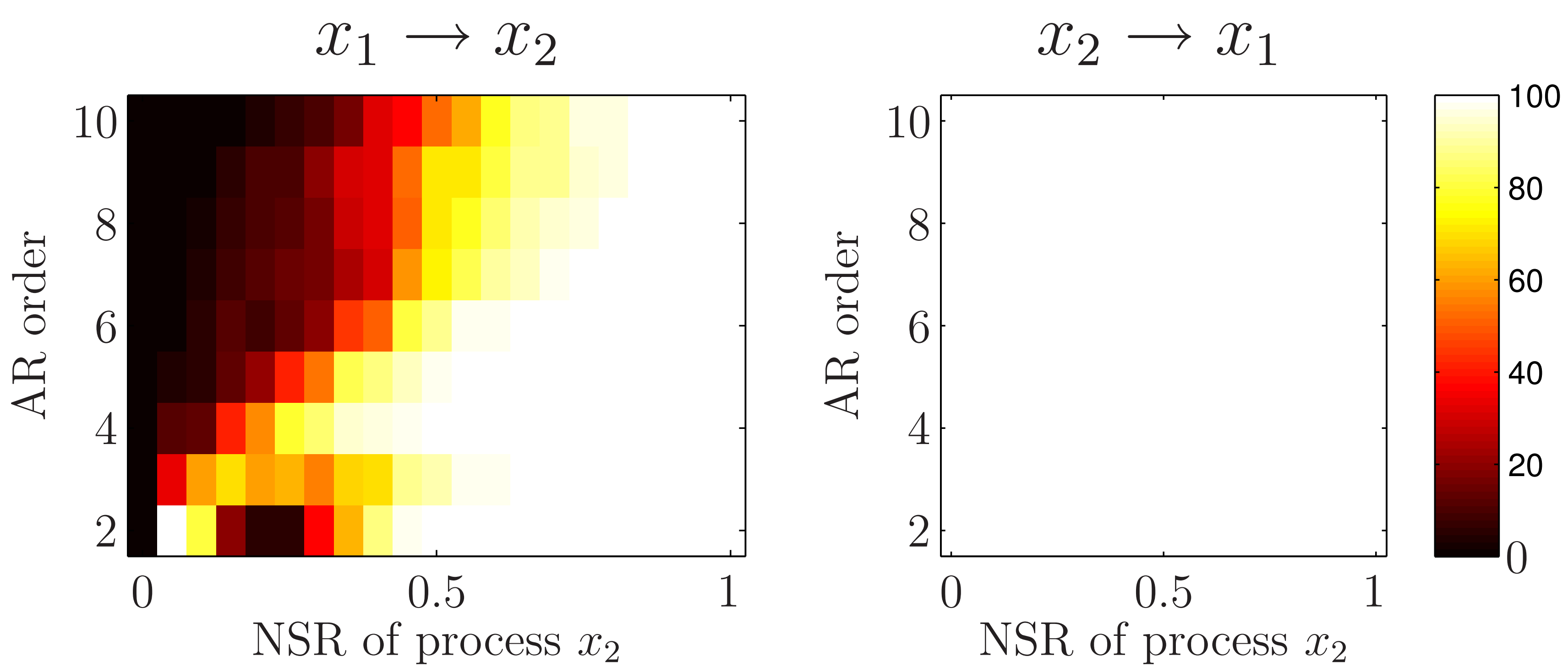
Fitted with order p = 2

$$x_2 \longrightarrow x_1$$



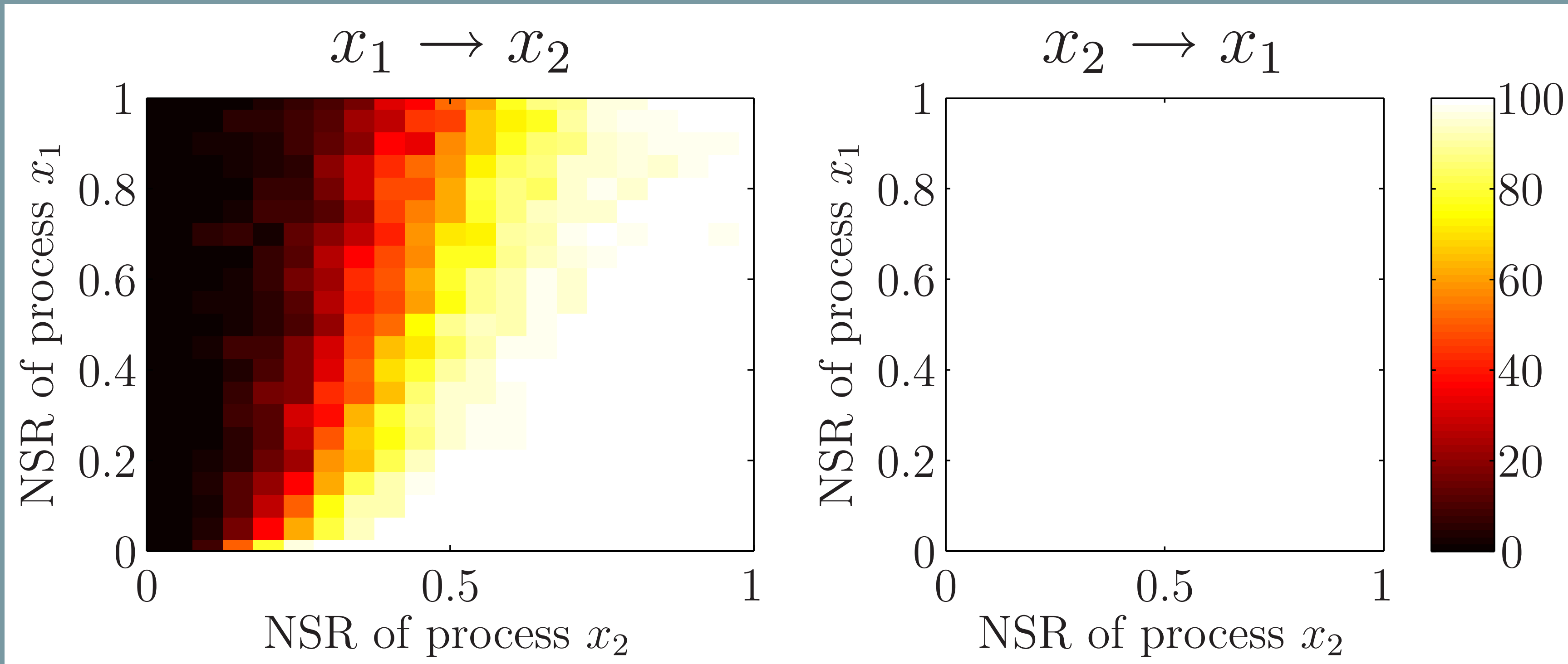
Simulation Results

Increasing the order p



Simulation Results

Fitted with order p = 10



State Space Model

$$\underbrace{\begin{pmatrix} \vec{x}(t) \\ \vec{x}(t-1) \\ \vdots \\ \vec{x}(t-p+1) \end{pmatrix}}_{\vec{u}(t)} = \underbrace{\begin{pmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \cdots & \mathbf{a}_p \\ \mathbf{I}_n & \mathbf{0}_n & \cdots & \mathbf{0}_n \\ \vdots & \ddots & \ddots & \vdots \\ \mathbf{0}_n & \cdots & \mathbf{I}_n & \mathbf{0}_n \end{pmatrix}}_{\mathbf{A}} \underbrace{\begin{pmatrix} \vec{x}(t-1) \\ \vec{x}(t-2) \\ \vdots \\ \vec{x}(t-p) \end{pmatrix}}_{\vec{u}(t-1)} + \underbrace{\begin{pmatrix} \vec{\varepsilon}_x(t) \\ \vec{0} \\ \vdots \\ \vec{0} \end{pmatrix}}_{\vec{\varepsilon}_u(t)}$$

$$\vec{y}(t) = \mathbf{C}_u \vec{u}(t) + \vec{\eta}(t)$$

Expectation-Maximisation (EM) algorithm

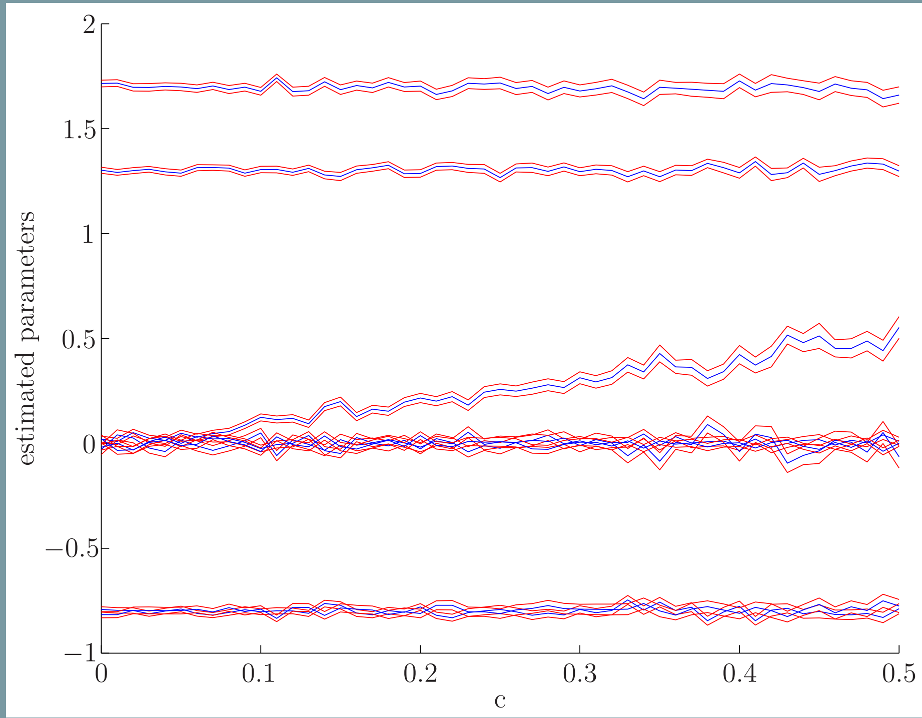
Covariance of Coefficients

- Derive second derivative of likelihood
- Arrange derivatives in matrix and invert
- Covariance Matrix

$$
\begin{aligned}
-\frac{\partial^2 \ln \mathrm{L}_{\vec{y}}(\Theta)}{\partial\Theta_i\partial\Theta_k} = & \frac{1}{2}\sum_{t=1}^{N}\Bigg(\mathrm{trace}(\Sigma(t,\Theta)^{-1}\frac{\partial^2\Sigma(t,\Theta)}{\partial\Theta_i\partial\Theta_k} \\
& - \Sigma(t,\Theta)^{-1}\frac{\partial\Sigma(t,\Theta)}{\partial\Theta_k}\Sigma(t,\Theta)^{-1}\frac{\partial\Sigma(t,\Theta)}{\partial\Theta_i})\Bigg) \\
& + \frac{1}{2}\sum_{t=1}^{N}\Bigg(\frac{\partial^2\epsilon(t,\Theta)}{\partial\Theta_i\partial\Theta_k}^T\Sigma(t,\Theta)^{-1}\epsilon(t,\Theta) \\
& - \frac{\partial\epsilon(t,\Theta)}{\partial\Theta_i}^T\Sigma(t,\Theta)^{-1}\frac{\partial\Sigma(t,\Theta)}{\partial\Theta_k}\Sigma(t,\Theta)^{-1}\epsilon(t,\Theta) \\
& + \frac{\partial\epsilon(t,\Theta)}{\partial\Theta_i}^T\Sigma(t,\Theta)^{-1}\frac{\partial\epsilon(t,\Theta)}{\partial\Theta_k} \\
& - \frac{\partial\epsilon(t,\Theta)}{\partial\Theta_k}^T\Sigma(t,\Theta)^{-1}\frac{\partial\Sigma(t,\Theta)}{\partial\Theta_i}\Sigma(t,\Theta)^{-1}\epsilon(t,\Theta) \\
& + \epsilon(t,\Theta)^T\Sigma(t,\Theta)^{-1}\frac{\partial\Sigma(t,\Theta)}{\partial\Theta_k}\Sigma(t,\Theta)^{-1}\frac{\partial\Sigma(t,\Theta)}{\partial\Theta_i}\Sigma(t,\Theta)^{-1}\epsilon(t,\Theta) \\
& - \epsilon(t,\Theta)^T\Sigma(t,\Theta)^{-1}\frac{\partial^2\Sigma(t,\Theta)}{\partial\Theta_i\partial\Theta_k}\Sigma(t,\Theta)^{-1}\epsilon(t,\Theta) \\
& + \epsilon(t,\Theta)^T\Sigma(t,\Theta)^{-1}\frac{\partial\Sigma(t,\Theta)}{\partial\Theta_i}\Sigma(t,\Theta)^{-1}\frac{\partial\Sigma(t,\Theta)}{\partial\Theta_k}\Sigma(t,\Theta)^{-1}\epsilon(t,\Theta) \\
& - \epsilon(t,\Theta)^T\Sigma(t,\Theta)^{-1}\frac{\partial\Sigma(t,\Theta)}{\partial\Theta_i}\Sigma(t,\Theta)^{-1}\frac{\partial\epsilon(t,\Theta)}{\partial\Theta_k} \\
& + \frac{\partial\epsilon(t,\Theta)}{\partial\Theta_k}^T\Sigma(t,\Theta)^{-1}\frac{\partial\epsilon(t,\Theta)}{\partial\Theta_i} \\
& - \epsilon(t,\Theta)^T\Sigma(t,\Theta)^{-1}\frac{\partial\Sigma(t,\Theta)}{\partial\Theta_k}\Sigma(t,\Theta)^{-1}\frac{\partial\epsilon(t,\Theta)}{\partial\Theta_i} \\
& + \epsilon(t,\Theta)^T\Sigma(t,\Theta)^{-1}\frac{\partial^2\epsilon(t,\Theta)}{\partial\Theta_i\partial\Theta_k}\Bigg)
\end{aligned}
$$

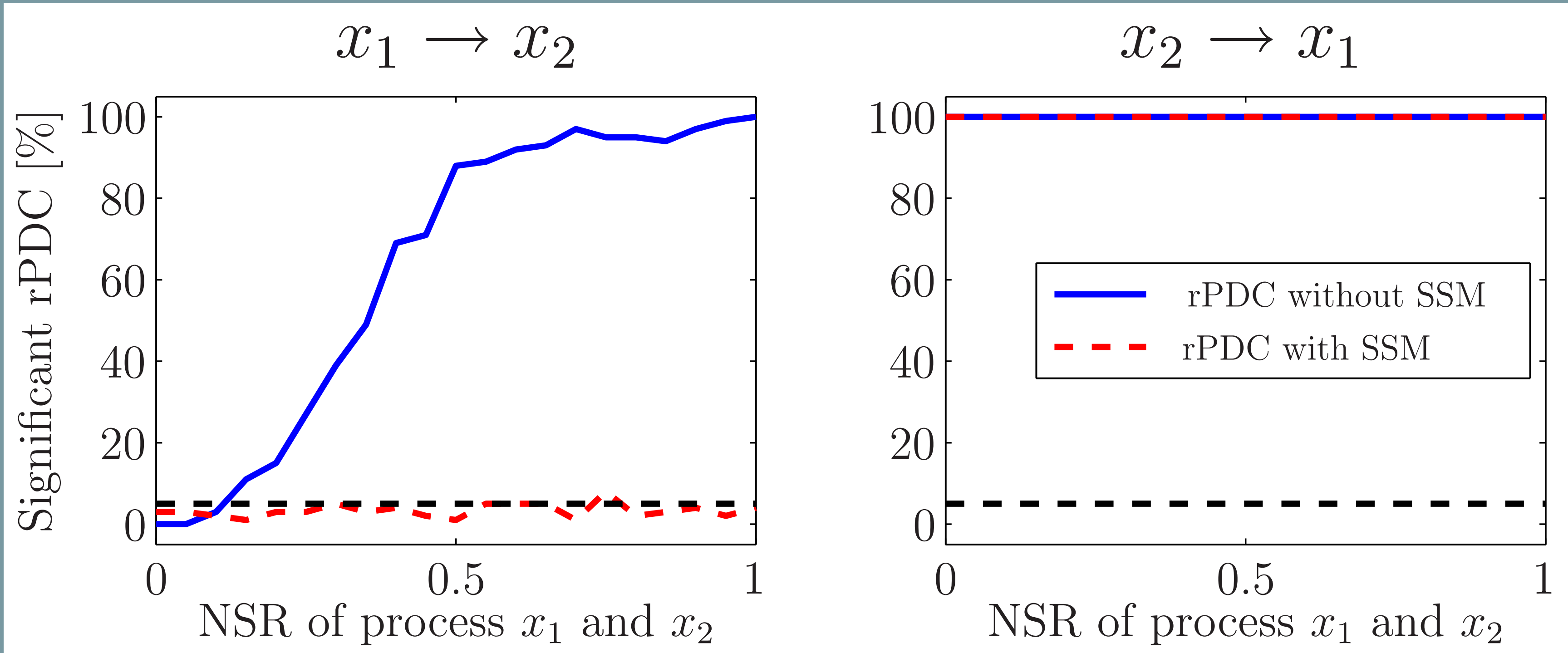
Simulation Results

Coefficients with Confidence



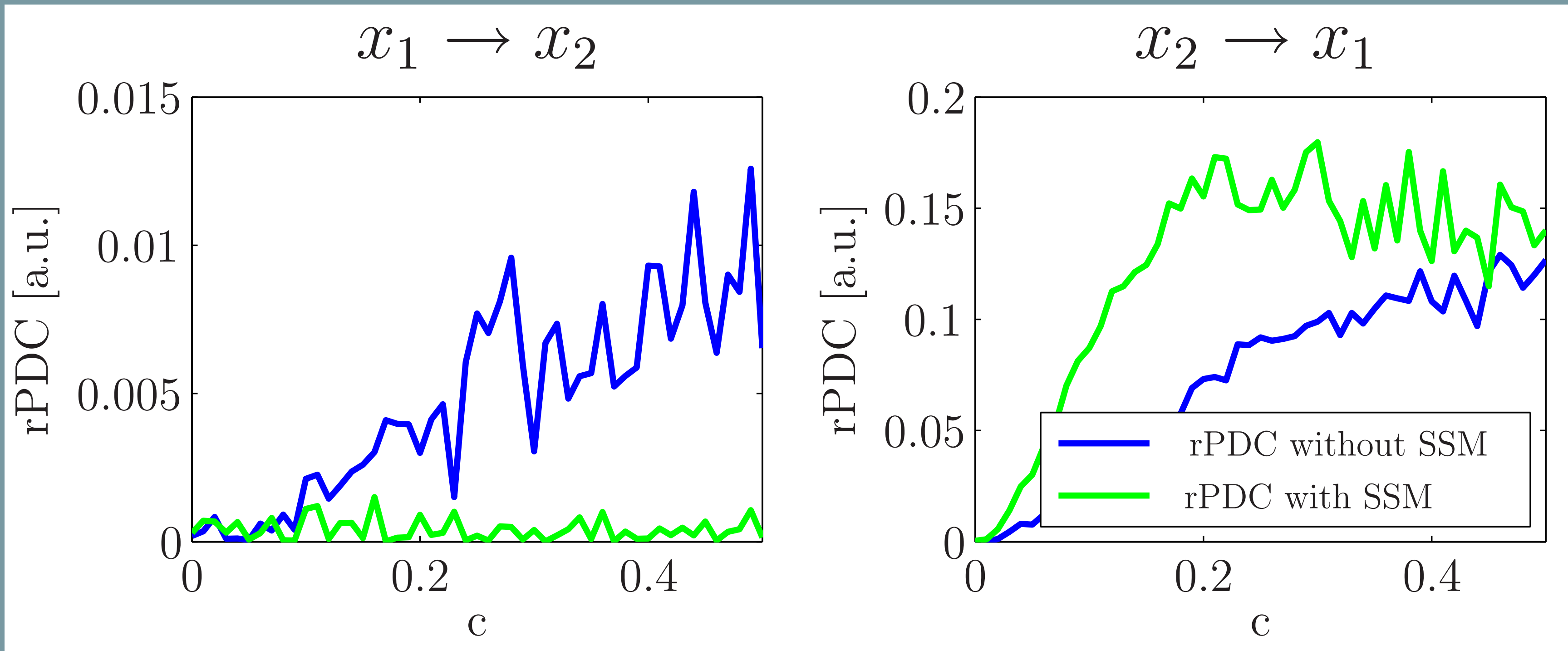
Simulation Results

With state space model



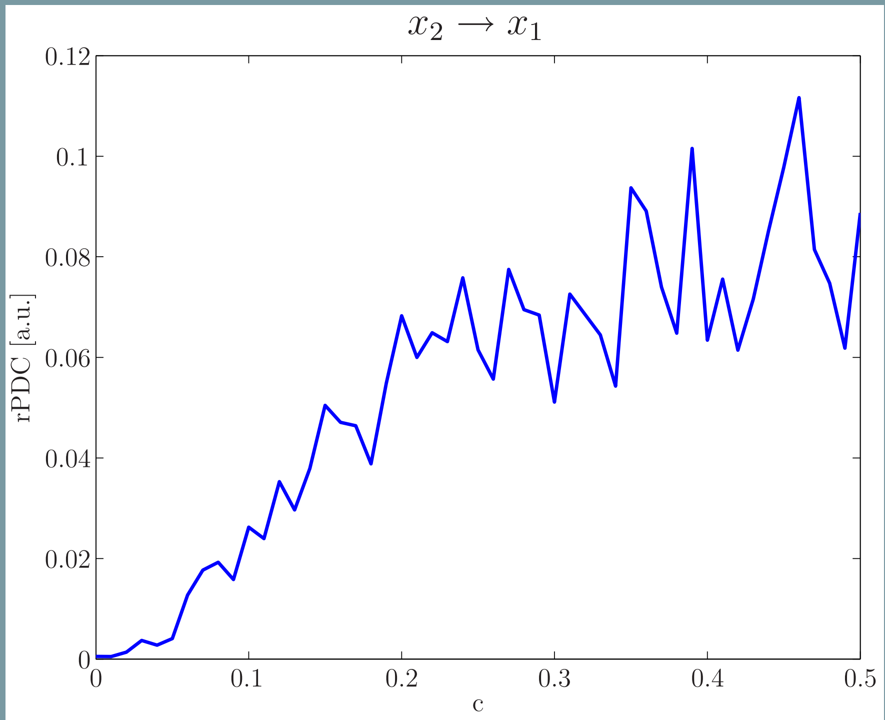
Simulation Results

Strength of the influence



Simulation Results

Strength of the influence



Application

- EEG from mice
- Right hippocampus and prefrontal cortex
- Quiet wake
- 100s segements

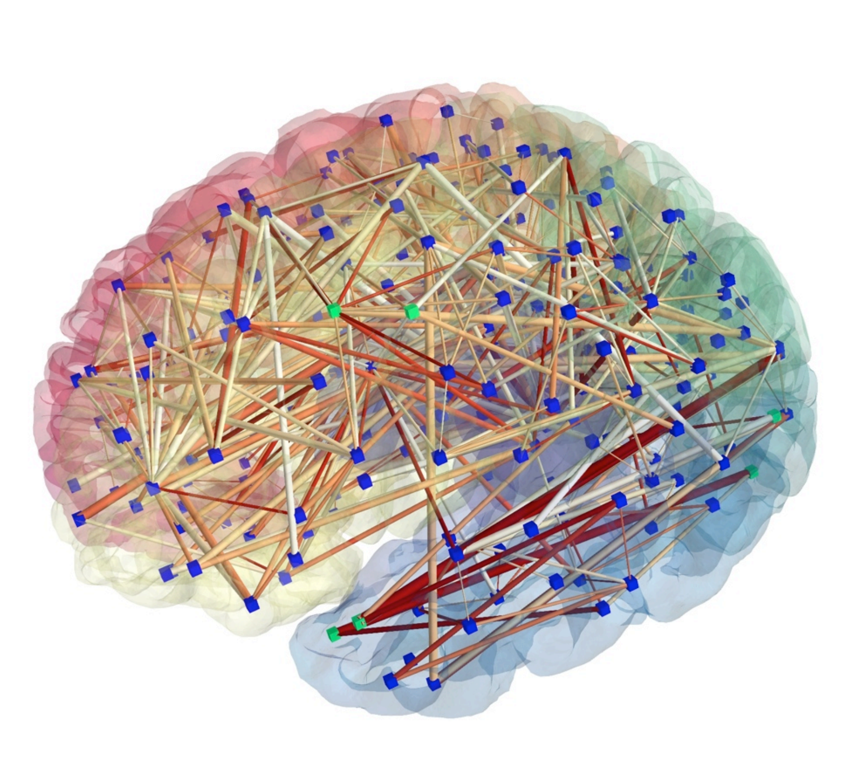
Application Results

Three mice

Two mice

Conclusion

- Networks from data
- Observational noise
- State space model
- Statistical inference



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Thank you for your attention!