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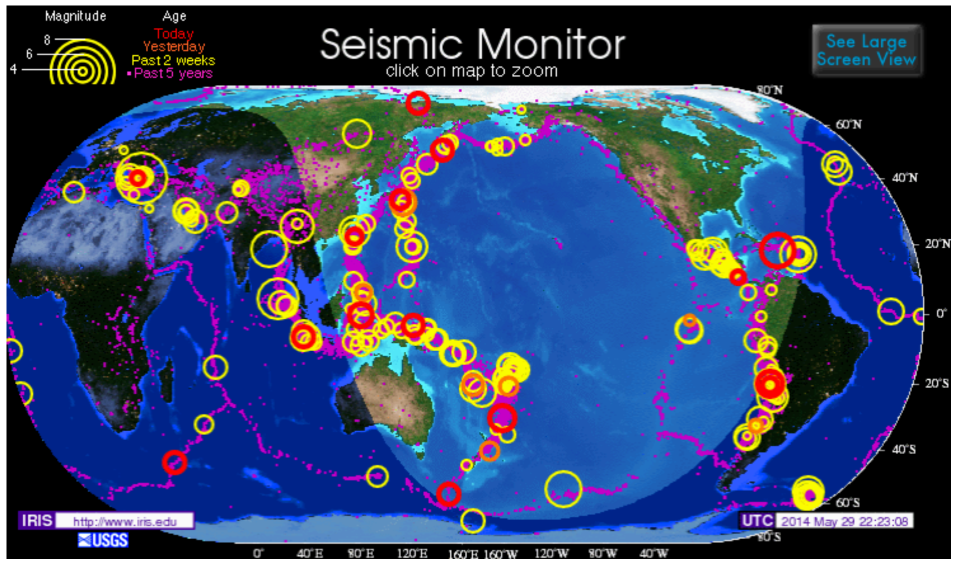
Insight into earthquake sequencing: A graph theoretic approach to modified Markov chain model

Kris Vasudevan and Michael S. Cavers
University of Calgary



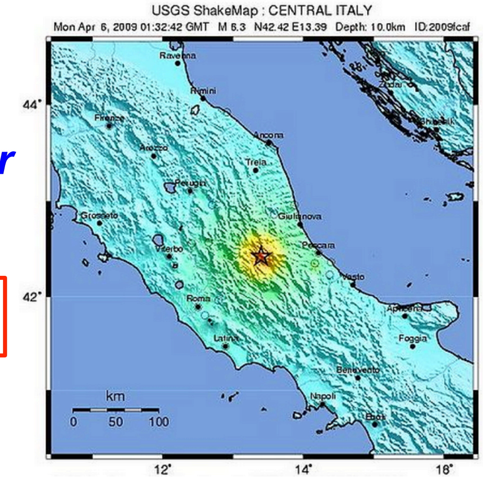
“Causality, Information Transfer and Dynamical Networks”

CIDNET14 seminar talk
Friday 13, June 2014

MAX PLANCK INSTITUTE FOR THE PHYSICS OF COMPLEX SYSTEMS
DRESDEN, GERMANY

Motivation

NATURE | NEWS | EDITORIAL



Italian court finds seismologists guilty of manslaughter

Six scientists and one official face six years in prison over L'Aquila earthquake.

Foreshocks

Nicola Nosengo

22 October 2012 Corrected: 23 October 2012, L'AQUILA, ITALY

"I'm not crazy. I know they can't predict earthquakes," the Italian public prosecutor Fabio Picuti told Nature last year. He was speaking as the manslaughter trial began in the ruined town of L'Aquila of six scientists and one government official for their alleged role in the deaths of 309 people in the quake of April 2009 (see Nature 477, 264–269; 2011). On Monday evening, the seven were found guilty and sentenced to six years in prison (see Nature http://doi.org/jkp; 2012). The verdict is perverse and the sentence ludicrous. Already some scientists have responded with warnings about the chilling effect on their ability to serve in public risk assessments.

Date	6 April 2009
Origin time	01:32:40.78 UTC[1]
Magnitude	6.3 M_w[2]
Depth	9.46 km (5.88 mi)[1]
Epicenter	42.3476°N 13.3800°E[1]
Countries or regions	Abruzzo, Italy
Total damage	$16 billion[3]
Casualties	297 dead[4] 1,500+ injured[5] 65,000+ homeless[4]

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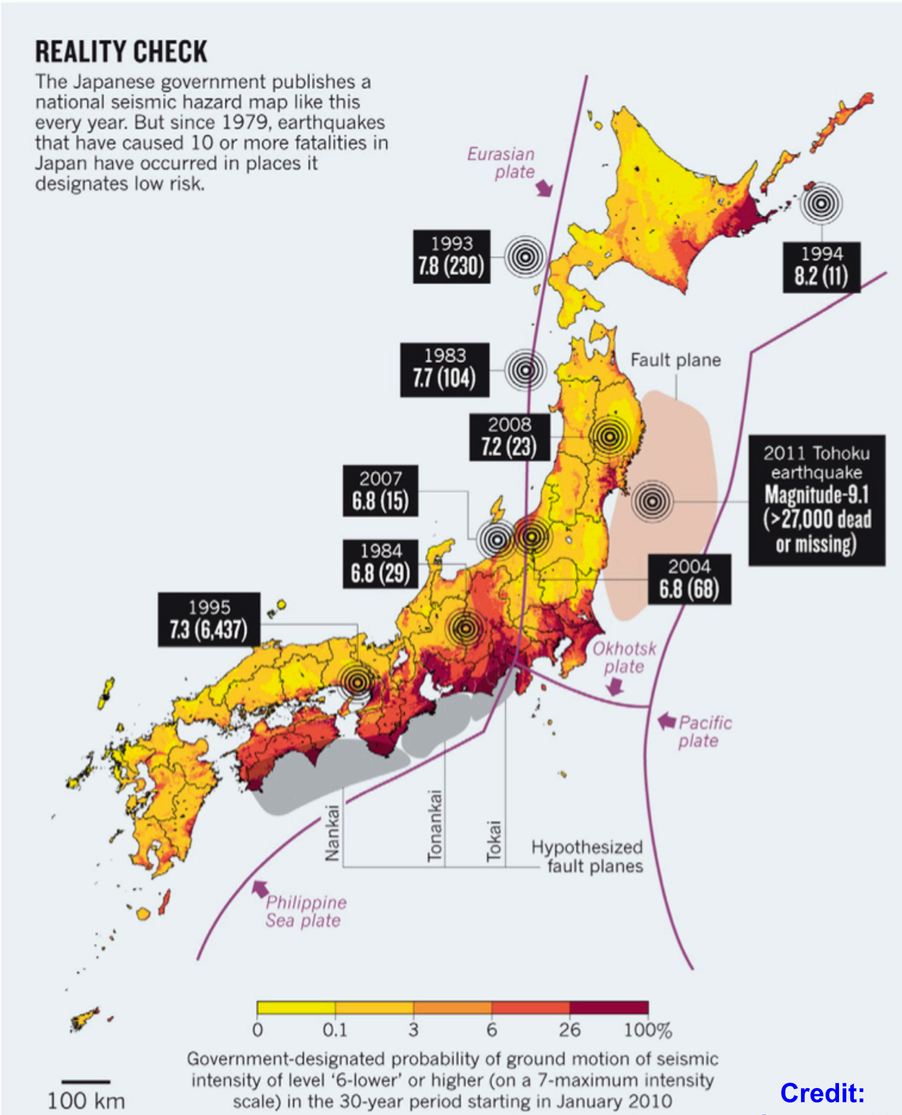
Comparison of Japanese government hazard map to the locations of earthquakes since 1979

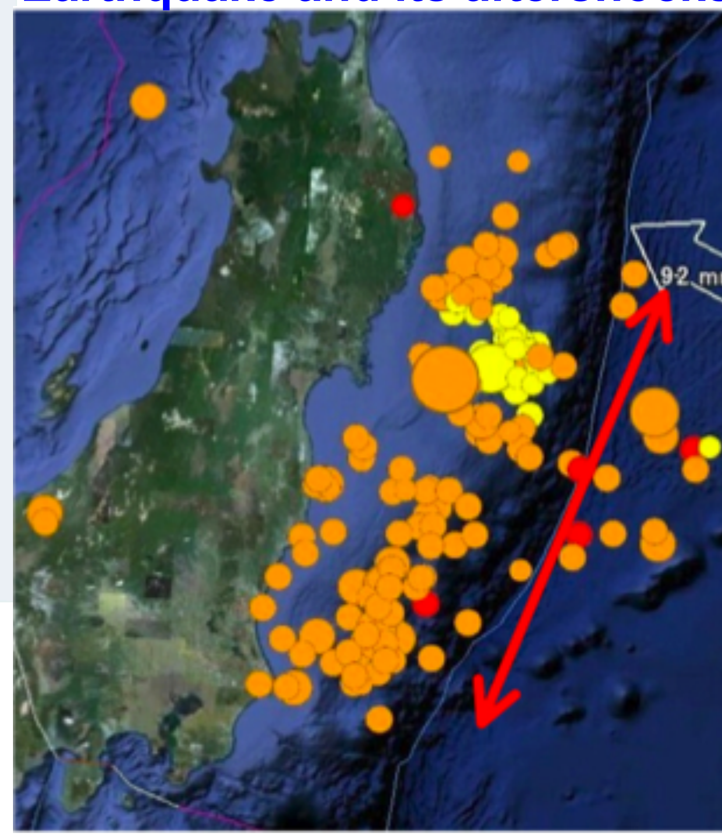
What can go wrong? Bad physics, bad assumptions, bad data and bad luck.



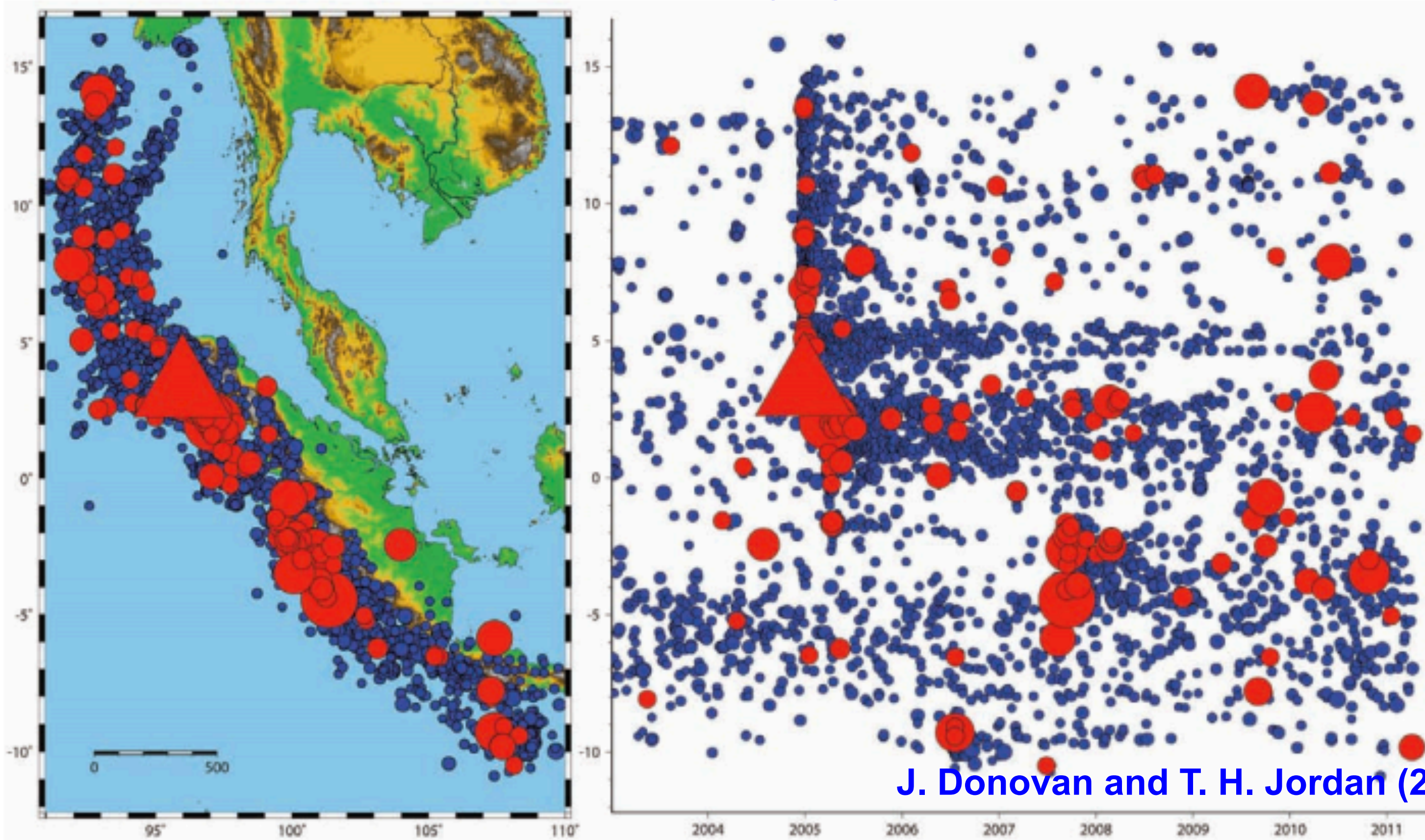
Credit: Geller,2011

Stein, S., Geller, R.J., and Liu, M., Tectonophysics, 562-563, 1-25 (2012)

March 11, 2011 Tohoku-Oki Earthquake and its aftershocks

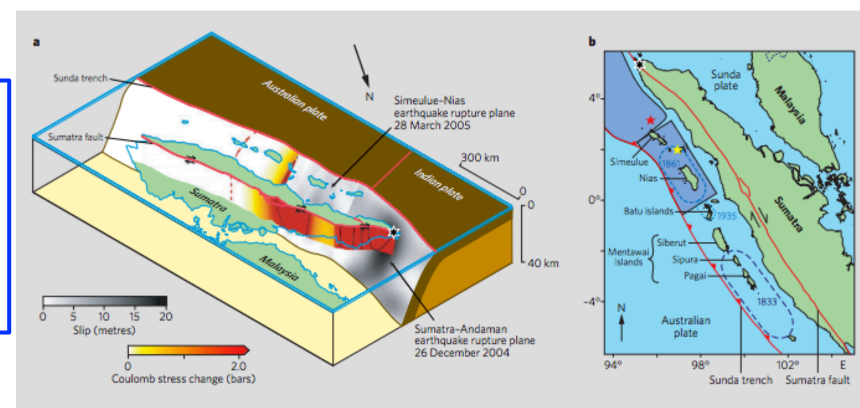


Sequence of earthquakes along the Indonesian subduction zone



J. Donovan and T. H. Jordan (2011)

McCloskey, J., Nalbant, S.S., And Steacy, S., Earthquake risk from co-seismic stress, Nature, 434, 291 (2005)



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Motivation

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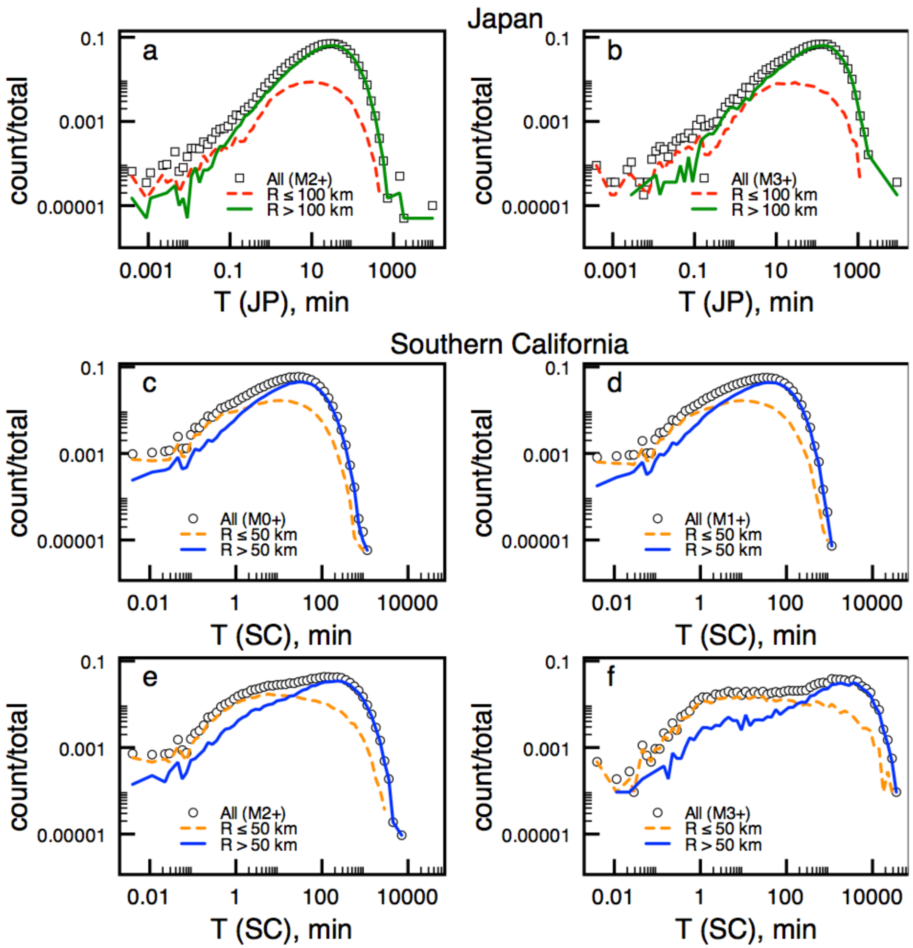
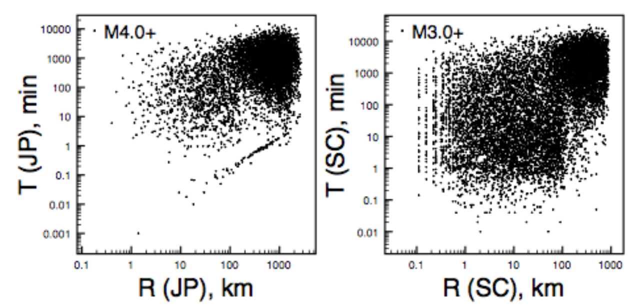
"Clustering of seismicity can be produced by various physical processes including, prominently, triggering of events over wide ranges of spatio-temporal scales. Triggering contributes to the complexity of earthquake patterns and is responsible for the effects of correlations and memory."
Ben-Zion, Y., Davidsen, J., and Shcherbakov, R., *Statistics and triggering of earthquakes*, BIRS 2013

Spatio-temporal clustering and separation in regional earthquakes



Conditional histograms (normalized with total number of events) $h(T|R\leq R^{*})$ (broken lines) and $h(T|R>R^{*})$ (solid lines) along with the histograms of all events $h(T)$ (symbols)



Batac and Kantz (2013)

Conceptualizing the forecasting problem

Motivation

- Forecasting: Essential ingredients
  - Plate-plate interactions
  - Earthquake cycle renewal
  - Earthquake clustering
  - Stress interaction among faults
  - Extreme events and the chaos theory
- Mathematical model
  - Simple and tractable
  - Should accommodate the static and dynamic components manifested by the processes
  - Easily expandable
- Our approach: Markov chain model expressed in a graph theoretic formalism that encompasses all aspects of the physical process.

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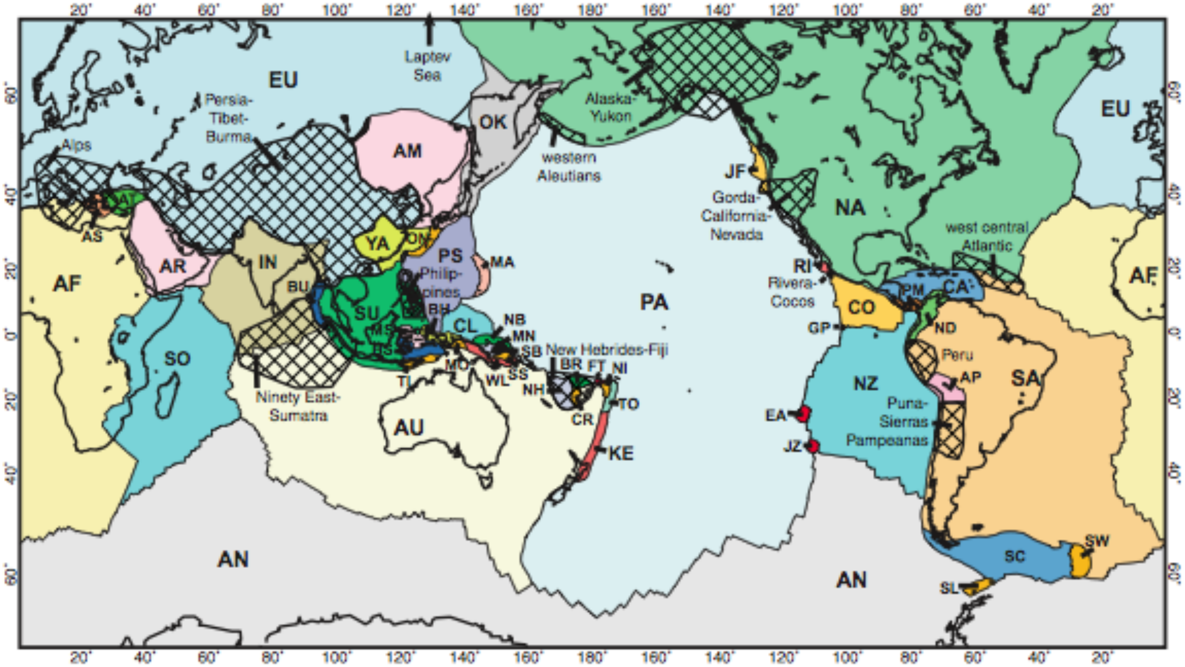
A graph theoretic approach to modified Markov chain model

Outline

- Methodology
  - Plate tectonics model
  - From earthquakes to directed graphs of Markov chain model
  - Weighted directed graphs
  - Determination of time interval
  - Network of recurrences
  - Visualization
- Analysis
  - Memoryless models
  - Graph properties
  - Comparison of limiting distributions
  - Probability of success
  - Time-series from the Markov chain model
  - Ensemble empirical mode decomposition
  - Fano factor, Allan factor
- Conclusions and questions

Methodology

Plate Tectonics Model



52-plate boundary zone definition

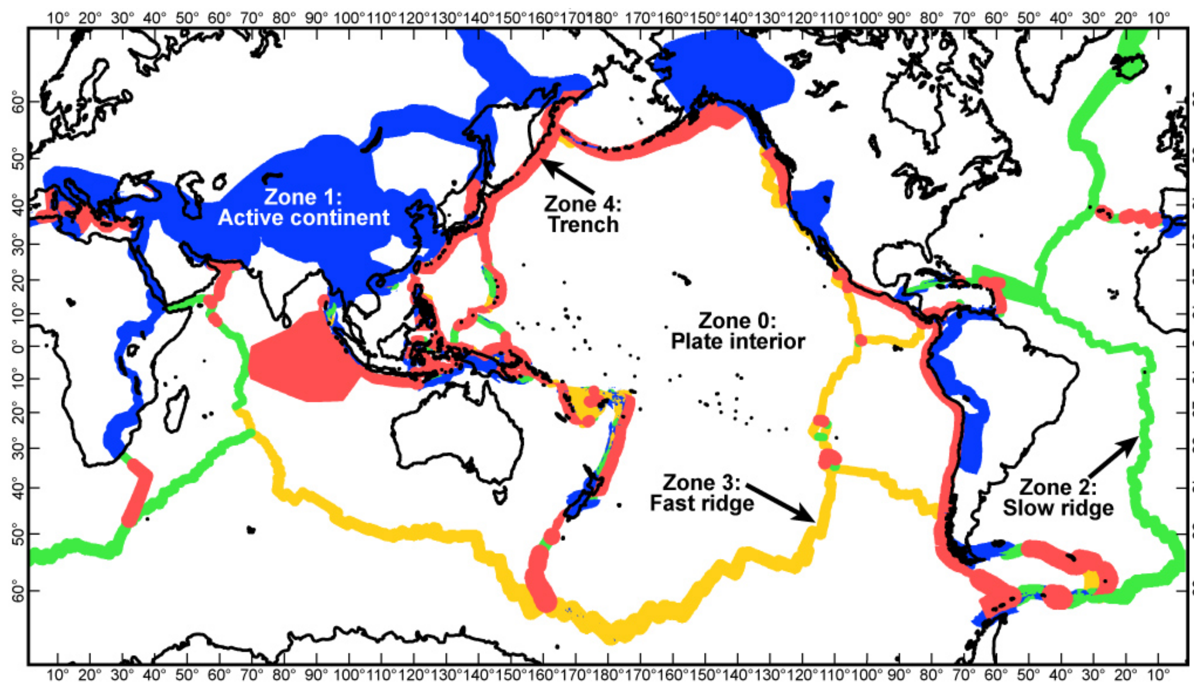
Bird(2003)

Definition of a "state"

Step 1

5-zone plate boundary template

Kagan, Bird and Jackson (2010)



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Methodology

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Questions to ask:

- What would be a simple mathematical representation of the earthquake sequencing? Markov chain
- Are there benefits in using the state-to-state transitions in the mathematical representation? Regional as well as global hazard analysis.

Global CMT catalogue for moment magnitude ≥ 5.6 and depth ≤ 70 km from 1982/01/01 to 2008/03/31

Zone	Tectonic Zone (Kagan, Bird and Jackson, 2010)	N	N/N_{total}
0	Plate-interior or the rest of the Earth's surface	237	0.0351
1	Active continent (including continental parts of all orogens and continental plate boundaries of PB2002)	898	0.1330
2	Slow-spreading ridges (oceanic crust, spreading rate < 40 mm/a; includes transforms)	487	0.0721
3	Fast-spreading ridges (oceanic crust, spreading rate > 40 mm/a; includes transforms)	723	0.1071
4	Trench that includes incipient subduction, and earthquakes in outer rise or upper plate	4407	0.6527
	Global (or N_{total})	6752	1.0000

Methodology

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Step 2

From earthquakes to directed graphs

Zones Z3	Z2	Z1	State
0	0	0	0
0	0	1	1
0	1	0	2
0	1	1	3
1	0	0	4
1	0	1	5
1	1	0	6
1	1	1	7

Cavers and Vasudevan (2014)

Example Earthquake Catalogue

EQ	Date	Magnitude	Zone
1	01-Jan-82	7.1	2
2	03-Jan-82	5.9	1
3	05-Jan-82	6.1	1
4	06-Jan-82	6.1	3
5	08-Jan-82	6	3
6	09-Jan-82	6.3	3
7	13-Jan-82	5.6	3
8	13-Jan-82	5.7	1
9	15-Jan-82	5.8	3
10	18-Jan-82	6.5	2
11	22-Jan-82	6	1
12	26-Jan-82	5.7	1
13	27-Jan-82	6.3	1
14	28-Jan-82	6.1	2
15	28-Jan-82	6.1	2
16	29-Jan-82	6	1
17	30-Jan-82	6.4	3

Hypothetical Example of a 3-zone Markov chain

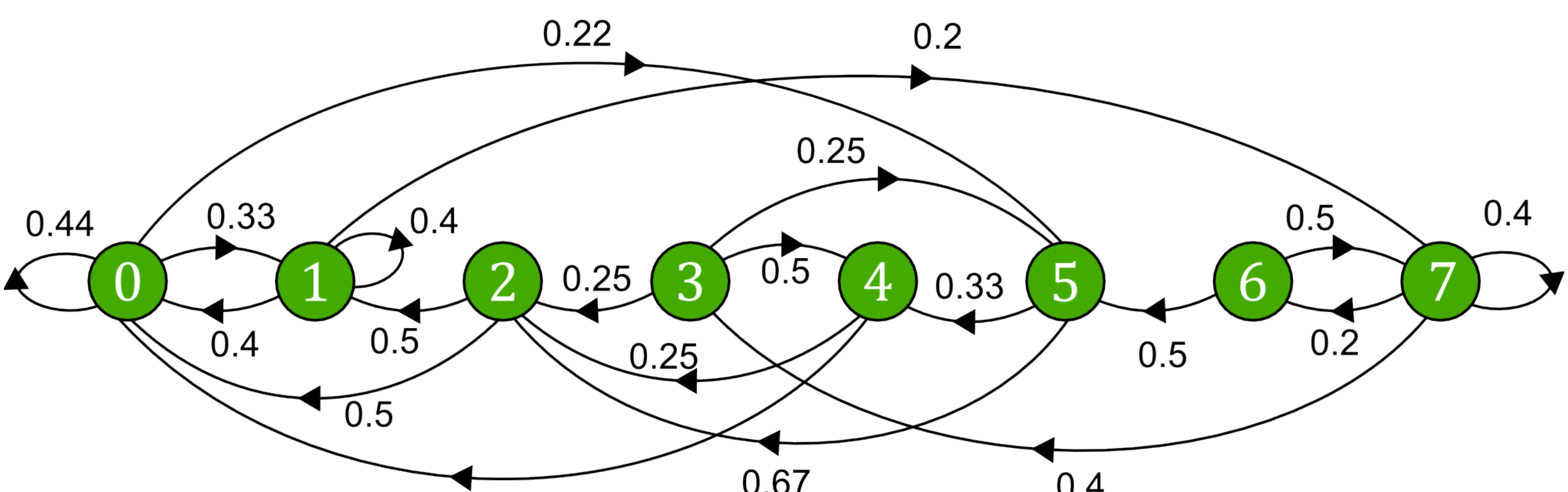
Methodology

Transition Frequency Matrix

	0	1	2	3	4	5	6	7
0	4	3	0	0	0	2	0	0
1	2	2	0	0	0	0	0	1
2	1	1	0	0	0	0	0	0
3	0	0	1	0	2	1	0	0
4	3	0	1	0	0	0	0	0
5	0	0	2	0	1	0	0	0
6	0	0	0	0	0	1	0	1
7	0	0	0	2	0	0	1	2

Probability Matrix

	0	1	2	3	4	5	6	7
0	0.44	0.33	0	0	0	0.22	0	0
1	0.40	0.40	0	0	0	0	0	0.20
2	0.50	0.50	0	0	0	0	0	0
3	0	0	0.25	0	0.50	0.25	0	0
4	0.75	0	0.25	0	0	0	0	0
5	0	0	0.67	0	0.33	0	0	0
6	0	0	0	0	0	0.50	0	0.50
7	0	0	0	0.40	0	0	0.20	0.40



Cavers and Vasudevan (2014)

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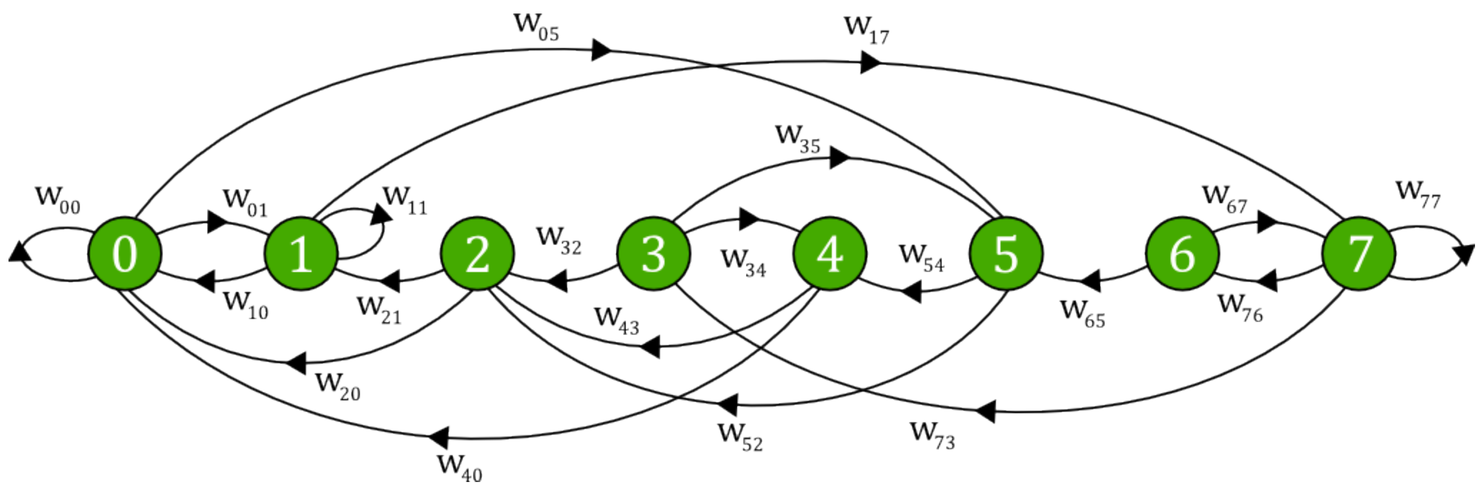
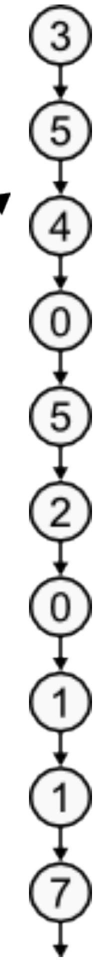
Step 3

Weighted directed graphs for earthquake sequences





Transition Frequencies

θ_{ij} is the number of occurrences from state *i* to state *j*

Transition Probabilities

$$p_{ij} = \Pr\{s(n+1) = j \mid s(n) = i\} = \Pr\{j \mid i\}$$

$$p_{ij} = \frac{\theta_{ij}}{\xi_i}, \text{ where } \xi_i = \sum_j \theta_{ij}$$

Cavers and Vasudevan (2014)

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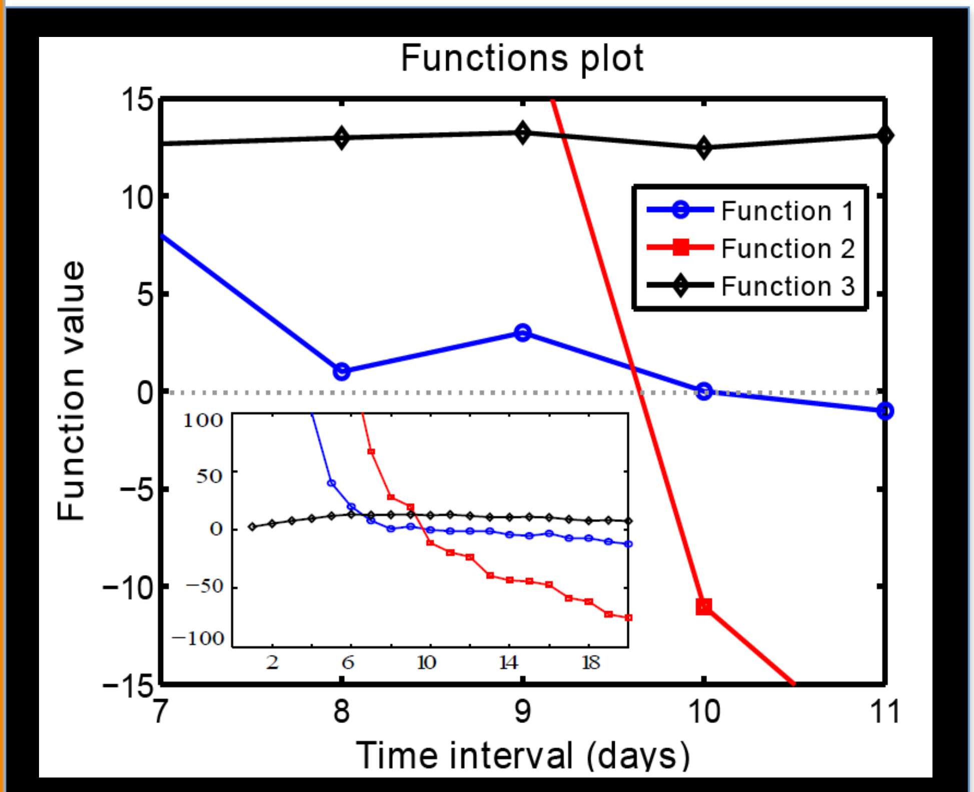
Determination of time interval

Step 4

Criteria:

•Δt should be small enough so that hazard estimations are useful.
•Δt should not be so small that state 0 to 0 transitions dominate.
•Δt should not be so large that state 31 to 31 transitions dominate.



Function 1: The difference between the number of transitions from state 0 to 0 and the number of transitions from state 31 to 31.

Function 2: The difference between the total number of transitions from state 0 and the total number of transitions from state 31.

Function 3: The entropy function given in the equation as based on the maximum entropy principle (*Jaynes*, 2003).

$$F_3(S) = -\sum_i \pi_i \sum_j p_{ij} \log_2 p_{ij}$$

Choose: Δt = 9

Transition probability matrix: $P = [p_{ij}]$

Stationary distribution: $\pi = [\pi_i]$

Cavers and Vasudevan (2014)

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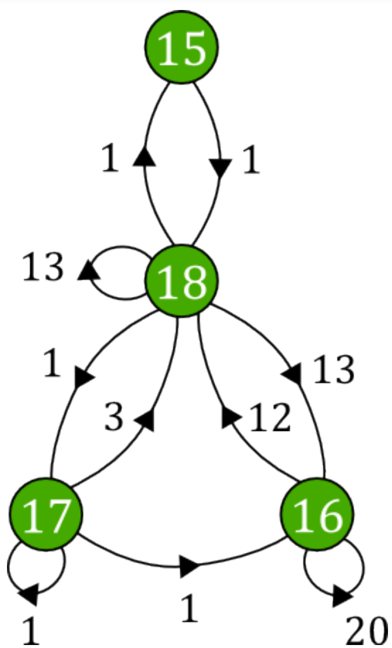
EQ	Time	Latitude	Longitude	Magnitude	Zone
1	723912.79	26.84	142.7	6.64	4
2	723914.59	-0.96	-21.83	7.16	2
3	723916.57	-0.49	-21.57	5.83	2
4	723918.36	-3.34	177.5	5.93	0
5	723922.26	13.91	124.35	7.13	4
6	723923.07	-52.37	28.5	6.26	2
7	723923.24	12.8	-87.3	6.17	4
8	723924	-9.83	152.47	6.16	4
9	723925.41	-9.26	151.8	6.13	4
10	723926.58	-9.69	152.45	6.11	4
11	723929.81	39.56	24.47	6.62	1
12	723931.18	6.5	93.78	6.31	4
13	723931.3	6.81	93.71	6.19	4
14	723932.91	19.22	-155.6	5.72	0
15	723932.94	19.2	-155.57	5.63	0
16	723934.59	23.84	121.65	6.08	4
17	723934.73	31.71	82.24	6.36	1
18	723935.26	14.07	124.53	6.68	4
19	723940.94	25.5	-45.29	5.94	2
20	723941.11	16.71	-61.47	6.11	0



In earthquake sequencing, certain unexpected long-range behaviour enters into forecasting problem. Long-range behaviour is an area of intense debate in seismology circles.

All earthquakes have recurrences. What role they play in state-to-state transitions is a topic of current interest.

Cavers and Vasudevan (2014)

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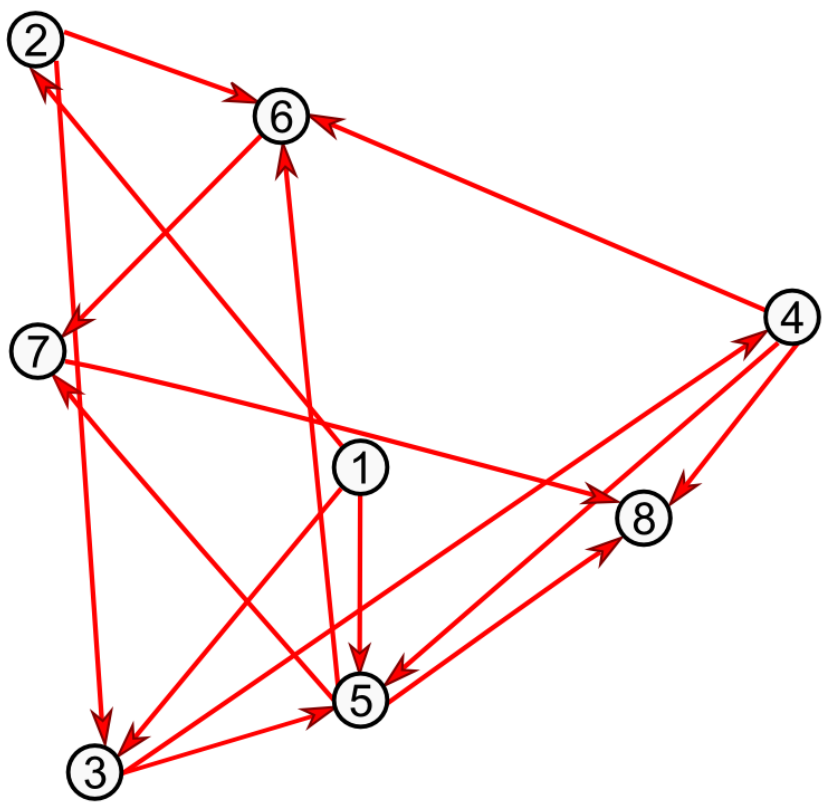
Network of recurrences

Step 5

Given:
- Series of events $a_1, a_2, \ldots, a_N$ ordered by time
- Metric d_{ij} is spatial distance between events a_i and a_j

An event Q is a **recurrence** of a previous event P if it is closer to P than any other event in the time interval between P and Q.



A recurrence is a new record ("**record breaking event**") with respect to distance.

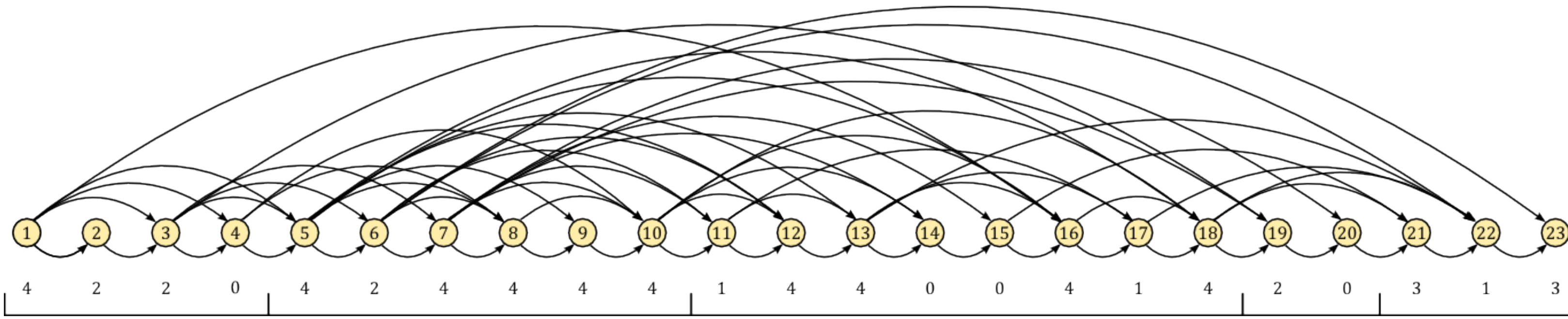
- Nodes: a_i
- Arcs: $a_i \rightarrow a_j$ if and only if
  1) $i < j$ and
  2) a_j is a recurrence of a_i

Davidsen, Grassberger and Paczuski (2008)

Cavers and Vasudevan (2014)

Network of recurrences

Step 6



$L_{jk}(r)$ denotes the number of record breaking events from zone j to zone k at distance at most r.

Each recurrence from an earthquake "*a*" to an earthquake "*b*" of distance *r* > 50 km is assigned a weight W_{ab} between 0 and 1:

$$W_{ab} = \frac{L_{jk}(20000) - L_{jk}(r)}{L_{jk}(20000) - L_{jk}(50)}$$

A Markov chain with the inclusion of spatio-temporal complexity (SCMC) of recurring events is derived by summing the weights of the recurrence arcs corresponding to occurrences from state i to state j in consecutive time intervals.

Cavers and Vasudevan (2014)

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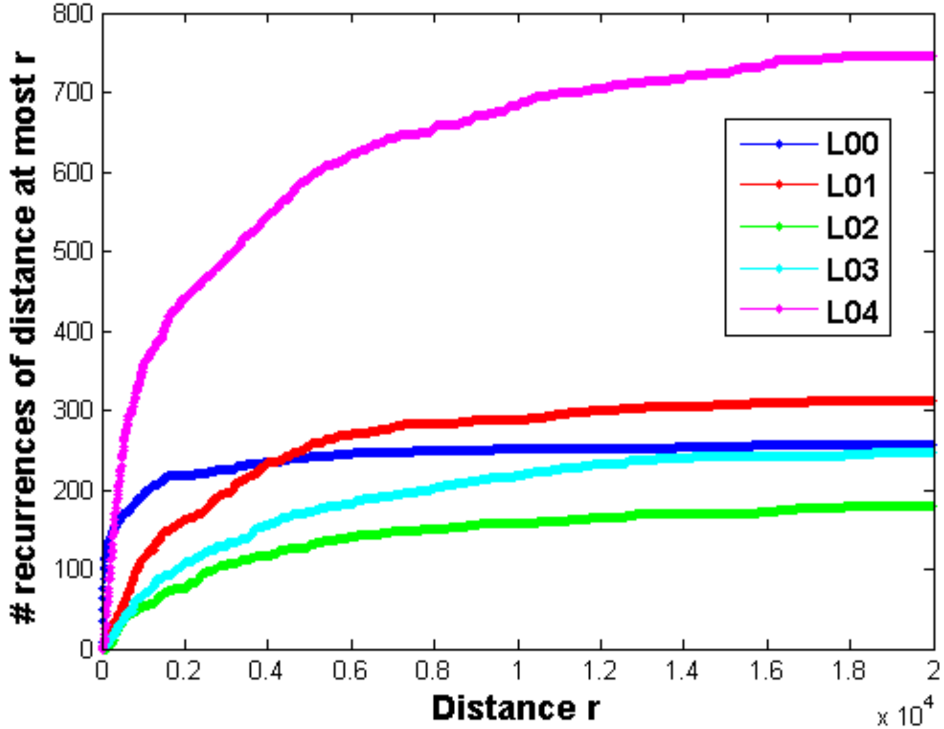
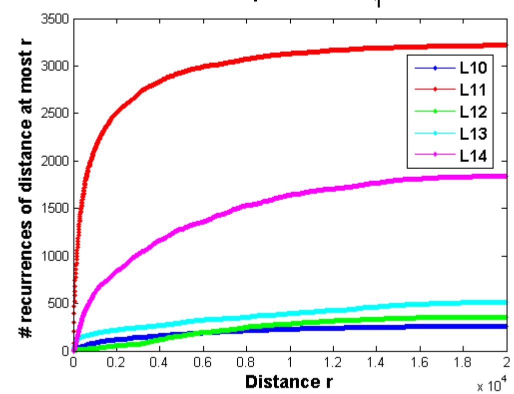
Plot of number of recurrences from zone *j* to zone *k* (*j*,*k*=0,1,2,3,4) vs *r*

Lookup table for L_{0*}

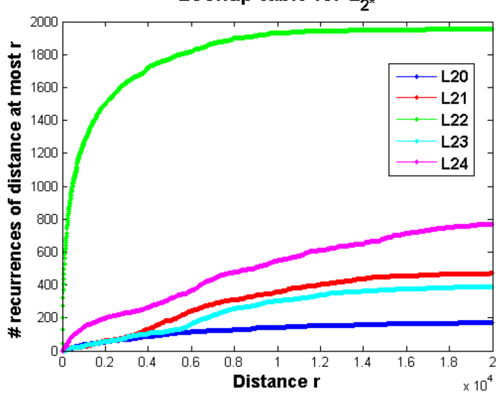


$L_{jk}(r)$ = # of record breaking events from zone j to k at distance at most r

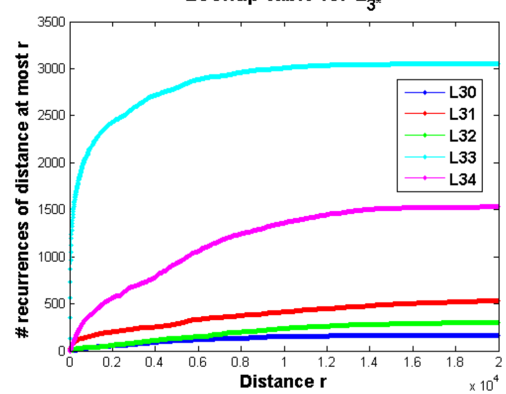
Lookup table for L_{1*}



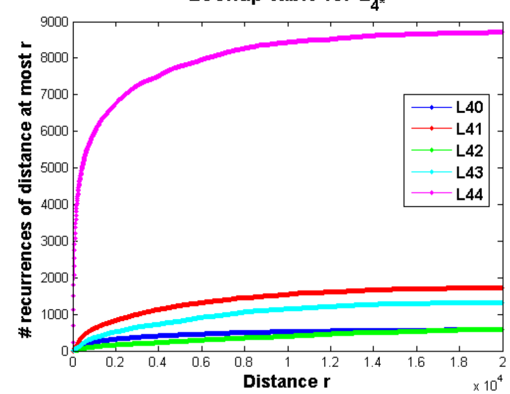
Lookup table for L_{2*}



Lookup table for L_{3*}



Lookup table for L_{4*}



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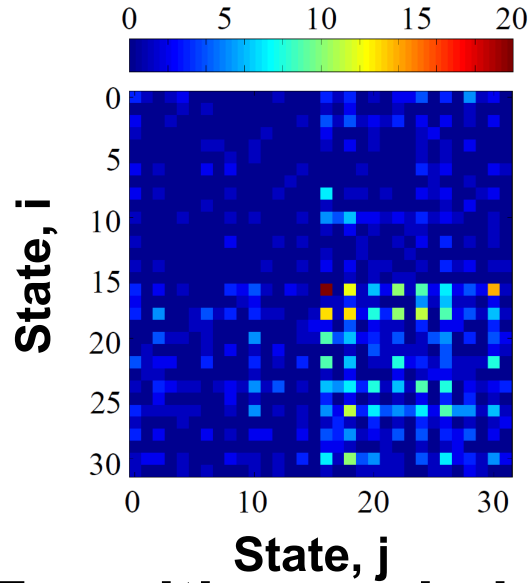
Methodology

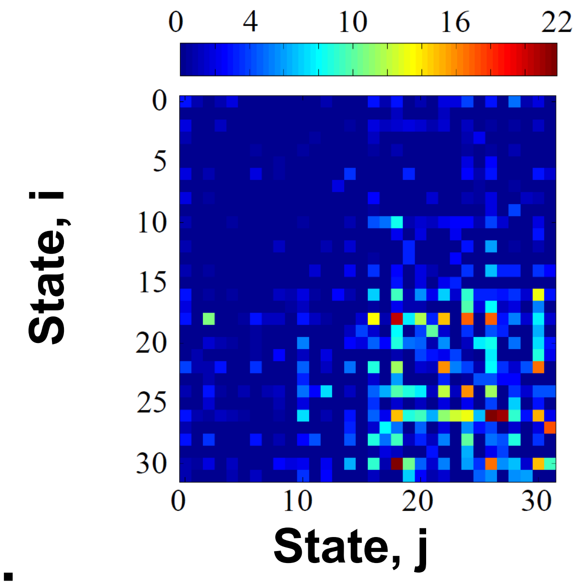
Transition-frequency matrix

Step 7

Without recurrences

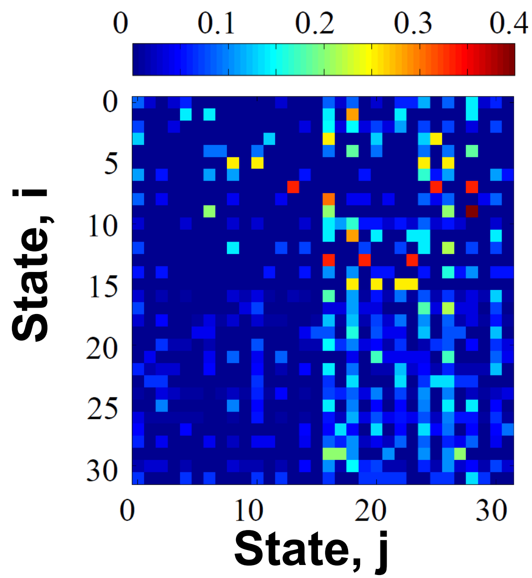


With recurrences

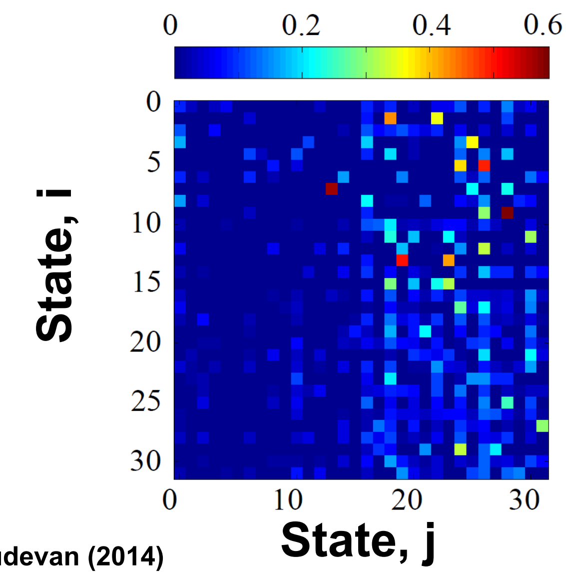


Transition-probability matrix

Without recurrences



With recurrences



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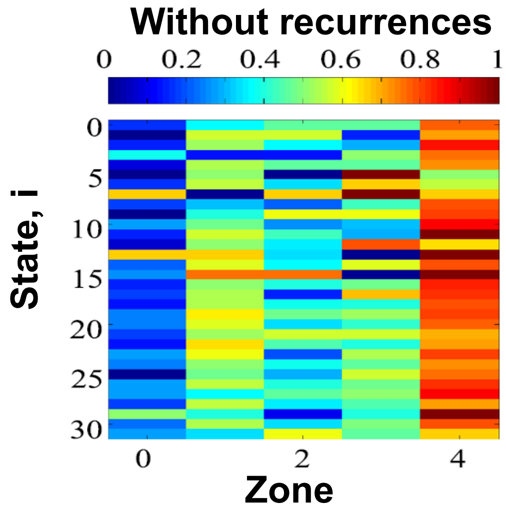
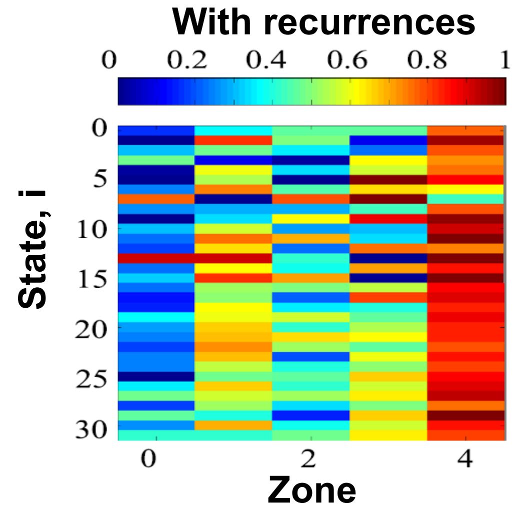
$$p_{iL} = \Pr\{L|i\} = \sum_{j \supset L} p_{ij}$$

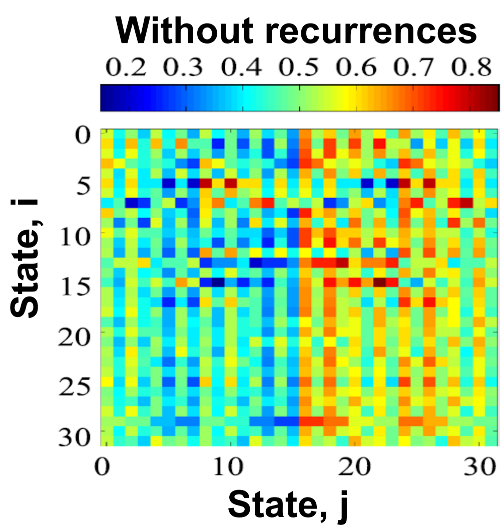
Conditional probabilities of earthquake occurrence in region *L*, given that the system is in state *i*.

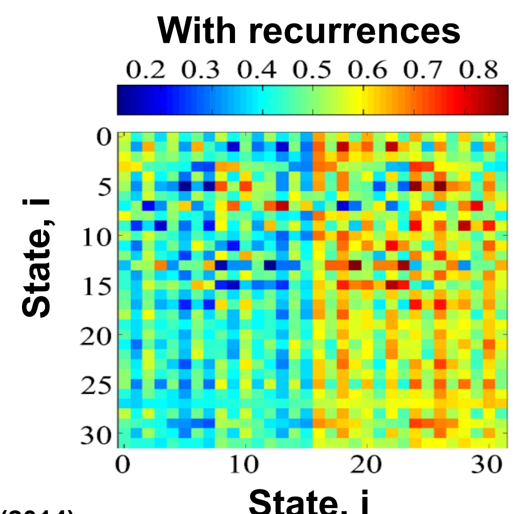




$$q_{ij} = \frac{1}{R}\left[\sum_{L \subset j} p_{iL} + \sum_{L \not\subset j} (1 - p_{iL})\right]$$

Region *L* features earthquakes for event *j* or does not feature *j*. *R* refers to the number of interrelated regions.





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Analysis

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Memoryless models: Transition probabilities

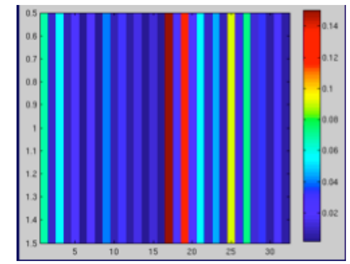
Uniform or average probability model

$$p_{ij}^U \equiv p^U = (S+1)^{-1} = \langle p_{ij} \rangle$$

$$p_{ij}^U = 0.0303$$

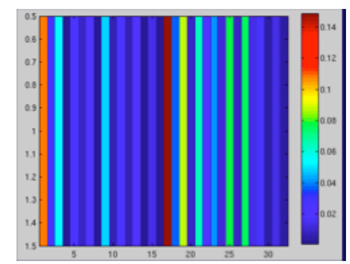
Poisson model

$$p_{ij}^P \equiv p_j^P = \prod_{L \subset j} (1 - e^{-\lambda_L \Delta t}) \prod_{L \not\subset j} e^{-\lambda_L \Delta t}$$



Markovian fixed model

$$p_{ij} \equiv p_j^0 = \frac{\xi_j}{\sum \xi_j}$$



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Analysis

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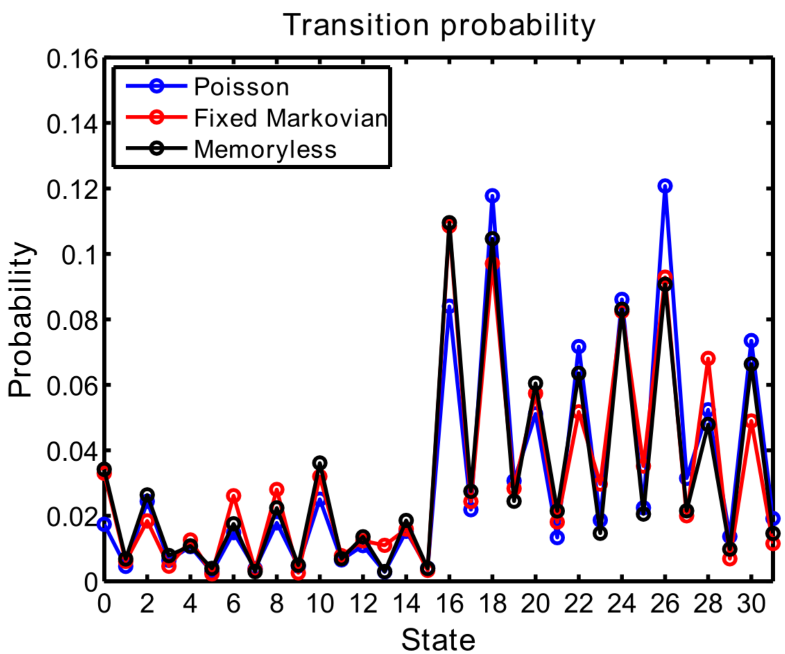
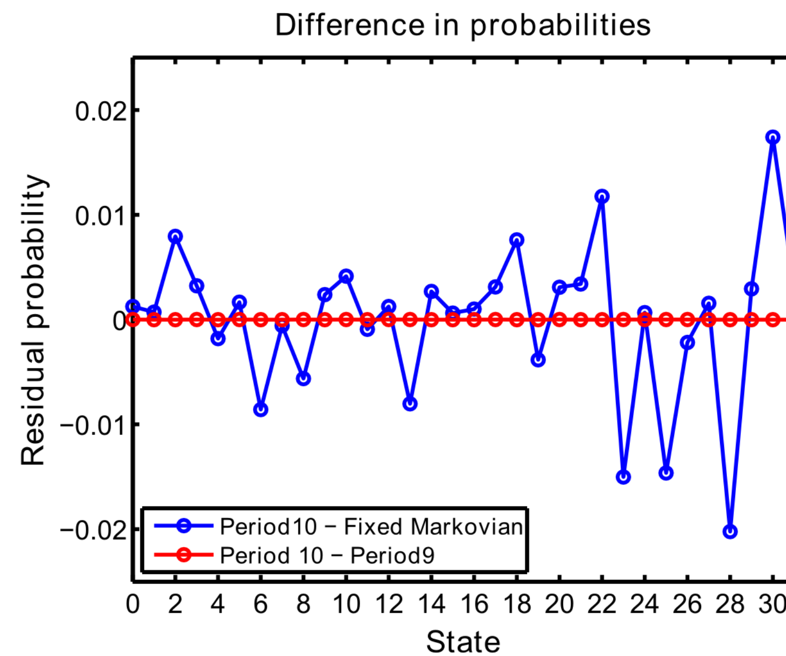
Transition probability associated with 32-states



(a)

A comparison of two residual probability plots



(b)

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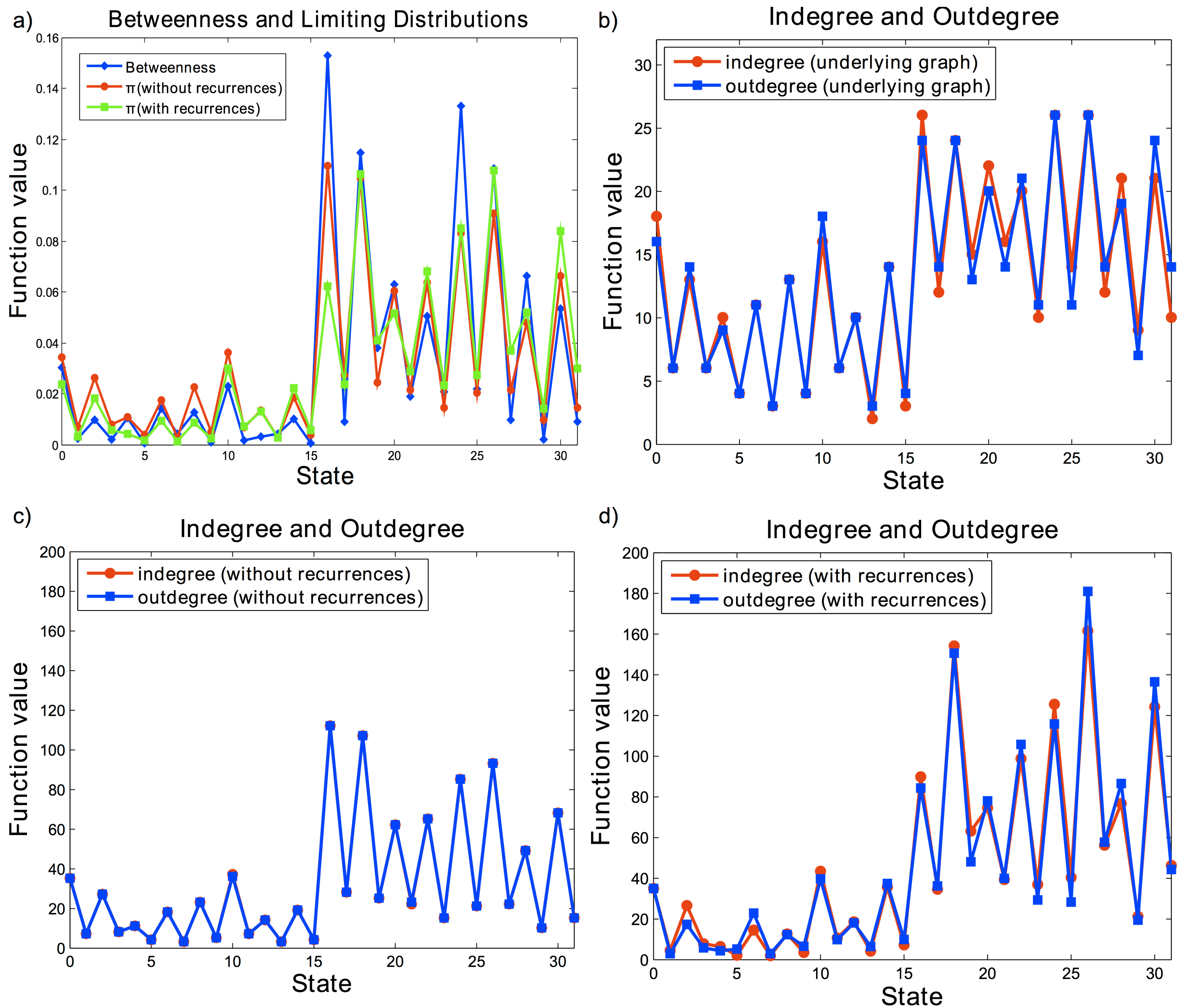
Traditional digraph properties



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Analysis

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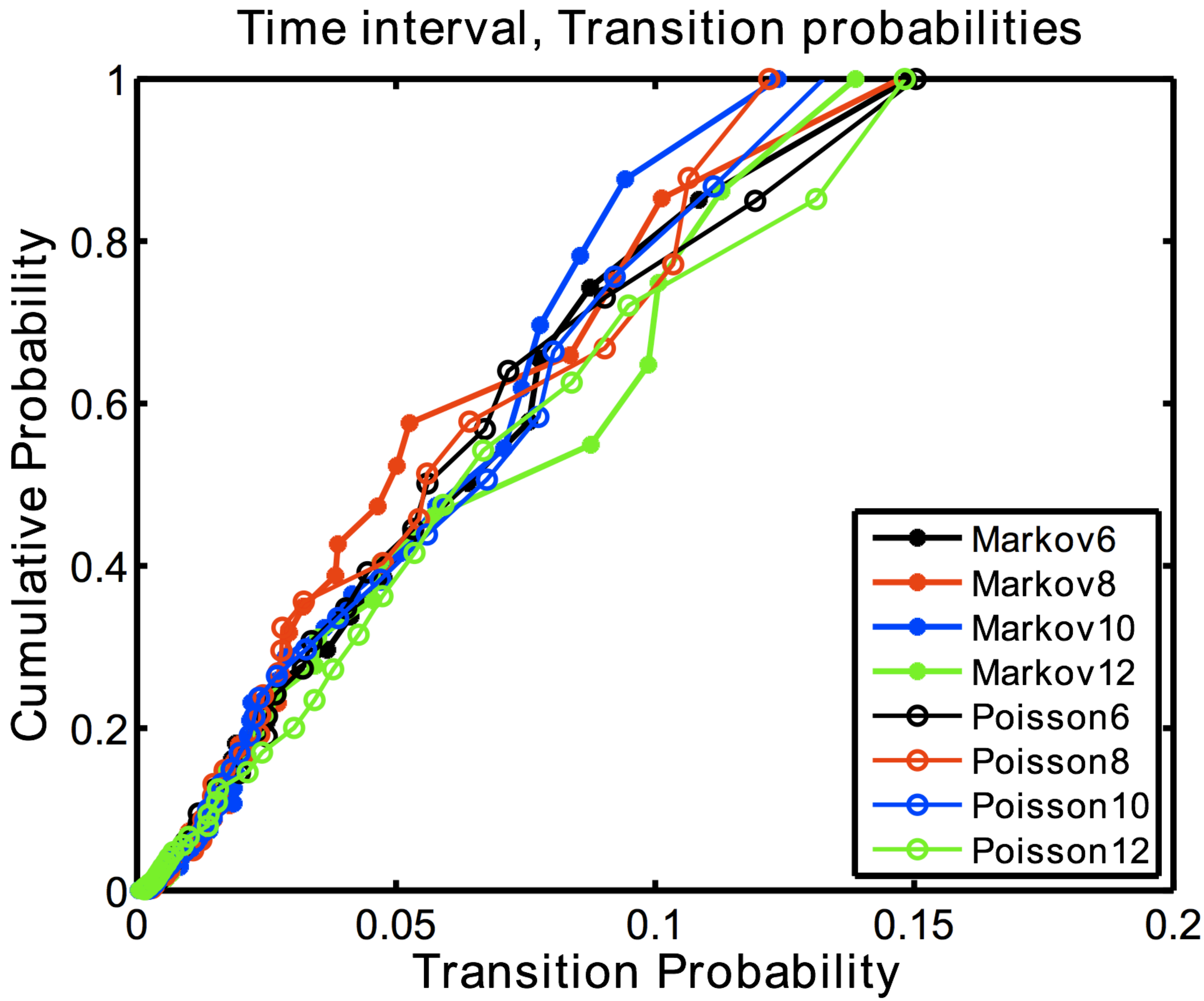
Analysis

A comparison of the Poisson model and the Markov chain model for different Δt values (without recurrences)



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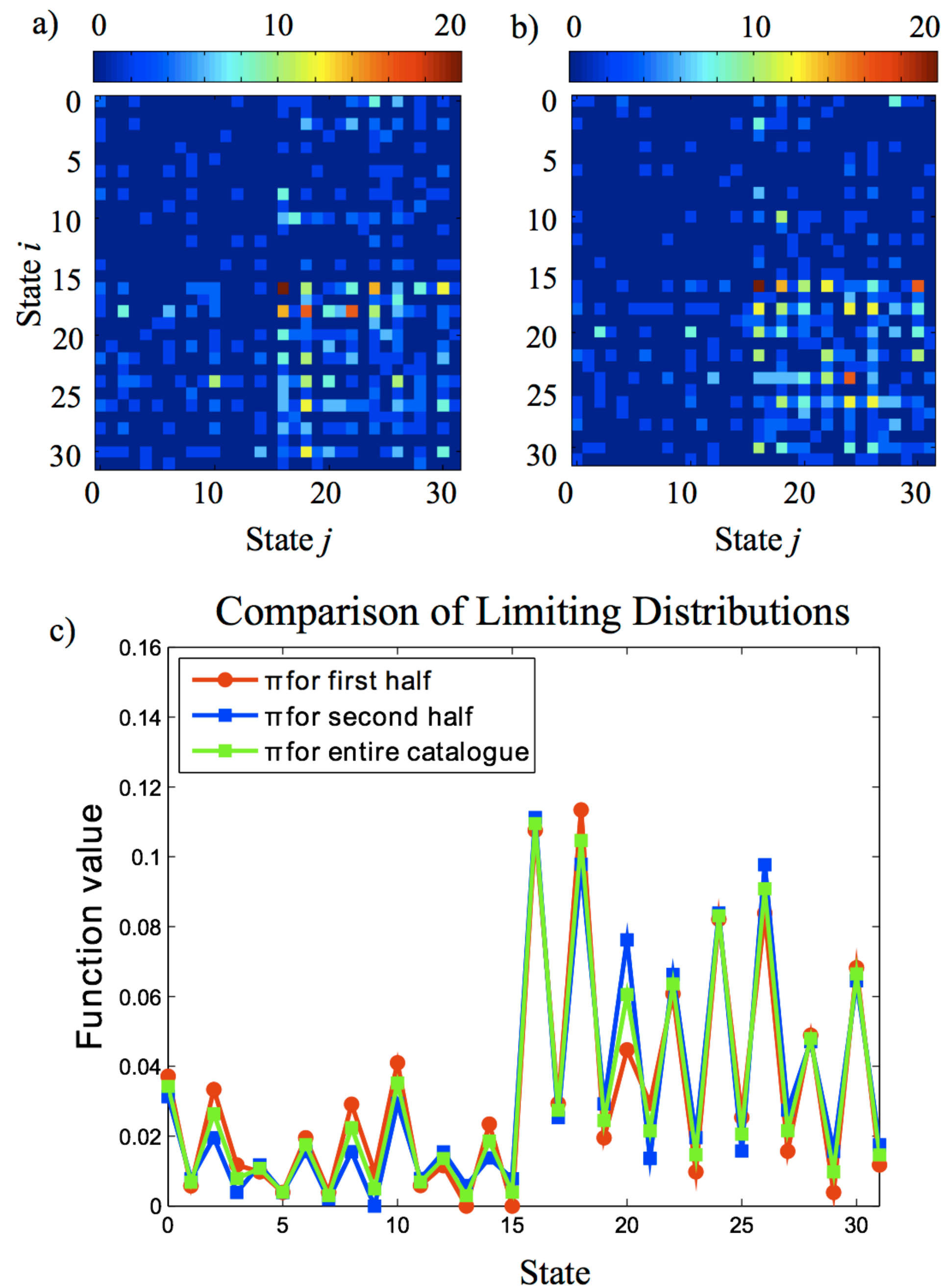
Analysis

Transition frequency matrices: First half and second half of the catalogue



Cavers and Vasudevan (2014)

KVMC2014

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Performance of the method by counting successes

Results of state aft- and forecasting for the Markov chain model without recurrences

n	m	m (%)	$\bar{\kappa}/S$	p^b
1024	872	85	9.75 / 32	~ 0
10	6	60	9.75 / 32	0.039272
50	28	56	9.75 / 32	0.000105
100	54	54	9.75 / 32	0.000000543

Results of state aft- and forecasting for the Markov chain model with recurrences

n	m	m (%)	$\bar{\kappa}/S$	p^b
1024	818	80	8.56 / 32	~ 0
10	6	60	8.56 / 32	0.02218
50	25	50	8.56 / 32	0.000255
100	52	52	8.56 / 32	0.000000051

n is the number of transitions, m is the number of successes, p (kappa bar over S) is the success probability in any trial, p^b is the binomial probability of observing m successes in n trials.

Cavers and Vasudevan (2014)

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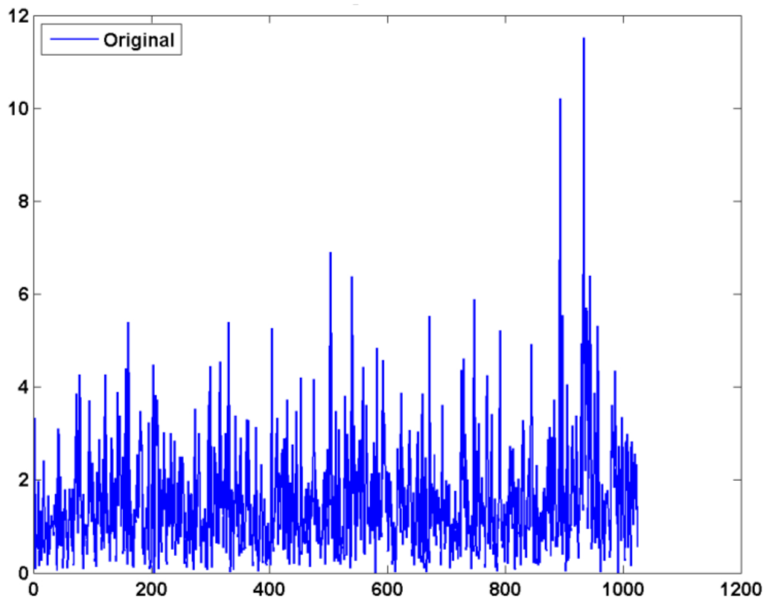
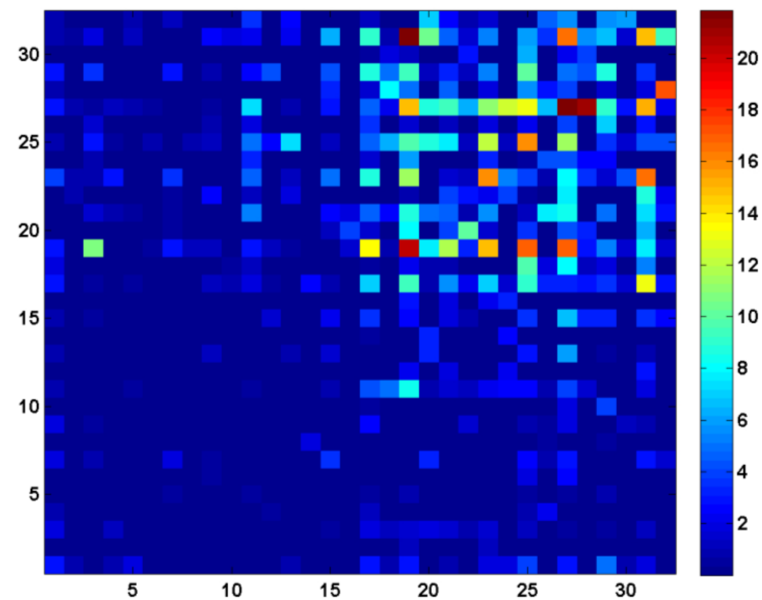
Analysis

Spatio-temporal complex Markov chain model

Non-linear time-series



Transition probabilities



Vasudevan and Cavers (2014)

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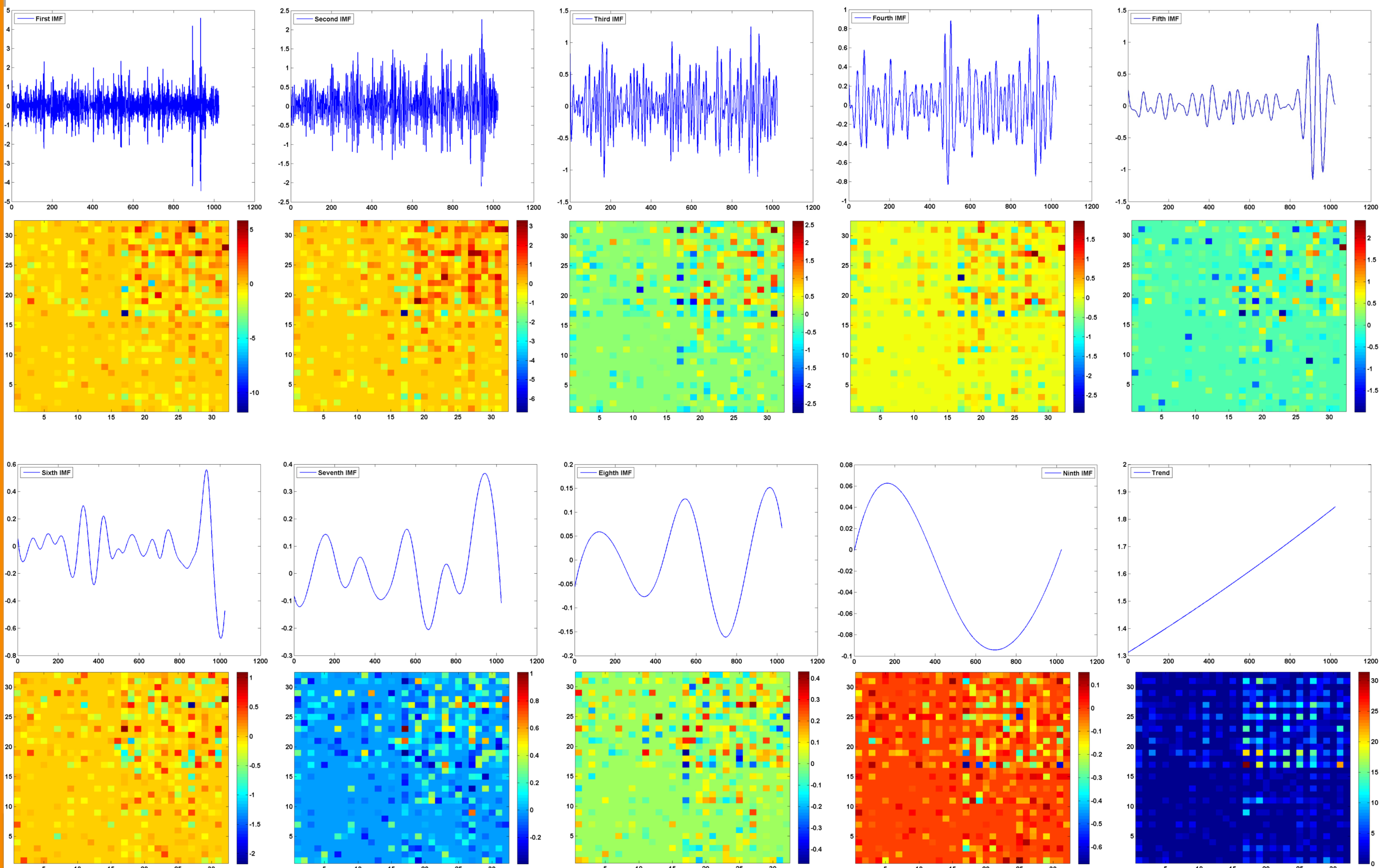
Ensemble Empirical Mode Decomposition Results

Intrinsic Mode Functions and their transition probability matrices



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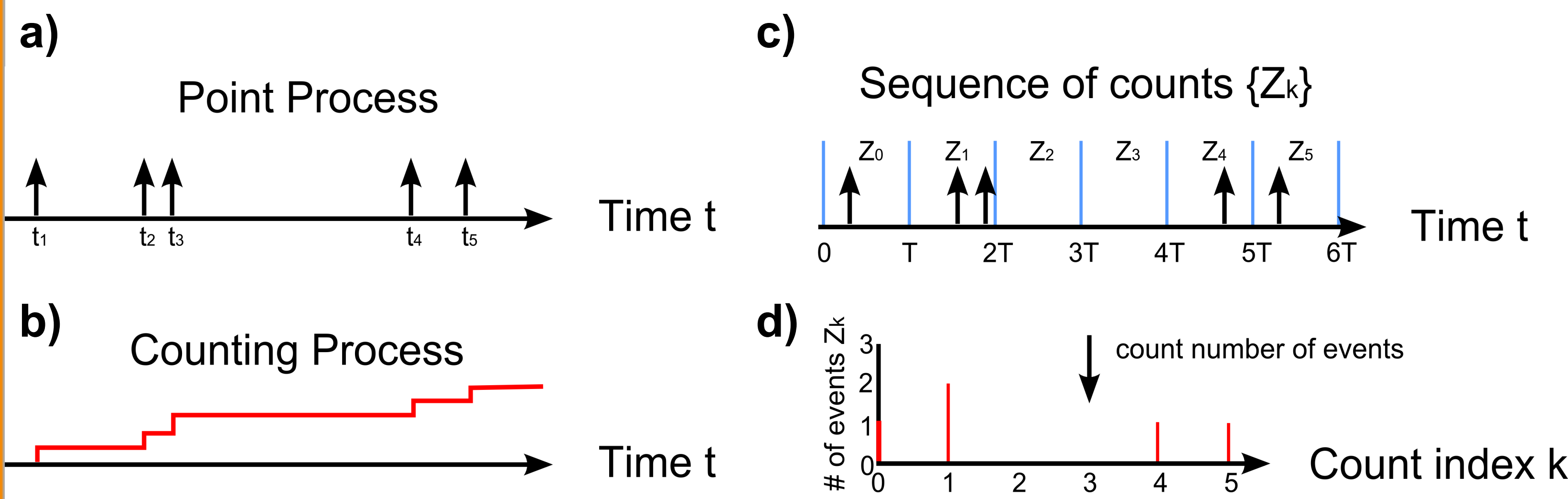
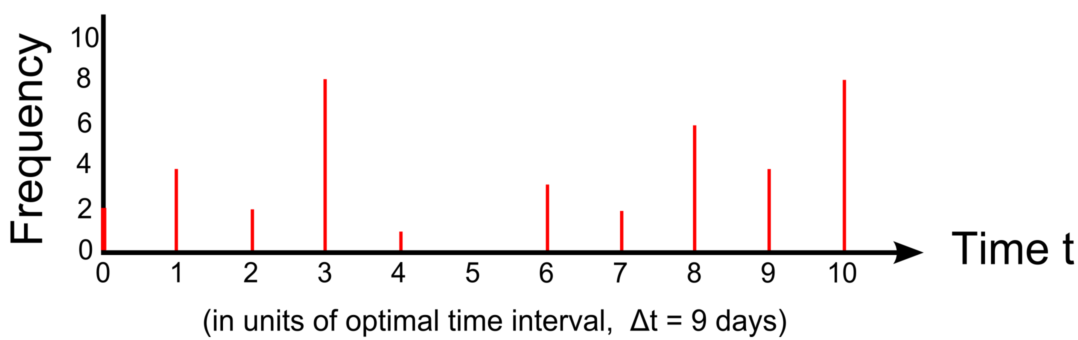
Vasudevan and Cavers (2014)

Representations of a point process



Thurner et al., Fractals (1996)

State to State Transitions



Vasudevan and Cavers (2014)

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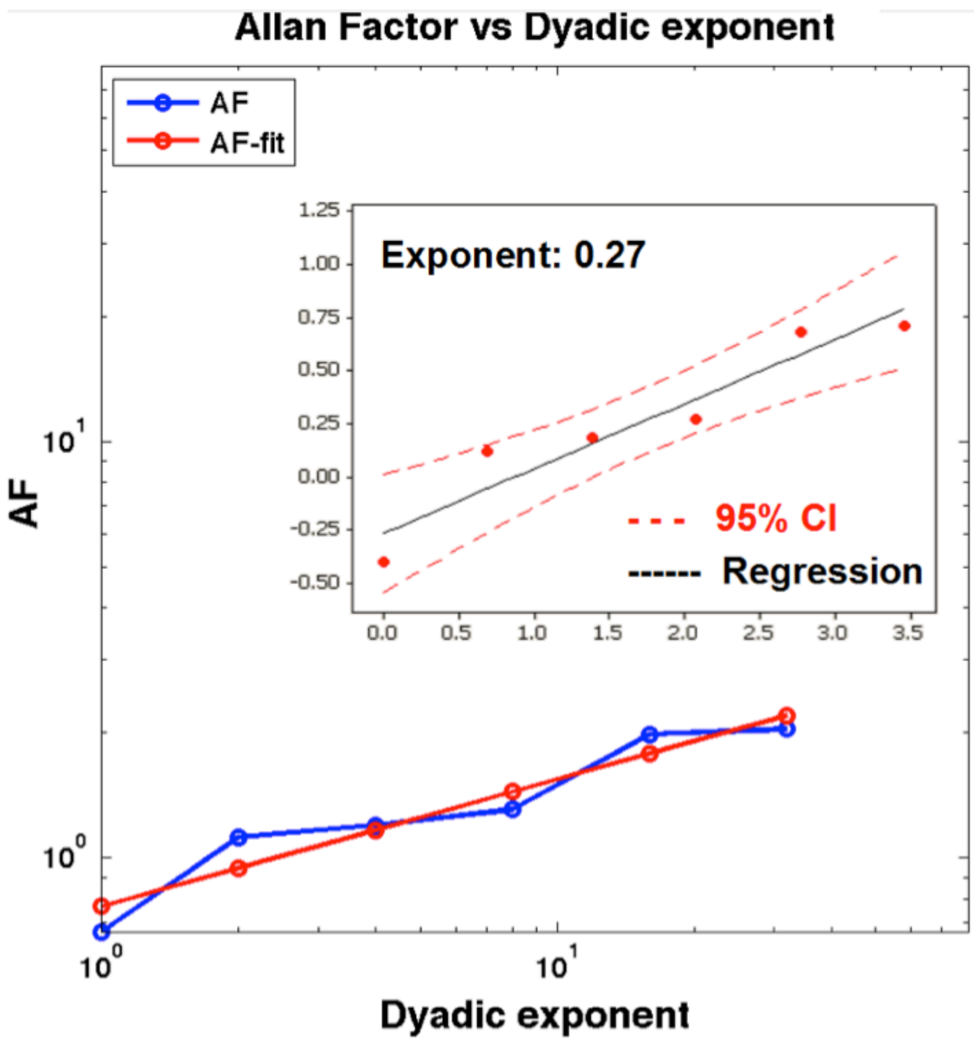
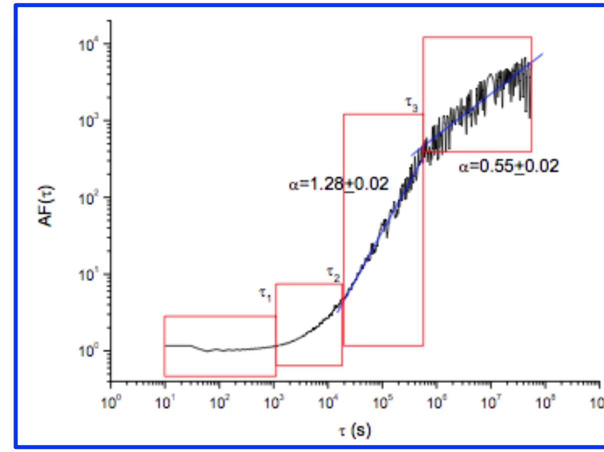
Allan factor

$$AF_{sstf}(\tau) = \frac{\langle N_{sstf,k+1}(\tau) - N_{sstf,k}(\tau)\rangle^2}{2\langle N_{sstf,k}(\tau)\rangle}$$



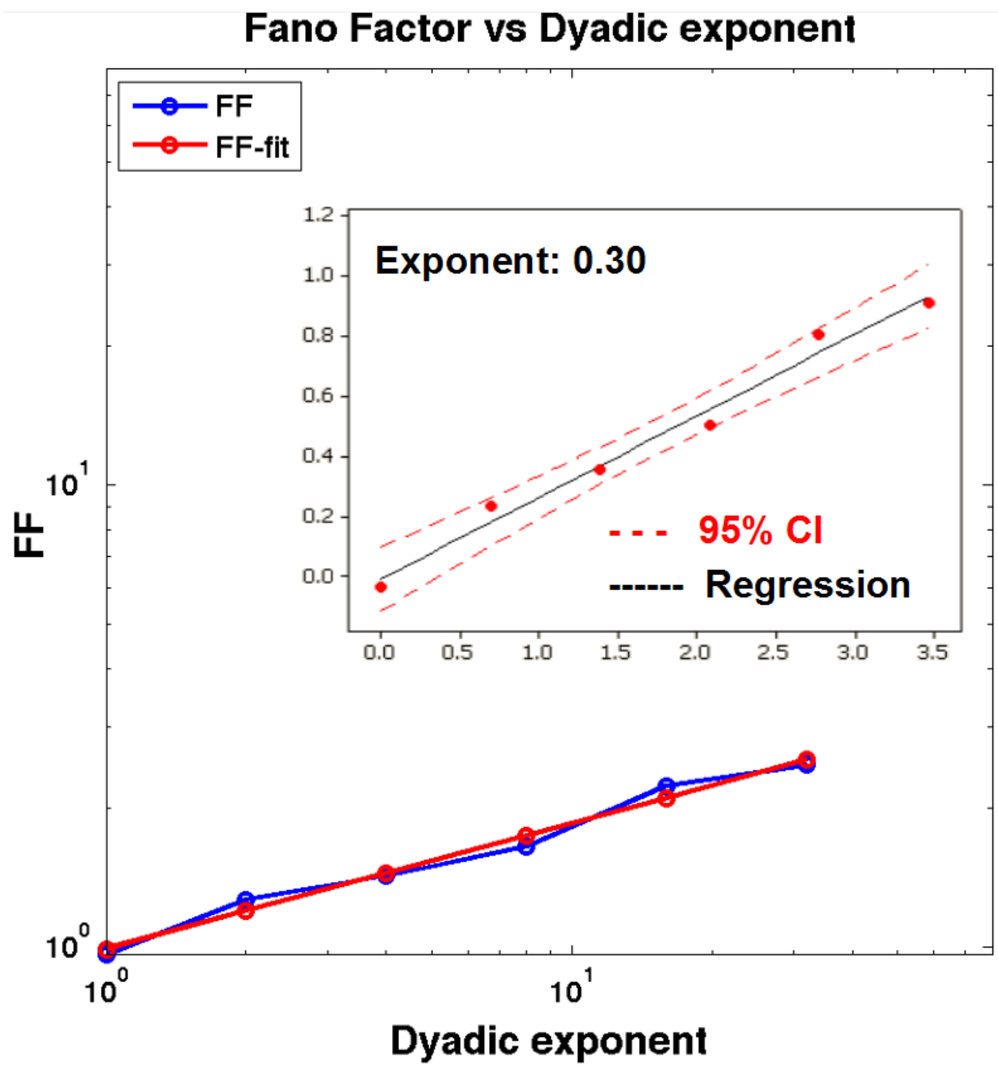


Telesca, Chen and Lee (2009)

Vasudevan and Cavers (2014)

Fano factor

$$FF_{sstf}(\tau) = \frac{\langle N^2_{sstf,k}(\tau) - N_{sstf,k}(\tau)\rangle^2}{\langle N_{sstf,k}(\tau)\rangle}$$



Analysis

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Vasudevan and Cavers (2014)

Conclusions

Spatio-temporal complex Markov chain model for earthquake sequencing is a new model.

A Markov chain model for 5-zone earthquake sequencing appears to differ from the Poisson model. For transitions involving small probabilities ($p_{ij} < 0.05$) with four different Δt values, there is no difference between the Poisson model and the Markov chain model.

Differences in transition probability values between the Markov chain model and the memoryless model are not negligible.

Markov chain model without recurrences and with recurrences reveal differences in transition probabilities but preserve the combinatorial structure of the graphs that depict the models.

Conclusions

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Spatio-temporal complex Markov chain models yield a non-linear time-series that is amenable to extensive analysis.

The ensemble empirical model decomposition of the time-series leads to nine intrinsic mode functions (IMFs) and a trend. Each one of the IMFs reveals the amplitude fluctuation of the state-to-state transitions.

The growth and decay of oscillations in easily identifiable packets in each IMF following certain periodicity is an intrinsic signature of the role of multiple zones in earthquake sequencing.

There is evidence for fractality of the multi-state modified Markov chain to represent the earthquake sequencing, as is revealed by the power-law scaling behavior present in the Fano and Allan factors with their respective exponents of 0.27 and 0.30.

Questions

Are there limits in quantifying the arc-weights in the spatio-temporal complex Markov chain model?

Is there any advantage in using the directed graph representation of the spatio-temporal complex Markov chain model?

Since one of the purposes of the present method is to examine earthquake forecasting problem, are there some forecasting metrics that can be computed using this model?

Is it meaningful to carry out the non-linear dynamics on the directed graph of the spatio-temporal complex Markov chain model? Would it lend itself to an understanding of the clustering patterns and the correlative and memory behaviour embedded in earthquake sequencing?

Insight into earthquake sequencing: A graph theoretic approach to modified Markov chain model

Kris Vasudevan and Michael S. Cavers
University of Calgary

"Causality, Information Transfer and Dynamical Networks"

CIDNET14 seminar talk
Friday 13, June 2014

MAX PLANCK INSTITUTE FOR THE PHYSICS OF COMPLEX SYSTEMS
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