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THERMAL COHERENT STATES, A BROADER CLASS OF MIXED COHERENT STATES AND GENERALISED THERMOFIELD DYNAMICS

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The pure Glauber (harmonic oscillator) coherent states provide a very useful basis for many purposes. They are complete since an arbitrary state in the Hilbert space H may be expanded in terms of them. The well-known *P*-representation provides a diagonal expansion of an arbitrary operator in H in terms of projection operators onto the coherent states. We discuss the extension of these results to the analogous mixed states introduced by us,[1] which describe comparable displaced harmonic oscillator systems in thermodynamic equilibrium at nonzero temperatures T. These thermal coherent states provide a very useful “random” (or “thermal” or “noisy”) basis in H, since the corresponding statistical density operator provides a probability measure on H. We prove a resolution of the identity for these states and use it to generalise the usual pure ($T = 0$) coherent state formalism to the mixed ($T \neq 0$) case. We show how our results relate to the Glauber-Lachs formalism in quantum optics for mixtures of coherent and incoherent radiation. Attention is focussed on the interplay between the quantal and thermodynamic uncertainties and on the entropy associated with such mixed states. The Bargmann representation of an operator is introduced, and its relationships to the usual *P*-, *Q*- and Weyl (*W*-) representations are studied.[2] We show how the thermal coherent states permit a generalisation to the $T \neq 0$ case of each of the *P*-, *Q*- and *W*- representations. A particularly important and unexpected result is that the present temperature-dependent *P*- and *Q*-representations are the analytic continuations to negative temperatures of each other. We discuss the possible physical applications of the thermal coherent states both to signal and image processing and to quantum optics or quantum information processing situations involving coherent signals in the presence of thermal noise.

The above formalism for thermal coherent states is further generalised to a broader class of so-called negative-binomial mixed states,[3] in terms of which we also show how to construct a resolution of the identity operator in H. The thermal coherent states are shown to be the particular limiting case $k = ½$ of this larger class which is characterised by the new parameter k. The negative-binomial distribution is itself intimately related to the discrete series of SU(1,1) representations. Indeed, the Hilbert space of a two-mode harmonic oscillator can be expressed as a direct sum of an infinite number of subspaces each of which is related to a particular representation in the discrete series of SU(1,1) representations. We have shown previously[4] how such states can be useful for processes involving general fluctuation-dissipation phenomena. We consider the pure SU(1,1) coherent states discussed by Perelomov in the two-mode harmonic oscillator Hilbert space, and show that the partial trace with respect to one of the two modes leads, rather miraculously, to our negative-binomial mixed states. This observation is then used to show how the formalism of thermofield dynamics may be generalised to a correspondingly much broader negative-binomial field dynamics, which we expect to have many uses.

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[1] R.F. Bishop and A. Vourdas, *J. Phys. A: Math. Gen.* **20** (1987), 3743.
[2] A. Vourdas and R.F. Bishop, *Phys. Rev. A* **50** (1994), 3331.
[3] A. Vourdas and R.F. Bishop, *Phys. Rev. A* **51** (1995), 2353.
[4] R.F. Bishop and A. Vourdas, *J. Phys. A: Math. Gen.* **20** (1987), 3727.