Modelling Traffic Flow

Using Cellular Automata

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Overview

- Introduction
- Cellular Automata for Traffic Flow
  - Definition
  - Basic Properties
  - Phase Transition
- Generalizations
  - "Defects"
  - Effects of Boundary Conditions
  - Metastable States, Hysteresis
  - Synchronized Traffic Flow
  - Multilane Traffic
  - City Traffic
- Summary

Traffic Flow: Problems

- "Phantom traffic jams"
  start-stop waves (stop and go)

- Fundamental diagram:
  flow $\Longleftrightarrow$ density

  Aim: Optimization

- Traffic control: Effects of
  - speed limits ?
  - multilane traffic ?
  - on-/off-ramps ?

Physical Aspects

- Non-equilibrium physics
  (systems far from equilibrium)

- driving force $\leftrightarrow$ dissipation

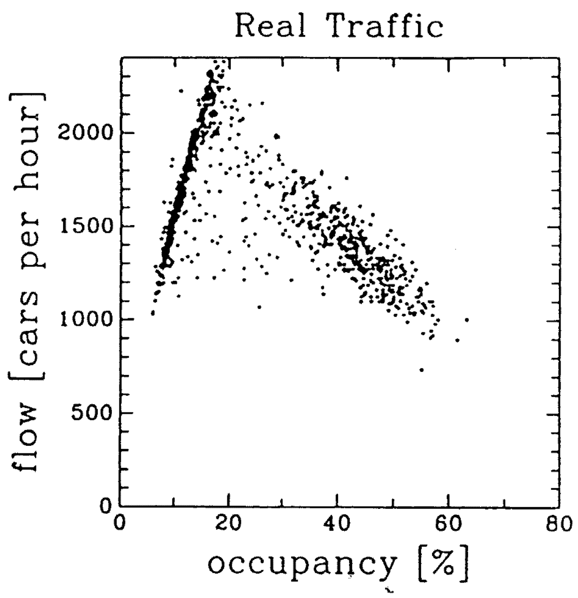
- phase transitions

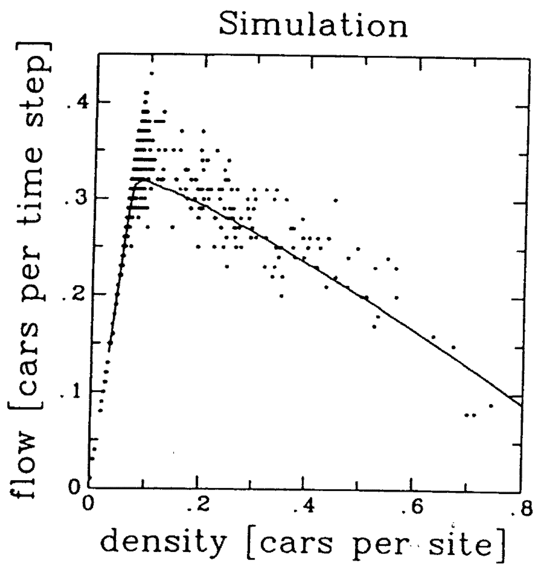
- dynamics

The Fundamental Diagram

Fundamental diagram:

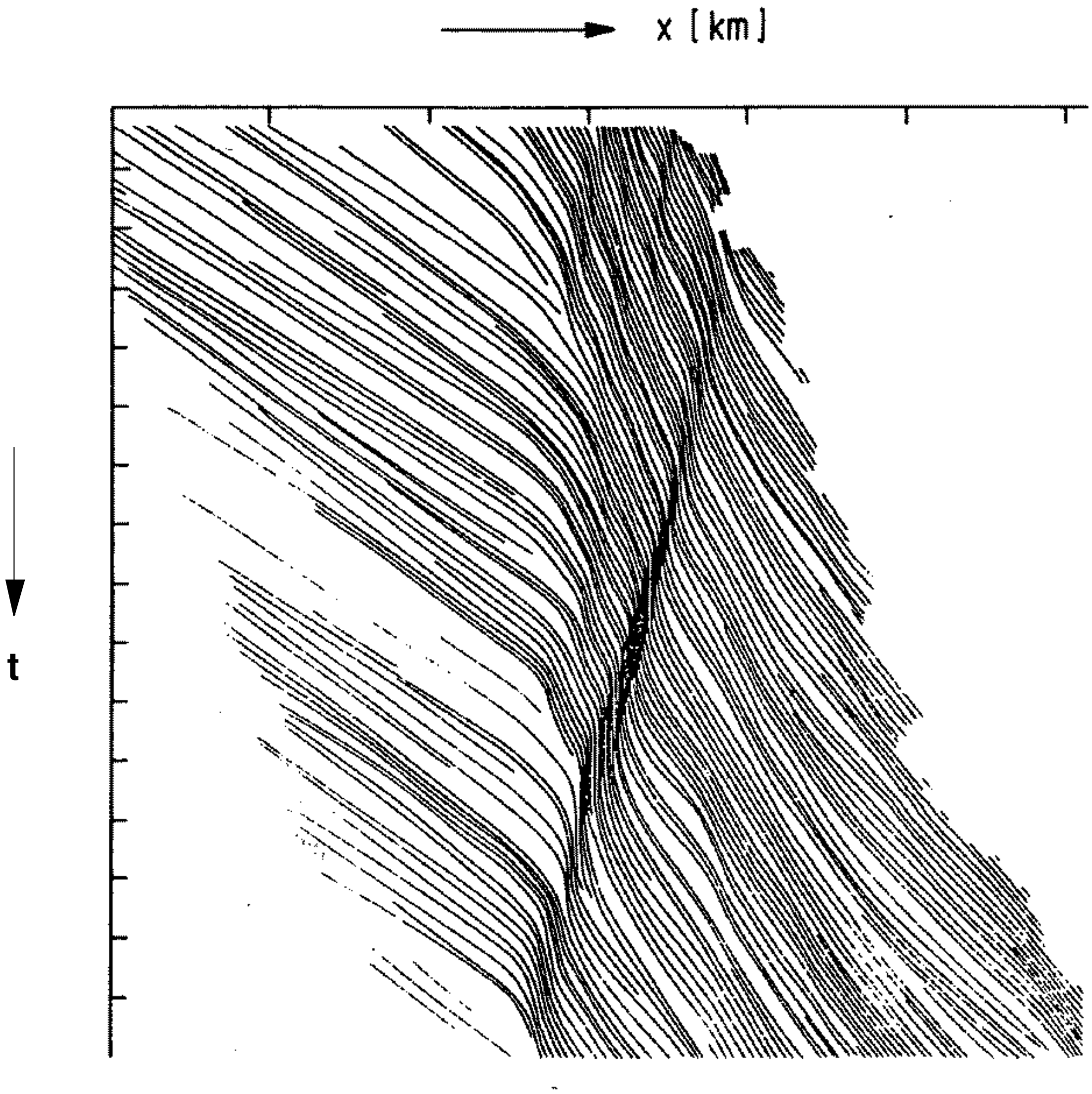
Flow (current) as function of car density





Spontaneous Formation of Jams

"Phantom Jams"



Methods of "Classical" Physics

Mechanics: Car-following models

microscopic modeling of equations of motion:

$$\boxed{\frac{dv_{j+1}}{dt} = \kappa \left[v_j(t) - v_{j+1}(t) \right]}$$

typical form: action = sensitivity · stimulus

Hydrodynamics: Lighthill-Whitham theory (1955)

macroscopic description:

continuity equation for a flow which depends only on ρ ($f = f(\rho)$):

$$\Rightarrow \qquad \boxed{\frac{\partial \rho}{\partial t} + f'(\rho)\frac{\partial \rho}{\partial x} = 0}$$

Solutions: kinematic waves

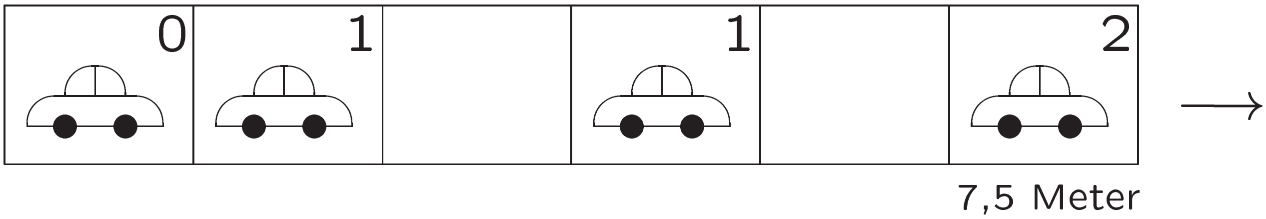
Related approach: (Prigogine-Herman '61)
Boltzmann equation for a distribution function $\Phi(x, v, t)$

Cellular Automaton for Traffic Flow

Cellular automata:

discrete time + discrete space

$\Rightarrow$ discrete velocities: $v = 0, 1, 2, \ldots, v_{max}$



Rules (Nagel-Schreckenberg 1992)

(d_j: Number of empty cells in front of car j)

a) Acceleration: $v_j \rightarrow \min(v_j + 1, v_{max})$

b) Braking: $v_j \rightarrow d_j$ (if $v_j > d_j$)

c) Randomization: $v_j \xrightarrow{p} v_j - 1$
with probability p (if $v_j > 0$)

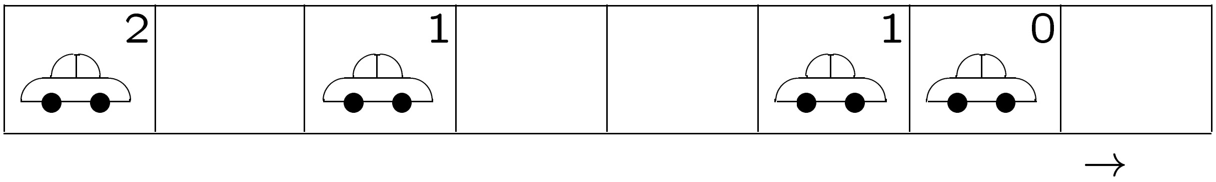
d) Driving: Car j moves v_j cells

Parallel dynamics: Rules are applied to all vehicles at the same time !

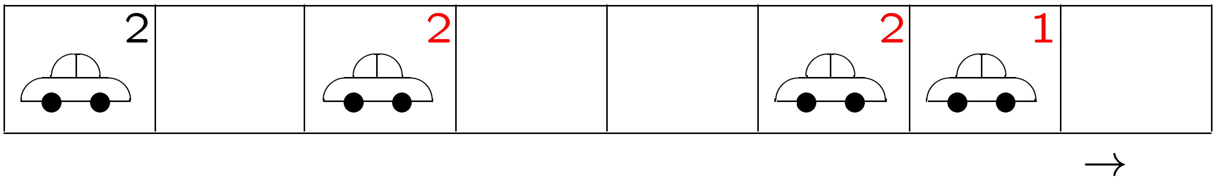
periodic boundary conditions (“Indianapolis”)

Example ($v_{max} = 2$)

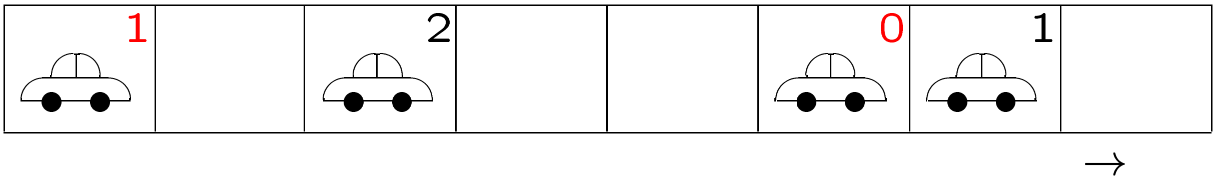
Configuration at time t:



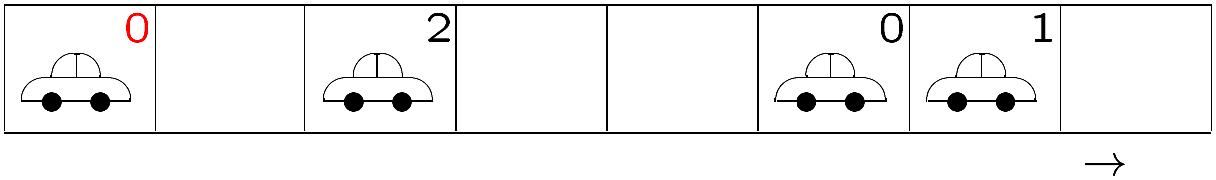
a) Acceleration:



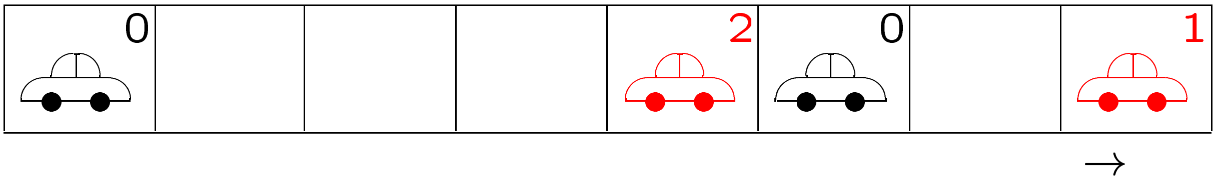
b) Braking:



c) Randomization ($p = 1/3$):



d) Driving (= configuration at time $t + 1$):



Interpretation of the Rules

a) Acceleration:

Drivers want to move forward as fast as possible (or allowed)

b) Braking:

no accidents

c) Randomization:

- Overreactions at braking
  delayed acceleration
- psychological effects (e.g. natural fluctuations in free driving)
- road conditions (e.g. weather, slope)

d) Driving:

motion of cars

Realistic Parameter Values

$$\boxed{v_{max} = \mathbf{5}, \quad p = \mathbf{1/2}}$$

$\Rightarrow$ free driving:

$$\bar{v} = v_{max} - p = 4.5 \text{ cells/timestep}$$

$$\Rightarrow \quad 120\,km/h \mathrel{\hat{=}} 4.5 \text{ cells/timestep}$$

(maximum velocity)

in densest jam:

$$7.5\,m/\text{car} \to \text{length of cell}$$

$$\Rightarrow \quad \frac{7.5\,m}{\text{cells}} \times \frac{4.5\,\text{cells}}{\text{timestep}} \times \frac{3.6\,s}{120\,m}$$

$$\approx \boxed{1 \text{ s / timestep}}$$

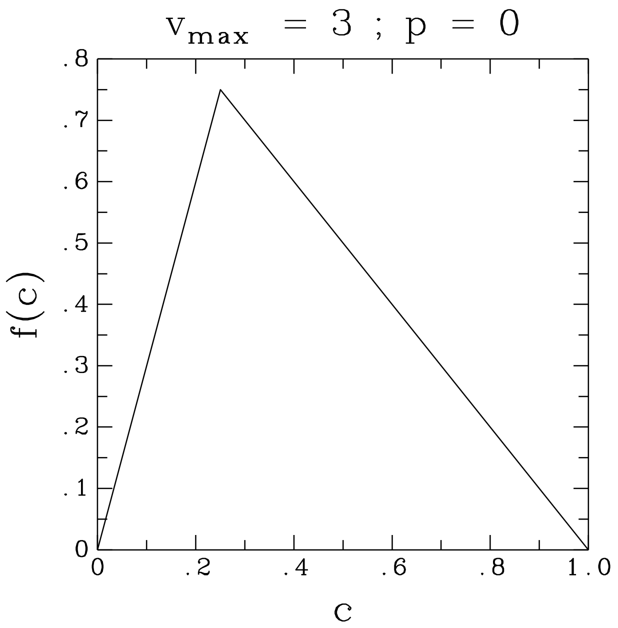
This is of the same order of magnitude as the smallest relevant timescale in real traffic, i.e. the reaction time.

Properties of the Model

a) $p = 0$: deterministic dynamics

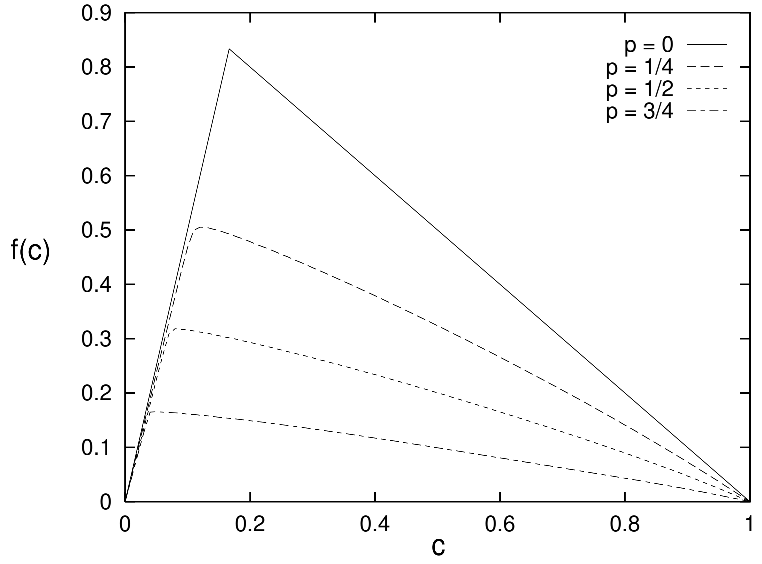
Fundamental diagram:

$$f(c) = \begin{cases} v_{max} c & \text{for } c < \frac{1}{v_{max}+1} \\ 1 - c & \text{for } c < \frac{1}{v_{max}+1} \end{cases}$$

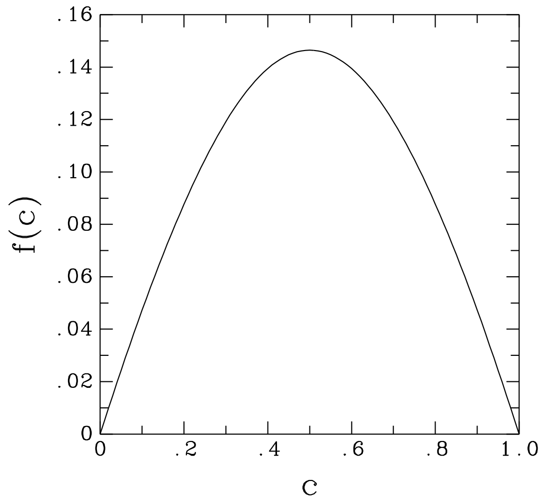


b) $p > 0$: stochastic dynamics

Fundamental diagram for $v_{max} = 5$:



special case: $v_{max} = 1$



"particle-hole symmetry"

Analytical Approaches

Master equation:

$$\boxed{\begin{aligned}\frac{\partial}{\partial t}P(\{\tau\},t) &= \sum_{\{\tau'\}}\left[w(\tau'\to\tau)P(\{\tau'\},t)\right.\\ &\qquad \left. -w(\tau\to\tau')P(\{\tau\},t)\right]\end{aligned}}$$

$w(\tau'\to\tau)$ = transition rate from τ' to τ

$P(\{\tau\},t)$ = weight of configuration

$$\{\tau\} = \{\tau_1,\ldots,\tau_L\}$$

stationary solution: $\boxed{\frac{\partial}{\partial t}P(\{\tau\},t) = 0}$

Mean-Field approx.: $P(\tau_1,\ldots,\tau_L) = \prod_{j=1}^{L} P(\tau_j)$

cluster approximation:

(AS-Schreckenberg '93; Schreckenberg-AS-Nagel-Ito '95)

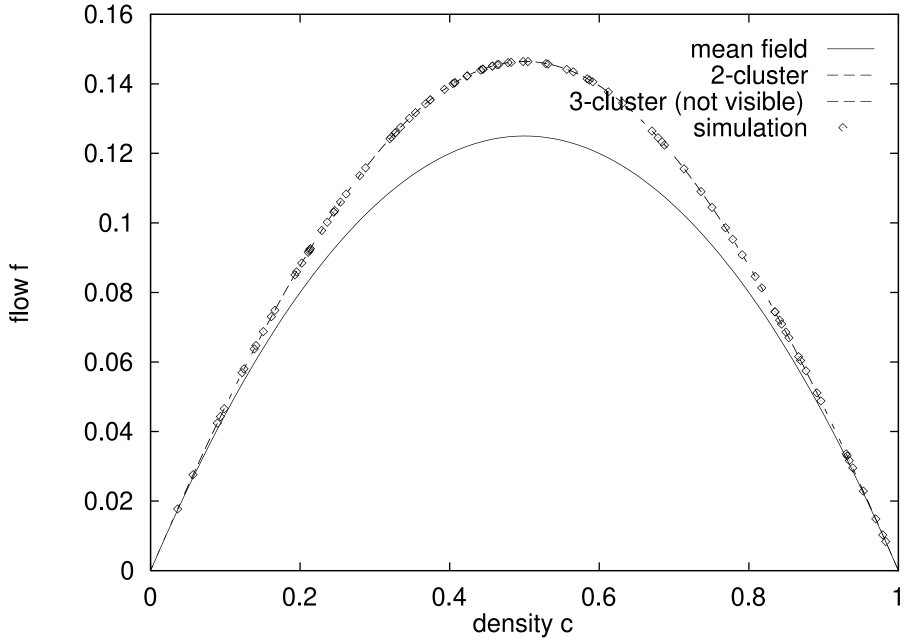
$$\boxed{\begin{aligned}P(\tau_1,\ldots,\tau_L) &\propto P(\tau_1,\tau_2)P(\tau_2,\tau_3)\cdots\\ &\quad \cdots P(\tau_{L-1},\tau_L)P(\tau_L,\tau_1)\end{aligned}}$$

Car-Oriented Mean-Field (COMF):

(AS-Schreckenberg '97)

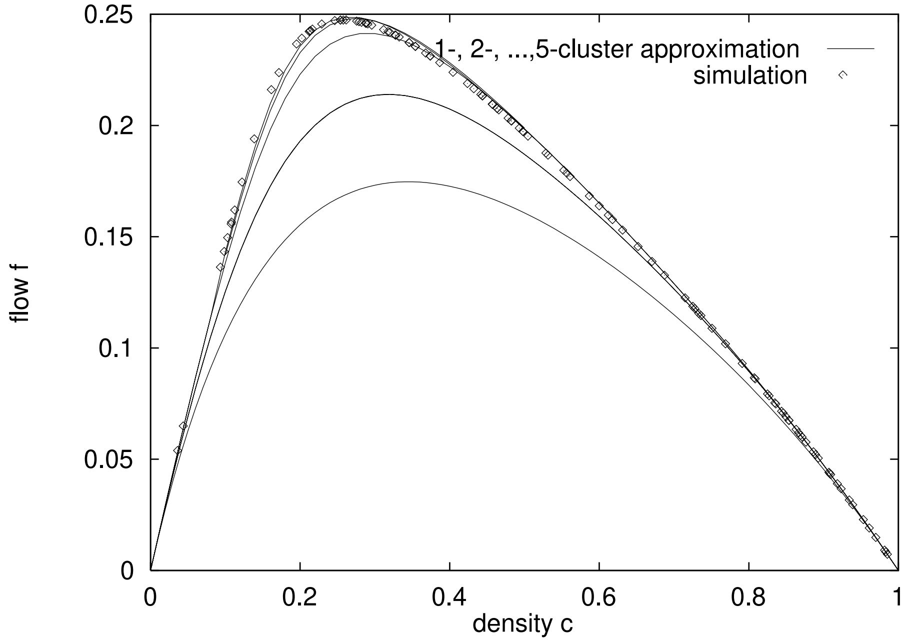
Distances between consecutive cars are assumed to be uncorrelated

$v_{max} = 1$:



"particle-hole attraction"

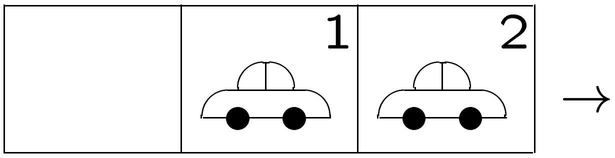
$v_{max} = 2$:



Garden of Eden States

Garden of Eden states (paradisical states):
can not be reached by the dynamics

e.g. for $v_{max} = 2$:

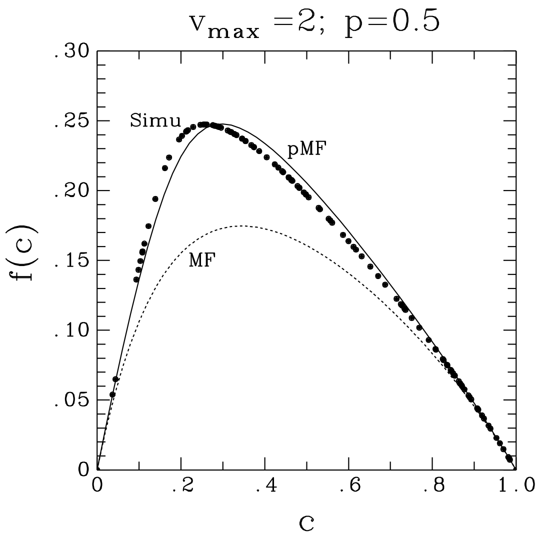


Mean-field theory in reduced configuration space:
(AS-Schreckenberg '98)

eliminate all Garden of Eden states !

$v_{max} = 1$: exact solution
Garden of Eden states are responsible for all correlations!

$v_{max} = 2$: there are "real" correlations, but considerable improvement over mean-field results



Phase Transition

Transition laminar flow $\rightarrow$ jammed phase ?

deterministic case: $p = 0$

continuous phase transition at $\rho^* = \frac{1}{v_{max}+1}$

with randomization: $p > 0$

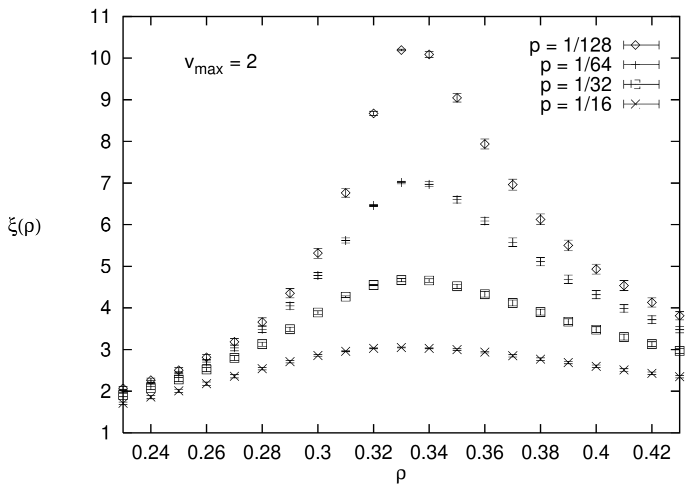
First guess: continuous transition at $\rho_c \lesssim \rho^*$

(Csányi-Kertész '95)

But: no "true" phase transition ($\rightarrow$ crossover)

(Sasvari-Kertész '97; Eisenblätter-Santen-AS-Schreckenberg '98)

correlation length ξ: $\boxed{\langle n_1 n_r \rangle - \rho^2 \propto e^{-r/\xi}}$

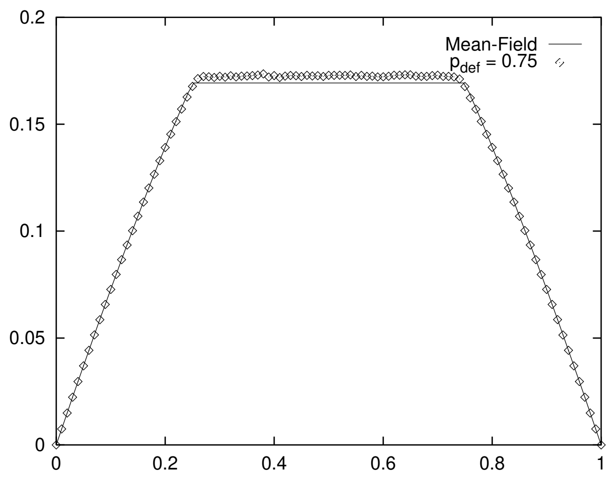


Maximum: $\xi_{max} \propto 1/\sqrt{p}$

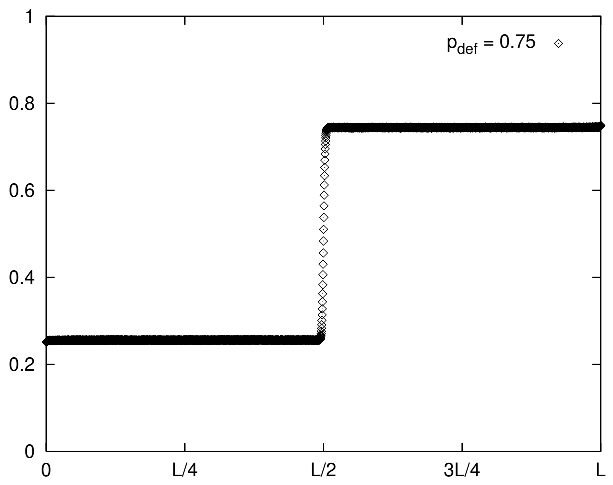
“Defects”

Defect: larger randomization p_{def} at one site j_{def}: $p_{def} > p$

Fundamental diagram:

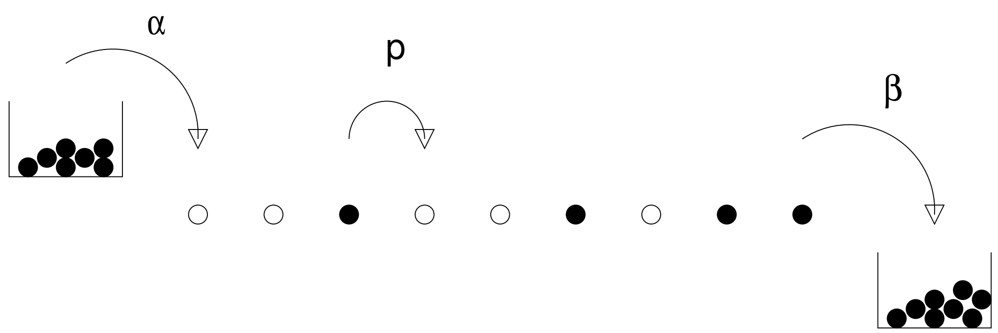


at intermediate densities: shock profil



Influence of Boundary Conditions

Asymmetric Simple Exclusion Process (ASEP):



Applications:

– "boundary induced phase transitions"

– traffic: Bottleneck

– Protein synthesis

– surface growth

ASEP can be solved exactly for $v_{max} = 1$ and arbitrary p, α, β

(Derrida-Evans-Hakim-Pasquier '93; Schütz-Domany '93; Rajewsky-Santen-AS-Schreckenberg '98; Tilstra-Ernst '98; Evans-Rajewsky-Speer '98; de Gier-Nienhuis '98)

elegant method: Matrixproduct Ansatz

Weight of configuration $\{\tau\}$:

$$\boxed{P(\tau_1,\ldots,\tau_L) = \langle W| \prod_{j=1}^{L} \left[\tau_j D + (1-\tau_j)E\right] |V\rangle}$$

($E \leftrightarrow$ empty cell, $D \leftrightarrow$ occupied cell)

translate configuration into product of matrices:

$$\boxed{0110001 \leftrightarrow EDDEEED = ED^2E^3D}$$

Master equation $\rightarrow$ matrix algebra $\rightarrow$ representations of algebra $\rightarrow$ calculation of expectation values

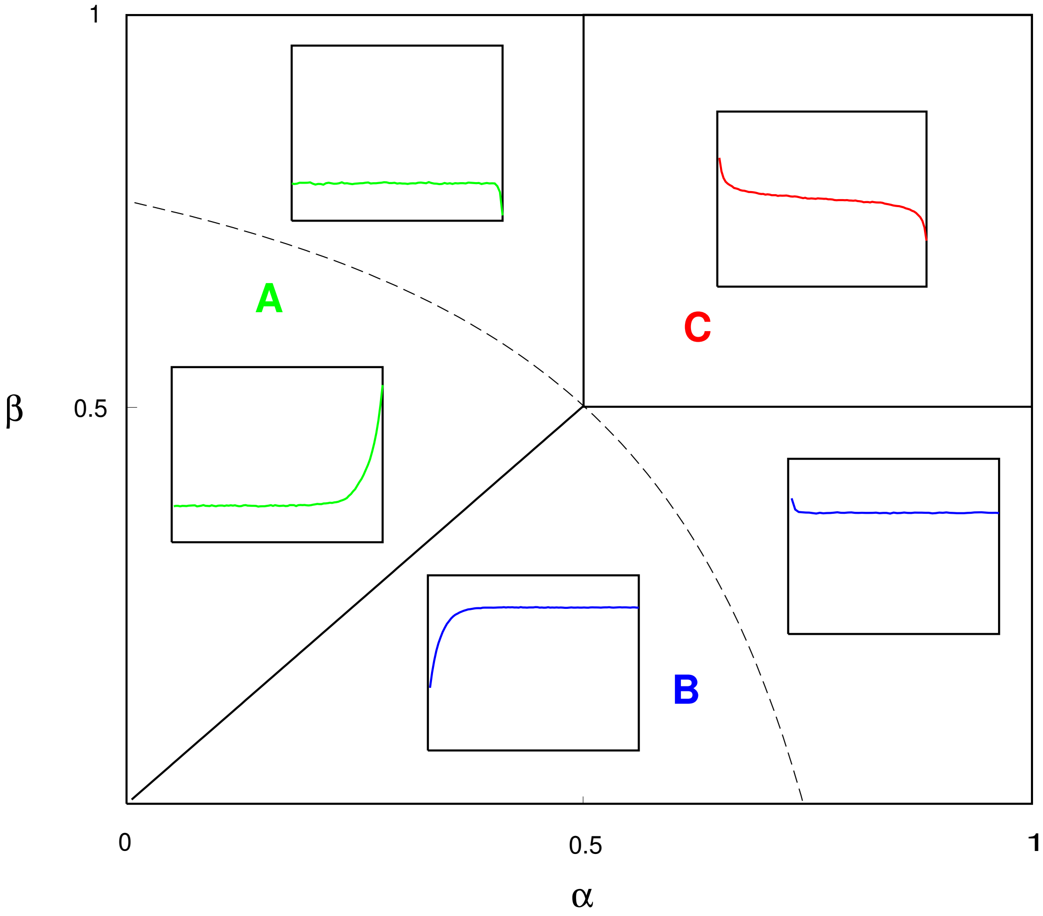
For stochastic systems the stationary states generically has matrix product form !

(Krebs-Sandow '97 ; Rajewsky-Schreckenberg '97 ; Klauck-AS '98)

spin systems: only in special cases

(Klümper-AS-Zittartz '91,'92,'93)

Phase diagram for $p = 0.75$



A. low-density phase:

current depends only on α

B. high-density phase:

current depends only on β

C. maximal current phase:

current independent of α and β

Metastable States and Hysteresis

Measurements on real traffic $\rightarrow$ existence of metastabile states with high flow

Modified model: VDR-model

(Barlovic-Santen-AS-Schreckenberg '98)

- **Randomization parameter:** determine $p = p(v(t))$.
- **Acceleration:** increase v by 1 (if possible)
- **Braking:** if the distance d to the next car is less than $v+1$, then reduce velocity to $v = d-1$.
- **Randomization**: $v \rightarrow \max(v-1, 0)$ with probab. p.
- **Driving:** $x \rightarrow x + v$

Slow-to-Start rule:

$$\boxed{p(v) = \begin{cases} p_0 & \text{falls } v = 0 \\ p & \text{falls } v > 0 \end{cases}} \qquad \text{with } p_0 > p$$

Hysteresis

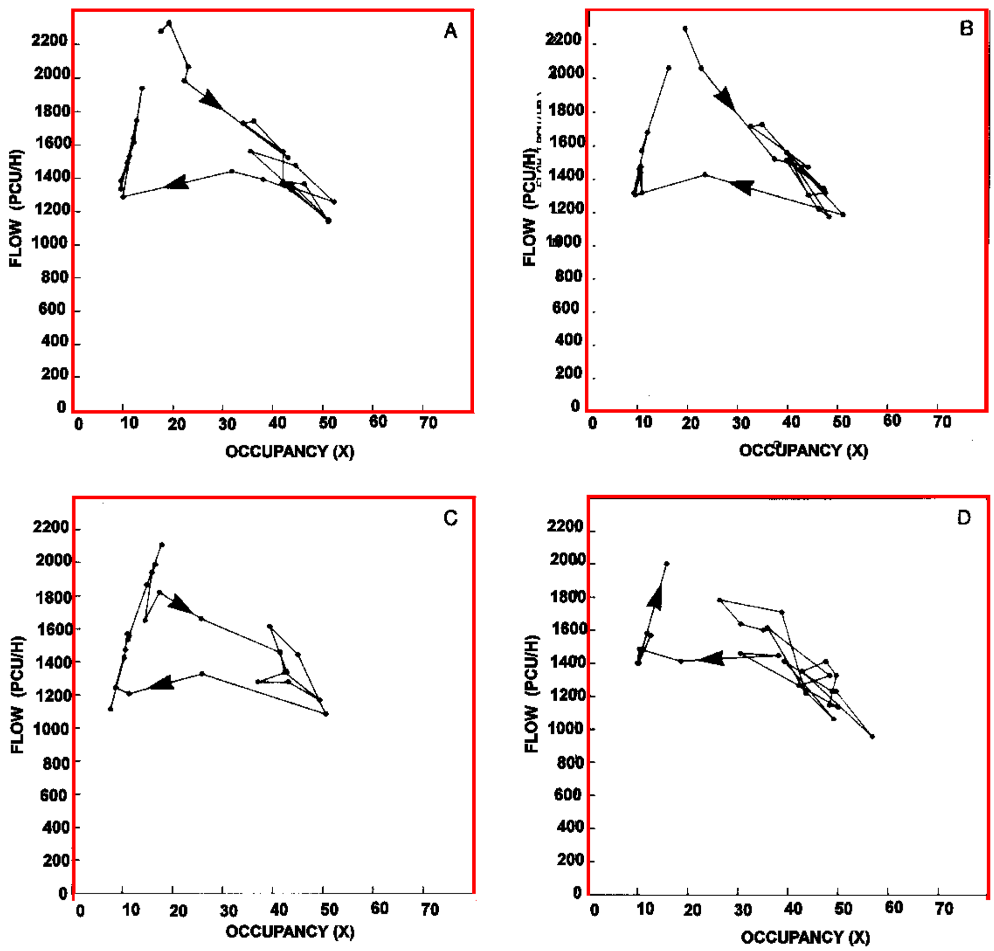
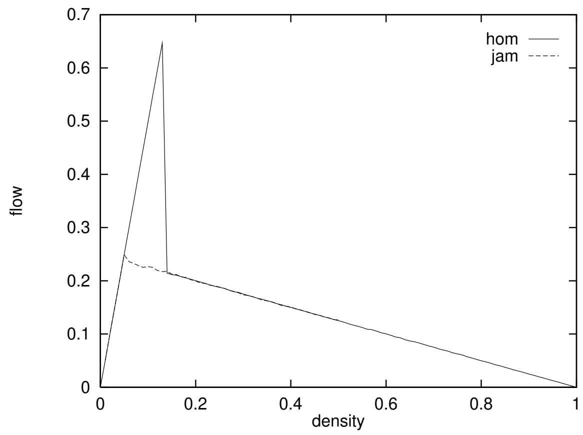


Figure 2.15
Four Days of Flow-Occupancy Data from Near Toronto (Hall et al. 1986).

Fundamental Diagram of the VDR-model



Two linear branches:

- low densities: $J(\rho) = \rho(v_{max} - p)$

- high densities: $J(\rho) = (1 - \rho)(1 - p_0)$

- metastable states with high flow

mikroscopic characterization of the states:

low-density branch:
– no jams
– cars move with velocity $v_{max} - p$

high-density branch:
– a macroscopic jam exists
– spontaneous formation of jams is suppressed

Slow-to-start rule
(e.g. in VDR-model)

$$\Downarrow$$

outflow from a jam is smaller than maximal flow:

$$\boxed{J_{out} < J_{max}}$$

$$\Downarrow$$

existence of metastable states

$$\Downarrow$$

hysteresis

Synchronized Traffic

new phase in traffic flow ?!?

states with

- high densities and high flow
- average velocity lower than in free traffic
- small variance of average velocity
- similar velocities on both lanes (synchronization of lanes)
- time series of flow show no correlations between consecutive points

possible explanation:

on- and off-ramps act as “defect”

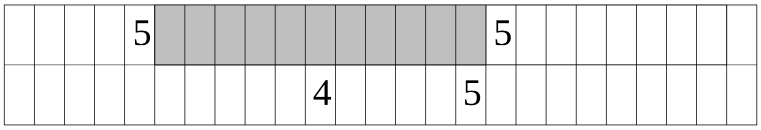
Two-Lane Traffic

Rules for lane-changes (symmetrical or asymmetric)

a) incentive criterion: situation on the other lane is better

b) safety criterion: avoid accidents which might occur through lane-change

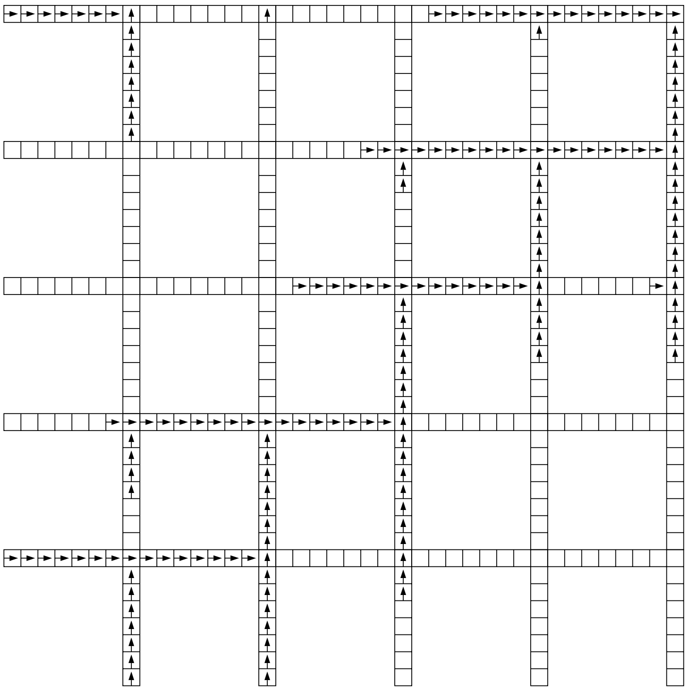
The safety criterion makes it necessary to look backwards:



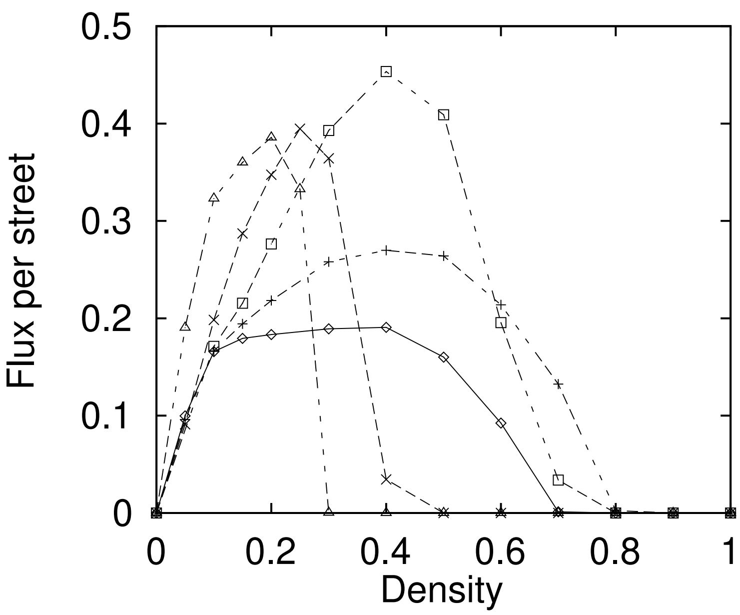
$\rightarrow$ interessing new phenomena, e.g. lane inversion for asymmetric rules

City Traffic

eastbound streets and northbound streets with crossings; controlled by traffic lights



optimal signal phase:



Summary

Cellular automata are ideal tools for modeling traffic flow:

- despite their simplicity one finds a good agreement with measurements

- simple implementation for simulations, even of complicated networks

- prototyp for systems far from equilibrium

- surprising effects: influence of defects and boundary conditions

- promising analytical results:
  new approximativ methods, matrix-product-Ansatz,...

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