# Coupled dynamics of a nanomechanical single electron transistor

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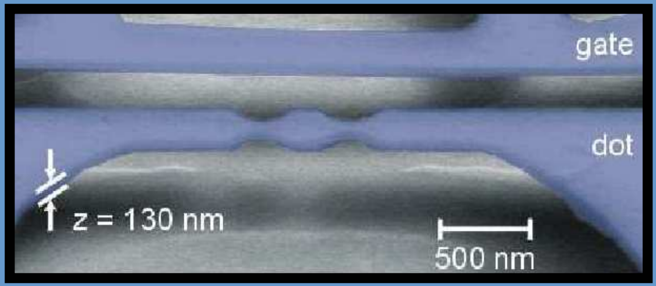
# Outline

- Nanoelectromechanical systems (NEMS) introduction:
  - Experimental devices
  - Classical to quantum transitions
  - NEMS as measuring devices
- Resonator coupled to single electron transistor
  - Model system
  - Resonator dynamics: intrinsic & extrinsic effects
  - Experimental progress
  - Twin-SET and resonator
  - Similar non-equilibrium systems
- Conclusions

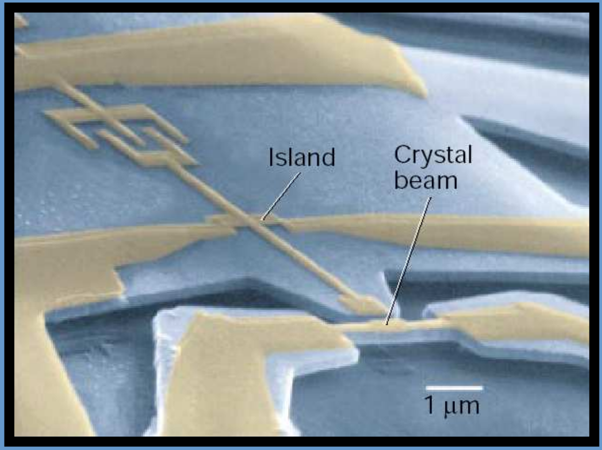
# Experimental Devices

- Resonators up to 1GHz
  - Huang et al., Nature **421**, 496

- Suspended Quantum Dot:
  - Weig et al. PRL **92**, 046804




- Resonator adjacent to mesoscopic conductor:
  - Knobel and Cleland, Nature **424**, 291



# Classical to Quantum Transition I

**Under what circumstances will a mechanical resonator display quantum properties?**

- Naive argument:
  - fundamental flexural mode frequency, *v*, quantum regime:

$$hv > k_B T$$

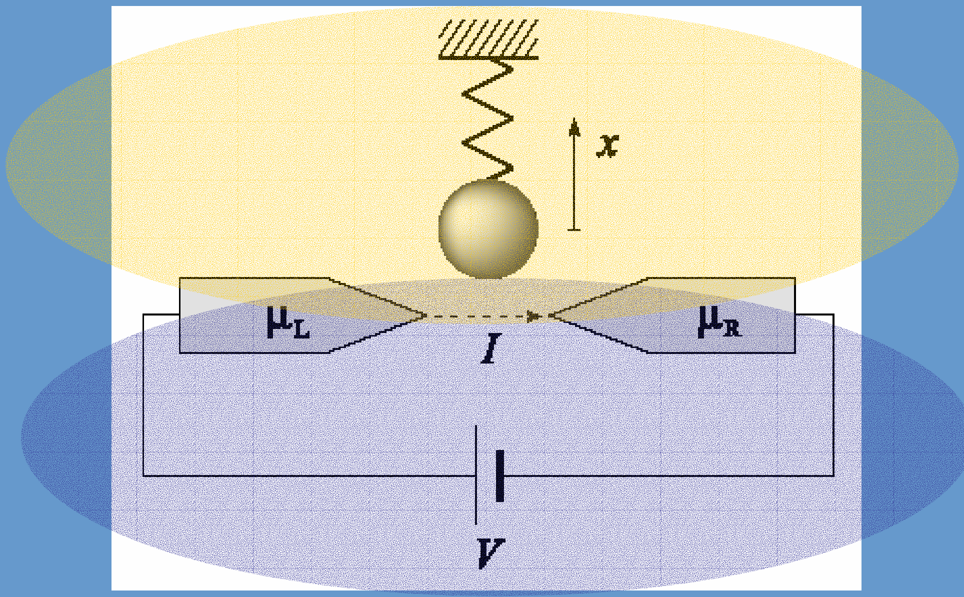
  - Requires cooling of 1GHz resonator to <50mK
- More generally, quantum regime set by environment
  - Temperature
  - Dissipation (Q-factor)
  - *Voltage applied*

- Active search for quantum signatures in micron-sized resonators:
  - Blencowe, Phys. Rep. **395**, 160; Bose et al., PRA **59**, 3204, AA, Blencowe and Schwab, PRL **88**, 148301; Wilson-Rae et al. PRL **92**, 075507,

# Classical to Quantum Transition II

**When will a resonator coupled to a mesoscopic conductor display quantum properties?**

Mechanical Resonator: freq. *ν*




Mesoscopic conductor: applied bias *V*

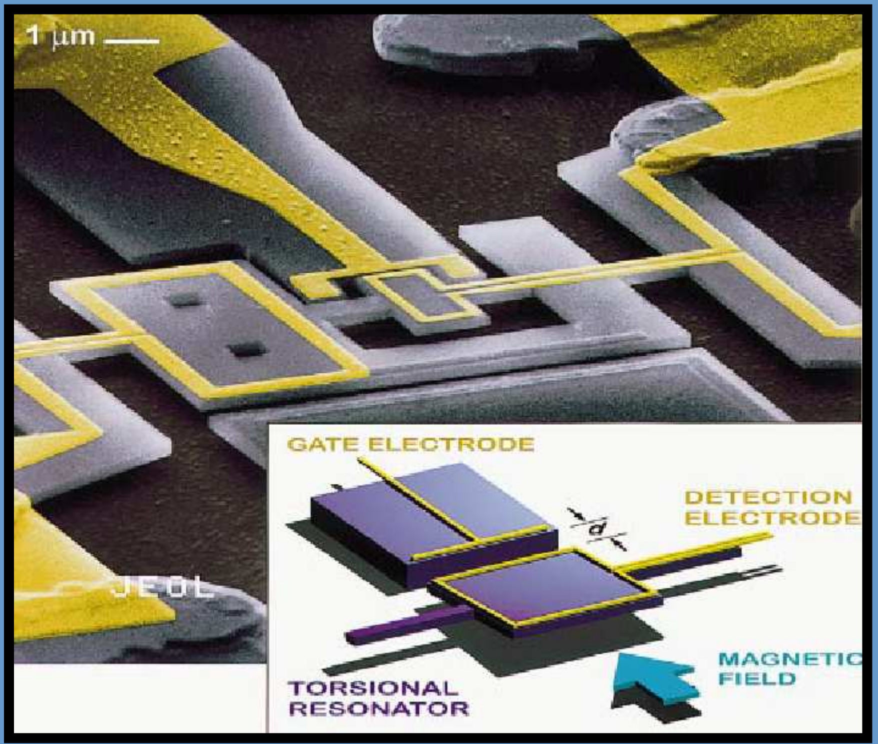
- Capacitive coupling between resonator and conductor
- Quantum regime set by voltage applied across conductor: *Quantum resonator only when* $eV<h\nu$

Mozyrsky and Martin PRL **89**, 018301

# NEMS as measuring devices

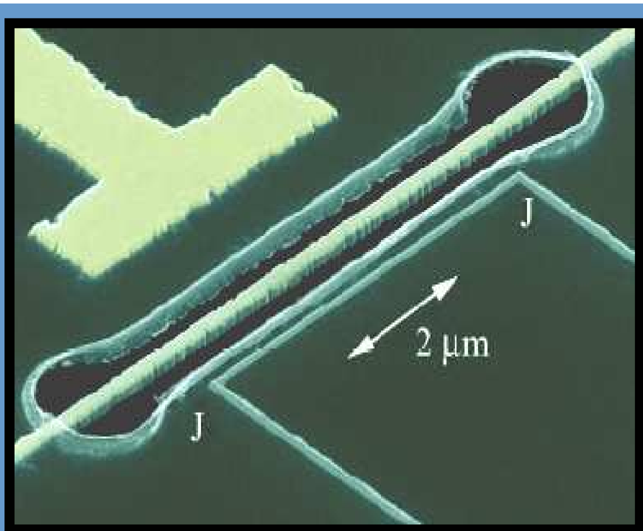
– *Smaller, faster MEMS*

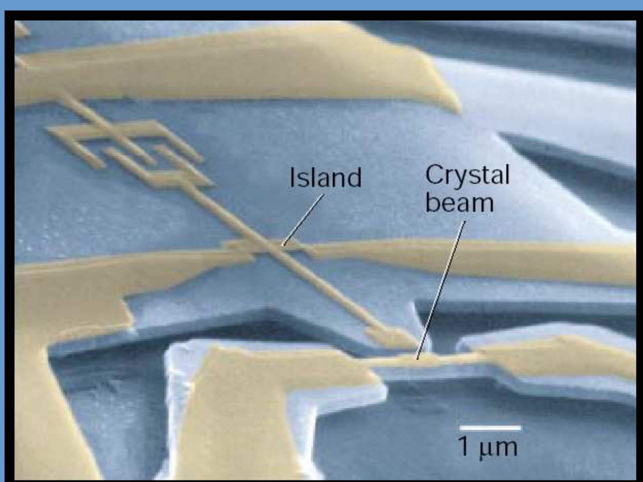
– Potential Applications as sensors of charge, spin, mass
– Sensitivity of measurement process depends on details, but common factors include:
  - Q-factor of resonator
  - Ultimate limits set by quantum dynamics of measuring device + measured system




Nanoelectromechanical electrometer
Cleland and Roukes, *Nature* **392** 160

# Resonator coupled to SET




- Experimentally accessible system
- Couple resonator to one(two) SETs
- Resonator mechanically compliant voltage gate

- Current through SET measures resonator position sensitively:
  - Force sensing applications
  - Quantum limited displacement detection

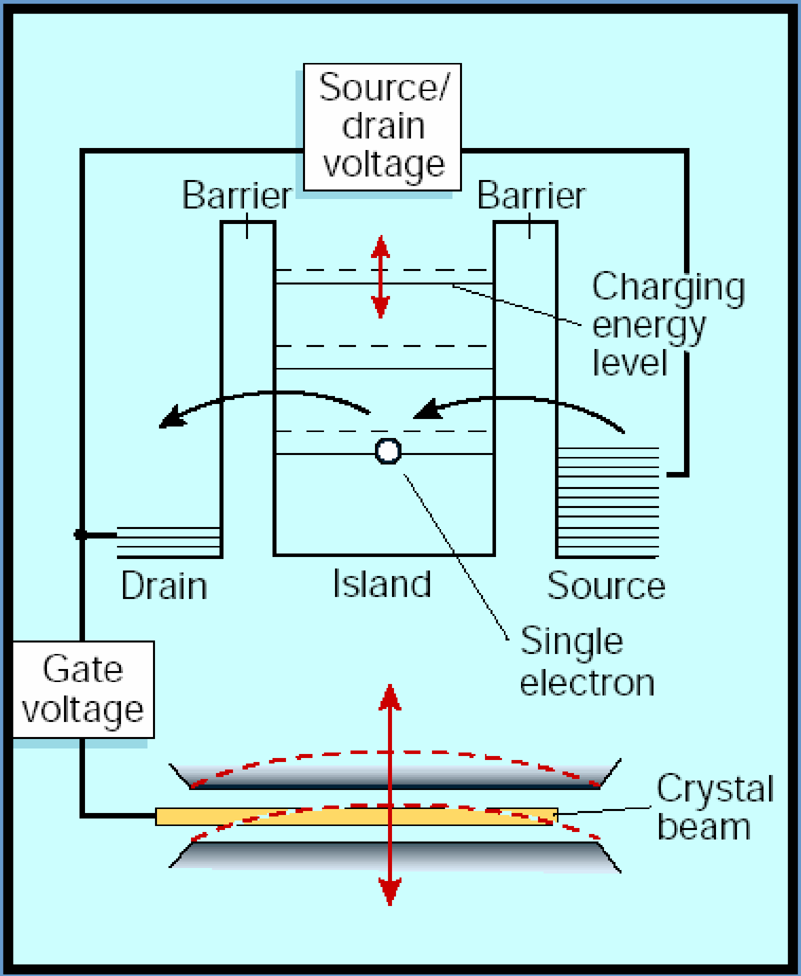
- *Resonator-SET coupled dynamics complex and interesting*

Pictures from Schwab and Cleland groups: Science **304**, 74; Nature **424**, 291

# Displacement detection with SET

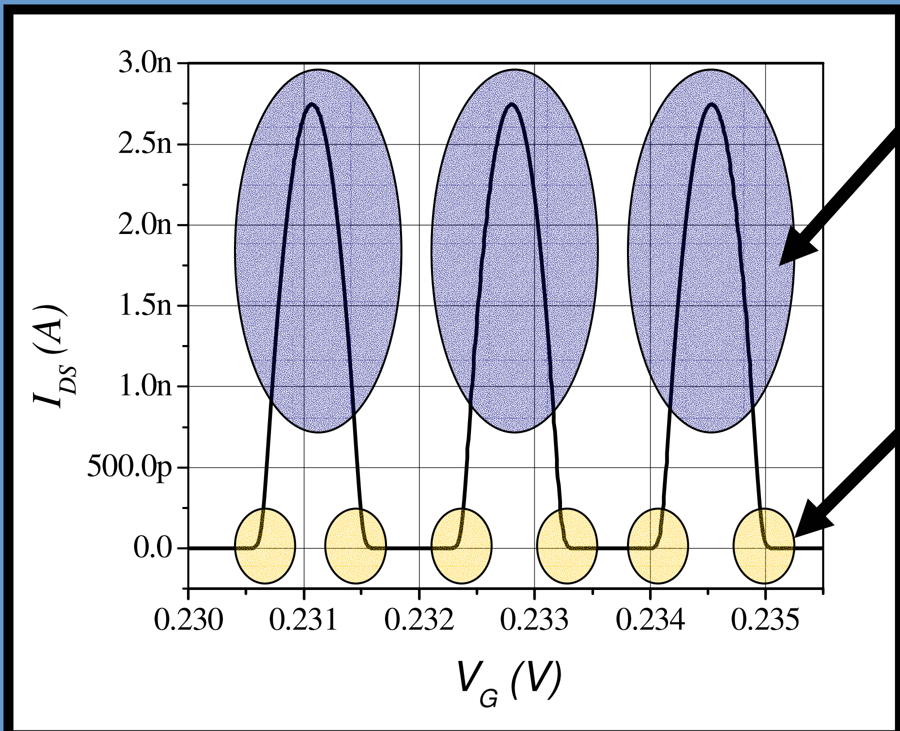
- Motion of resonator modulates effective gate voltage
- SET suggested as ultra-sensitive displacement detector
  - White Jap. J. Appl. Phys. Pt2 32, L1571
  - Blencowe and Wybourne APL **77, 3845**
- Possibility of reaching quantum limited-displacement detection

- fluctuations in island charge act back on resonator causing heating



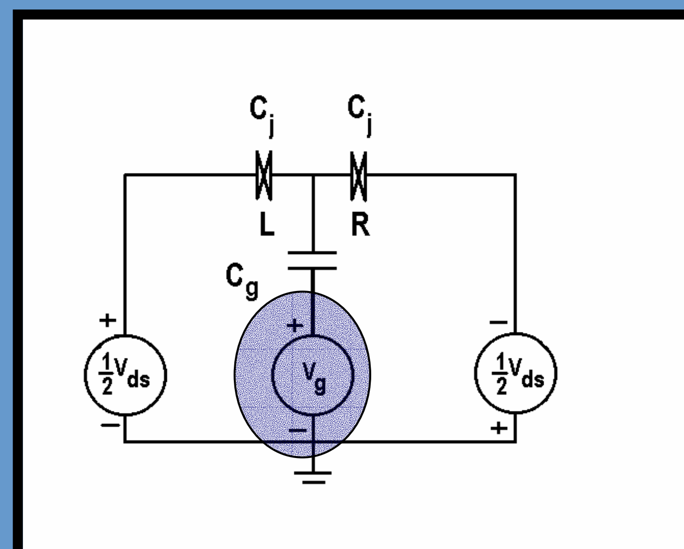

Blencowe, Nature (N&V) **424**, 262

# SET essentials



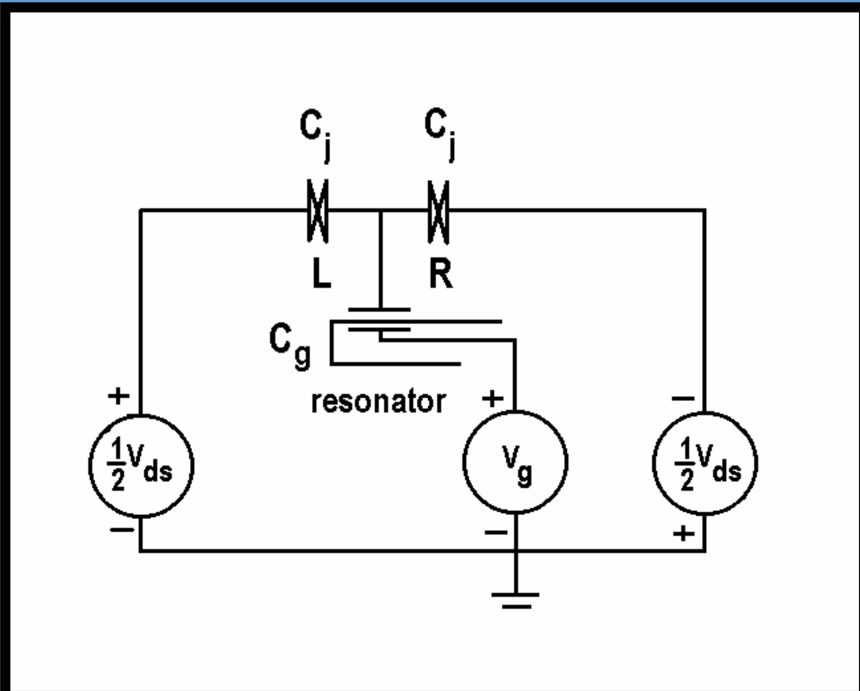

Sequential Tunnelling: Orthodox model

Co-Tunnelling: Quantum regime




Electron tunnelling follows 'classical' rate laws controlled by electrostatic energy differences except low current regions

# Resonator-SET Model System




Resonator-SET island distance: $d$>> any motion of resonator

- Work in regime where co-tunnelling negligible

- Resonator acts as movable gate capacitor: electron tunnel rates depend on resonator position

- typical $V_{ds}$ such that:

$$eV_{ds} >> \hbar\omega_0$$

Resonator modelled as classical harmonic oscillator,

# Master Equation formulation

• Orthodox regime: charge dynamics modelled by master equations for charge number with simple tunnel rates
• At low-T only SET states with N and N+1 electrons accessible
• Include resonator by expanding $C_g$ to lowest order:

$$C_g(x) = C_g(1 - x/d)$$

• Generalise master equations to include resonator dynamics:

1. Electron tunnel rates depend on resonator position
2. Eqm. position of resonator shifted by $x_0$ for each electron added to SET

# Master Equations

Coupled equations for probability distributions:
$P_{N(N+1)}(x,u;t)$:

Mechanical part

Electron tunnelling

$$\frac{\partial P_N}{\partial t} = \omega_0^2 x \frac{\partial P_N}{\partial u} - u \frac{\partial P_N}{\partial x} + \frac{\partial}{\partial u}\left(\gamma_e u P_N + \frac{G}{2m^2}\frac{\partial P_N}{\partial u}\right) + \left(\Gamma_L P_{N+1} - \Gamma_R P_N - \frac{m\omega_0^2 x_0}{Re^2} x P\right)$$

$$\frac{\partial P_{N+1}}{\partial t} = \omega_0^2 (x - x_0) \frac{\partial P_{N+1}}{\partial u} - u \frac{\partial P_{N+1}}{\partial x} + \frac{\partial}{\partial u}\left(\gamma_e u P_{N+1} + \frac{G}{2m^2}\frac{\partial P_{N+1}}{\partial u}\right) - \left(\Gamma_L P_{N+1} - \Gamma_R P_N - \frac{m\omega_0^2 x_0}{Re^2} x P\right)$$

Back-action of electrons on resonator

Linear modulation of
Electronic tunnel rates

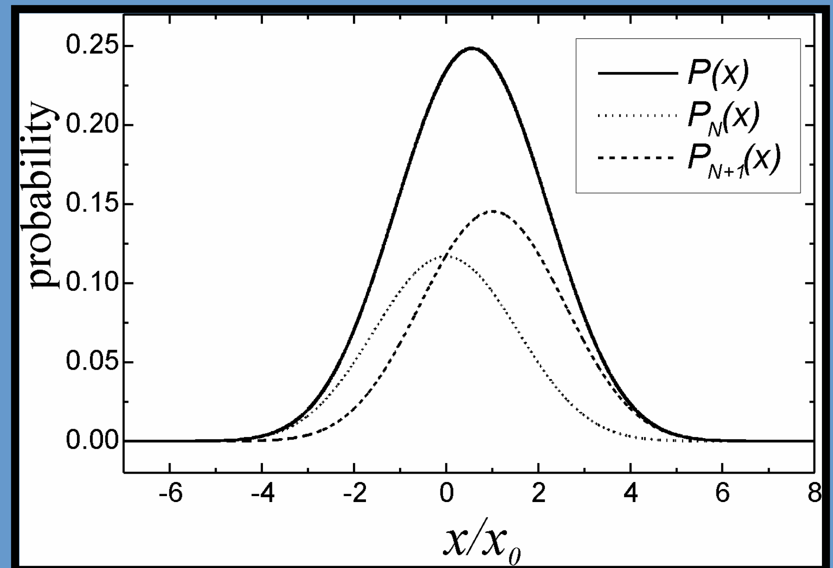
Characterise system by dimensionless parameters:

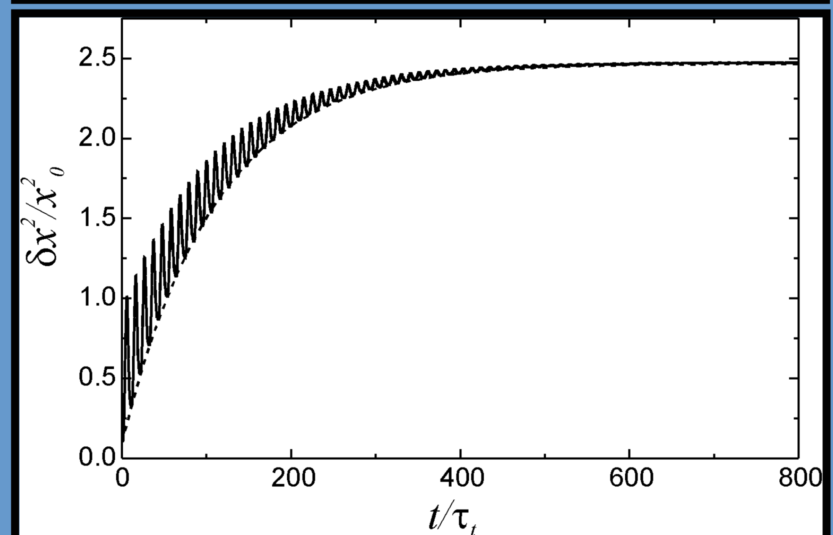
- SET-resonator coupling, $\kappa = m\omega_0^2 x_0^2/(Re^2)$
- resonator frequency $\varepsilon = \omega_0 \tau_t$

# Steady-state

- Steady-state probability distributions almost Gaussian: thermal state

- System reaches steady-state at rate given by an effective damping constant



# Resonator Dynamics: Intrinsic Effects

Undamped resonator at *T=0* coupled to SET: for κ,ε<<1

1. Resonator frequency renormalized by SET electrons:

$$\varepsilon' = (1-\kappa)^{1/2}\varepsilon$$

2. Resonator like damped harmonic oscillator with damping constant:

$$\gamma_i = \kappa\varepsilon^2$$

3. Steady-state like thermal state with *intrinsic* temperature:

$$k_B T_i = eV_{ds}\overline{P}_N\overline{P}_{N+1}$$

$\overline{P}_{N(N+1)}$ average probabilities of *N(N+1)* electrons on SET island.

Very similar to results obtained for quantum calculation of QPC

Mozyrsky and Martin PRL **89**, 018301

# Effects of temperature and damping

Include damping and temperature, $T_r$, due to resonator's coupling to substrate

Effective temperature:

$$\frac{T_{eff}}{Q_{eff}} = \frac{T_r}{Q_e} + \frac{T_i}{Q_i}$$

$$Q_{eff}^{-1} = Q_i^{-1} + Q_e^{-1}$$

$Q_i = \omega_0 / \gamma_i$: quality factor due to electron interactions
$Q_e$: quality factor due to substrate interactions

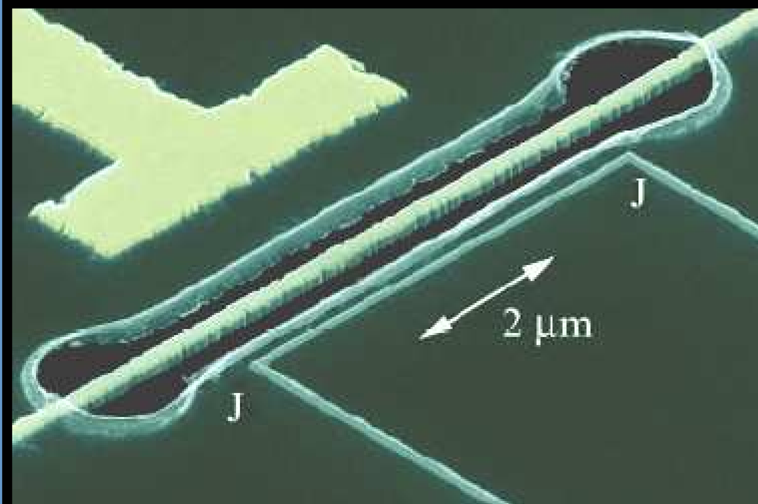
**In practice:** intrinsic temperature relatively large ~1K; intrinsic damping relatively small, $Q_i >> Q_e$

- Current noise: interesting and complex

AA, Blencowe and Zhang, PRB **69** 125313
AA cond-mat/0401387

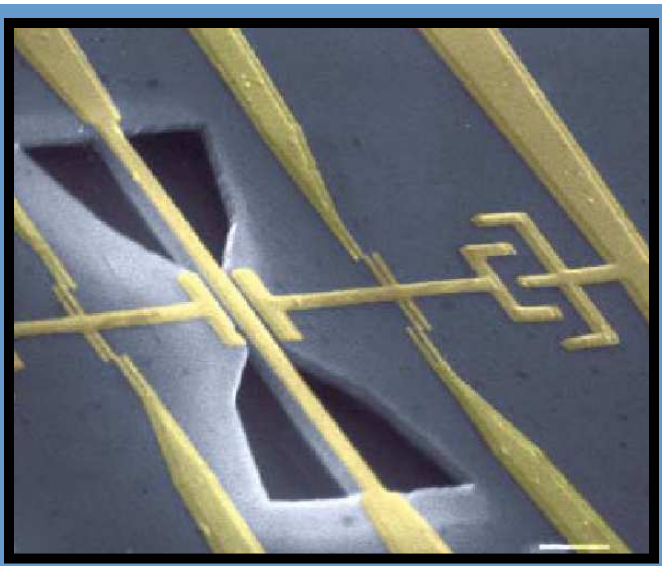
# Experimental progress

- Most recent experiments in Schwab group, LPS (UMD): LaHaye et al, Science **304**, 74
- Coupling between superconducting rfSET and 19.7MHz resonator
- Cooling to 56mK (=58 quanta) achieved



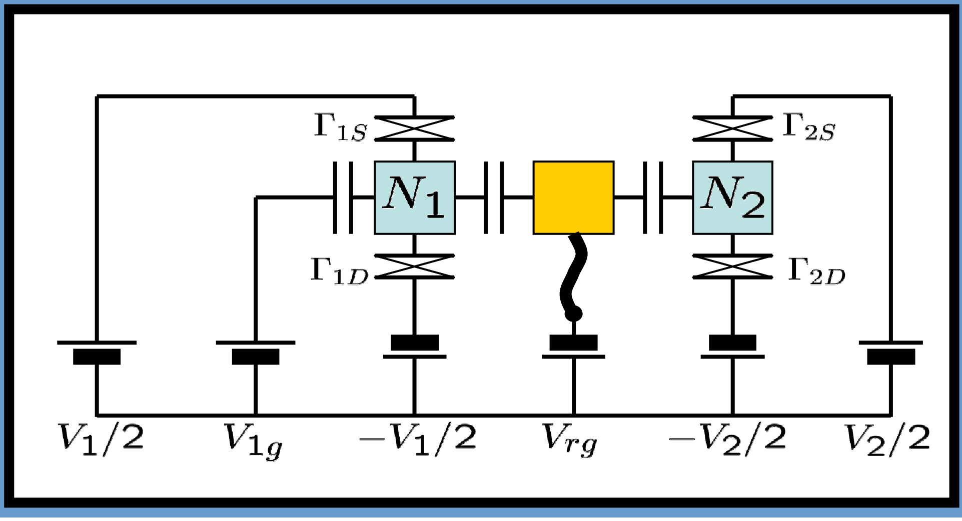

- Displacement sensitivity within a factor of 10 of quantum limit; resonator temp. set by heating from electronic circuitry
- Current experiments with tightly-coupled devices & normal state SETs to examine back-action effects

# Twin-SET and resonator



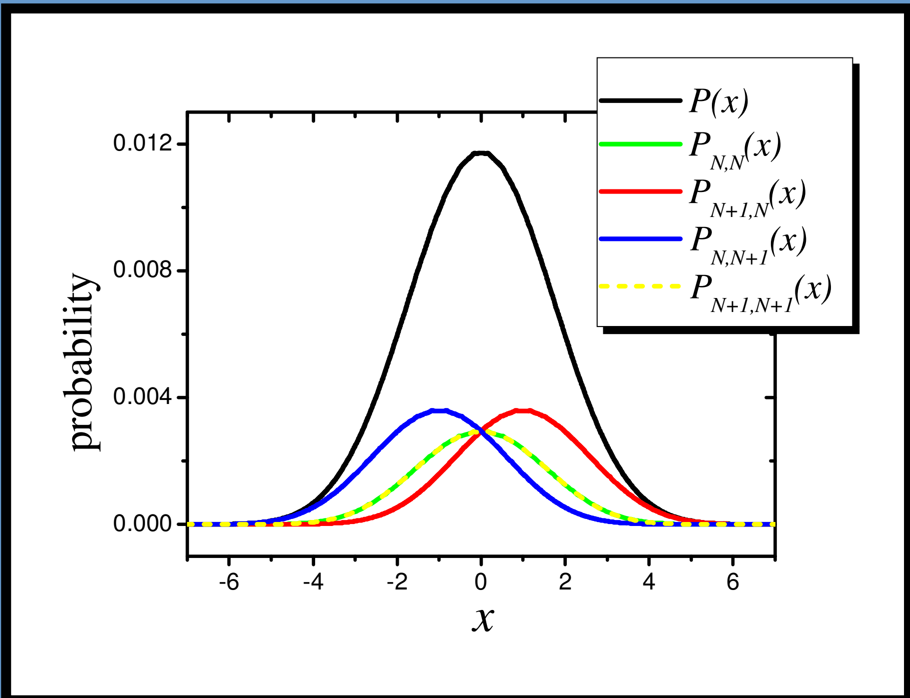
Cleland group, Science **299**, 36

- Can also make "twin-SET" devices
- Correlations between two conductors would allow more precise measurements




More complex equivalent circuit, but resonator behaviour still captured by thermal bath model in weak-coupling limit

# Twin-SET: Resonator dynamics




1 SET: $$\frac{\delta x^2}{x_0^2} = \frac{1}{4}\frac{(1-2\kappa)}{\kappa(1-\kappa)}$$

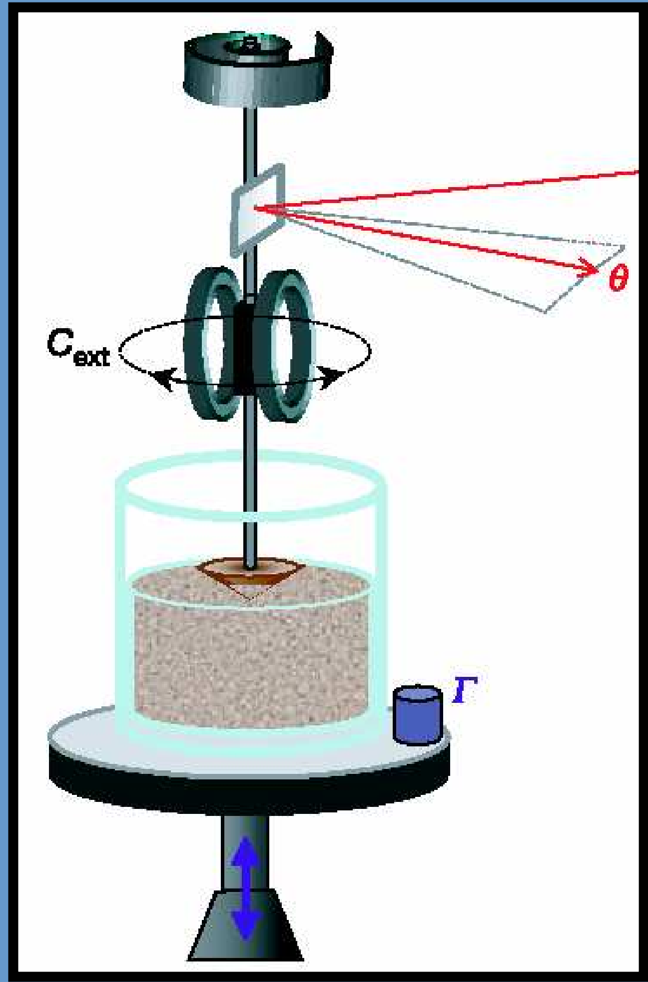
2 SET: $$\frac{\delta x^2}{x_0^2} = \frac{1}{4}\frac{1}{\kappa(1-\kappa)}$$

Ratio: $$R_{2/1} = \frac{(1-\kappa)}{(1-2\kappa)}$$

- Adding a second SET with identical properties hardly alters state of resonator: effective temperature unchanged
- In general, second SET acts like an additional thermal bath

# Comparison with other systems

- Several other non-equilibrium systems shown to map onto thermal bath model:
  - gas-fluidized particle, i.e. sphere placed on fine screen in gas stream [Ojha et al., Nature **427**, 521]
  - Torsional wire immersed in vibration-fluidized granular matter [D'Anna et al., Nature **424**, 909]
- Features in common:
  - Separation of slow and fast time-scales
  - Large number of non-equilibrium particles
- Generality of mapping remains open question




D'Anna et al., Nature **424**, 909

# Conclusions

- Nanoelectromechanical systems combine a range of interesting physics with potential device applications

- Dynamics of resonator-SET system rich
  - SET acts as effective thermal bath
  - Overall state of resonator determined by competetion between intrinisc & extrinsic effects
- Thermal bath model extends to Twin-SET and resonator
- Experiments in progress to test validity of theoretical approaches [Schwab group,UMD]