Quantum Hall Breakdown
a microscopic model and a hydrodynamic analogy

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Talk Outline

Breakdown of the Integer Quantum Hall Effect (IQHE).

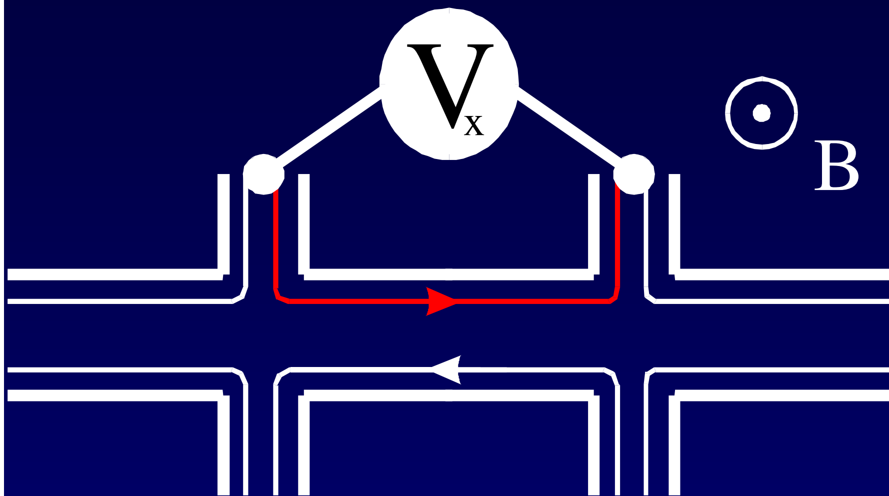
(1) Experimental observations

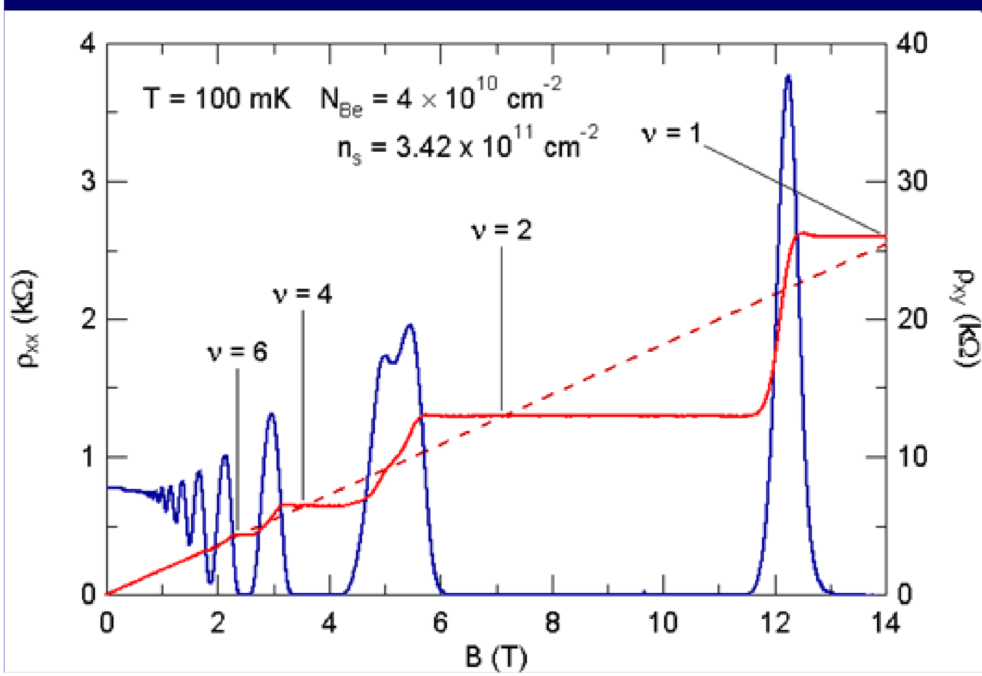
(2) Microscopic model of the breakdown of the IQHE

(3) Fluid analogy

(4) Future work

Observations of Quantum Hall Breakdown



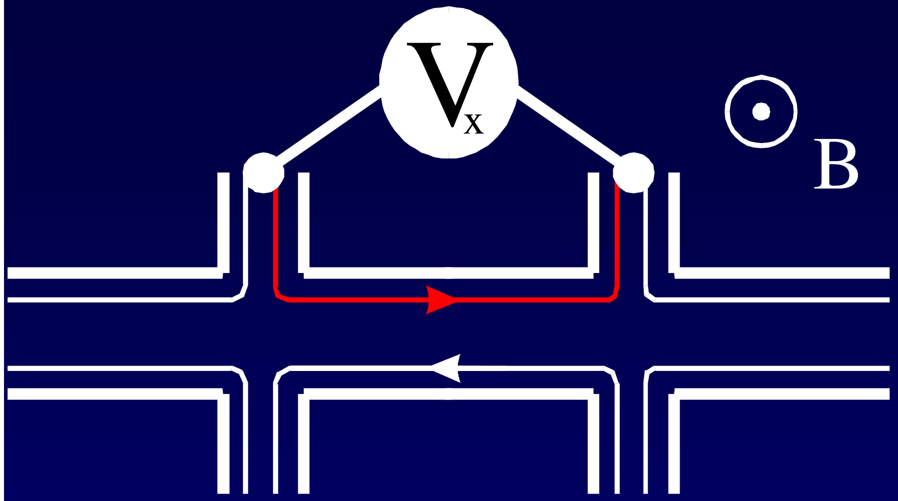


IQHE - almost dissipationless flow of current along sample

- $V_x \cong 0$

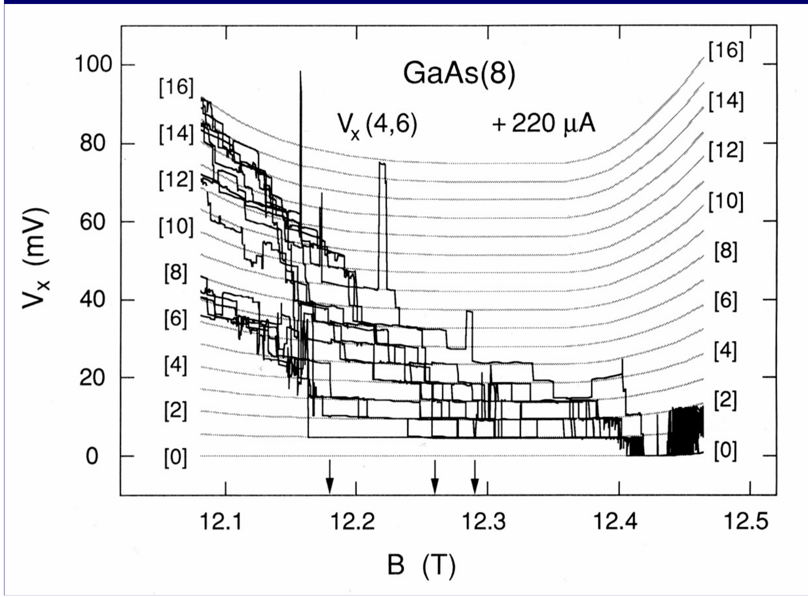
- Hall resistance quantised, $R_H = h/\nu e^2$

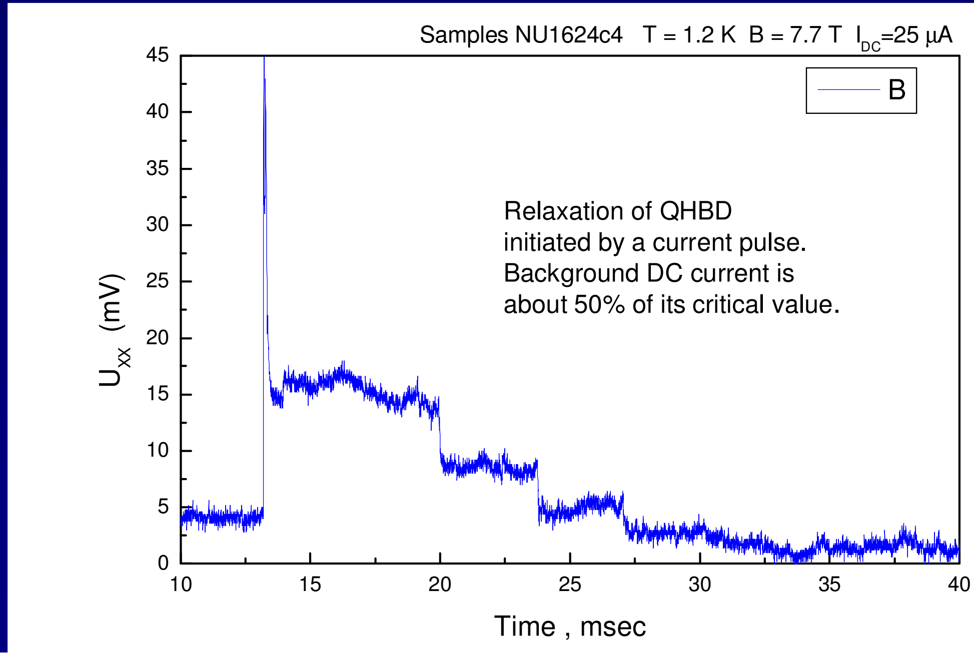
Observations of Quantum Hall Breakdown



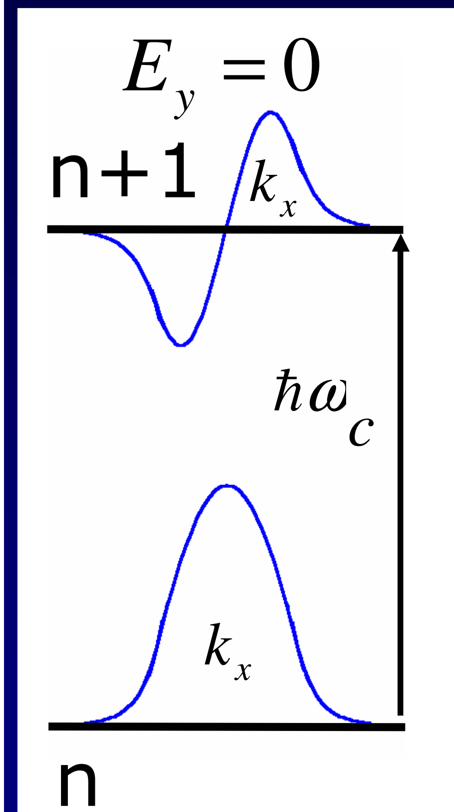
Above a critical current the flow is no longer dissipationless.

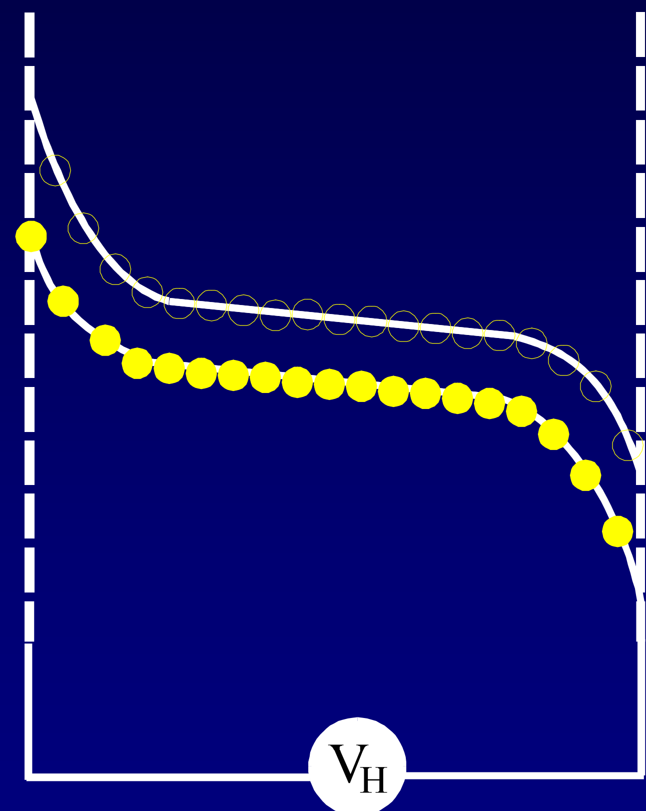
NIST Experiment;
US Resistance Standard

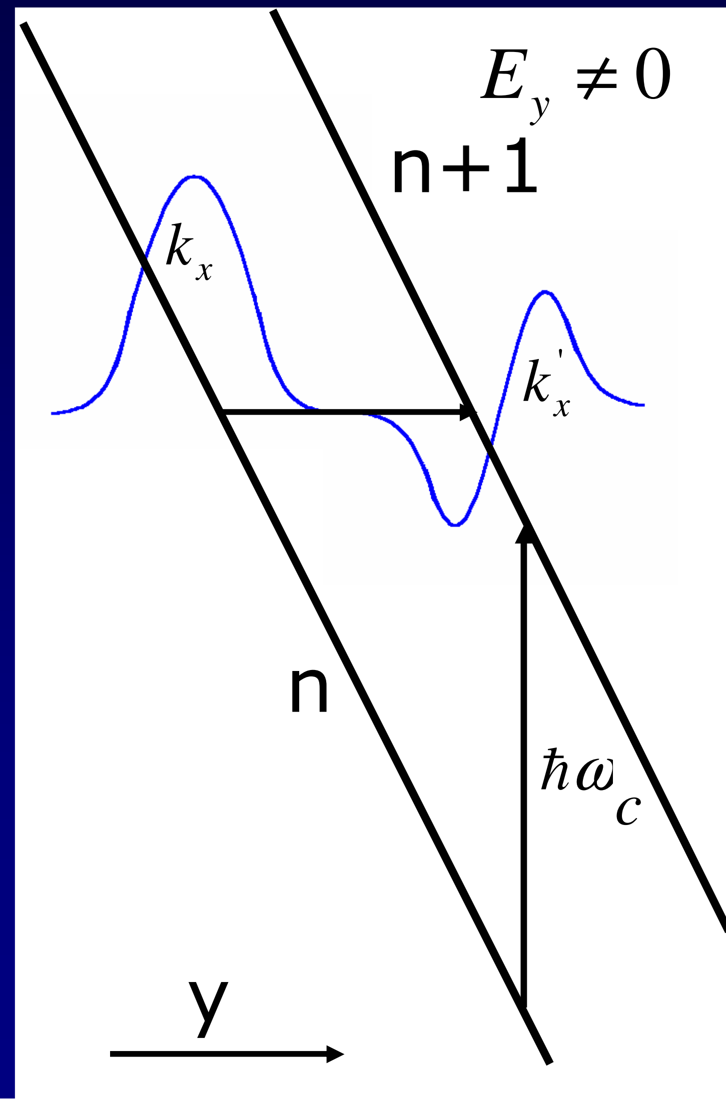




Quantum Picture of Generation of Electron-Hole Pairs







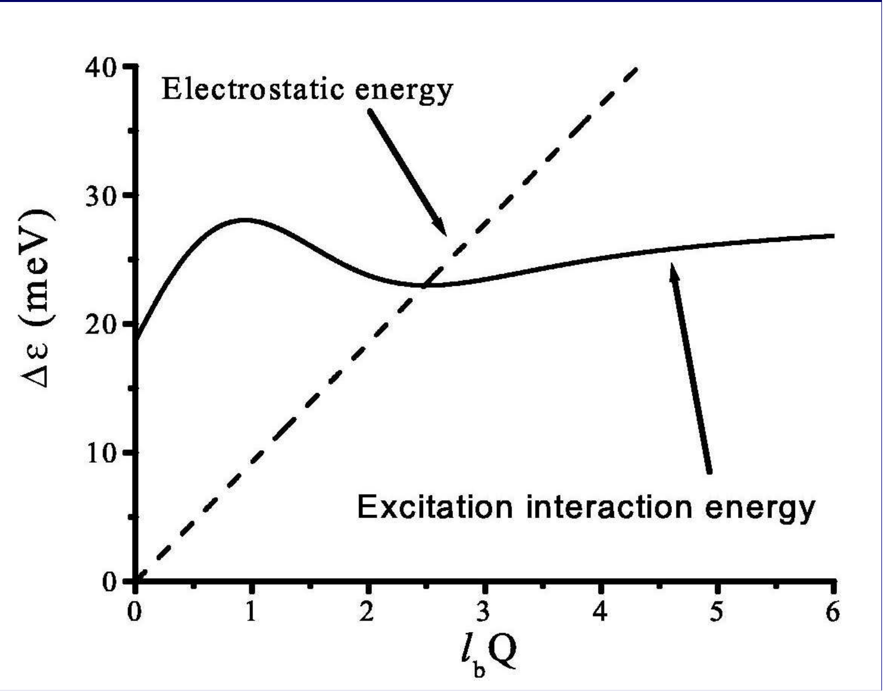
Quantum Picture of Generation of Electron-Hole Pairs

Magneto-exciton - electron in *(n+1)*th Landau level bound to hole in *n*th Landau level;
Momentum, ***q***, is good quantum number;
Electric Dipole $e l_c^2 \mathbf{q}$;
Dispersion relation:

$$\varepsilon(\mathbf{q}) = \hbar\omega_c + \frac{e^2}{4\pi\varepsilon_0 \kappa\, l_c}\Delta_{n,(n+1)}(\mathbf{q})$$

$$\omega_c = \frac{eB}{m^*} \qquad l_c = \sqrt{\frac{\hbar}{eB}} = \sqrt{\frac{\hbar}{m^*\omega_c}}$$

Lerner and Lozovik (1980), Kallin and Halperin (1984), MacDonald (1985).



Quantum Picture of Generation of Electron-Hole Pairs

Spontaneous Magnetoexciton creation rate due to an ionised impurity- Golden rule

$$W = \frac{2\pi}{\hbar}\sum_{\mathbf{q}}\left|\left\langle \mathbf{q}\right|V_{imp}\left(r\right)\left|0\right\rangle\right|^2 \delta\left(\varepsilon\left(q\right) - eE_y l_c{}^2 q_x\right)$$

|**q**> is a 1-magneto-exciton state

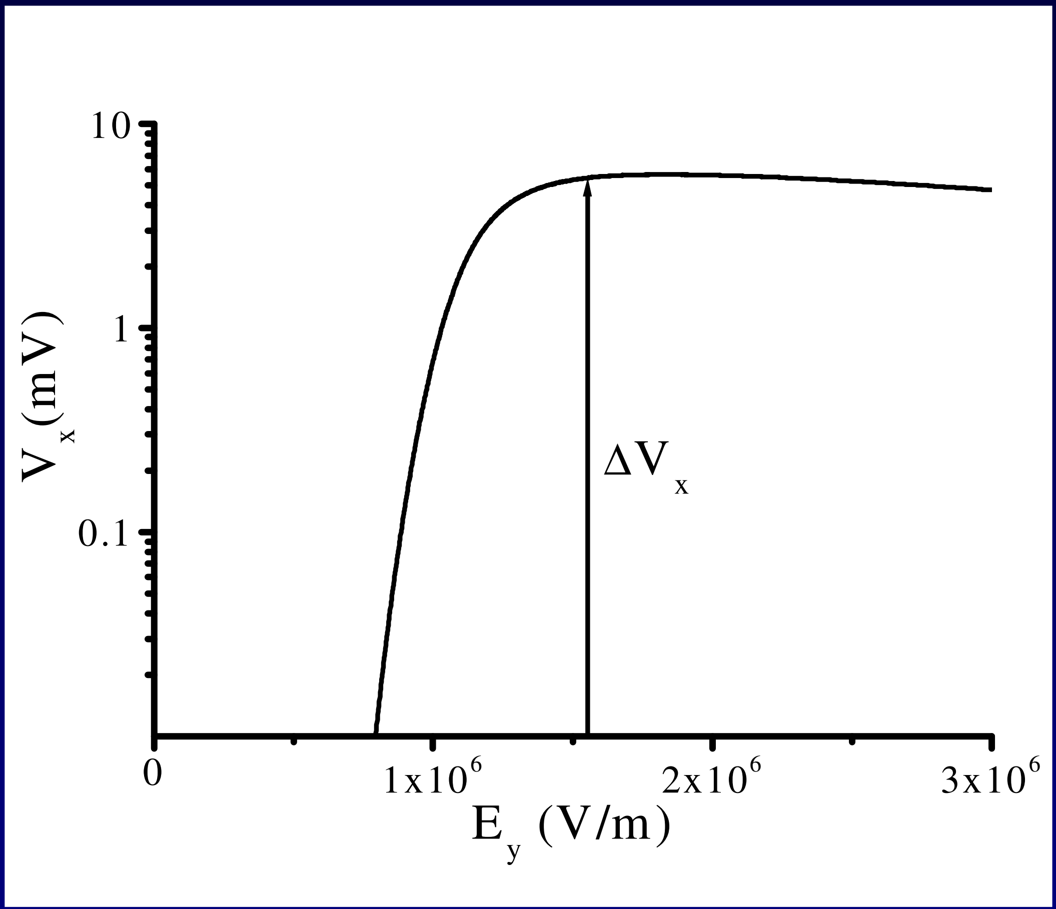
$V_{imp}(r)$ impurity potential, averaged over vertical sub-band wavefunction.

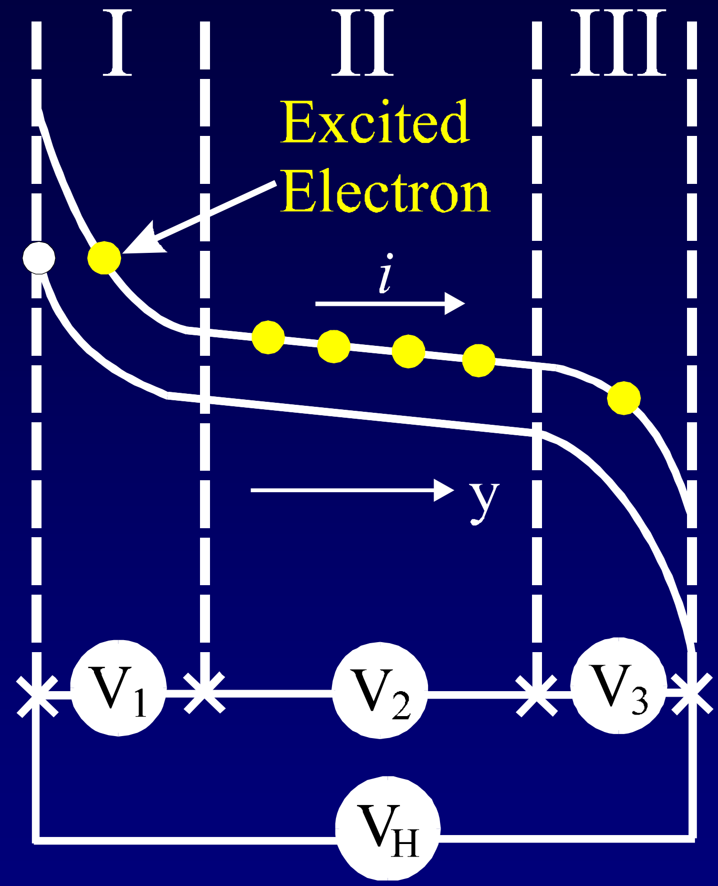
Quantum Picture of Generation of Electron-Hole Pairs

Close to a suitably located impurity, local electric field is enhanced;
impurity potential enables spontaneous creation of magneto-excitons;

Rate of generation, *W*, leads to a net flow of charge across device, $i=eW$, and hence a dissipative voltage, $V_x=hW/\nu e$.

Final Quantum Result





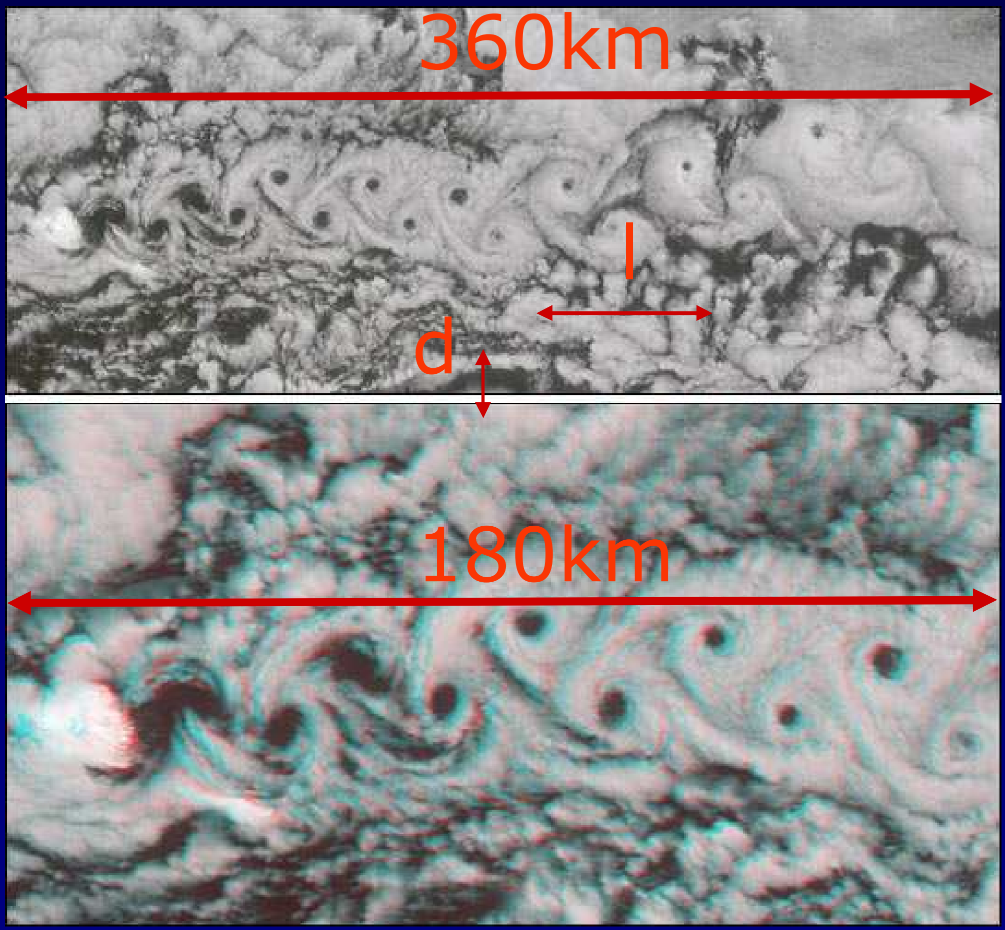
$\nu = 2 \qquad V_x = hW/2e$

$$\Delta V_x = 5.6\text{mV}$$

Also see agreement between theory and experiment for step heights in hole gas devices with *$\Delta V_x \cong 1$mV.*

An analogy with hydrodynamics: Von-Karman Vortex Street

$$d / l = 0.28$$



Fluid Model of the Breakdown of the IQHE

$$m^*\left[\dot{\mathbf{v}} - \left[\mathbf{v}\times\boldsymbol{\Omega}\right]\right] = e\left[\mathbf{E} + \mathbf{v}\times\mathbf{B}\right] - \nabla\left(\frac{m^*}{2}\left|\mathbf{v}\right|^2 + \mu\right)$$

$$m^*\Omega + eB = 2\pi\rho$$

M. Stone, Phys Rev. B **42**, 212 (1990)

$$\dot{\rho} + \nabla\cdot\left(\rho\,\mathbf{v}\right) = 0$$

Continuity Equation

$$v_x = \frac{E_y}{B} + \varepsilon_1\cos\left(Qx - \omega t\right) \qquad v_y = \varepsilon_2\sin\left(Qx - \omega t\right)$$

Excitation

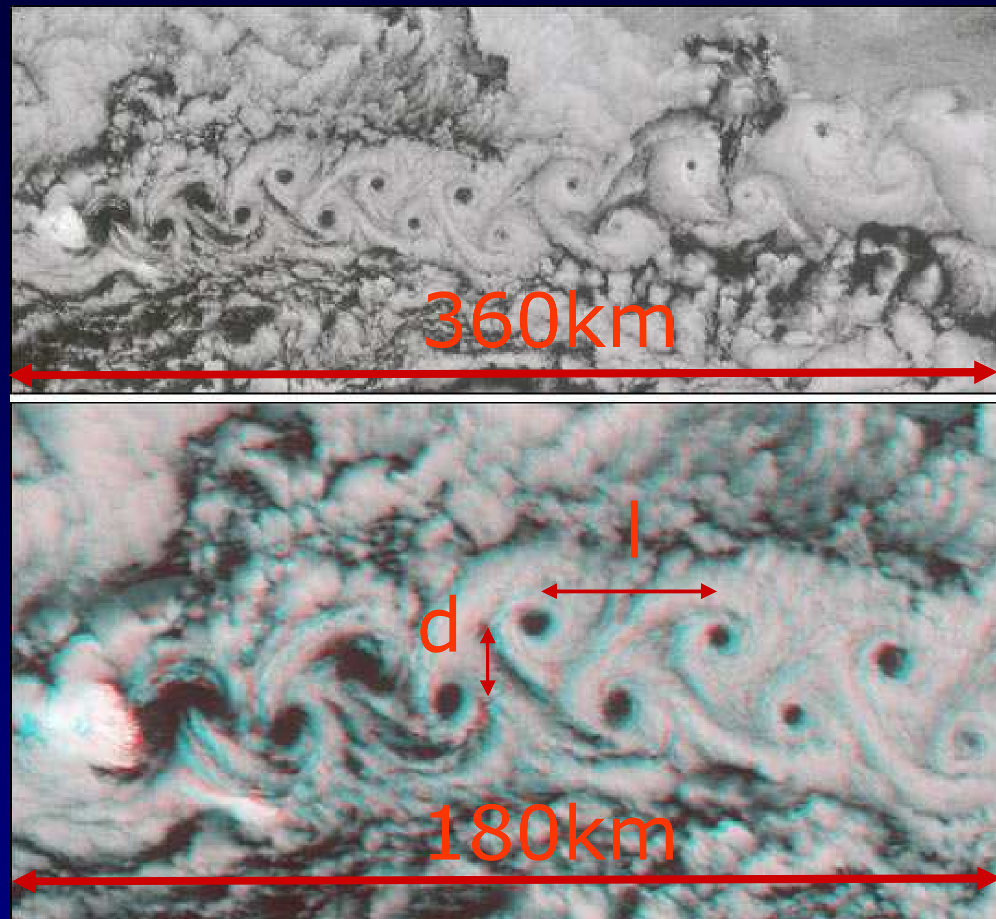
$$\hbar\omega = \hbar\omega_c + \lambda Q^2 + \ldots - eE_y l_B\left(l_B Q\right)$$

Energy Dispersion for excitation

$$\Delta E = \hbar\omega_c + \frac{e^2}{4\pi\kappa\, l_B}\Delta_{n,(n+1)}\left(\mathbf{q}\right) - eE_y\, l_B^2\left(q_x\right) = 0$$

Quantum Result

Von-Karman Vortex Street



$$d/l = 0.28$$

$$v_x = E_y / B$$

$$V_x = \frac{0.28\pi\hbar\omega_c}{e(l_B Q)^2}$$

$$l_B Q = 1.9$$

$$\hbar\omega = \Delta\varepsilon = \hbar\omega_c - eE_y l_B (l_B Q)$$

$$\Delta V_x = 4.9\text{mV}$$

Future work

1. More sophisticated treatment of the electrostatics within device;

2. More careful treatment of fluid analogy - Bogoliubov type analysis of Composite Bosons in a field.

3. Analysis of more complex experimental geometries

4. Low temperature scanning probe microscopy of quantum Hall devices under breakdown conditions (CJ Mellor).

Future work

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Tunneling between edge states in the quantum Hall regime limited by a mesoscopic island: A current-plateau phenomenon

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