Luttinger Liquid at the Edge of a Graphene Vacuum

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I. Introduction: Graphene Edge States (Non-Interacting)

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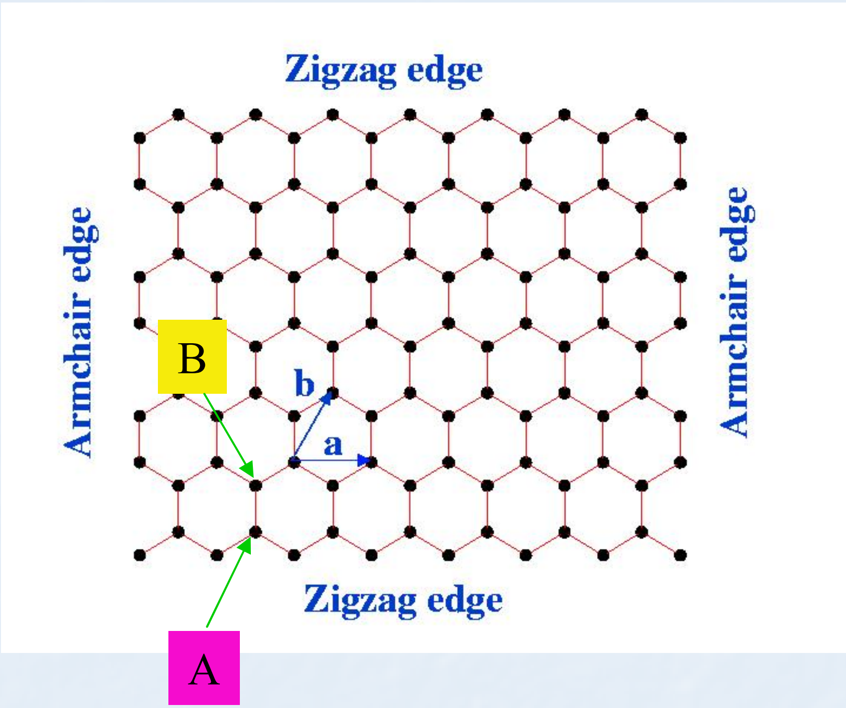
(with Drew Iyengar and Jianhui Wang)

V. Summary

Funding: NSF

I. Edge States for Graphene



Honeycomb lattice, two atoms per unit cell
Lattice constant: 2.46Å
Nearest neighbor distance: 1.42Å

Simple tight-binding model for p_z orbitals:

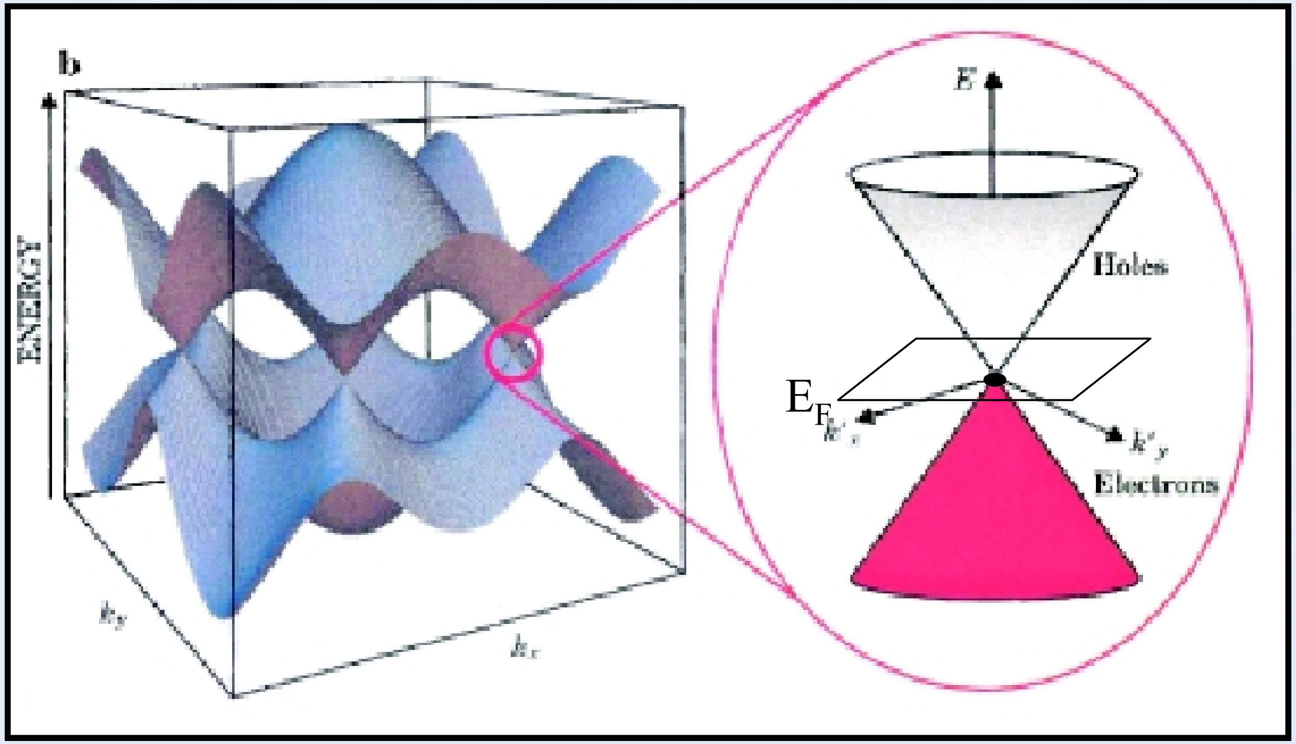
$$H = -t \sum_{n_1 n_2 = n.n.} |n_1\rangle\langle n_2|$$

$$t \approx 2.5\text{-}3 \text{ eV}$$

$$-t\begin{pmatrix} 0 & 1+2\cos\frac{k_x a_0}{2} e^{i\frac{\sqrt{3}}{2}k_y a_0} \\ 1+2\cos\frac{k_x a_0}{2} e^{-i\frac{\sqrt{3}}{2}k_y a_0} & 0 \end{pmatrix}\begin{pmatrix}\psi_A \\ \psi_B\end{pmatrix} = \varepsilon\begin{pmatrix}\psi_A \\ \psi_B\end{pmatrix}$$

A and B sublattice sites in unit cell

- For each **k** there are eigenvalues at ±|ε| ⇒ particle-hole symmetry
- Fermi energy at ε=0



Wavefunctions in a magnetic field:

$$\Psi(K,n) = e^{ik_x x}\begin{pmatrix} \pm\phi_{n-1}(y-k_x\ell^2) \\ \phi_n(y-k_x\ell^2) \end{pmatrix} \qquad \Psi(K',n) = e^{ik_x x}\begin{pmatrix} \pm\phi_{n}(y-k_x\ell^2) \\ \phi_{n-1}(y-k_x\ell^2) \end{pmatrix}$$

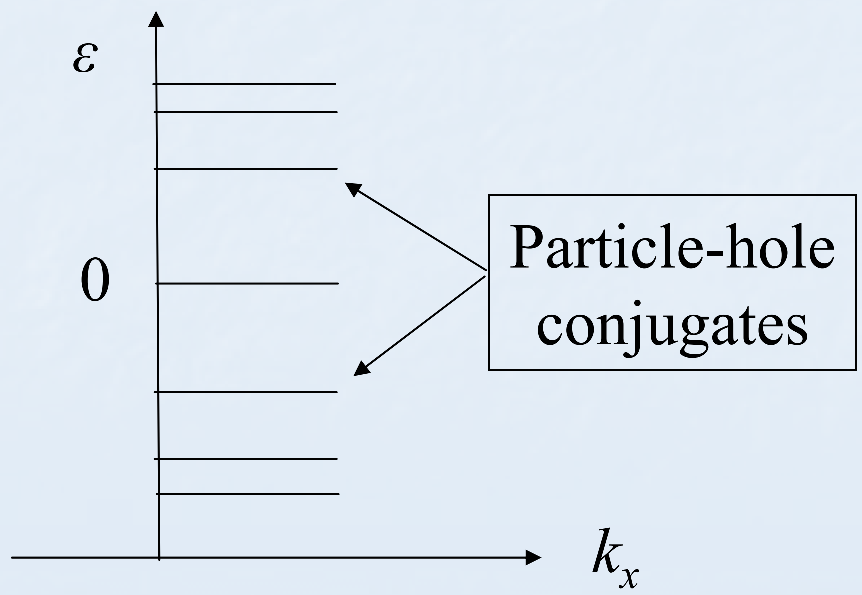
$$\Psi(K,0) = e^{ik_x x}\begin{pmatrix} 0 \\ \phi_0 \end{pmatrix} \qquad \Psi(K',0) = e^{ik_x x}\begin{pmatrix} \phi_0 \\ 0 \end{pmatrix}$$

$$\phi_n = \text{harmonic oscillator state}$$

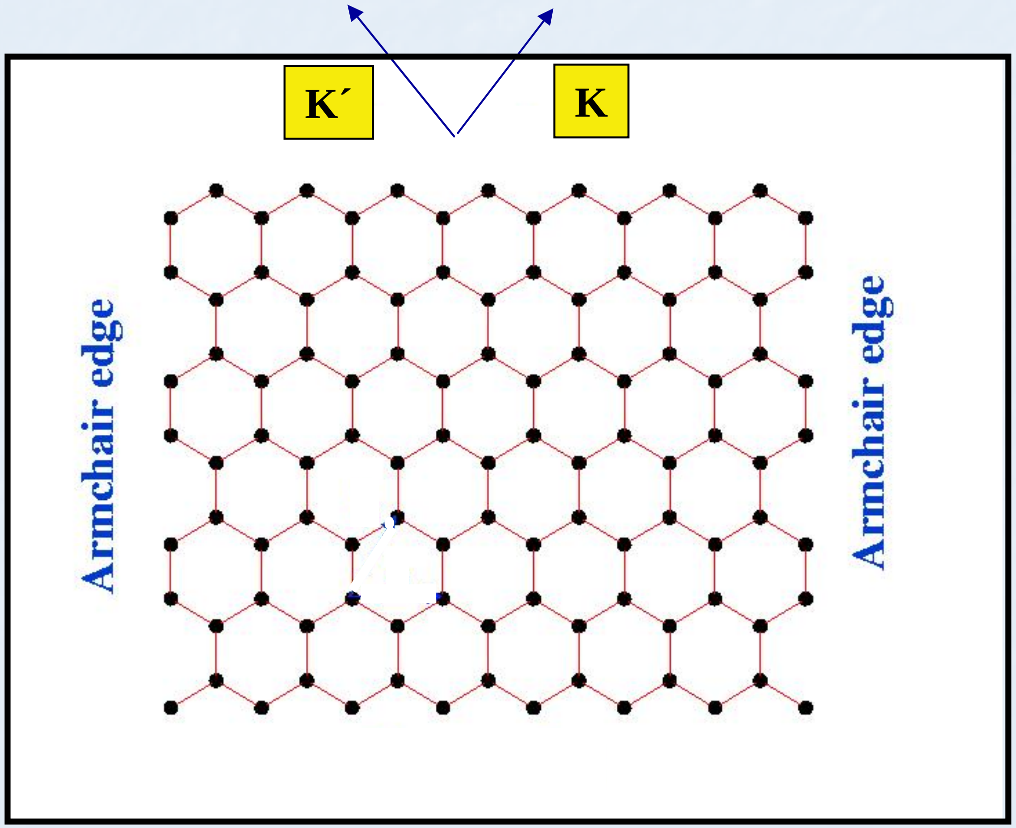
Energies:

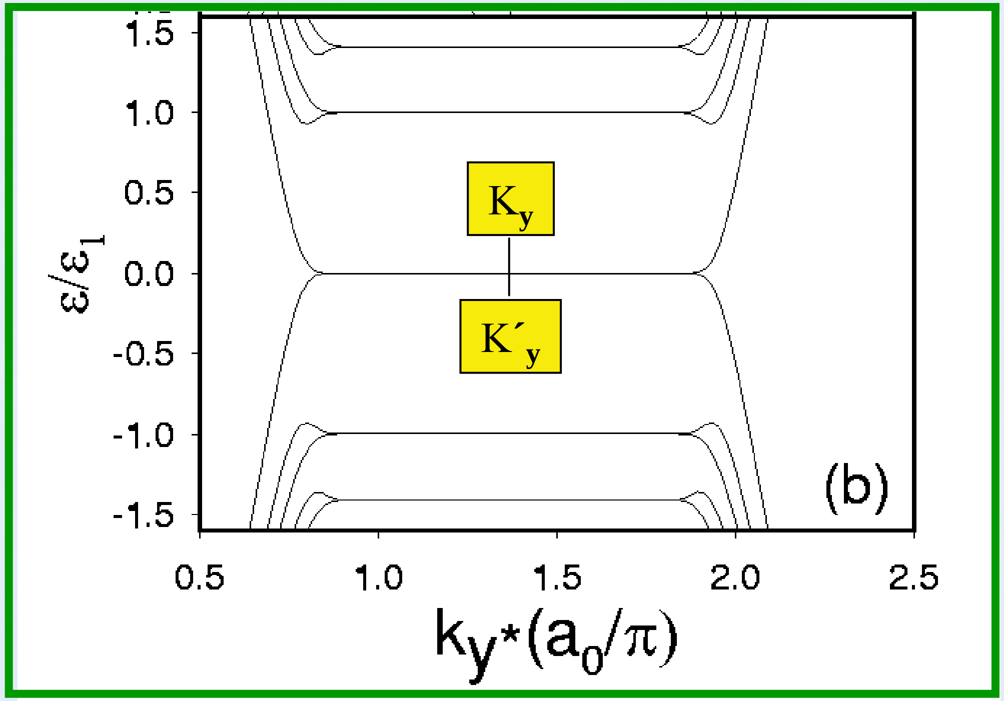
$$\boxed{\varepsilon(\tau,n) = \pm\sqrt{3\,n}\,\frac{a}{\ell}t}$$

With valley and spin indices, each Landau level is 4-fold degenerate



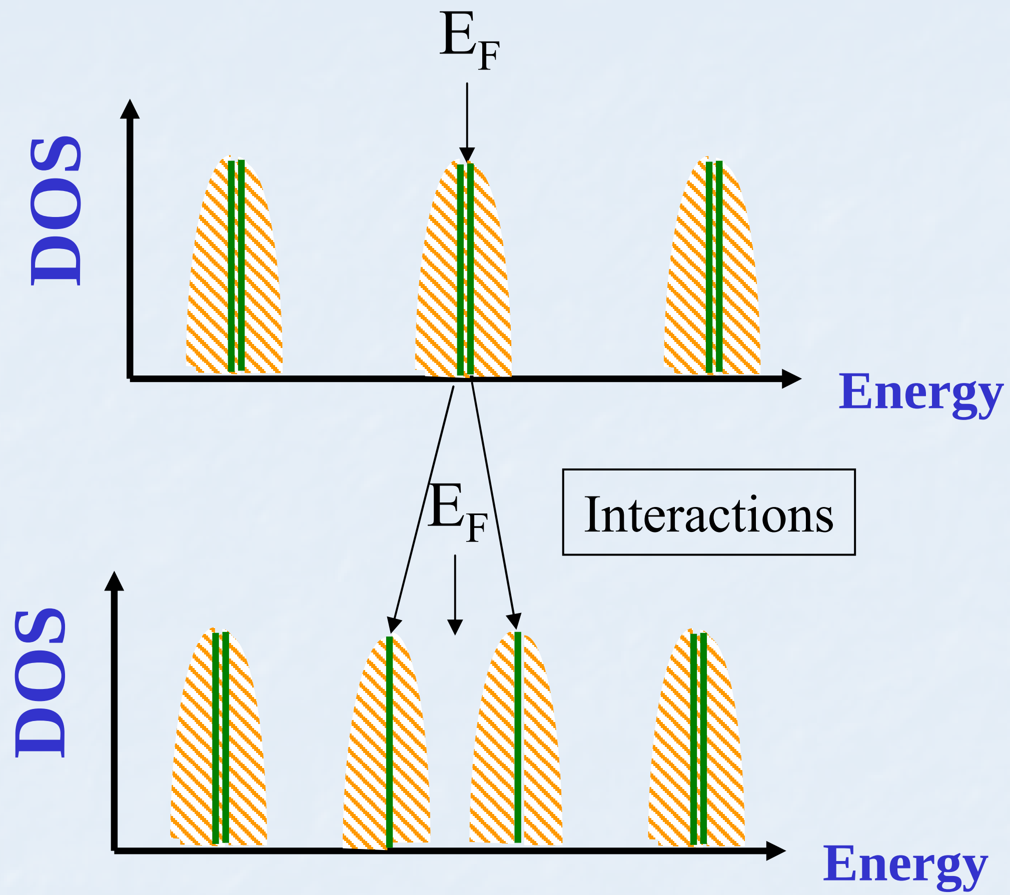
- Real samples in experiments are very narrow (.1-1μm) ⇒ edges can have a major impact on transport
- Can get a full description of QHE within Dirac equation
- Edge structure can be probed directly via STM at very small length scales. Nothing comparable is possible in standard 2DEG's (GaAs samples, Si MOSFET's)





Tight-binding results, armchair edge

II. Quantum Hall Ferromagnetism and the Graphene Edge



- Exchange tends to force electrons into the same level even when bare splitting between levels is small
- Renormalizes gap to much larger value than expected from non-interacting theory (even if bare gap is zero!)

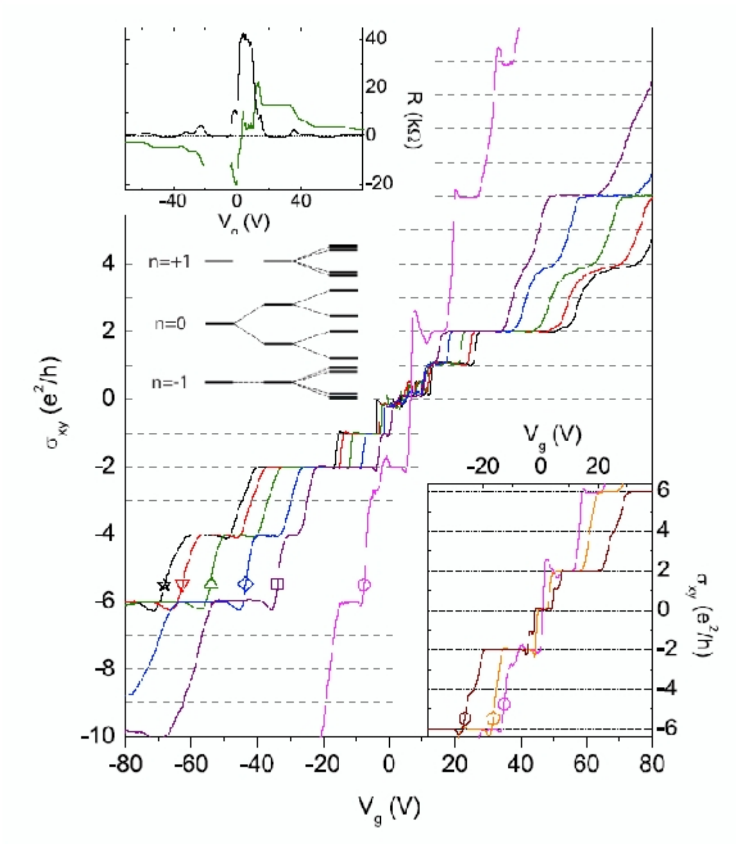


FIG 2. (color online) σ_{xy}, as a function of V_g at different magnetic fields: 9 T (circle), 25 T (square), 30 T (diamond), 37 T (up triangle), 42 T (down triangle), and 45 T (star). All the data sets are taken at $T = 1.4$ K, except for the $B = 9$ T curve, which is taken at $T = 30$ mK. Left upper inset: R_{xx} and R_{xy} for the same device measured at $B = 25$ T. Left lower inset: a schematic drawing of the LLs in low (left) and high (right) magnetic field. Right inset: detailed σ_{xy} data near the Dirac point for B = 9 T (circle), 11.5 T (pentagon) and 17.5 T (hexagon) at $T = 30$ mK.

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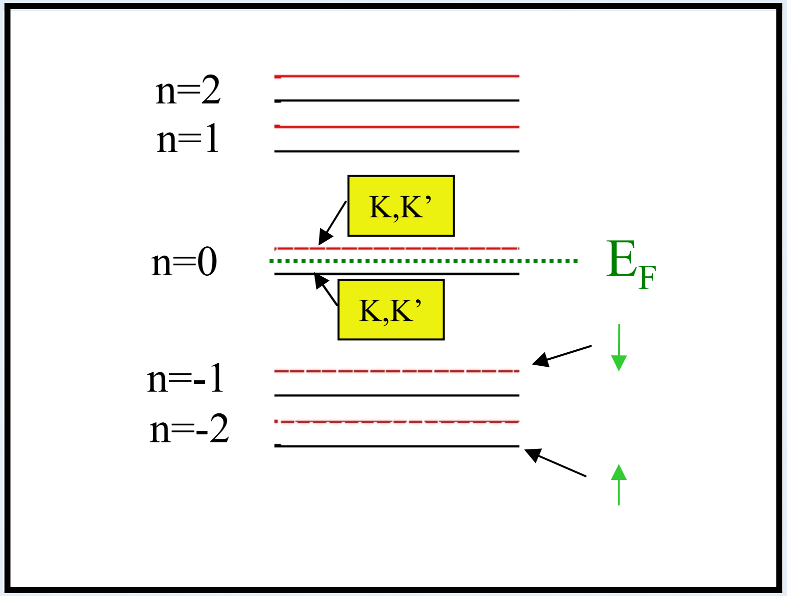
This does happen in graphene (Zhang et al., 2006).

- Plateaus at ν=0?,±1.
- System may be a *quantum Hall ferromagnet.*

cf. Alicea and Fisher, 2006
Nomura and Macdonald, 2006
Fuchs and Lederer, 2006

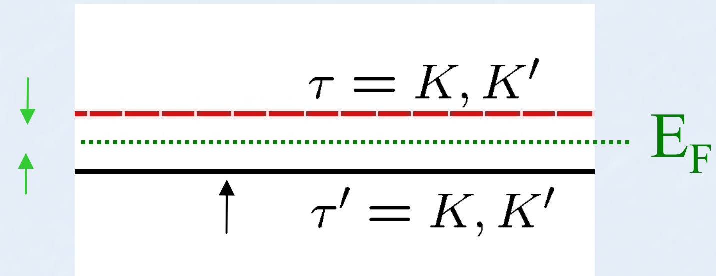
"Vacuum" state (undoped graphene):



$$|\mathrm{Vac}>=\prod_X C^\dagger_{K\uparrow X}\, C^\dagger_{K'\uparrow X}|n<0\rangle$$

Low-lying excitations: 2 (+2) spin (+valley) waves

$$|\mathbf{q}>=\rho_{\tau,\tau'}(\mathbf{q})|\mathrm{Vac}>$$



$$\rho_{\tau,\tau'}(\mathbf{q}) = \frac{1}{N_\phi}\sum_X e^{-\frac{i}{2}q_x(2X+q_y)} C^+_{\tau\downarrow X} C_{\tau'\uparrow X+q_y}$$

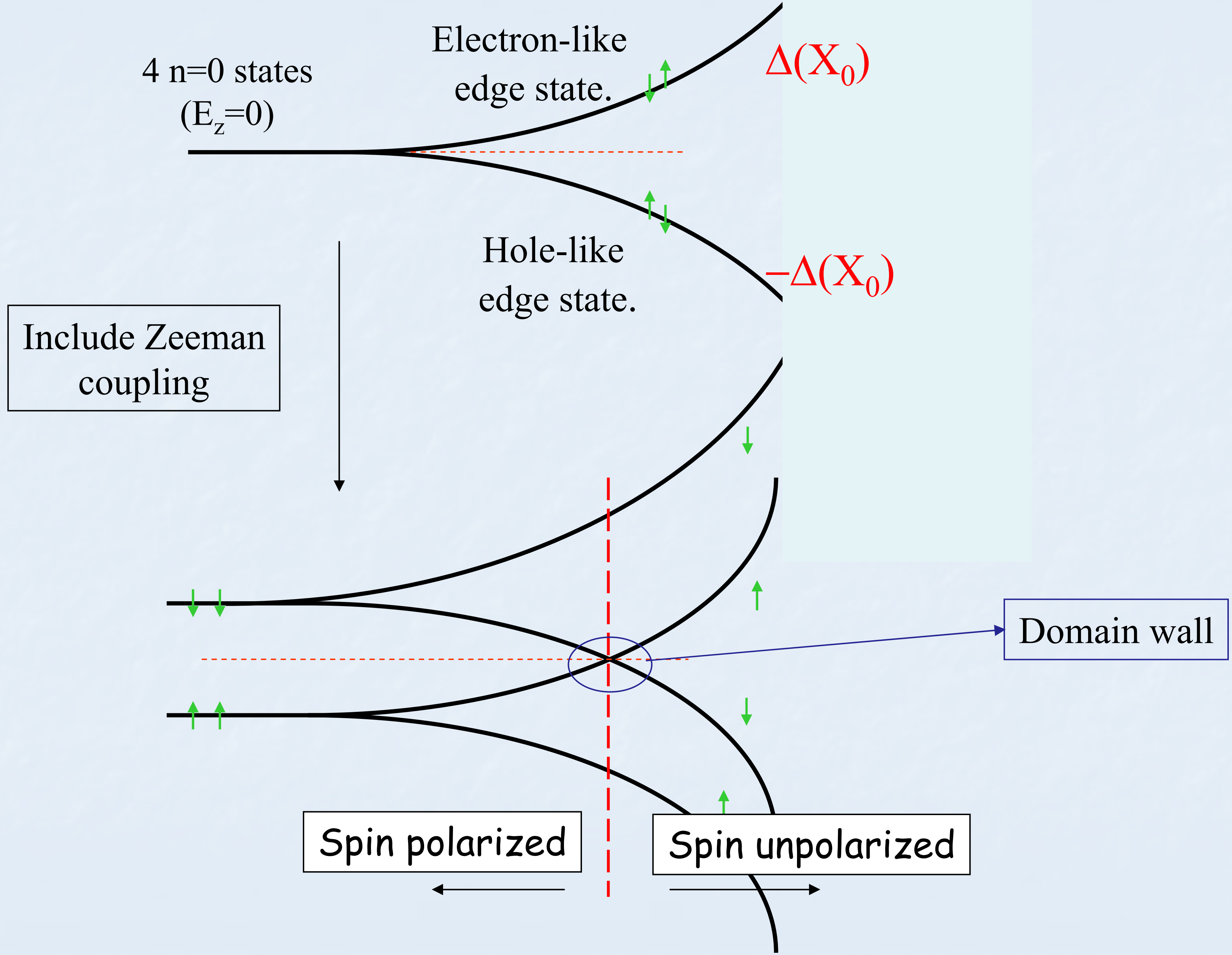
$$\boxed{\omega_0(q) \approx 2E_z + 4\pi\rho_s q^2\ell^2}$$

$$\boxed{\rho_s = 1/16\sqrt{2\pi}\frac{e^2}{\epsilon_0\ell}}$$

Spin stiffness

$$\boxed{\Rightarrow \text{Analogy with Heisenberg ferromagnet.}}$$

Consequences for edge states:



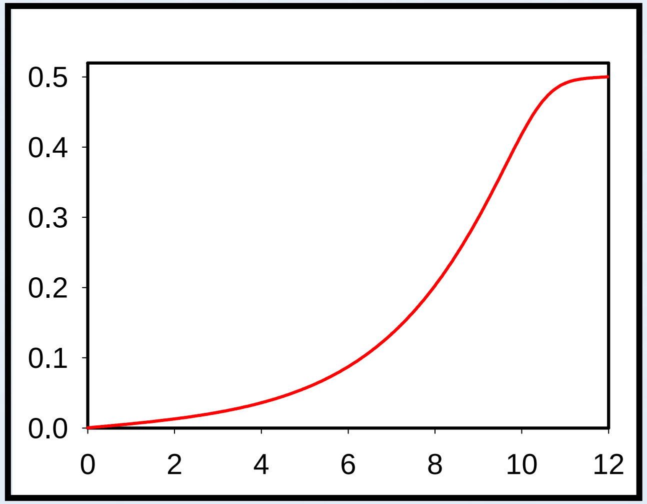
Description of the domain wall:

$$\boxed{|\Psi\rangle = \prod_{X_0<L}\left[\cos\frac{\theta(X_0)}{2}C^{+}_{+,X_0,\uparrow} + \sin\frac{\theta(X_0)}{2}e^{i\varphi}C^{+}_{-,X_0,\downarrow}\right]C^{+}_{-,X_0,\uparrow}|0>}$$

$$X_0 \to -\infty \quad \theta = 0; \quad X_0 \to L \quad \theta = \pi; \quad \varphi = 0$$

$$E = \pi\ell^2\rho_s \sum_{X_0<L}\left(\frac{d\theta}{dX_0}\right)^2 + \sum_{X_0<L}\big(E_z - \Delta(X_0)\big)\cos\theta(X_0)$$

Pseudospin stiffness

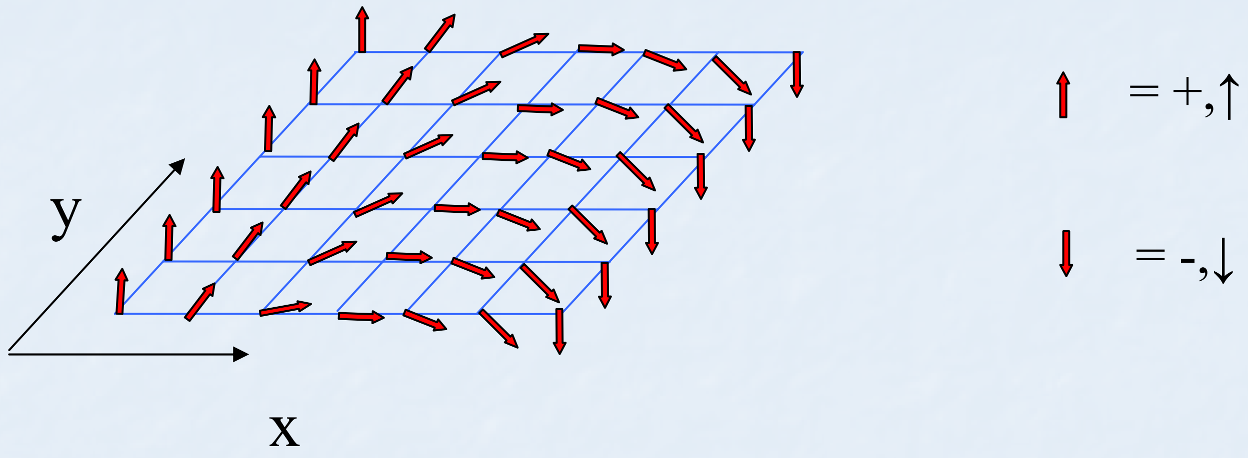


$\theta(X_0)/2\pi$

X_0/ℓ

Result of minimizing energy. Width of domain wall set by strength of confinement.

III. Properties of the Domain Wall

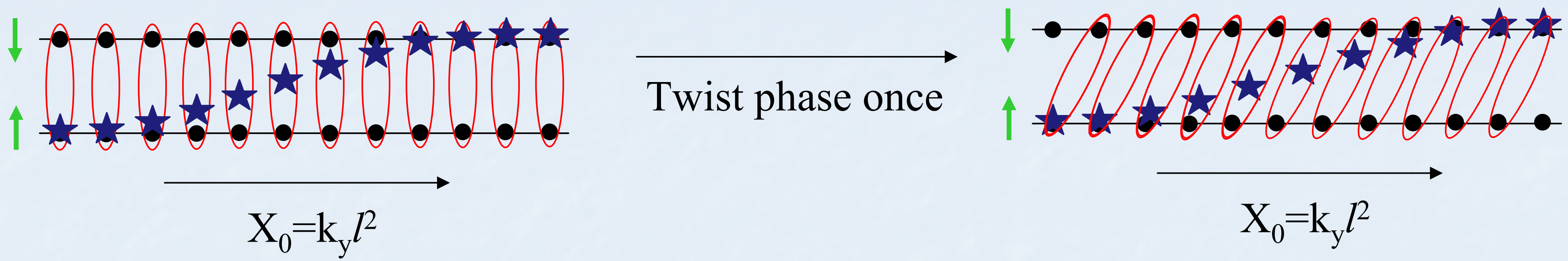


1. φ=0: Broken U(1) symmetry ⇒ linearly dispersing collective mode

$$S_0 = \int d\tau dy \left[\frac{1}{2}\Gamma m(y,\tau)^2 + \frac{1}{2}\tilde{\rho}\left(\frac{\partial\phi}{\partial y}\right)^2 + i m(y,\tau)\left(\frac{\partial\phi}{\partial\tau}\right)\right]$$

φ ~ in-plane angle of "spins"
m ~ position of domain wall

2. Spin-charge coupling ⇒ gapless charged excitations!



★ = weight in w/f

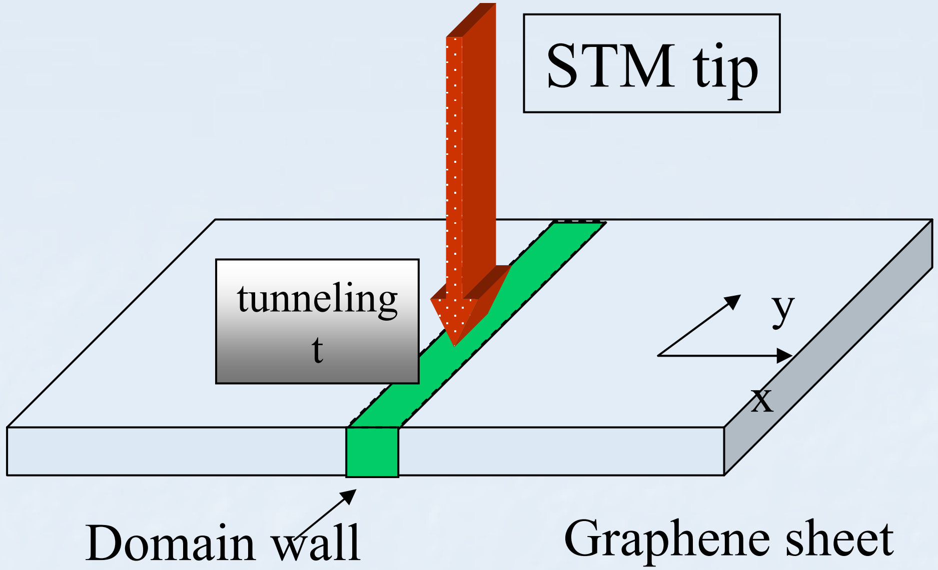
Fermion operator:

$$\psi(y,\tau) \sim e^{\pm\frac{i}{2}\phi(y,\tau)} e^{i2\pi \int_{-\infty}^{y} dy' m(y',\tau)}$$

3. Tunneling from STM tip: power law IV

$\Rightarrow$ not a Fermi liquid!

Power law exponent a function of confinement potential



$$I \sim t^2 \int dE \left[G_{tip}^{adv}(E) G_{DW}^{ret}(E - eV) - G_{tip}^{ret}(E - eV) G_{DW}^{adv}(E) \right]$$

$$G(\tau) \sim \left\langle T_\tau \psi(y = 0; \tau) \psi^+(y = 0; 0) \right\rangle \sim \frac{1}{\tau^\kappa}$$

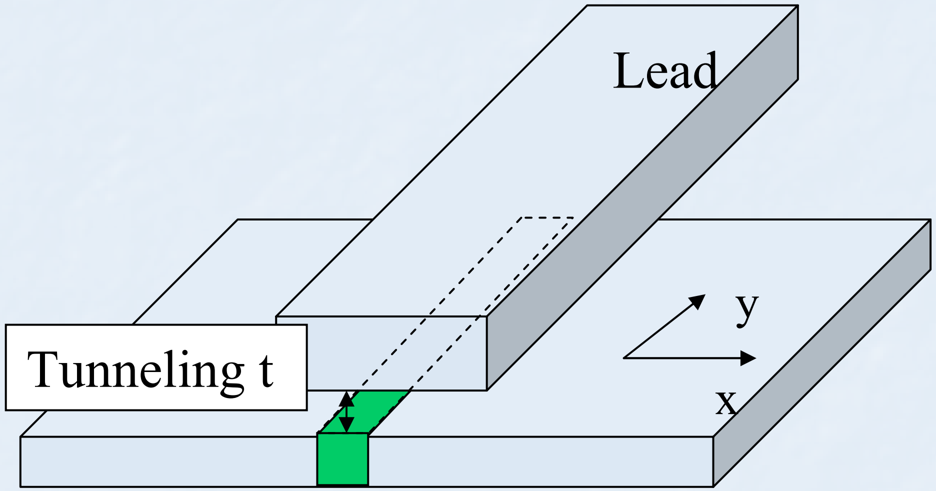
$$\kappa = (x + 1/x)/2; \quad x = 4\pi\sqrt{\tilde{\rho}/\Gamma}$$

U(1) spin stiffness

$\Gamma \sim$ confinement potential

$\Rightarrow$ Exponent sensitive to edge confinement!

4. Tunneling from a bulk lead: possibility of a quantum phase transition (into 3D metal).



Model lead as non-interacting electrons in a magnetic field

$$\Rightarrow \quad S = S_0 + \tilde{S}$$

with

$$\boxed{\tilde{S} = -t^2 \int_0^\beta d\tau_1 d\tau_2 \int dy \psi^*(y,\tau_1) K(\tau_1 - \tau_2) \psi(y,\tau_2).}$$

$$K \sim 1/(\tau_1 - \tau_2) \text{ for large } |\tau_1 - \tau_2|$$

Perturbative RG:

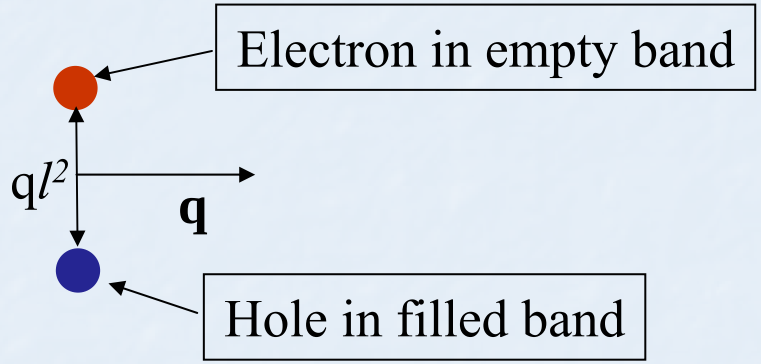
$$\frac{dt^2}{dl} = -(\kappa - 2)t^2$$

Shrinking t $\Rightarrow$ DW a Luttinger liquid
Growing t $\Rightarrow$ DW + lead = Fermi liquid?

IV. Inter-Landau Level Excitations (Magnetoplasmons)

Low-lying excited states = particle-hole pairs

Standard 2DEG:



cf. Kallin and Halperin, 1984

• Measurable in cyclotron resonance, inelastic light scattering.
• This picture is largely the same for graphene, just need to be careful about spinor structure of particle and hole states.

<u>Two-Body Problem</u>

$$\boxed{H_0 = \underbrace{(\mathbf{p}_1 - \mathbf{A}_1)\cdot\sigma}_{\text{Electron}} \otimes 1 + 1 \otimes \underbrace{(\mathbf{p}_2 + \mathbf{A}_2)\cdot\sigma}_{\text{Hole}}}$$

($\hbar = \omega_c = l = 1$)

To diagonalize **(A**= -*By***x**)**:**

1. Adopt center and relative coordinate $\mathbf{R}=(\mathbf{r}_1+\mathbf{r}_2)/2$, $\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2$
2. Apply unitary transformation $H'_0 = U^{+}H_0U$ with

$$U = e^{i\vec{p}\cdot(\hat{z}\times\vec{P})}e^{-ixY}$$

$\vec{P}$ = center of mass momentum

$$\boxed{\rightarrow H_0' = \sqrt{2}\left[-\mathbf{1}\otimes\begin{pmatrix} 0 & c_- \\ c_-^\dagger & 0 \end{pmatrix} + \begin{pmatrix} 0 & c_+^\dagger \\ c_+ & 0 \end{pmatrix}\otimes\mathbf{1}\right]}$$

with

$$\boxed{\begin{aligned} c_+^\dagger &= \frac{i}{\sqrt{2}}(-2\partial_z + \bar{z}/2) \\ c_-^\dagger &= \frac{i}{\sqrt{2}}(-2\partial_{\bar{z}} + z/2) \end{aligned}}$$

$z=x+iy$

Wavefunctions constructed from:

$$\boxed{\varphi_{n_+,n_-}(z,\bar{z}) = \frac{(c_+^\dagger)^{n_+}}{\sqrt{n_+!}}\frac{(c_-^\dagger)^{n_-}}{\sqrt{n_-!}}\varphi_{0,0}(z,\bar{z})}$$

with

$$\boxed{\varphi_{0,0}(z,\bar{z}) = (2\pi)^{-1/2}e^{-\frac{1}{4}z\bar{z}}}$$

Wavefunctions are 4-vectors $|n_+, n_->$ constructed from φ_{n_+,n_-} with energies

$$\boxed{E = \sqrt{2}[s_+\sqrt{|n_+|} - s_-\sqrt{|n_-|}]}$$

$s_+ = 1,\quad s_- = -1$

Electron Hole

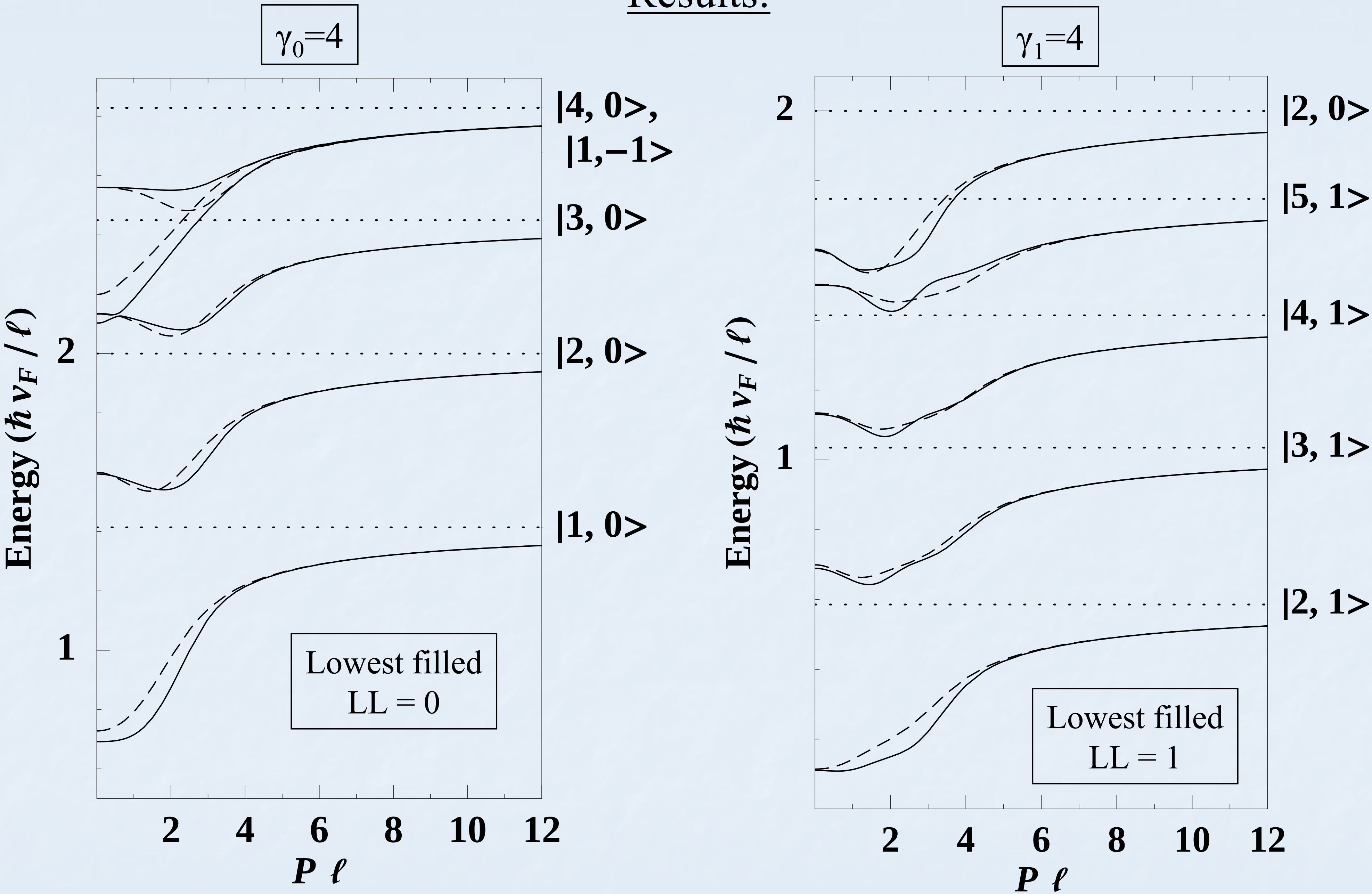
3. Apply unitary transformation to interaction H_1:

$$\boxed{H_1' = -e^2/(\epsilon\,|\mathbf{r} - \hat{\mathbf{z}} \times \mathbf{P}|)\,\mathbf{1} \otimes \mathbf{1}}$$

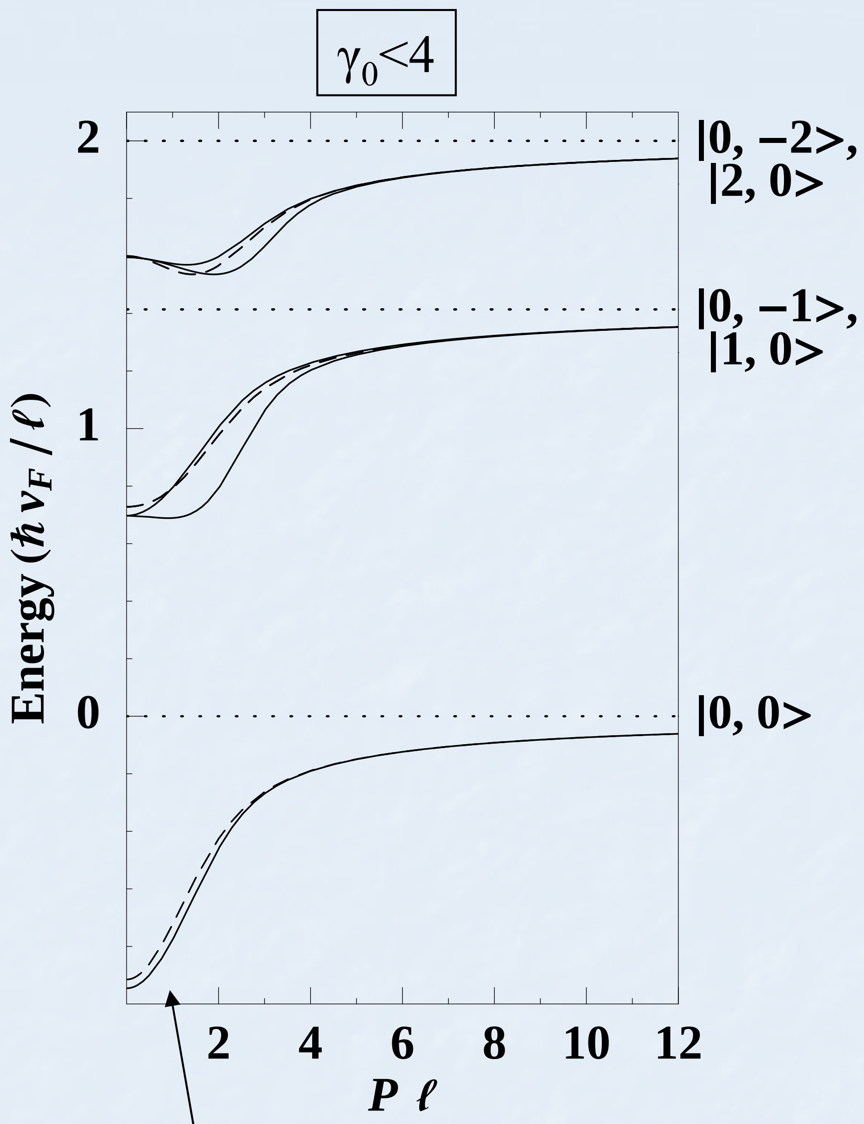
4. Compute eigenvalues of $< n'_+, n'_- | H_0' + H_1' | n_+, n_- >$

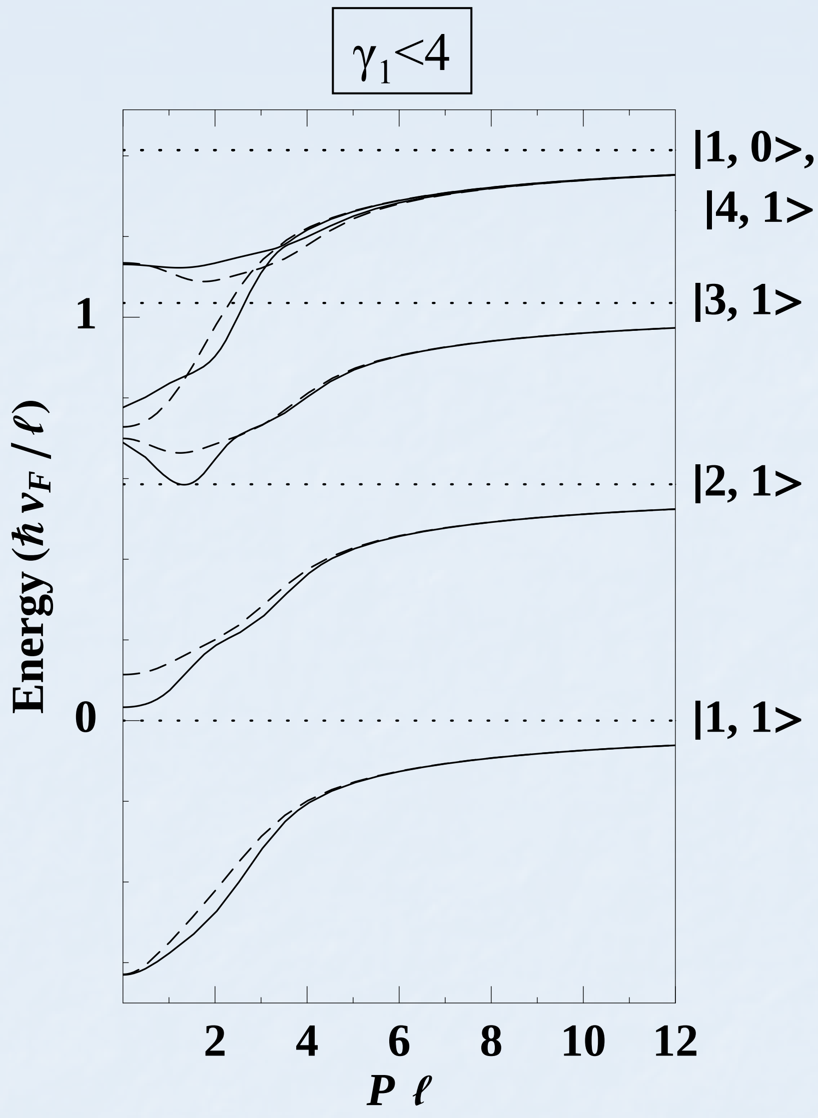
$\Rightarrow$ two-body eigenenergies with fixed **P**

Results:



Interaction scale: $\beta = (e^2/\varepsilon l)/(\hbar v_F/l\,) \approx (c/v_F\varepsilon)/137 = 0.73$



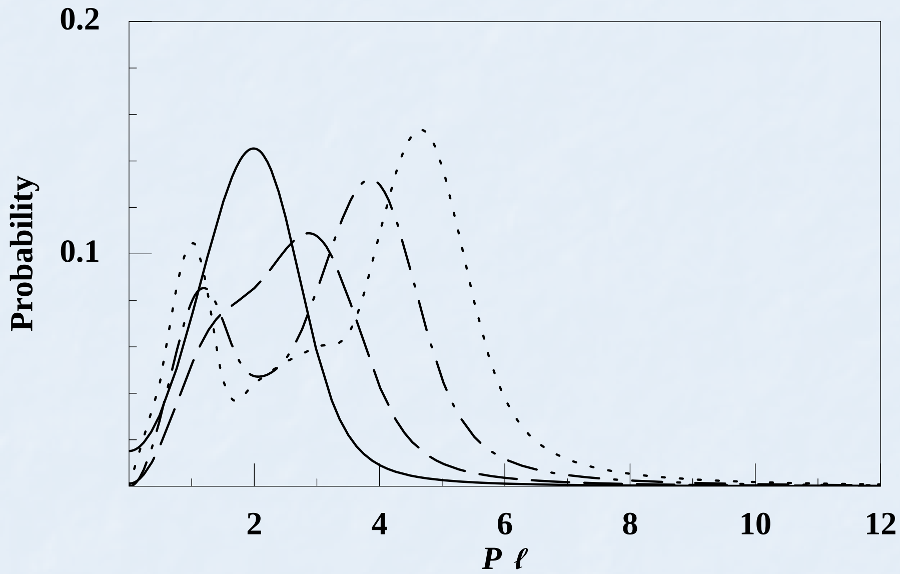


Note negative energy

Comments:

1. Negative energies because we have not included loss of exchange self-energy $\Rightarrow$ many-body approach needed

2. Landau level mixing relatively small



Note however for $\beta \approx 1$, LL mixing becomes much more pronounced $\Rightarrow$ system on cusp between weakly and strongly interacting

Many-Body Particle-Hole Approach

- A generalization of spin-wave calculation

$$|\mathbf{q}> = \rho_{\tau,\tau'}(\mathbf{q})|\text{Vac}>$$

$$\rho_{\tau,\tau'}(\mathbf{q}) = \frac{1}{N_\phi}\sum_X e^{-\frac{i}{2}q_x(2X+q_y)} C^+_{\tau\downarrow X} C_{\tau'\uparrow X+q_y}$$

$$\downarrow$$

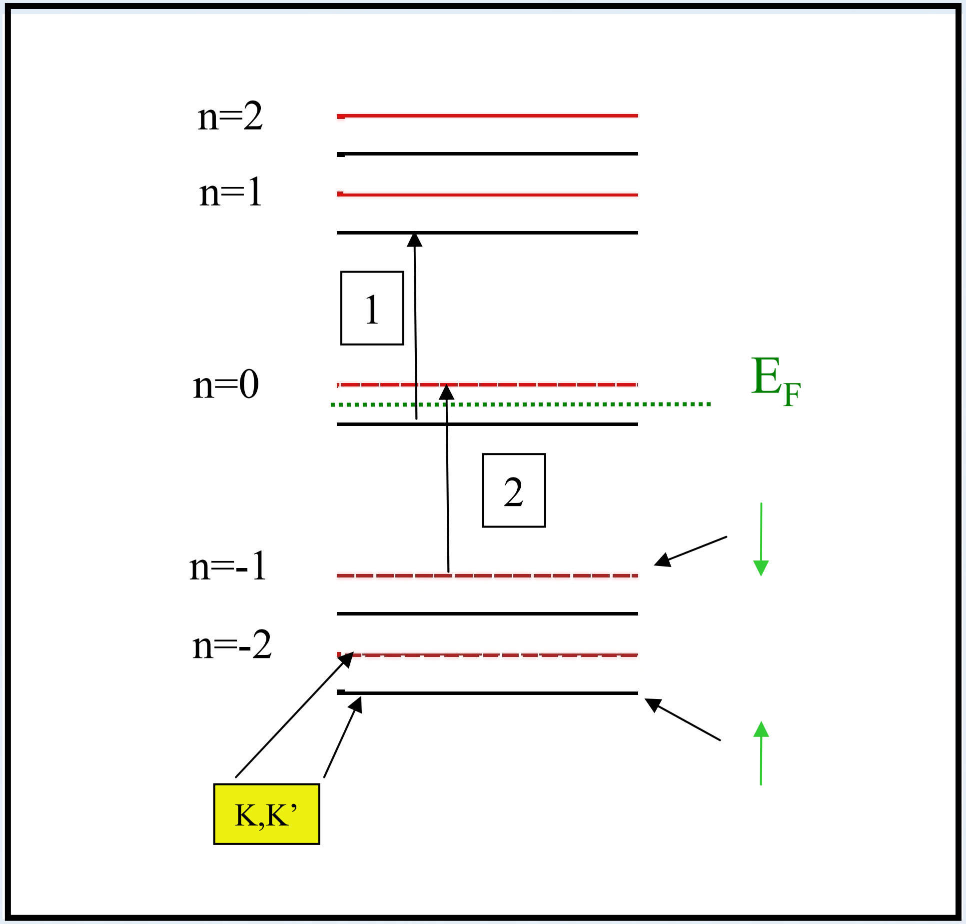
$$|\mathbf{q}> = \rho_{\tau,\sigma,n;\tau',\sigma',n'}(\mathbf{q})|\text{GS}>$$

$$\rho_{\tau,\sigma,n;\tau',\sigma',n'}(\mathbf{q}) = \frac{1}{N_\phi}\sum_X e^{-\frac{i}{2}q_x(2X+q_y)} C^+_{\tau,\sigma,n,X} C_{\tau',\sigma',n',X+q_y}$$

$$\boxed{\Delta E = <\mathbf{q}|H|\mathbf{q}> - E_0 \ ?}$$

Almost, but not quite.

Must watch out for degeneracies

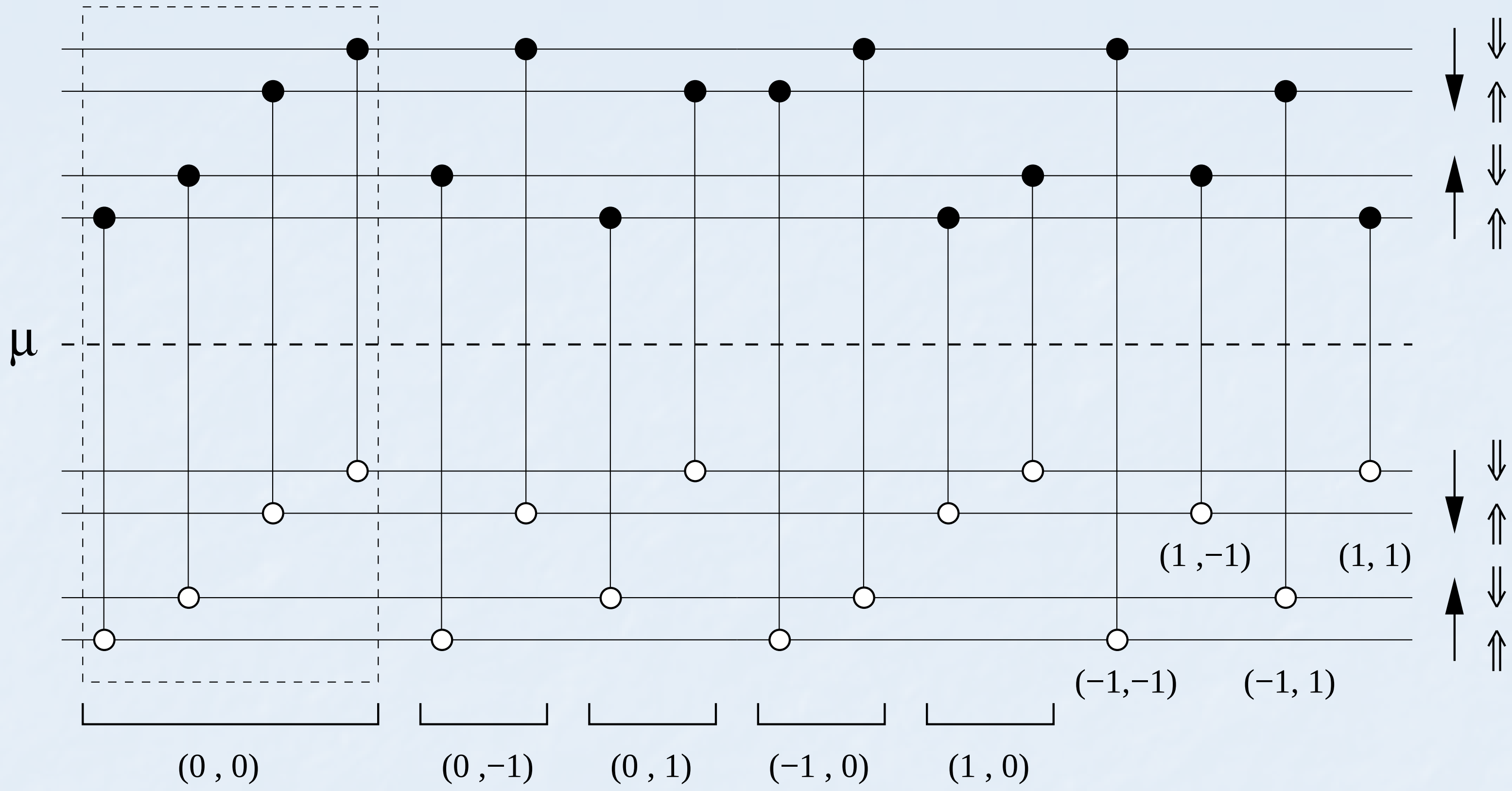


$$\boxed{|\mathbf{q}>=[c_1\rho_1(\mathbf{q})+c_2\rho_2(\mathbf{q})]|\mathrm{GS}>}$$

$$\boxed{|c_1|^2+|c_2|^2=1}$$

- Excitations characterized by ΔS_z and $\Delta\tau_z$

<u>Also</u>: Exchange energy with "infinite" number of filled hole levels leads to (logarithmically) divergent self-energy. Fix this with an explicit cutoff in number of filled Landau levels.



γ=4

↑,↓=spin, double arrows=pseudospin

(m,n)=(s_z,t_z)

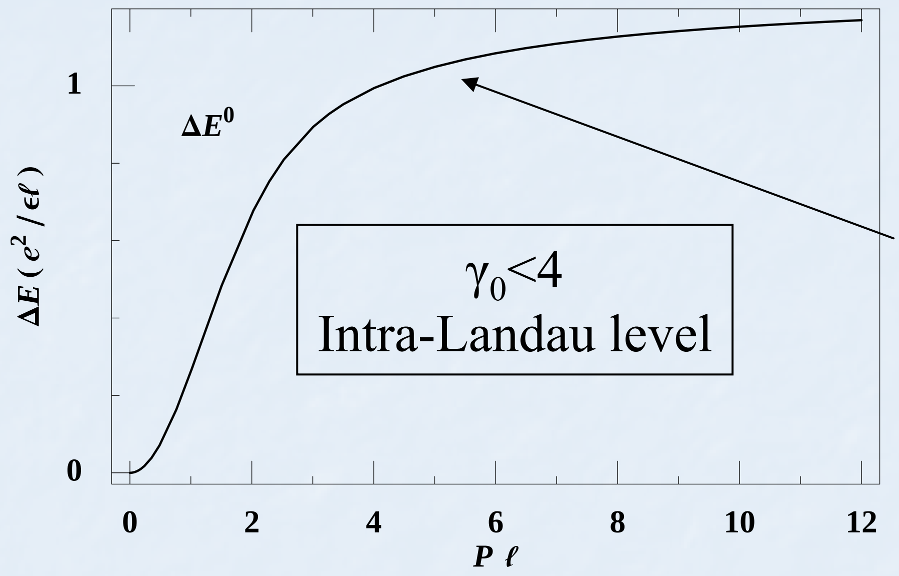
Energy generically involves four terms:

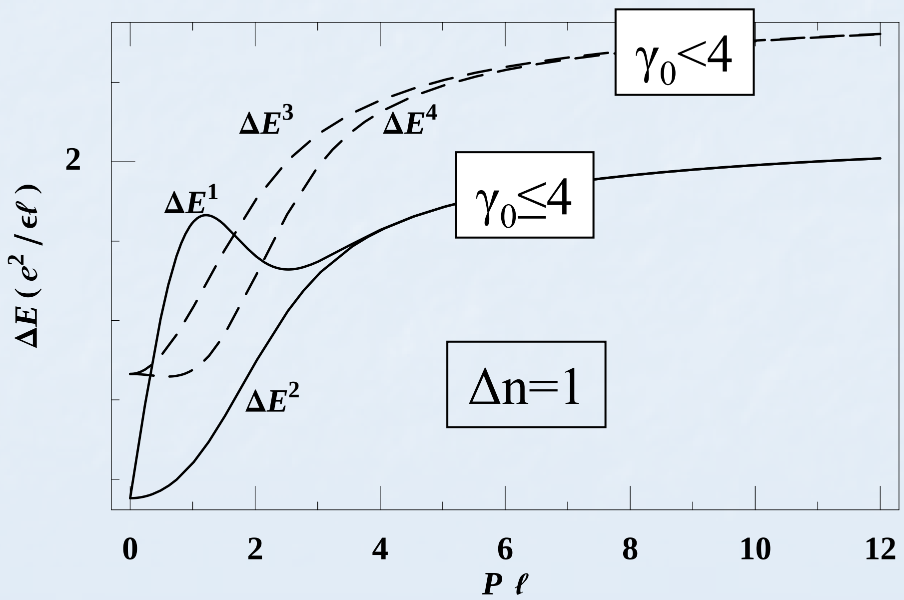
$$M_{1,2;1',2'} \equiv < \Omega|(a^{\dagger}_{1'}a_{2'})^{\dagger}\hat{H}a^{\dagger}_{1}a_{2}|\Omega > - < \Omega|\hat{H}|\Omega > \delta_{1,1'}\delta_{2,2'}$$

$$\hat{M} = \hat{M}^{0} + \hat{M}^{\text{dir}} + \hat{M}^{\text{exch}} + \hat{M}^{\Omega}$$

$$\begin{aligned}
M^{0}_{1,2;1',2'} &= \delta_{1,1'}\delta_{2,2'}(\varepsilon_1 - \varepsilon_2) && (1) \quad \text{Noninteracting} \\
M^{\text{dir}}_{1,2;1',2'} &= -V_{1',2,2',1} && (2) \quad \text{Direct (Ladders)} \\
M^{\text{exch}}_{1,2;1',2'} &= V_{1',2,1,2'} && (3) \quad \text{Exchange (Bubbles - RPA)} \\
M^{\Omega}_{1,2;1',2'} &= \sum_{3}\Theta(\mu - \varepsilon_3)[\delta_{1,1'}V_{3,2,3,2'} && (4) \quad \text{Exchange self-energy} \\
&\quad -\delta_{2,2'}V_{1',3,1,3}].
\end{aligned}$$

Results: N=0



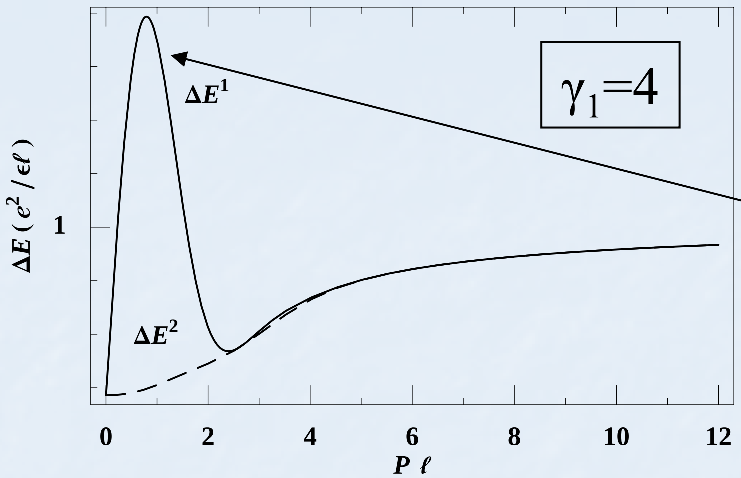


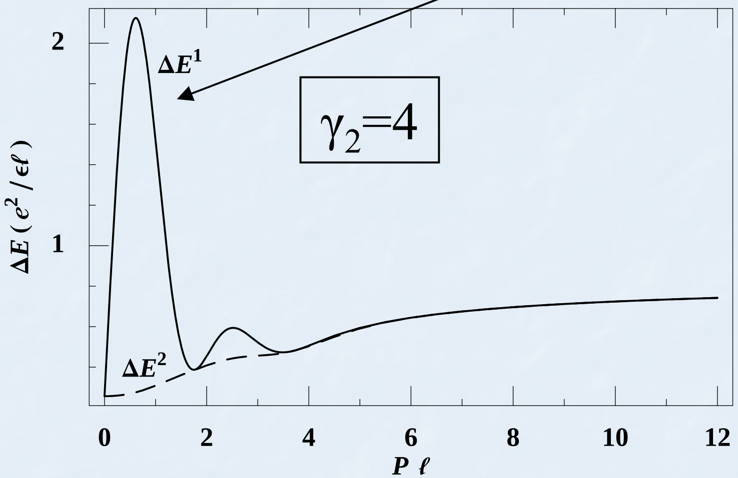
Comments:

1. Change in kinetic energy and Zeeman energy must be added in
2. Gapless excitations for ν=-1,1
3. Excitation spectra *identical* for ν=-1,1: particle-hole symmetry

Dashed lines equivalent to two-body result.

Very large many-body correction!





- Minima/maxima may be visible in inelastic light scattering or microwave absorption.

Summary

- Graphene: a new and interesting material both for fundamental and applications reasons.
- Clean system is likely a quantum Hall ferromagnet.
- Armchair edges: oppositely dispersing spin up and down bands $\Rightarrow$ domain wall
- Domain wall supports gapless collective excitations, and gapless charged excitations through pseudospin texture.
- Domain wall supports power law IV (Luttinger liquid).
- Domain wall may undergo quantum phase transition when coupled to a bulk lead.

- Collective inter-Landau level excitations = excitons
- Many-body corrections split and distort dispersions found in two-body problem