Interference in presence of Dissipation

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cond-mat/0602391 PRB (in press)

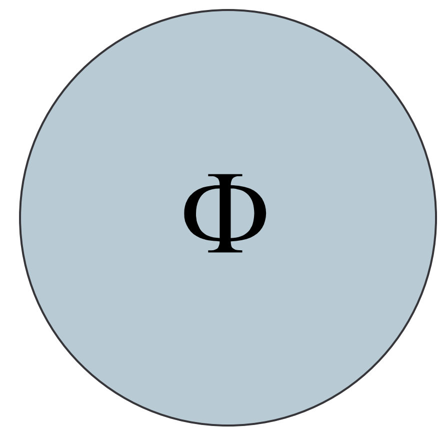
Motivation

Variational and RG solution

Monte Carlo

Dynamics and Relaxation in Complex Quantum and Classical Systems and Nanostructures, Dresden 2006

Particle on a ring

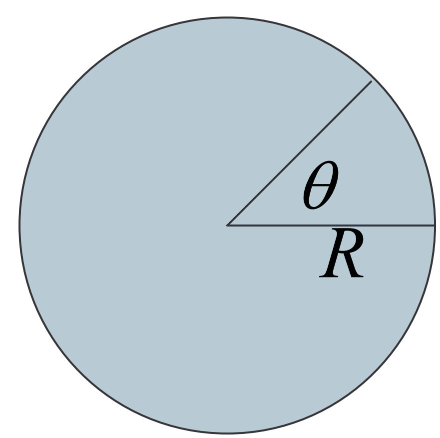


Energy spacing $\approx \frac{\hbar^2}{MR^2}$

Flux Φ is magnetic flux for charged particle

-- in general a measure of interference capacity

Position of particle on the circle: $\vec{R}(\tau) = R[\cos\theta(\tau), \sin\theta(\tau)]$



Kinetic Lagrangian:

$$L_K = \int d\tau \; \frac{1}{2} M (\frac{d\vec{R}}{d\tau})^2 = \int d\tau \frac{1}{2} MR^2 (\frac{d\theta}{d\tau})^2$$

Amplitude for AB oscillations $\approx$ energy level spacing $=1/(2MR^2)$

Theoretical motivations:

Possible behaviors for renormalized mass or AB amplitude with coupling to environment [Guinea, 2002] :

$\sim e^{-(R/\ell^*)^2}$ for Caldeira Legget bath; ℓ^* a dephasing length?

$\sim 1/R^{2+\mu}$ for charged particles in a metallic environment.

F. Guinea, PRB 65, 205317 (2002): $\mu << 1$ non-universal

D. S. Golubev, C. P. Herrero and A. D. Zaikin, EPL (2002): $\mu \approx 1.8$

At finite T, however, $\sim T e^{-R/\ell^*}$ (instantons) with ℓ^* finite at $T \to 0$

-- dephasing?

Experimental motivation: Electric dipoles

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Coherence-Preserving Trap Architecture for Long-Term Control of Giant Ryberg Atoms

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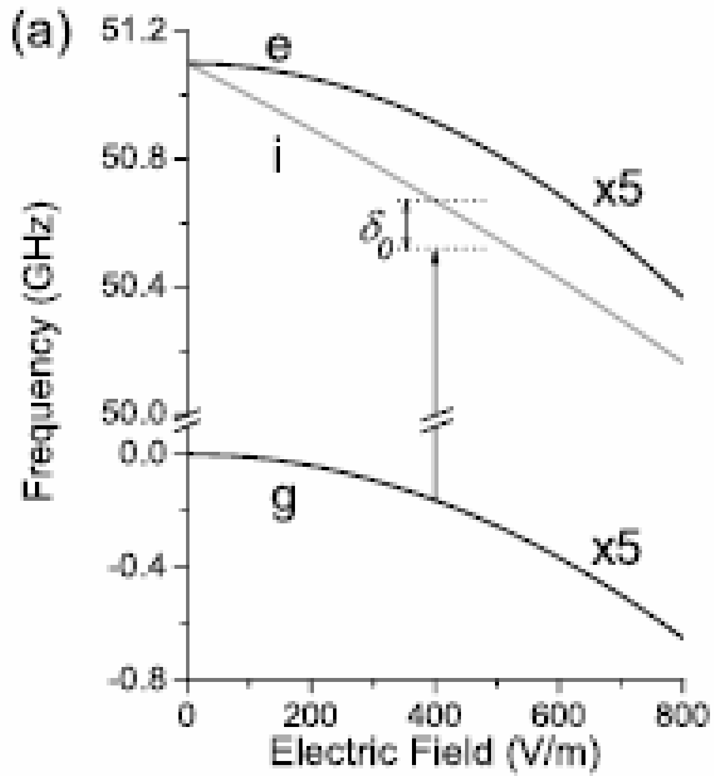
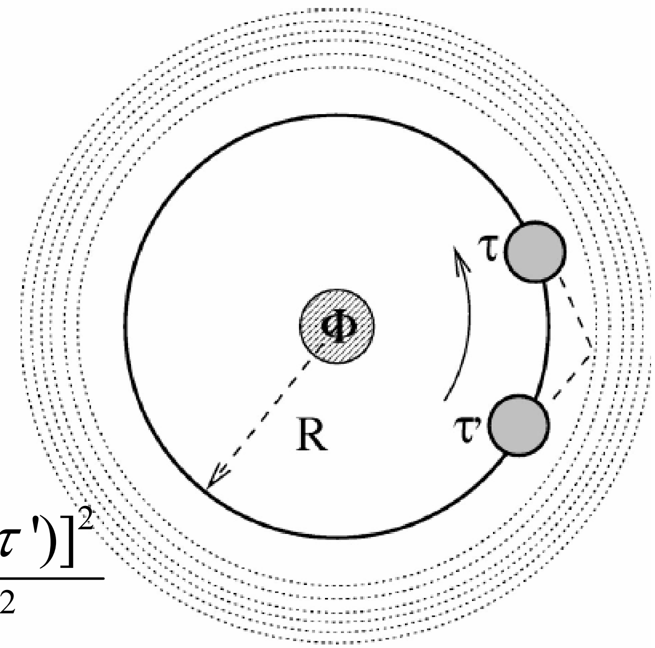


n=50 → n=51

Caldeira-Legget environment



Expand in particle coordinate $\vec{R}(t)$

$$\tilde{S} = \int d\tau \{ \frac{1}{2} M \left(\frac{d\vec{R}}{d\tau} \right)^2 + \sum_i \vec{R}(\tau) \lambda_i Q_i(\tau) + L_{bath}[Q_i(\tau)] \}$$

Integrate bath coordinates, dissipation is obtained if

$$S_{\text{int}} = \bar{\alpha} \int d\omega \, |\omega| \, \vec{R}(\omega) \vec{R}(-\omega) = \bar{\alpha} \int d\tau \int d\tau' \frac{[\vec{R}(\tau) - \vec{R}(\tau')]^2}{(\tau - \tau')^2}$$

Since $\vec{R}(\tau) = R[\cos\theta(\tau), \sin\theta(\tau)]$

$$S = \int d\tau \frac{1}{2} M R^2 \left(\frac{d\theta}{d\tau} \right)^2 + 2\bar{\alpha} R^2 \int d\tau \int d\tau' \frac{1 - \cos[\theta(\tau) - \theta(\tau')]}{(\tau - \tau')^2}$$

[$\approx$ Coulomb blockade]

$\Rightarrow$ Long range interaction

pre-exponent:

In terms of the dissipation strength $\alpha = 4\bar{\alpha}R^2$:

AB oscillation amplitude is $\sim \alpha^{\mu} e^{-\pi^2 \alpha}$

RG : $\mu = 2$ (Hofstetter & Zwerger, 1997)

real time RG $\mu = 6.5$ (König & Schoeller, 1998)

instantons $\mu = 2$ (Panyukov & Zaikin, 1991)

$\mu = 3$ (Wang & Grabert, 1996)

$\mu = 4$ (Beloborodov, Andreev & Larkin, 2003)

instantons+CFT $\mu = 3$ (Lukyanov & Zamolodchikov, 2004)

Monte Carlo: $\mu = 5$ (Werner & Troyer, 2005)

$\mu = 3$ (Lukyanov & Werner, 2006)

Metallic environment

$$L_{\text{int}} = -\int d\tau \int d^3 r V[R(\tau) - r]\rho(r)$$

Gaussian average

$$< e^{-iL_{\text{int}}} >_{\rho} = e^{-S_{\text{int}}}$$

$$S_{\text{int}} = \frac{1}{2} \iint d\tau d\tau' \iint d^3 r d^3 r' V[R(\tau) - r] V[R(\tau') - r'] < \rho(r)\rho(r') >$$

$$\sim \int d\omega d^3 q \,\mathrm{Im}[\frac{1}{\varepsilon(\omega, q)}] e^{-i\omega(\tau - \tau') - iq\cdot[R(\tau) - R(\tau')]} f(q)$$

$f(q) = \dfrac{1}{q^2}$ charged particle

$= \dfrac{\mathrm{q}_{\mathrm{z}}^2}{q^2}$ electric dipole

General action:

General form of dissipative environment

$$\mathrm{S}=\frac{1}{2}MR^2\int_0^\beta d\tau\left(\frac{\partial\theta}{\partial\tau}\right)^2+\gamma\int_0^\beta d\tau\int_0^\beta d\tau'\frac{\pi^2\beta^{-2}K[\theta(\tau)-\theta(\tau')]}{\sin^2[\pi\beta^{-1}(\tau-\tau')]}$$

$$\gamma K(z)=\sum_{\mathrm{n}}\alpha_n\sin^2(\frac{n}{2}z)$$

Caldeira Legget: $K(z)=\sin^2(\frac{n}{2}z)$ single n=1, $\gamma=\alpha=4\bar{\alpha}R^2$

Metallic environment:

Charge: $K(z)=1-\frac{1}{[4\mathrm{r}^2\sin^2(z/2)+1]^{1/2}}$ $\gamma=\frac{3}{8k_F^2\ell^2}$, $r=\frac{R}{\ell}$

Electric dipole: $K(z)=1-\frac{1}{[4\mathrm{r}^2\sin^2(z/2)+1]^{3/2}}$ $\gamma=\frac{3}{8k_F^2\ell^2}\frac{p^2}{e^2\ell^2}$

Winding numbers:

$$\theta_{\mathrm{m}}(\tau) = \theta(\tau) + 2\pi m\tau / \beta \qquad Z = \sum_m e^{2\pi i m\phi_x} Z_m$$

Variational method: For Z_m find optimal action $S_0 = \int \frac{d\omega}{4\pi} G^{-1}(\omega) \,|\, \theta(\omega) \,|^2$

so that $\beta F_{\mathrm{var}} = \beta F_0 + \langle S - S_0 \rangle_0$ is minimized.

$$\mathrm{G}^{-1}(\omega) = MR^2\omega^2 + 2\sum_n \alpha_n n^2 \int_{1/\omega_c}^{\infty} d\tau \frac{1-\cos(\omega\tau)}{\tau^2} \cos(2\pi n m\tau / \beta) e^{-n^2 \int_{\omega_1} G(\omega_1)(1-\cos\omega_1\tau)}$$

At $\beta \to \infty$ and $\omega \to 0$ $\mathrm{G}^{-1}(\omega) = B\omega^2$,

m independent since integrand $\sim \exp[-n^2\tau / B]$

Result: $\frac{1}{B} = \frac{\partial^2 E_0}{\partial \phi_x^2}\Big|_0 = 4\pi^2 \langle m^2 \rangle_0 / \beta$ measures the AB amplitude.

At finite $\beta < B / n^2$ small m dominates $\langle \mathrm{S}_{\mathrm{int}} \rangle = \pi^2 \sum_{\mathrm{n}} \alpha_n n \,|\, m \,|$

$$F \sim T\cos(2\pi\phi_x) e^{-\pi^2 \sum_n \alpha_n n} \quad \rightarrow \quad \text{instantons.}$$

Variational procedure

$G^{-1}(\omega) = B\omega^2 \qquad \omega < \omega_0 \qquad [B = MR^2$ if no interactions$]$

$G^{-1}(\omega) = f(\omega) \qquad \omega_0 < \omega < \omega_c$

Variational equation, assuming low frequency dominance of $1/f(\omega)$

$$f'(\omega) = \pi\eta \sum_n \alpha_n n^2 e^{-n^2 \int_\omega^{\omega_c} d\omega_1 / \pi f(\omega_1)} \quad ; \quad f(\omega_c) = \pi\omega_c \sum_n \alpha_n n^2$$

Within perturbation theory $\eta = 1 + \dfrac{\sum_n \alpha_n n^4}{(\pi \sum_n \alpha_n n^2)^2}$.

$f(\omega_0)$=Bω_0^2 and the amplitude of AB oscillations is $\sim$ 1/B.

Renormalization group

Consider the single $\alpha = \alpha_1 R^2$ problem:

$$f'(\omega) = \pi\eta\alpha e^{-\int_{\omega}^{\omega_c} d\omega_1/\pi f(\omega_1)}$$

Change cutoff to $\omega_c' = \omega_c + d\omega_c$

and change couplings to $\alpha' = \alpha + \mathrm{d}\alpha$ so that equation is invariant

$$(\eta - \frac{d\eta}{d\alpha} d\alpha)(\alpha' - d\alpha)(1 + \frac{d\omega_c}{\pi f(\omega_c)}) = \alpha'\eta$$

$$\Rightarrow \quad \frac{d\alpha}{d\ln\omega_c} = \frac{1}{\pi^2} + \frac{1}{\pi^4\alpha} + O(\alpha^{-2})$$

same as 2 loop RG [Hofstetter and Zwerger, PRL 78, 3737 (1997)]

Integration to $\alpha^{\mathrm{R}} \approx 1$ yields $\mathrm{B} \sim 1/\omega_c^R \sim \alpha^{-2} e^{\pi^2\alpha}$

Variational equation

$$f'(\omega) = \pi\eta\alpha e^{-\int_{\omega}^{\omega_c} d\omega_1 / \pi f(\omega_1)}$$

$$f''(\omega) = \frac{f'(\omega)}{\pi f(\omega)} \quad \Rightarrow \qquad f'(\omega) = \pi^{-1} \ln[K f(\omega)]$$

K is integration constant, from boundary condition $K = \dfrac{e^{\pi^2 \alpha\eta}}{\pi\alpha\omega_c}$

Asymptotic expansion of Log integral $\int \frac{\mathrm{df}}{\mathrm{f}}$: $\quad f(\omega) = \omega g(K\omega)$ with $g(x)$ α independent

$g(K\omega_c) = \pi\alpha$ and with $d/d\alpha \Rightarrow$

$$\eta = 1 + \frac{1}{\pi^2\alpha\eta - 1 + \pi^2\alpha^2 \frac{d\eta}{d\alpha}} \Rightarrow \qquad \text{expansion in large } \alpha \quad !$$

For $\omega < \omega_c$ choose $\bar{\alpha}(\omega)$ such that

$$\mathrm{K}\omega = K(\alpha)\omega = K(\bar{\alpha}(\omega))\omega_c = \frac{e^{\pi^2 \bar{\alpha}(\omega)\eta(\omega)}}{\pi\bar{\alpha}(\omega)}$$

hence $f(\omega) = \pi\omega\bar{\alpha}(\omega) = \dfrac{\omega}{\pi\eta} \ln[\dfrac{\omega K}{\pi\eta} \ln(\dfrac{\omega K}{\pi\eta} .....)]$

Charge and electric dipole

$$\alpha_n \approx \frac{2\gamma}{\pi r}\ln(r/n) \quad \text{for n<r} \;;\; \alpha_n = 0 \;\; \text{otherwise : charge}$$

$$\approx \frac{p^2}{e^2\ell^2}\frac{2\gamma}{\pi \mathrm{r}}(1-\frac{n^2}{r^2}) \quad \text{for n<r} \;;\; \alpha_n = 0 \;\; \text{otherwise: electric dipole}$$

$\mathrm{r}=\frac{\mathrm{R}}{\ell}$ $\quad \gamma=\frac{3}{8\mathrm{k}_\mathrm{F}^2\ell^2}$ $\quad \ell$ is the mean free path in the metal.

For r>>1 replace $\sum_{\mathrm{n}} \rightarrow r\int_0^1 dx$ where $x = n/r$. $\quad \bar{\theta}(\tau)=\mathrm{r}\theta(\tau)$

$$\Rightarrow \quad S_{\mathrm{int}} = \sum_n \frac{1}{r}\alpha^*(n/r)\bar{S}\{n\theta(\tau)\} \rightarrow \int_0 dx\alpha^*(x)\bar{S}\{x\theta(\tau)\}$$

action is r independent! hence AB oscillations $\sim 1/\mathrm{R}^2$

as free particles.

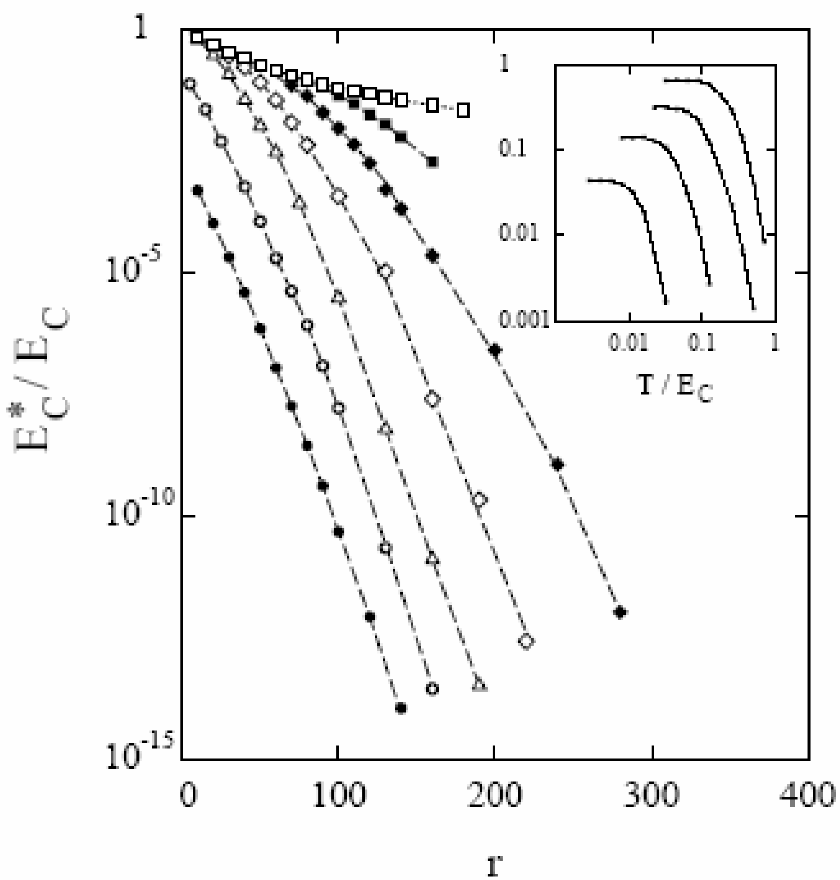
Assumption:

The $m=0$ sector determines B.

Proof within variational scheme.

Monte Carlo

D. S. Golubev, C. P. Herrero and A. D. Zaikin, Europhys. Lett. 63, 426 (2003)



r=10,30,60,120

γ=0.019

$$MR^2 = 1/2E_c, \quad B = 1/2E_c^*$$

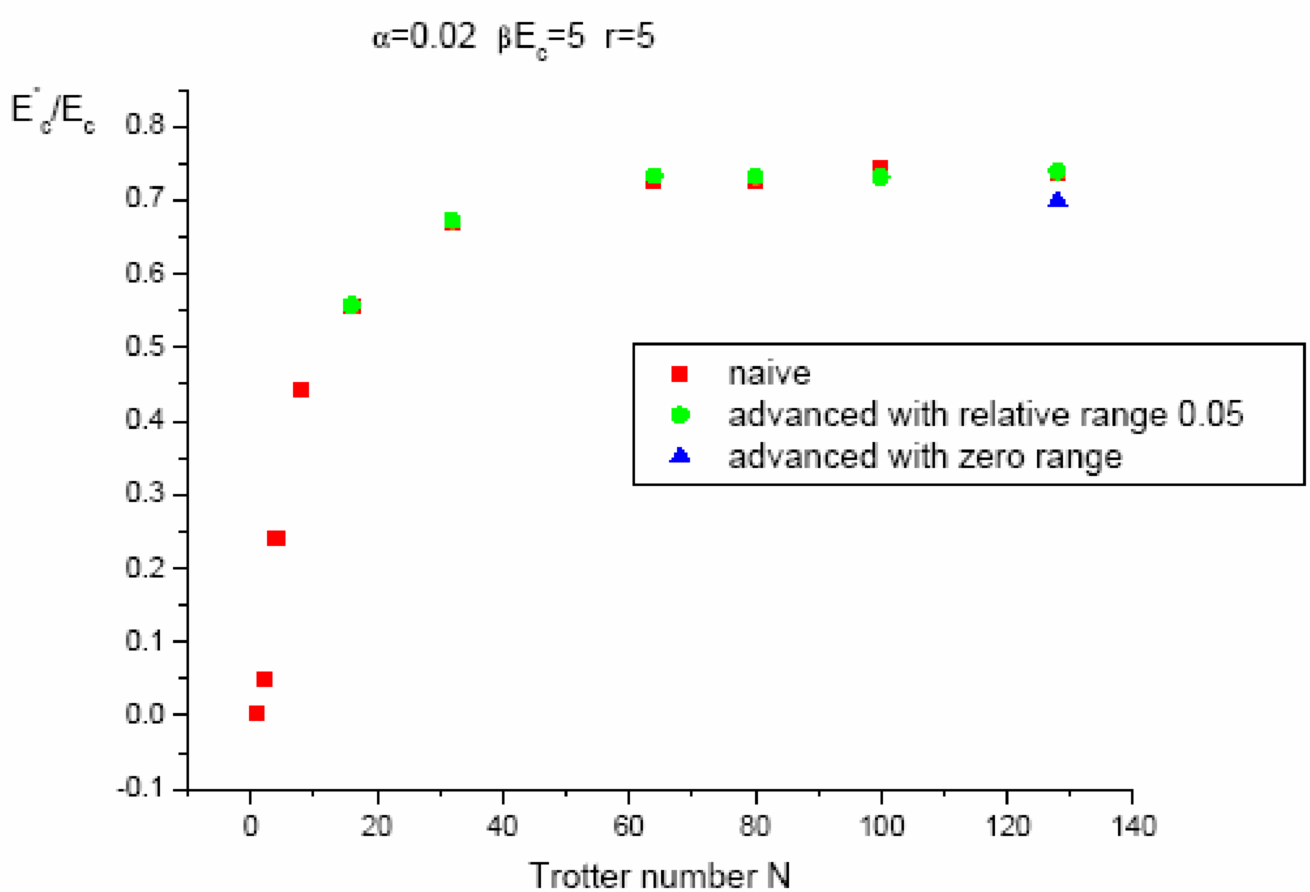
$[0,\beta]$ is divided into N_T segments.

Diagonal term in action:

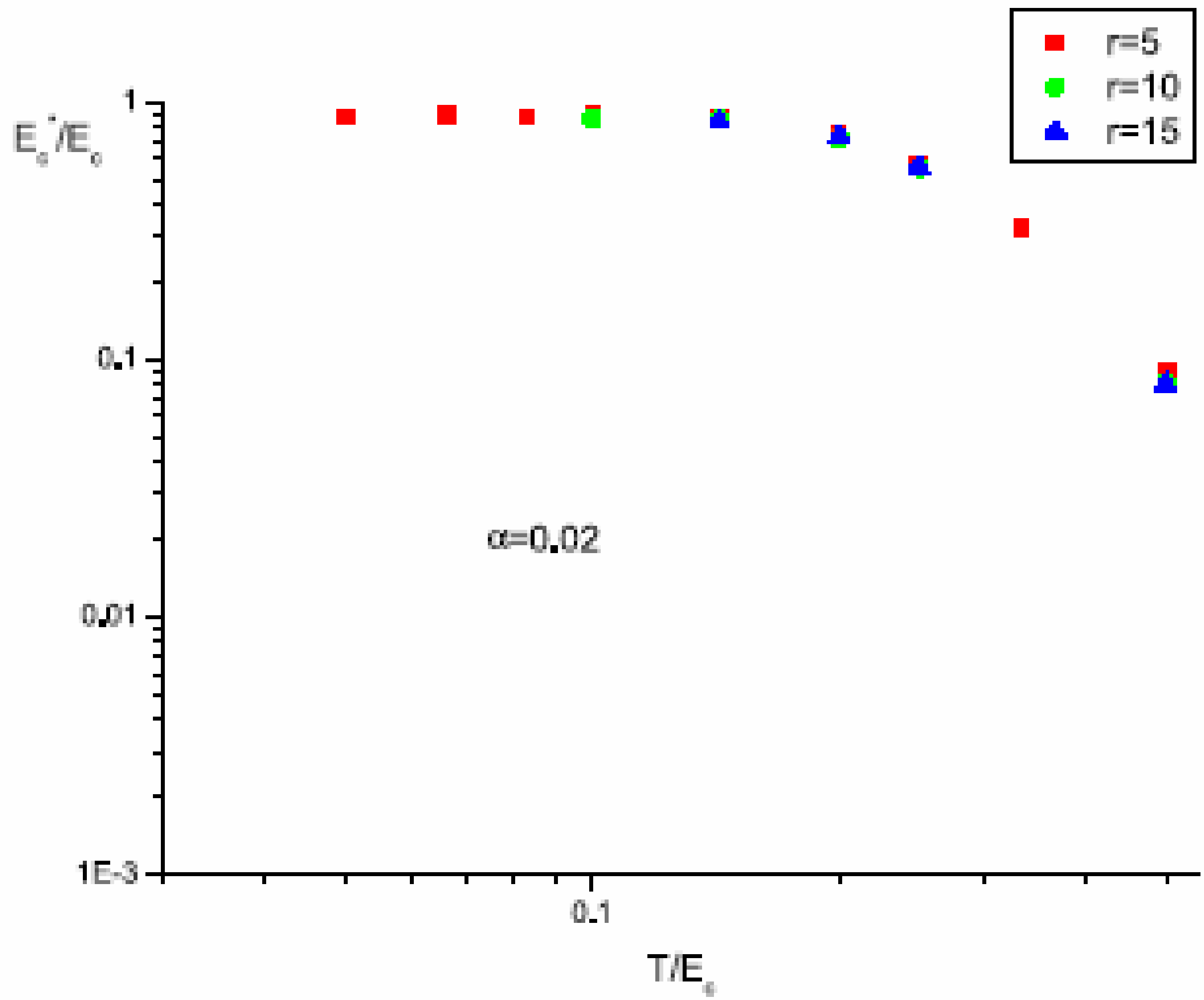
$$[N_T M/2\beta + 2r^2\gamma](\theta_i - \theta_{i+1})^2$$

$$\Rightarrow \text{ need } N_T >> 4r^2\gamma\beta/M$$

Monte Carlo (with V. Kagalovski):



Dipoles – Monte Carlo



Conclusions

- The variational method reproduces RG at least to 2^{nd} order.
- Various environments can lead to distinct R dependence, distinct dephasing behavior at T=0?
- Large electric dipoles allow crossover from C-L exponenetial behavior to a power law, for a sufficiently large electric dipole.