Dynamics and Relaxation in Complex Quantum and Classical Systems and Nanostructures (Dresden, 2006 summer)

Quantum Fluctuation of Conductivities in Quantum Hall Effect

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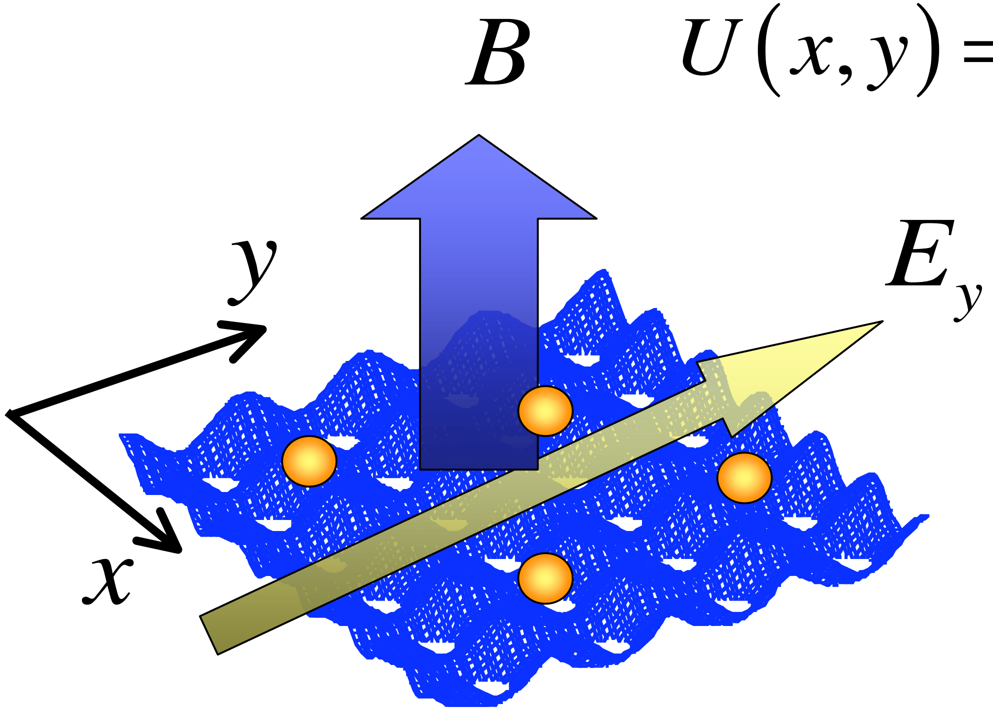
Conductivity in a periodic potential

$$H(t)=\frac{1}{2m}\left(\vec{p}+e\vec{A}(t)\right)^2+U(x,y)$$



$$U(x,y)=U_1\cos\left(\frac{2\pi x}{a}\right)+U_2\cos\left(\frac{2\pi y}{b}\right)$$

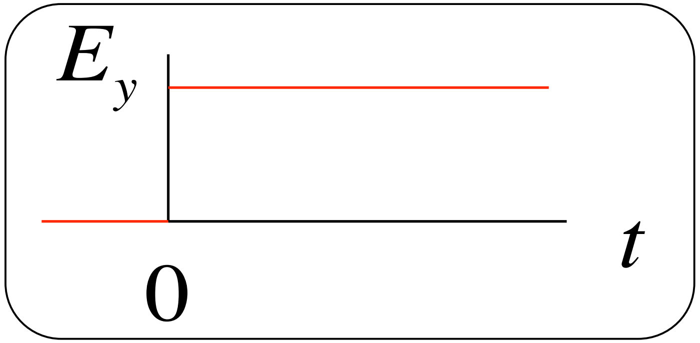
$$\vec{A}(t)=\begin{pmatrix}0\\ Bx-E_y t\end{pmatrix}$$

$$J_x=\sigma_{xy}E_y,\quad J_y=\sigma_{yy}E_y$$

[D. J. Thouless, M. Kohmoto, M.P. Nightingale, and M. den Nijs, PRL **9** (1982) 405.]

My calculation

Switch on E_y abruptly at $t = 0$



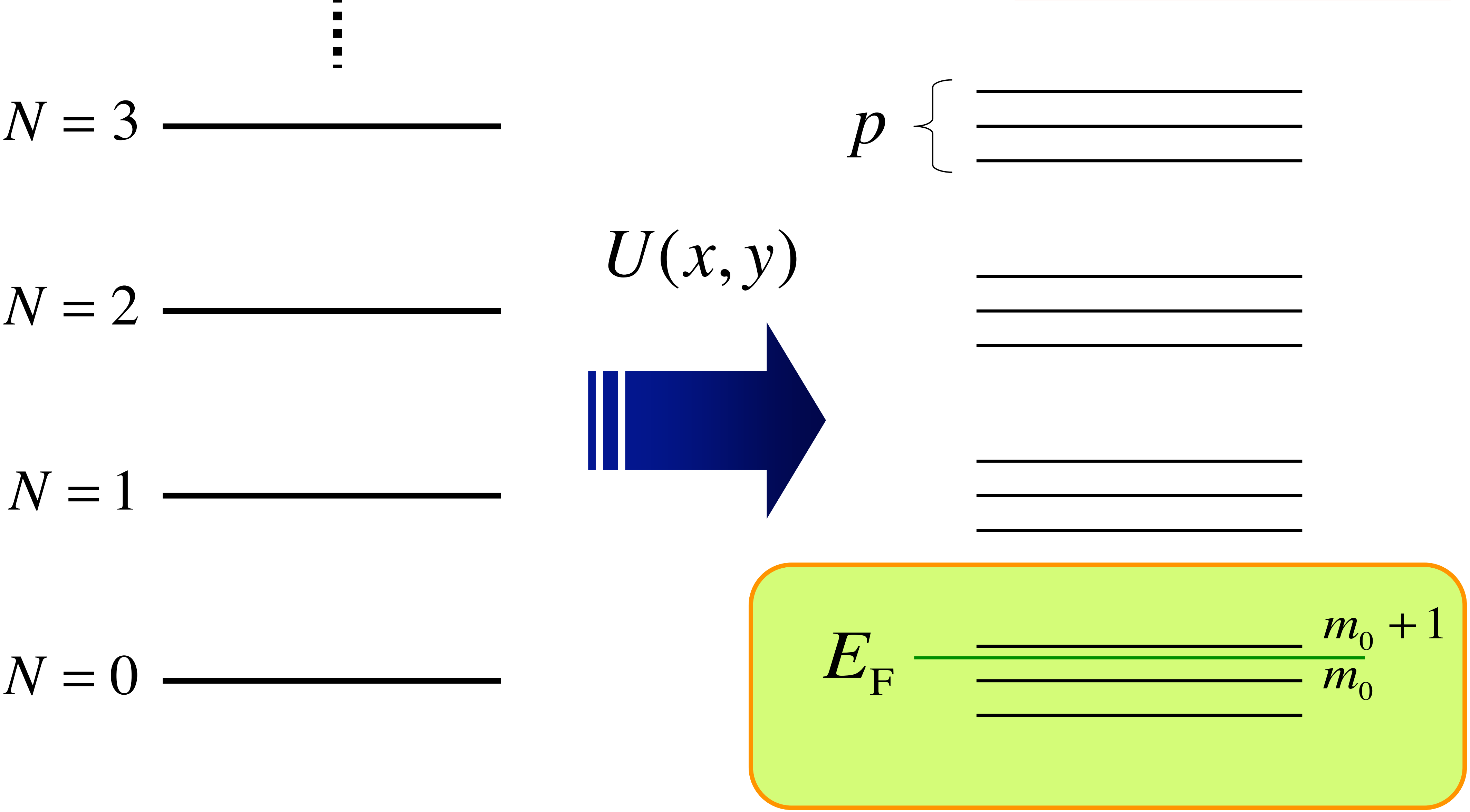
Linear response for finite time

$$\sigma_{xy} = \frac{e^2}{h} N_{\mathrm{Ch}} + [\text{Oscillation in time}]$$

$$\sigma_{yy} = 0 + [\text{Oscillation in time}]$$

Energy levels

$$\frac{\Phi}{\Phi_0} = \frac{abB}{2\pi\hbar / e} = \frac{p}{q}$$



Wavefunction in the weak potential

$$u_{m\vec{k}}(x,y)=\frac{1}{\sqrt{N}}\sum_{n'=1}^{p}d_m^{n'}\sum_{\ell=-\infty}^{\infty}\mathrm{e}^{-\frac{eB}{2\hbar}\left(x+\frac{\hbar k_y}{eB}-\ell qa-\frac{n'qa}{p}\right)^2}$$

$$\times\,\mathrm{e}^{-\mathrm{i}k_x(x-\ell qa-n'qa/p)}\mathrm{e}^{-2\pi\mathrm{i}y\frac{\ell p+n'}{b}}$$

$$\beta d_m^{n'-1}+\alpha_{n'}d_m^{n'}+\beta^* d_m^{n'+1}=\varepsilon_{m\vec{k}}^{(1)}d_m^{n'}$$

$$\alpha_{n'}=U_1\mathrm{e}^{-\frac{\pi qb}{2pa}}\cos\left(-qbk_y/p+2\pi n'q/p\right)$$

$$\beta=\frac{U_2}{2}\mathrm{e}^{-\frac{\pi qa}{2pb}}\mathrm{e}^{-\mathrm{i}qak_x/p}$$

[D. J. Thouless, M. Kohmoto, M.P. Nightingale, and M. den Nijs, PRL **9** (1982) 405.]

Greenwood's linear response theory

[D. A. Greenwood, Proc. Phys. Soc. **71** (1958) 585.]

$$\rho \simeq \rho^{(0)} + E_y \rho^{(1)}$$

$$\rho^{(0)} = \int_{\mathrm{MBZ}} \frac{d^2\vec{k}}{(2\pi)^2} \sum_{m=1}^{p} \left| m\vec{k} \right\rangle f\left(E_{m\vec{k}}\right) \left\langle m\vec{k} \right|$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho(t) = \frac{1}{\mathrm{i}\hbar}\left[H(t), \rho(t)\right]$$

$$\left\langle J_x(E_y) \right\rangle = \mathrm{Tr}\, \rho(t) \frac{\partial H(t)}{\partial A_x}$$

$$J_x = \sigma_{xy} E_y$$

$$J_y = \sigma_{yy} E_y$$

Density Matrix

$$\rho_{mn\vec{k}} = \left\langle u_{m\vec{k}} \middle| \mathrm{e}^{-\mathrm{i}\vec{k}\cdot\vec{x}} \rho \, \mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}} \middle| u_{n\vec{k}} \right\rangle, \quad \rho^{(0)}_{mn\vec{k}} = f_m \delta_{mn}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho^{(1)}_{mn\vec{k}} = \frac{1}{\mathrm{i}\hbar}\left(\varepsilon_{m\vec{k}} - \varepsilon_{n\vec{k}}\right)\rho^{(1)}_{mn\vec{k}} + \frac{e}{\hbar}\left(f_m - f_n\right)\left\langle \frac{\partial u_{m\vec{k}}}{\partial k_y} \middle| u_{n\vec{k}} \right\rangle$$

$$(m \neq n),$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho^{(1)}_{nn\vec{k}} = 0$$

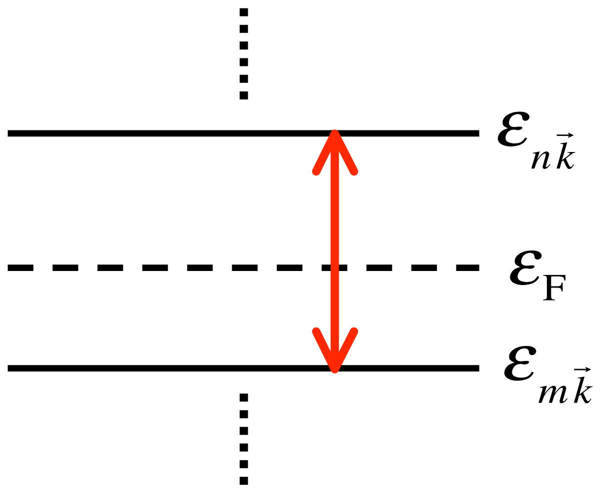
$$\rho^{(1)}_{mn\vec{k}} = \mathrm{i}e\left\langle \frac{\partial u_{m\vec{k}}}{\partial k_y} \middle| u_{n\vec{k}} \right\rangle \frac{f_m - f_n}{\varepsilon_{m\vec{k}} - \varepsilon_{n\vec{k}}}\left[1 - \mathrm{e}^{-\mathrm{i}\left(\varepsilon_{m\vec{k}} - \varepsilon_{n\vec{k}}\right)t/\hbar}\right]$$

Conductivity

$$\sigma_{xy} = \frac{e^2}{2\pi\hbar} N_{\mathrm{Ch}}$$

$$+ \frac{e^2}{\hbar} \int_{\mathrm{MBZ}} \frac{\mathrm{d}^2\vec{k}}{(2\pi)^2} \sum_{m\neq n} g_{mn}^{xy}\left(\vec{k}\right)$$

$$\times \sin\left(\left(\varepsilon_{m\vec{k}} - \varepsilon_{n\vec{k}}\right)t/\hbar - \theta_{mn}\left(\vec{k}\right)\right)$$



$$\sigma_{yy} = \frac{e^2}{\hbar} \int_{\mathrm{MBZ}} \frac{\mathrm{d}^2\vec{k}}{(2\pi)^2} \sum_{m\neq n} g_{mn}^{yy}\left(\vec{k}\right) \sin\left(\left(\varepsilon_{m\vec{k}} - \varepsilon_{n\vec{k}}\right)t/\hbar\right)$$

Details

$$N_{\mathrm{Ch}} = \sum_m f_m \int_{\mathrm{MBZ}} \frac{\mathrm{d}^2 \vec{k}}{\pi} \mathrm{Im} \left\langle \frac{\partial u_{m\vec{k}}}{\partial k_y} \middle| \frac{\partial u_{m\vec{k}}}{\partial k_x} \right\rangle$$

$$g_{mn}^{xy}\left(\vec{k}\right) = \left(f_m - f_n\right) r_{mn}\left(\vec{k}\right)$$

$$g_{mn}^{yy}\left(\vec{k}\right) = \left(f_m - f_n\right) \left| \left\langle \frac{\partial u_{m\vec{k}}}{\partial k_y} \middle| u_{n\vec{k}} \right\rangle \right|^2$$

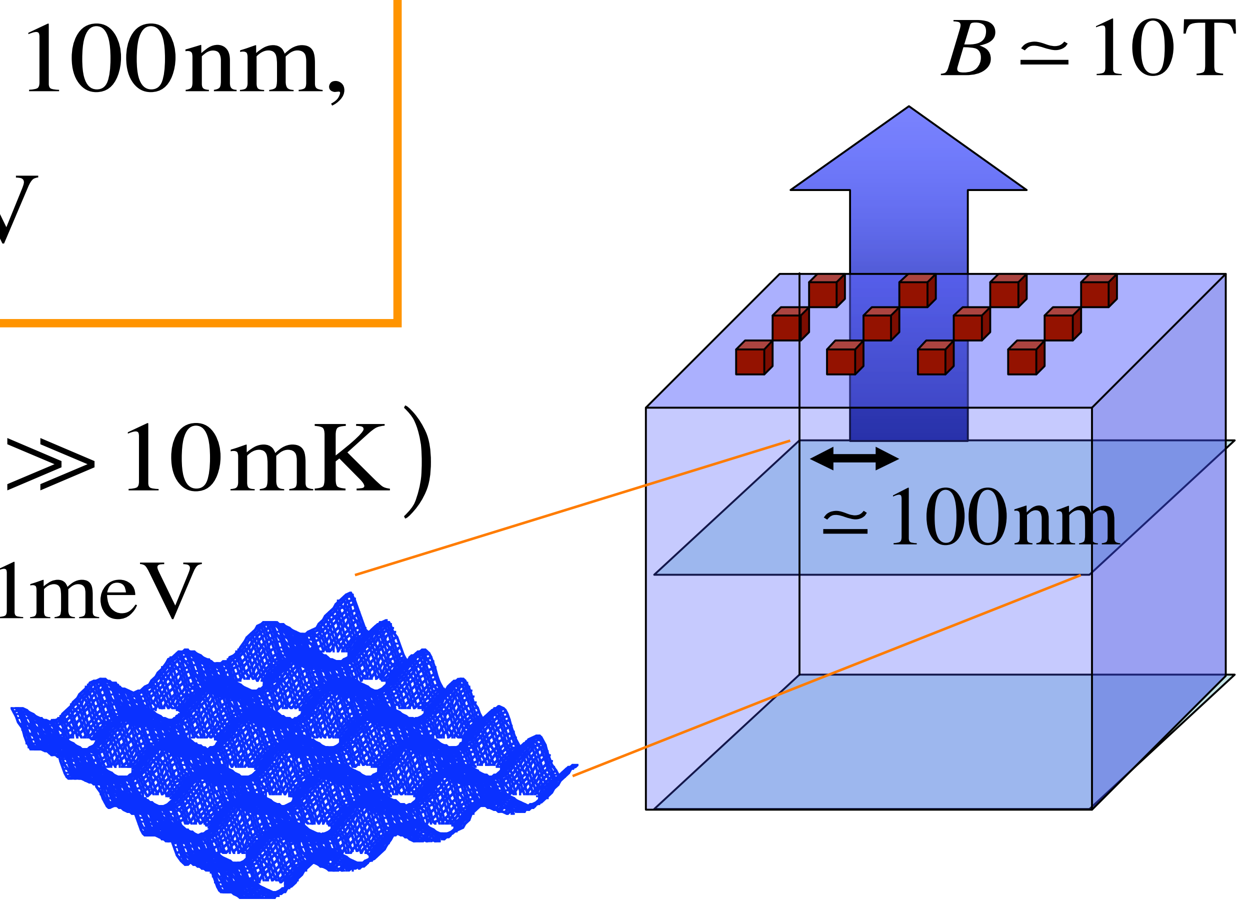
$$r_{mn}\left(\vec{k}\right) \mathrm{e}^{\mathrm{i}\theta_{mn}\left(\vec{k}\right)} = \left\langle \frac{\partial u_{m\vec{k}}}{\partial k_y} \middle| u_{n\vec{k}} \right\rangle \left\langle u_{n\vec{k}} \middle| \frac{\partial u_{m\vec{k}}}{\partial k_x} \right\rangle$$

Parameters

$$B \simeq 10\mathrm{T},\ \ a,b \simeq 100\mathrm{nm},$$
$$U_1, U_2 \simeq 0.1\mathrm{meV}$$

$$\therefore \Delta \sim U \sim 1\mathrm{K} \quad (\gg 10\mathrm{mK})$$

$U_1, U_2 \simeq 0.1\mathrm{meV}$

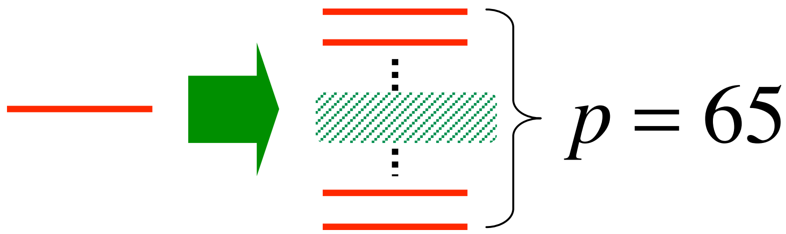


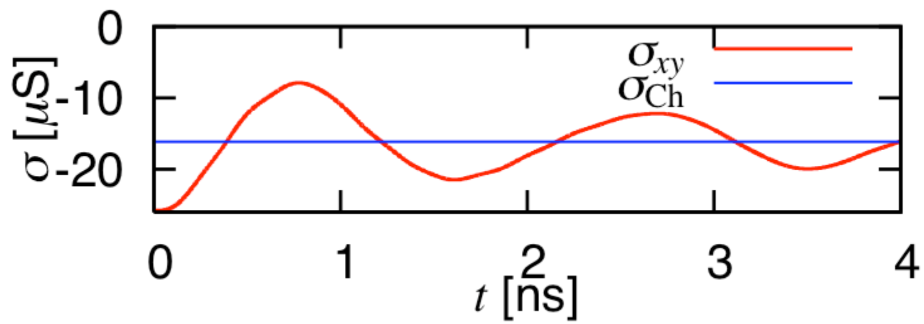
[C.T. Liu, *et al*., Appl. Phys. Lett. **58** (1991) 2945]

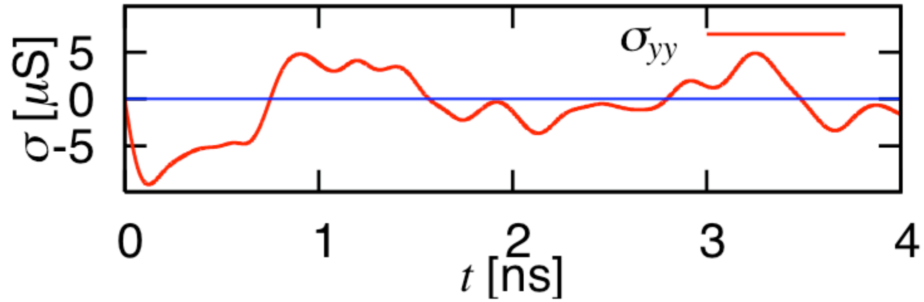
[M.C. Geisler, *et al*., PRL **92** (2004) 256801 and PRB **72** (2005) 045320]

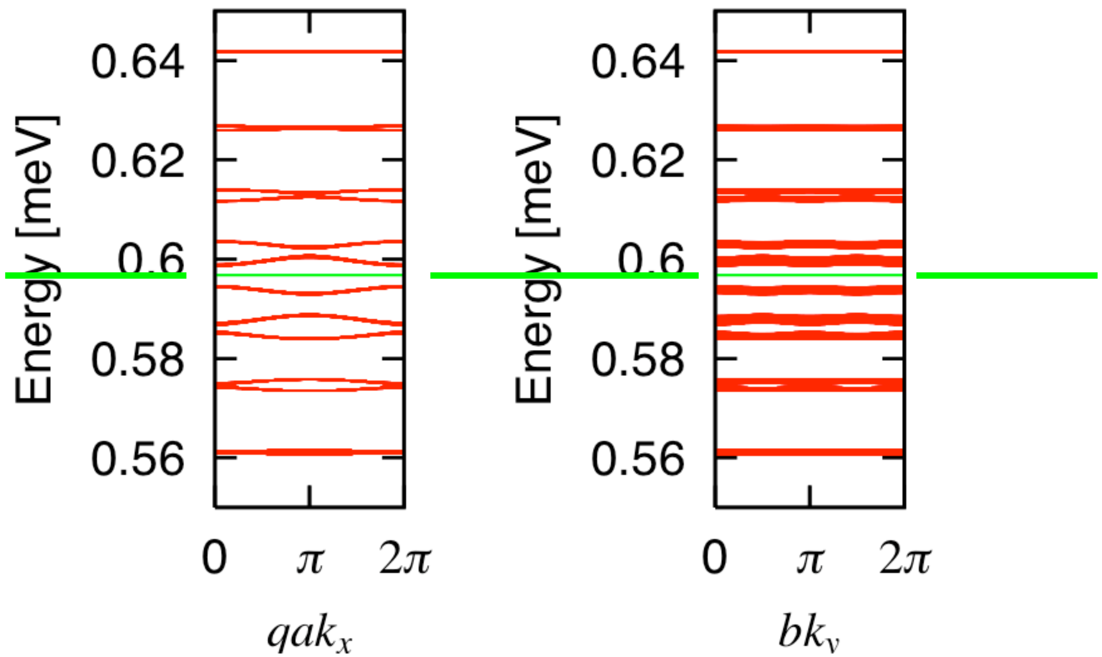
Example 1

$$\frac{\Phi}{\Phi_0} = \frac{p}{q} = \frac{65}{2}$$

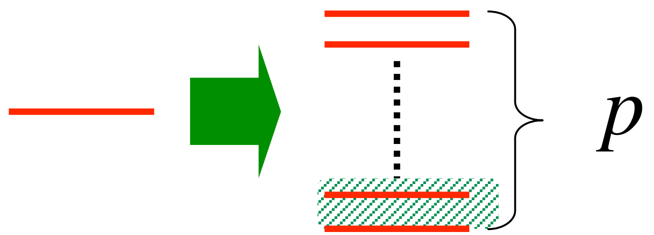




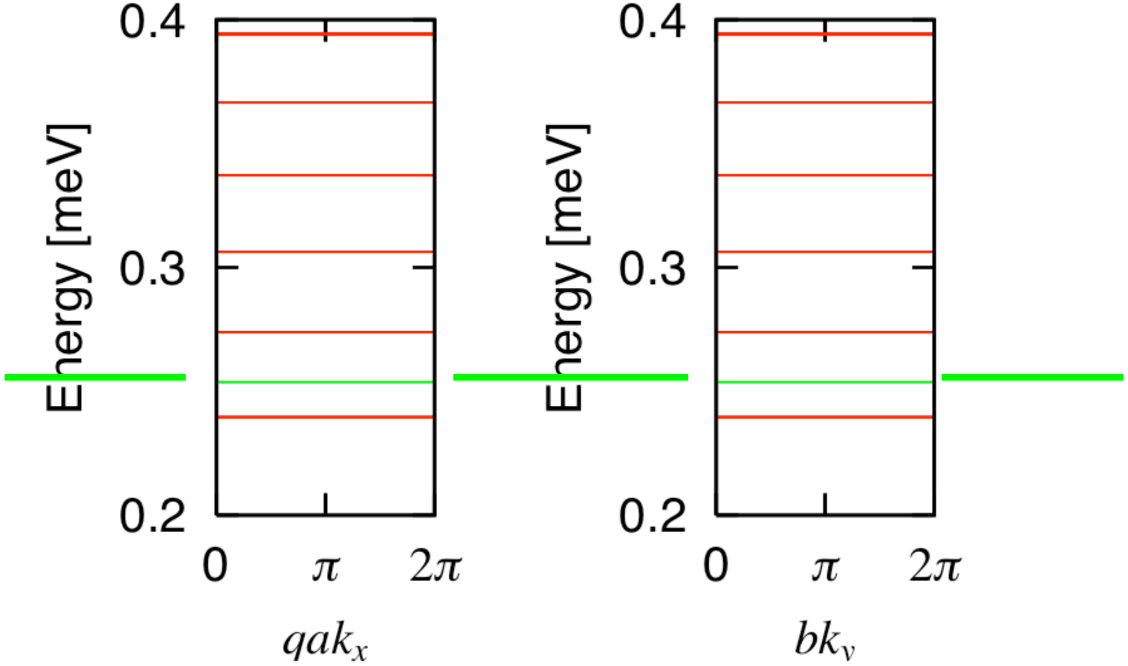




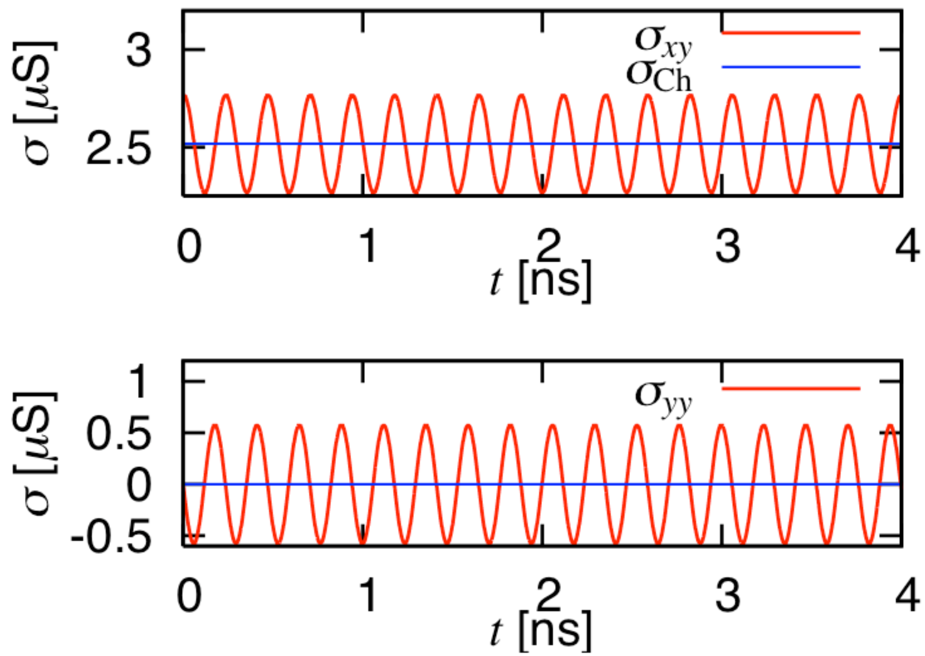
Example 2



p



0.4
0.3
0.2
Energy [meV]
0
π
2π
qak_x
bk_y



σ_xy
σ_Ch
σ [μS]
3
2.5
1
0.5
0
-0.5
σ_yy
0
1
2
3
4
t [ns]

Long-time average

σ_{xy} and σ_{yy}

$$\sigma_{xy} = \sigma_{\mathrm{Ch}} + \Delta\sigma_{xy}, \quad \sigma_{\mathrm{Ch}} = \frac{e^2}{2\pi\hbar} N_{\mathrm{Ch}}$$

$$\Delta\sigma_{xy} = \frac{e^2}{\hbar}\int_{\mathrm{MBZ}} \frac{\mathrm{d}^2\vec{k}}{(2\pi)^2} \sum_{m\neq n} g_{mn}^{xy}(\vec{k}) \sin\left(\left(\varepsilon_{m\vec{k}} - \varepsilon_{n\vec{k}}\right)t/\hbar - \theta_{mn}(\vec{k})\right)$$

$$\sigma_{yy} = \frac{e^2}{\hbar}\int_{\mathrm{MBZ}} \frac{\mathrm{d}^2\vec{k}}{(2\pi)^2} \sum_{m\neq n} g_{mn}^{yy}(\vec{k}) \sin\left(\left(\varepsilon_{m\vec{k}} - \varepsilon_{n\vec{k}}\right)t/\hbar\right)$$

$$\frac{1}{L_x L_y} \sum_{j_x=-(M_x-1)/2q}^{M_x/2q} \sum_{j_y=-(M_y-1)/2q}^{M_y/2q}$$

$$\overline{\sigma_{yy}} \equiv \lim_{T\to\infty} \frac{1}{T}\int_0^T dt\, \sigma_{yy} = 0, \quad \overline{\sigma_{xy}} = \sigma_{\mathrm{Ch}}$$

Long-time variance

$$\mathrm{var}\left(\sigma_{yy}\right) \equiv \overline{\left(\sigma_{yy}\right)^2} - \left(\overline{\sigma_{yy}}\right)^2$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_0^T dt \left(\frac{1}{L_x L_y}\right)^2 \sum_{j_x j_y} \sum_{m\neq n} \left(\frac{e^2}{\hbar} g_{mn}^{yy}\left(\vec{k}\right)\right)^2 \sin^2\left(\left(\varepsilon_{m\vec{k}} - \varepsilon_{n\vec{k}}\right) t / \hbar\right)$$

$$\leq \left(\frac{1}{L_x L_y}\right)^2 \max\left(\frac{e^2}{\hbar} g_{mn}^{yy}\left(\vec{k}\right)\right)^2 \sum_{j_x j_y} \sum_{m\neq n}$$

$$= \frac{1}{qabL_x L_y} \max\left(\frac{e^2}{\hbar} g_{mn}^{yy}\left(\vec{k}\right)\right)^2 \sum_{m\neq n} \xrightarrow{L\to\infty} 0$$

$$\mathrm{var}\left(\sigma_{yy}\right) \xrightarrow[T\to\infty]{L\to\infty} 0, \quad \mathrm{var}\left(\sigma_{xy}\right) \xrightarrow[T\to\infty]{L\to\infty} 0$$

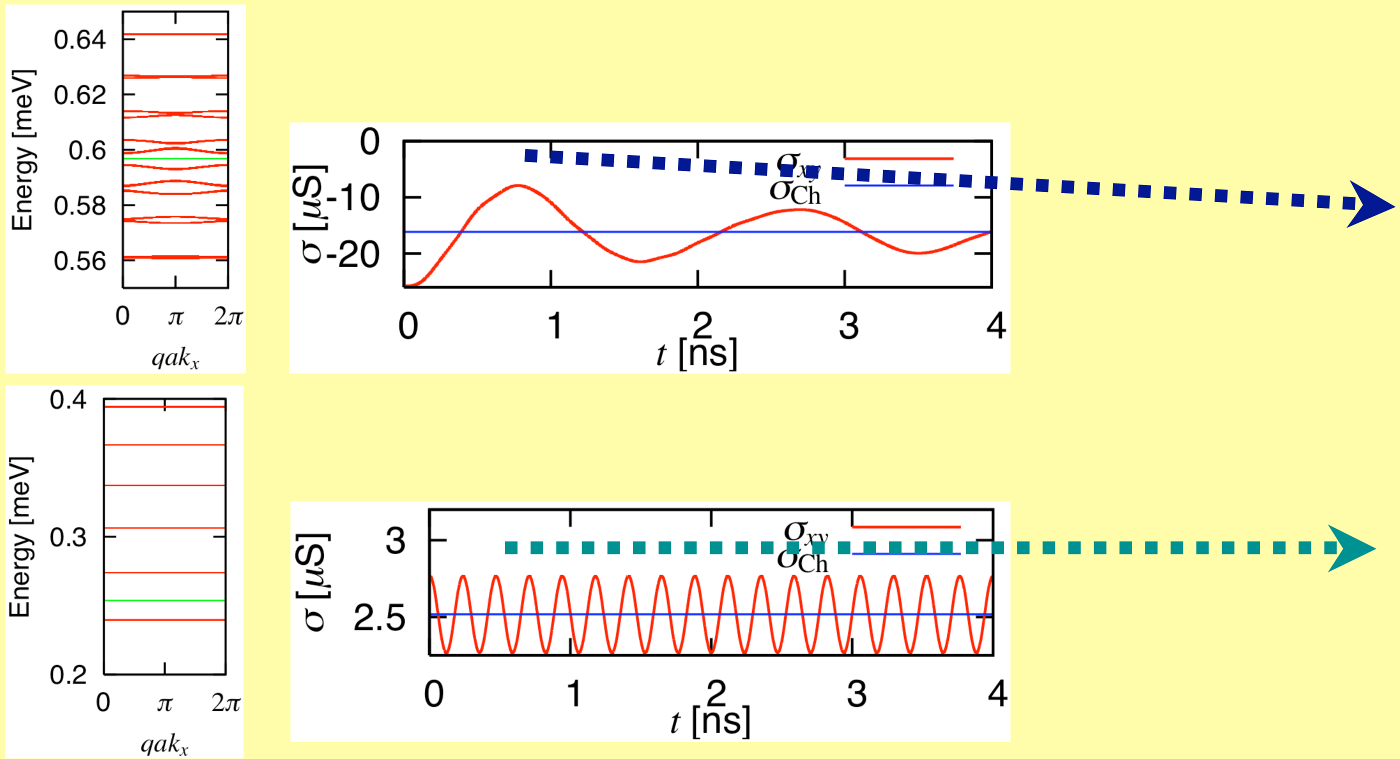
Long-time behavior

If bands are flat, and

only a pair of m_0th and (m_0+1)th bands contributes,

For $T \to \infty, L \to \infty,$

$$\overline{\sigma_{xy}} = \sigma_{\mathrm{Ch}}, \quad \overline{\sigma_{yy}} = 0, \quad \mathrm{var}(\sigma_{xy}) = \mathrm{var}(\sigma_{yy}) \neq 0$$



Decays fast

Decays slow

Summary

■ Quantum Hall effect of 2D Bloch electrons in a periodic potential

Conductivity for finite time

⇨ Conductivities oscillate in short time.

⇨ Eventually the time dependence will cease.

⇨ Behavior depends on the Fermi energy.

Manabu Machida, Naomichi Hatano, and Jun Goryo, J. Phys. Soc. Jpn. **75** (2006) 063704.