Weak localisation magnetoresistance in graphene

Edward McCann

Lancaster University, UK

with

K. Kechedzhi, V.I. Fal'ko,

H. Suzuura, T. Ando,

B.L. Altshuler

Outline

1. **Introduction – chiral particles in graphene, Berry's phase π, absence of backscattering, antilocalisation (?)**

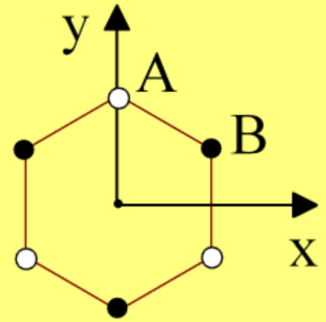
2. **Weak localisation in graphene – trigonal warping and "hidden" valley symmetry [high density $\varepsilon_F \tau >> 1$]**

3. **Weak localisation in bilayer graphene**

Electronic dispersion of a monolayer

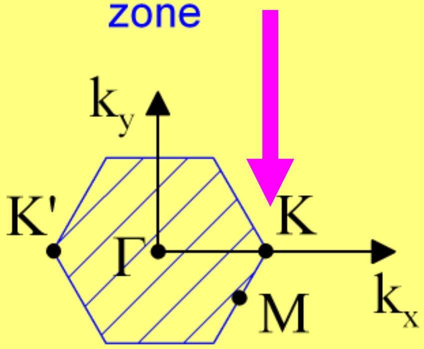
Saito *et al*, "Physical Properties of Carbon Nanotubes" (Imperial College Press, London, 1998)

Symmetrical unit cell

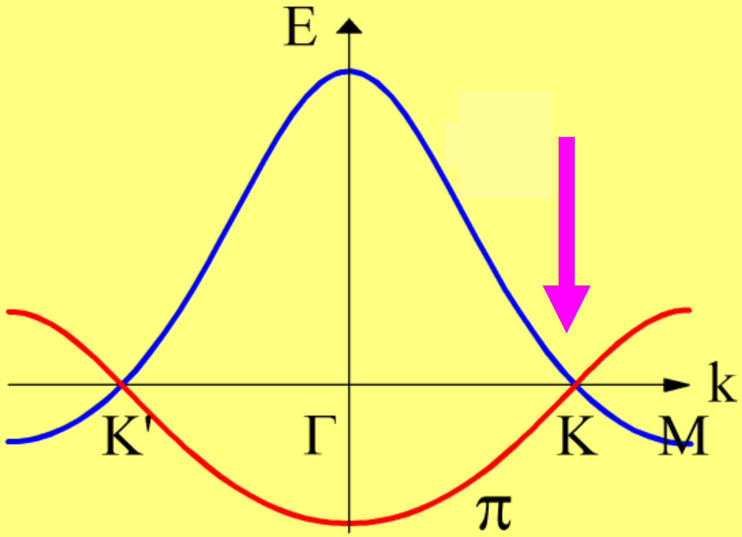


Two non-equivalent carbon positions

Brillouin zone



Two non-equivalent K-points



Two bands: no energy gap at the K-points

Tight binding model of a monolayer

Saito *et al*, "Physical Properties of Carbon Nanotubes" (Imperial College Press, London, 1998): Chapter 2.

Bloch function

$$\Phi_j(\mathbf{k},\mathbf{r}) = \frac{1}{\sqrt{N}} \sum_{\mathbf{R}_j}^{N} e^{i\mathbf{k}.\mathbf{R}_j}\, \phi_j(\mathbf{r}-\mathbf{R}_j)$$

sum over N atomic positions

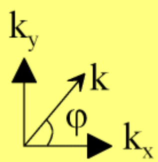
j^{th} atomic orbital: j = A or B

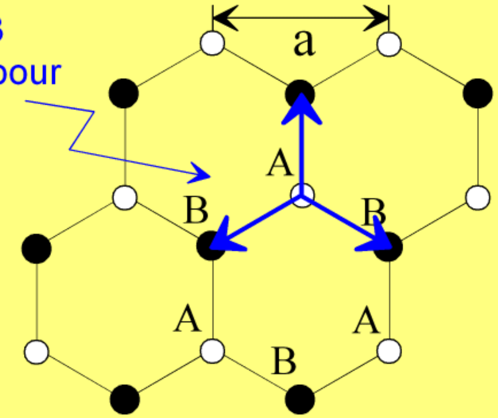
Transfer integral on a hexagonal lattice

$$\mathcal{H}_{AB} = \langle\Phi_A|H|\Phi_B\rangle$$

$$\mathcal{H}_{AB} = \frac{1}{N} \sum_{\mathbf{R}_A}^{N}\sum_{\mathbf{R}_B}^{N} e^{i\mathbf{k}.(\mathbf{R}_B - \mathbf{R}_A)} \underbrace{\langle\phi_A(\mathbf{r}-\mathbf{R}_A)|H|\phi_B(\mathbf{r}-\mathbf{R}_B)\rangle}_{\gamma_0}$$

sum over 3 nearest neighbour positions





Dirac Hamiltonian of a monolayer

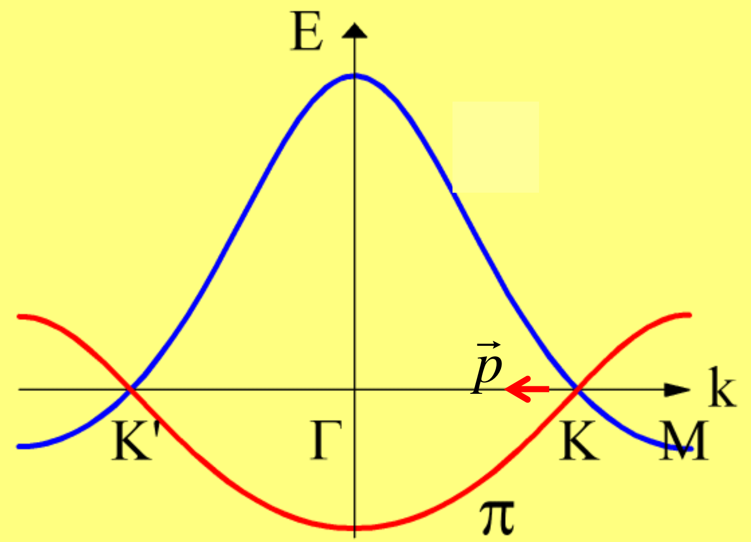
written in a 2 component basis of A and B sites

B to A hopping given by $\pi^{+} = p_x - ip_y$

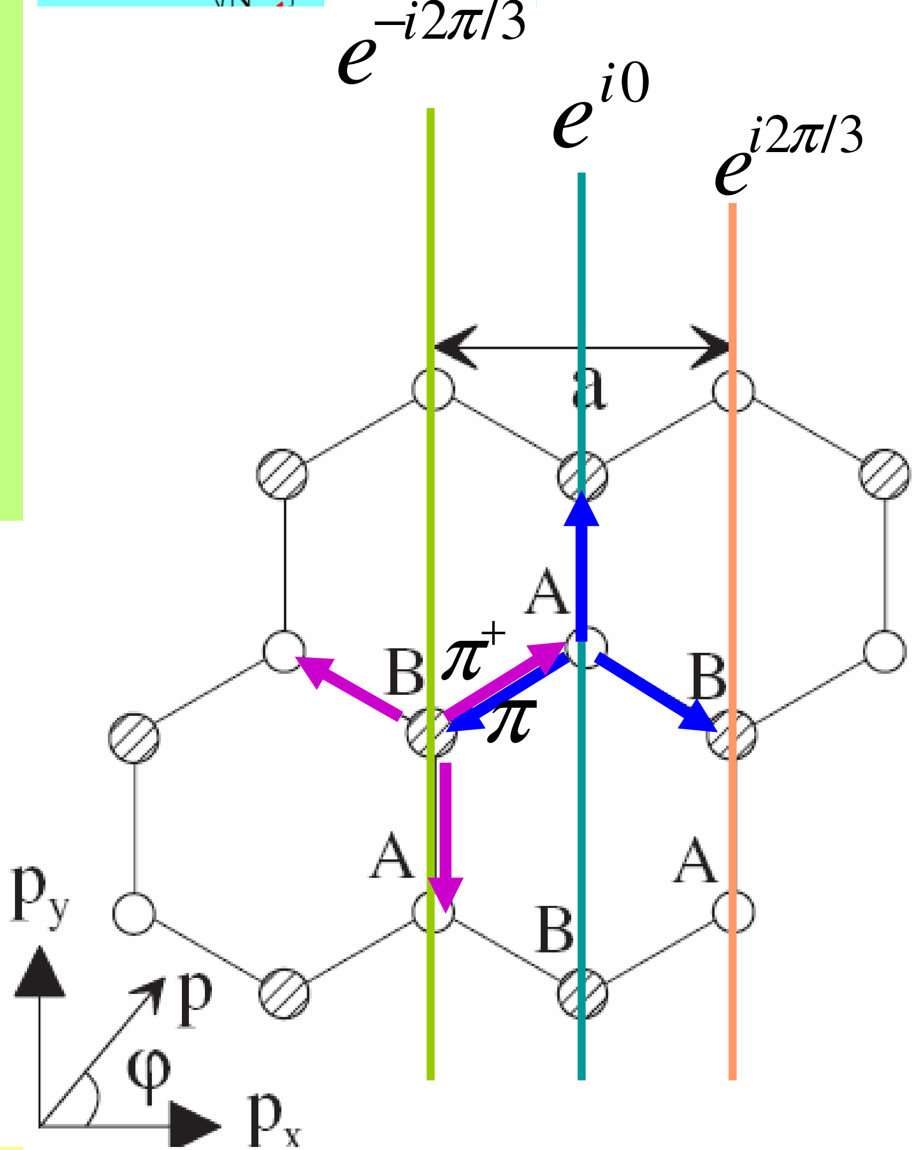
$$H = v\xi \begin{pmatrix} 0 & \pi^{+} \\ \pi & 0 \end{pmatrix} = v\xi(\sigma_x p_x + \sigma_y p_y)$$

A to B hopping given by $\pi = p_x + ip_y$

$$\Phi_j(\mathbf{k}, \mathbf{r}) = \frac{1}{\sqrt{N}} \sum_{\mathbf{R}_j}^{N} e^{i\mathbf{k}.\mathbf{R}_j} \phi_j(\mathbf{r}-\mathbf{R}_j)$$



Two bands: no energy gap at the K-points



Dirac-like equation

For one K point (e.g. $\xi=+1$) we have a 2 component wave function,

$$\psi = \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix}$$

with the following effective Hamiltonian:

$$H = v\begin{pmatrix} 0 & p_x - ip_y \\ p_x + ip_y & 0 \end{pmatrix} = v\begin{pmatrix} 0 & \pi^+ \\ \pi & 0 \end{pmatrix} = v\left(\sigma_x p_x + \sigma_y p_y\right) = v\vec{\sigma}.\vec{p}$$

$$\pi = p_x + ip_y = pe^{i\varphi}$$
$$\pi^+ = p_x - ip_y = pe^{-i\varphi}$$

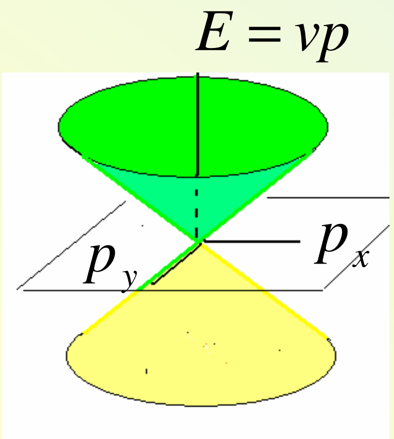
Bloch function amplitudes on the AB sites ('pseudospin') mimic spin components of a relativistic Dirac fermion.

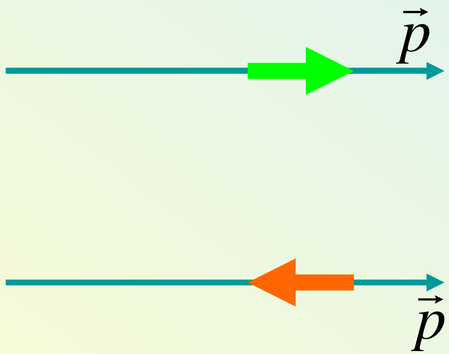
Dirac-like equation

$$H = v\begin{pmatrix} 0 & \pi^{+} \\ \pi & 0 \end{pmatrix} = v\vec{\sigma}\cdot\vec{p} = vp\ \vec{\sigma}\cdot\vec{n}$$

Chiral electrons

pseudospin direction is linked to an axis determined by electronic momentum.





for conduction band electrons,

$$\vec{\sigma}\cdot\vec{n} = 1$$

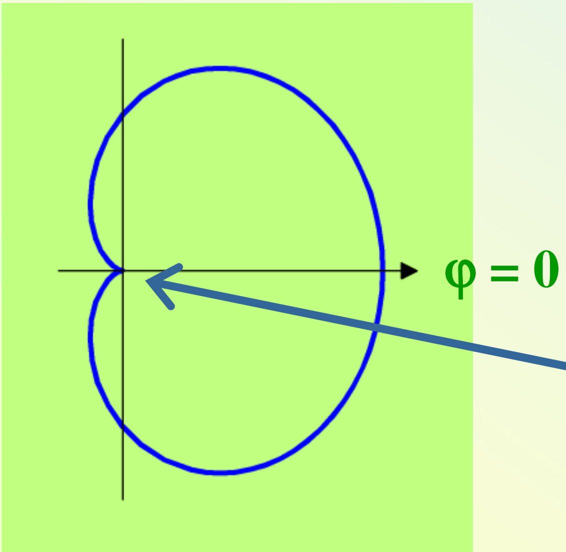
$$\vec{\sigma}\cdot\vec{n} = -1$$

valence band ('holes')

Absence of backscattering

$$H = v\begin{pmatrix} 0 & \pi^+ \\ \pi & 0 \end{pmatrix} = vp\begin{pmatrix} 0 & e^{-i\varphi} \\ e^{i\varphi} & 0 \end{pmatrix}; \qquad E = vp \quad \Leftrightarrow \quad \psi(\varphi) = \tfrac{1}{\sqrt{2}}\begin{pmatrix} e^{-i\varphi/2} \\ e^{i\varphi/2} \end{pmatrix}$$

angular scattering probability:



$$\left|\langle \psi(\varphi) | \psi(\varphi = 0) \rangle\right|^2 = \cos^2(\varphi/2)$$

under pseudospin conservation, chirality suppresses backscattering in a monolayer

Absence of backscattering [carbon nanotubes]

Journal of the Physical Society of Japan
Vol. 67, No. 8, August, 1998, pp. 2857–2862

Berry's Phase and Absence of Back Scattering in Carbon Nanotubes

Tsuneya ANDO, Takeshi NAKANISHI,[1] and Riichiro SAITO[2]

The absence of back scattering in carbon nanotubes is shown to be ascribed to Berry's phase which corresponds to a sign change of the wave function under a spin rotation of a neutrino-like particle in a two-dimensional graphite. Effects of trigonal warping of the bands appearing in a higher order $\boldsymbol{k}\cdot\boldsymbol{p}$ approximation are shown to give rise to a small probability of back scattering.

Absence of backscattering [carbon nanotubes]

VOLUME 83, NUMBER 24 PHYSICAL REVIEW LETTERS 13 DECEMBER 1999

page 5098

Disorder, Pseudospins, and Backscattering in Carbon Nanotubes

Paul L. McEuen, Marc Bockrath, David H. Cobden,* Young-Gui Yoon, and Steven G. Louie

Department of Physics, University of California, and Materials Science Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720

(Received 7 June 1999)

We address the effects of disorder on the conducting properties of metal and semiconducting carbon nanotubes. Experimentally, the mean free path is found to be much larger in metallic tubes than in doped semiconducting tubes. We show that this result can be understood theoretically if the disorder potential is long ranged. The effects of a pseudospin index that describes the internal sublattice structure of the states lead to a suppression of scattering in metallic tubes, but not in semiconducting tubes. This conclusion is supported by tight-binding calculations.

Weak localisation in graphene

VOLUME 89, NUMBER 26 PHYSICAL REVIEW LETTERS 23 DECEMBER 2002

article 266603

Crossover from Symplectic to Orthogonal Class in a Two-Dimensional Honeycomb Lattice

Hidekatsu Suzuura* and Tsuneya Ando*

Institute for Solid State Physics, University of Tokyo, 5-1-5 Kashiwanoha, Kashiwa 277-8581, Japan

(Received 29 March 2002; published 12 December 2002)

We have calculated the weak-localization correction to the conductivity for disordered electrons in a two-dimensional honeycomb lattice and shown that it can be either positive or negative depending on the interaction range of impurity potentials. From symmetry considerations, the symplectic class turns out to be realized at nonzero temperatures and crossover to the orthogonal class is predicted with decreasing temperature.

long-range potential:
weak anti-localisation

short-range potential:
weak localisation

Weak localisation in graphene 2006

Experiment

"Strong suppression of weak localization in graphene"
SV Morozov, KS Novoselov, MI Katsnelson, F Schedin, LA Ponomarenko, D Jiang, and AK Geim,
Phys Rev Lett. **97**, 016801 (2006)

Theory

"Electron Localization Properties in Graphene"
DV Khveshchenko, Phys Rev Lett. **97**, 036802 (2006)

"Intervalley scattering, long-range disorder, and effective time reversal symmetry breaking in graphene"
AF Morpurgo and F Guinea, cond-mat/0603789

"Weak localisation magnetoresistance and valley symmetry in graphene"
E McCann, K Kechedzhi, VI Fal'ko, H Suzuura, T Ando, and BL Altshuler, Phys Rev Lett. **97**, 146805 (2006)

"Effect of disorder on transport in graphene"
IL Aleiner and KB Efetov, cond-mat/0607200

"Low energy theory of disordered graphene"
A Altland, cond-mat/0607247

$$H = v\begin{pmatrix} 0 & \pi^{+} \\ \pi & 0 \end{pmatrix}$$

Berry phase π
suppressed backscattering

weak anti-localisation?

Berry phase romantics

role of different types of disorder?
…. of trigonal warping?

Trigonal warping

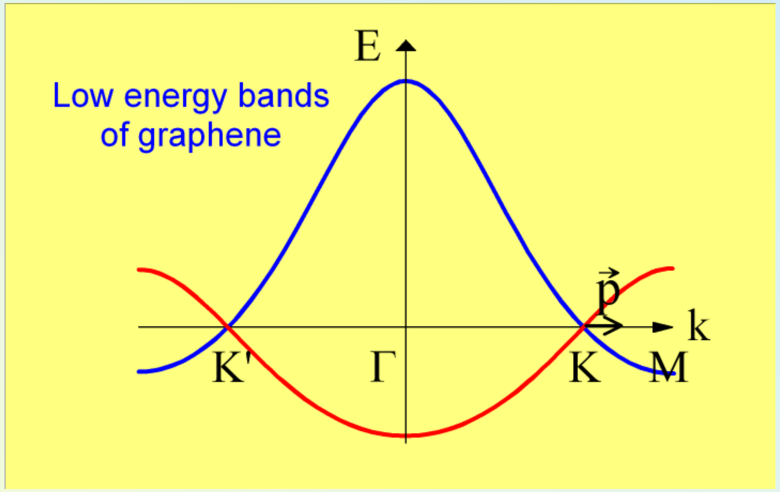
Journal of the Physical Society of Japan
Vol. 67, No. 8, August, 1998, pp. 2857–2862

Berry's Phase and Absence of Back Scattering in Carbon Nanotubes

Tsuneya ANDO, Takeshi NAKANISHI,[1] and Riichiro SAITO[2]

The absence of back scattering in carbon nanotubes is shown to be ascribed to Berry's phase which corresponds to a sign change of the wave function under a spin rotation of a neutrino-like particle in a two-dimensional graphite. Effects of trigonal warping of the bands appearing in a higher order $\boldsymbol{k}\cdot\boldsymbol{p}$ approximation are shown to give rise to a small probability of back scattering.

Trigonal warping



$$\pi = p_x + ip_y \equiv pe^{i\phi}$$

$$\pi^+ = p_x - ip_y \equiv pe^{-i\phi}$$

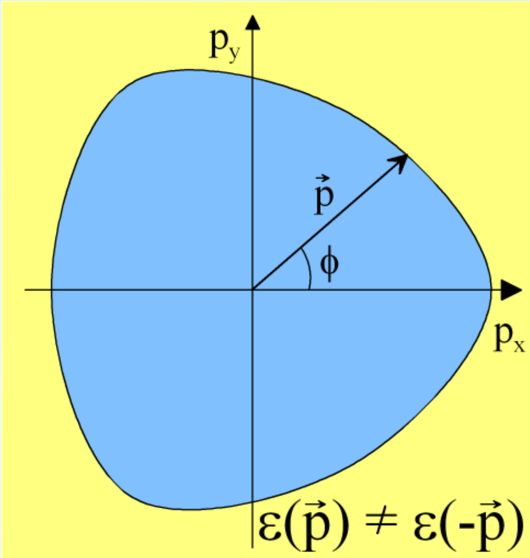
$$H_1 = \xi v \begin{pmatrix} 0 & \pi^+ \\ \pi & 0 \end{pmatrix} - \mu \begin{pmatrix} 0 & \pi^2 \\ (\pi^+)^2 & 0 \end{pmatrix}$$

$$\begin{pmatrix} A \\ B \\ B \\ A \end{pmatrix} \begin{array}{l} \xi = +1 \\ \\ \xi = -1 \end{array}$$

valley index ξ=+1,-1

trigonal warping

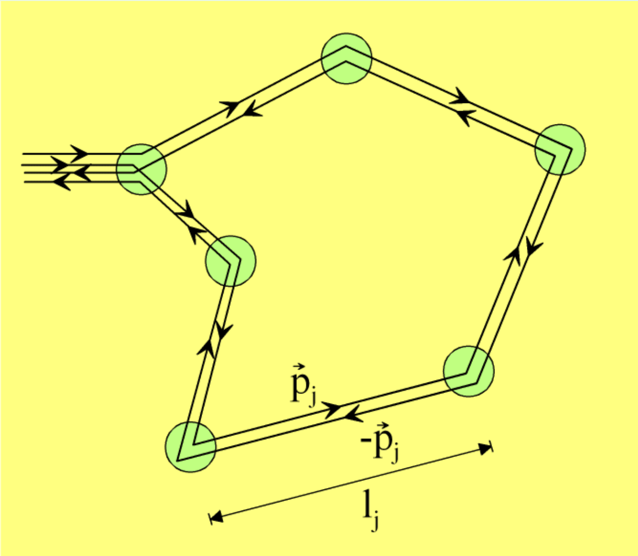
Trigonal warping



$$H_1 = \xi v \begin{pmatrix} 0 & \pi^+ \\ \pi & 0 \end{pmatrix} - \mu \begin{pmatrix} 0 & \pi^2 \\ (\pi^+)^2 & 0 \end{pmatrix} \qquad \begin{pmatrix} A \\ B \end{pmatrix}_{\xi=+1} \begin{pmatrix} B \\ A \end{pmatrix}_{\xi=-1}$$

$$\varepsilon^2 = v^2 p^2 - 2\xi\mu v p^3 \cos 3\phi + \mu^2 p^4 \; ; \quad \mu^2 p^2 / v^2 << 1$$

Trigonal warping



After time $t >> \tau$ there are t/τ segments of length $l_j \sim v_F \tau$

Trigonal warping leads to an additional phase difference between electrons travelling in opposite directions around closed loop:

$$\delta = \sum_j \left[\varepsilon(\vec{p}_j) - \varepsilon(-\vec{p}_j)\right] l_j / \hbar v_F$$

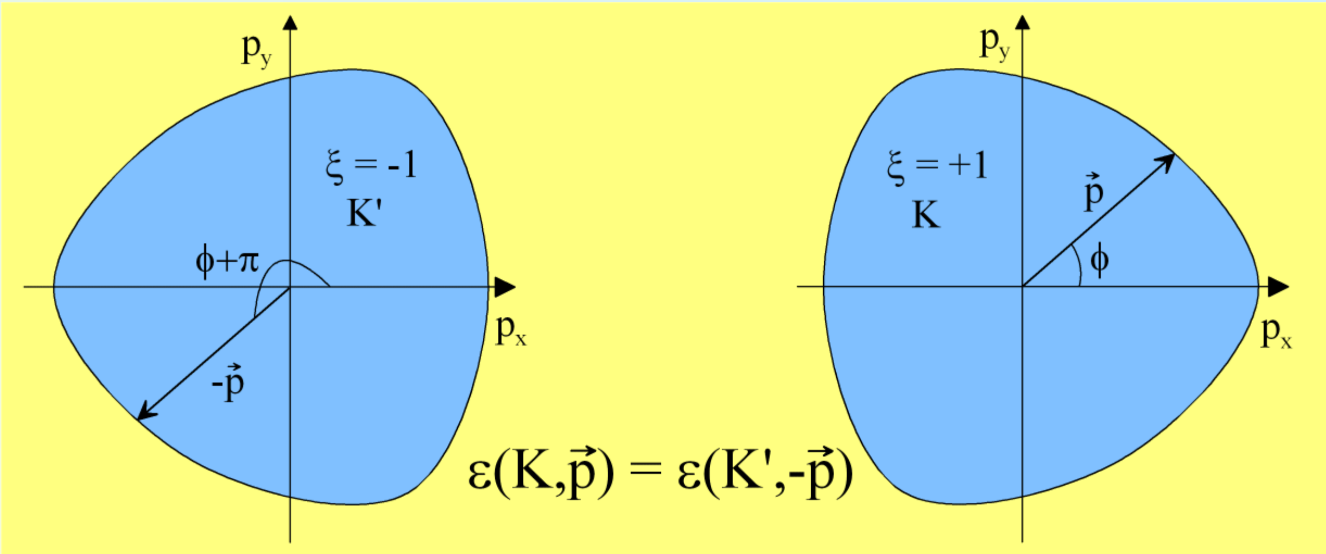
Produces dephasing when

$$\langle \delta^2 \rangle \sim \langle Tr\, h_w^2(\vec{p}) \rangle_\phi t\tau / \hbar^2 \sim 1$$

Trigonal warping relaxation rate:

$$\tau_w^{-1} \sim \tau \langle Tr\, h_w^2(\vec{p}) \rangle / \hbar^2 \sim 2\tau \left(\varepsilon^2 \mu / \hbar v^2\right)^2$$

Trigonal warping



$$H_1 = \xi v \begin{pmatrix} 0 & \pi^+ \\ \pi & 0 \end{pmatrix} - \mu \begin{pmatrix} 0 & \pi^2 \\ (\pi^+)^2 & 0 \end{pmatrix}$$

$$\begin{pmatrix} A \\ B \end{pmatrix}_{\xi=+1} \qquad \begin{pmatrix} B \\ A \end{pmatrix}_{\xi=-1}$$

$$\varepsilon^2 = v^2 p^2 - 2\xi\mu v p^3 \cos 3\phi + \mu^2 p^4 \;; \quad \mu^2 p^2 / v^2 << 1$$

Model of disorder

$$\hat{V}(\vec{r}) = u(\vec{r})\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} + \text{symmetry breaking disorder}$$

charges lying a distance from the sheet

$$\langle u(\vec{r})u(\vec{r}\,')\rangle = u^2\delta(\vec{r}-\vec{r}\,')$$

[e.g. due to atomically sharp defects or edges]

we assume that elastic scattering is dominated by 'diagonal' disorder, rate τ_0^{-1}

‘Hidden’ valley symmetry

Usually write 4 by 4 matrix using two sets of Pauli matrices:

$$\Pi_x = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}; \quad \Pi_y = \begin{pmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & 0 & -i \\ i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{pmatrix}; \quad \Pi_z = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\sigma_x = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}; \quad \sigma_y = \begin{pmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{pmatrix}; \quad \sigma_z = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

valley: $$\left[\Pi_{l_1}, \Pi_{l_2}\right] = 2i\varepsilon^{l_1 l_2 l_3}\Pi_{l_3}$$

lattice: $$\left[\sigma_{s_1}, \sigma_{s_2}\right] = 2i\varepsilon^{s_1 s_2 s_3}\sigma_{s_3}$$

two sets commute $$\left[\sigma_s, \Pi_l\right] = 0$$

'Hidden' valley symmetry

Instead, we introduce two sets of 4 by 4 Hermitian matrices:

$$\Lambda_x = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}; \quad \Lambda_y = \begin{pmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & 0 & i \\ i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \end{pmatrix}; \quad \Lambda_z = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\Sigma_x = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix}; \quad \Sigma_y = \begin{pmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \end{pmatrix}; \quad \Sigma_z = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

valley 'pseudospin' $\left[\Lambda_{l_1}, \Lambda_{l_2}\right] = 2i\varepsilon^{l_1 l_2 l_3} \Lambda_{l_3}$

lattice 'isospin' $\left[\Sigma_{s_1}, \Sigma_{s_2}\right] = 2i\varepsilon^{s_1 s_2 s_3} \Sigma_{s_3}$

two sets commute $\left[\Sigma_s, \Lambda_l\right] = 0$

'Hidden' valley symmetry

We introduce two sets of 4 by 4 Hermitian matrices:

$$\Lambda_x = \Pi_x \otimes \sigma_z; \quad \Lambda_y = \Pi_y \otimes \sigma_z; \quad \Lambda_z = \Pi_z \otimes \sigma_0$$

$$\Sigma_x = \Pi_z \otimes \sigma_x; \quad \Sigma_y = \Pi_z \otimes \sigma_y; \quad \Sigma_z = \Pi_0 \otimes \sigma_z$$

valley 'pseudospin' $\left[\Lambda_{l_1}, \Lambda_{l_2}\right] = 2i\varepsilon^{l_1 l_2 l_3} \Lambda_{l_3}$

lattice 'isospin' $\left[\Sigma_{s_1}, \Sigma_{s_2}\right] = 2i\varepsilon^{s_1 s_2 s_3} \Sigma_{s_3}$

two sets commute $\left[\Sigma_s, \Lambda_l\right] = 0$

‘Hidden’ valley symmetry

Why?

$$\Lambda_x = \Pi_x \otimes \sigma_z; \quad \Lambda_y = \Pi_y \otimes \sigma_z; \quad \Lambda_z = \Pi_z \otimes \sigma_0$$

$$\Sigma_x = \Pi_z \otimes \sigma_x; \quad \Sigma_y = \Pi_z \otimes \sigma_y; \quad \Sigma_z = \Pi_0 \otimes \sigma_z$$

they all change sign under time inversion

$$\Sigma_y \Lambda_y \Lambda_l^* \Sigma_y \Lambda_y = -\Lambda_l$$

$$\Sigma_y \Lambda_y \Sigma_s^* \Sigma_y \Lambda_y = -\Sigma_s$$

<u>16 possible Hermitian matrices:</u>

6 not time-reversal invariant

$$\Sigma_x, \Sigma_y, \Sigma_z,$$

$$\Lambda_x, \Lambda_y, \Lambda_z$$

10 time-reversal invariant

$$\hat{I}, \Sigma_x \Lambda_x, \Sigma_y \Lambda_x, \Sigma_z \Lambda_x,$$

$$\Sigma_x \Lambda_y, \Sigma_y \Lambda_y, \Sigma_z \Lambda_y,$$

$$\Sigma_x \Lambda_z, \Sigma_y \Lambda_z, \Sigma_z \Lambda_z$$

basis for non-magnetic, static disorder

Model of disorder

$$\hat{V}(\vec{r}) = \hat{I}\,u(\vec{r}) + \sum_{s,l=x,y,z} u_{sl}(\vec{r})\,\Sigma_s \Lambda_l$$

charges lying a distance from the sheet

$$\langle u(\vec{r})u(\vec{r}\,')\rangle = u^2 \delta(\vec{r}-\vec{r}\,')$$

we assume that elastic scattering is dominated by 'diagonal' disorder, rate τ_0^{-1}

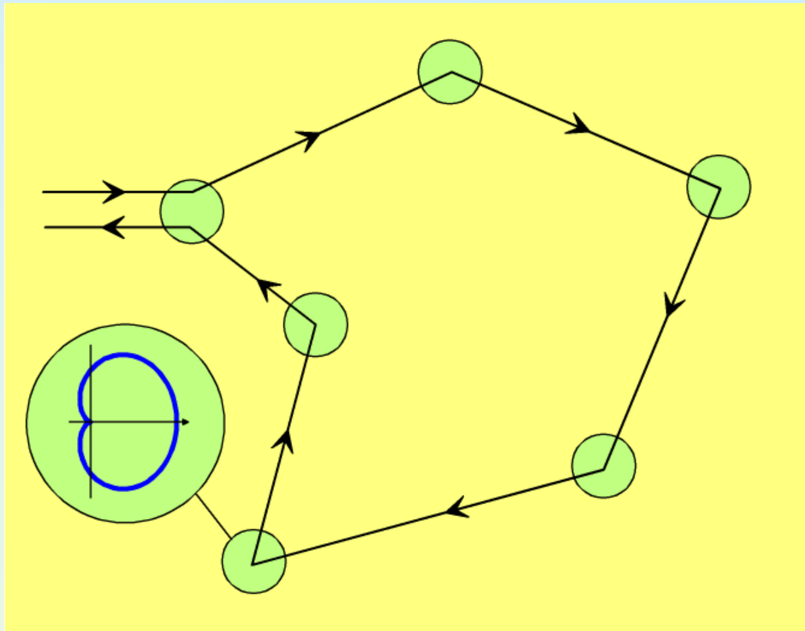
different A/B on-site energies	$\Sigma_z \Lambda_z$
valley antisymmetric vector potential	$\Sigma_x \Lambda_z, \Sigma_y \Lambda_z$
inter-valley scattering	$\Sigma_x \Lambda_x, \Sigma_y \Lambda_x, \Sigma_z \Lambda_x,$ $\Sigma_x \Lambda_y, \Sigma_y \Lambda_y, \Sigma_z \Lambda_y$

$$\langle u_{sl}(\vec{r})u_{s'l'}(\vec{r}\,')\rangle = u_{sl}^2\, \delta_{ss'}\, \delta_{ll'}\, \delta(\vec{r}-\vec{r}\,')$$

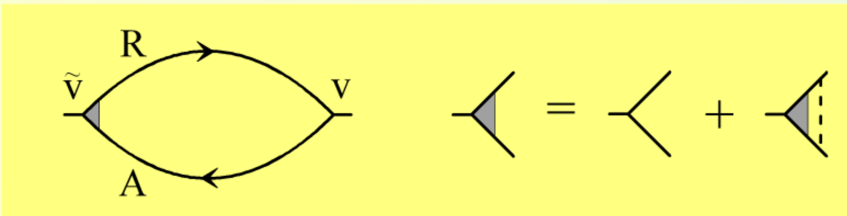
Diagonal disorder - Drude conductivity

under isospin conservation,
chirality suppresses
backscattering in a monolayer

$\tau_{tr} = 2\tau_0$



Drude conductivity



current operator is momentum-independent

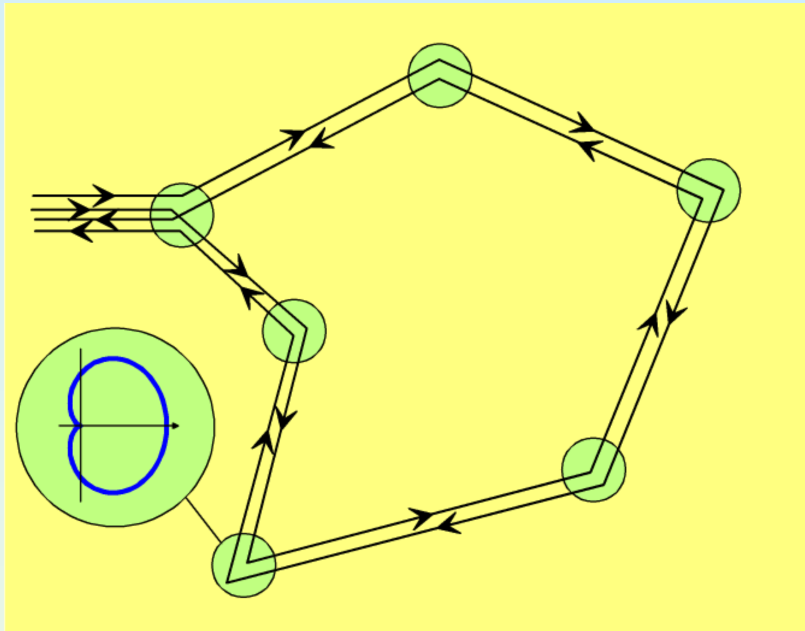
$$g_{xx} = 4e^2 \upsilon D$$

density of states per spin in one valley

diffusion coefficient

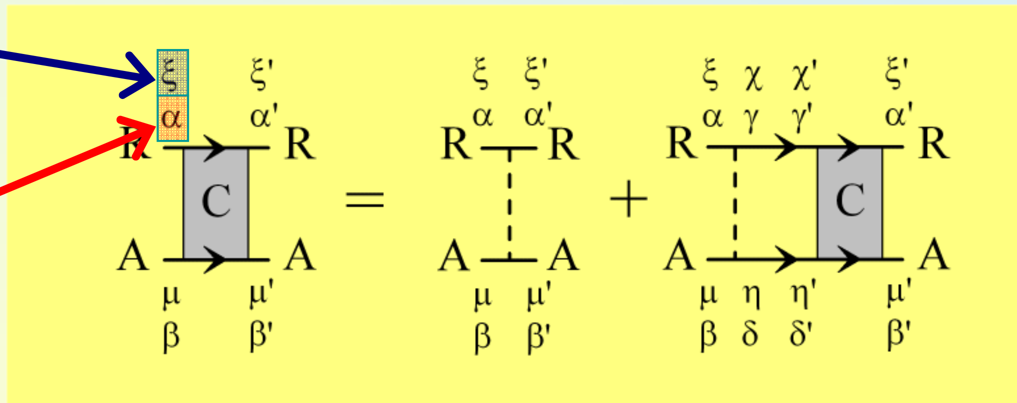
$$D = \tfrac{1}{2} v^2 \tau_{tr}$$

Diagonal disorder – weak localisation



valley component ξ = K or K'

lattice component α = A or B



Diagonal disorder – weak localisation

group Cooperons into isospin (Σ) and pseudospin (Λ) singlets (0) and triplets (x,y,z):

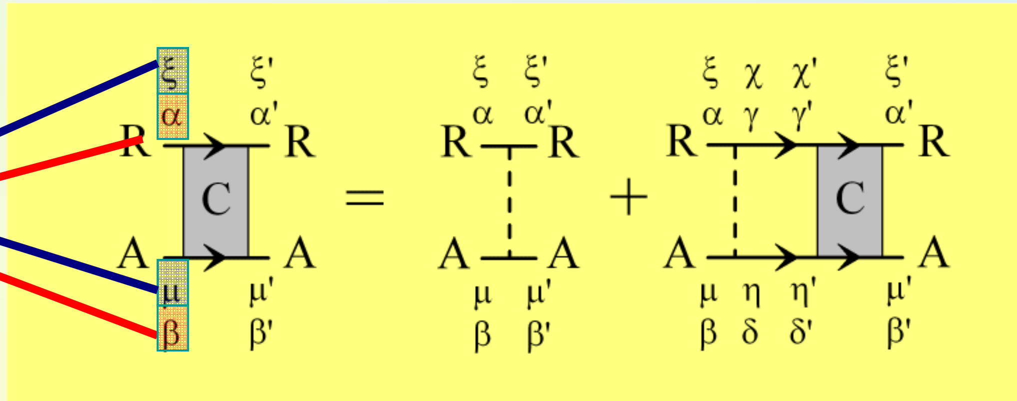
$$C^{l_1 l_2}_{s_1 s_2} = \tfrac{1}{4} \sum_{\alpha,\beta,\alpha',\beta'} \sum_{\xi,\mu,\xi',\mu'} \left(\Sigma_y \Sigma_{s_1} \Lambda_y \Lambda_{l_1}\right)^{\xi\mu}_{\alpha\beta} C^{\xi\mu,\xi'\mu'}_{\alpha\beta,\alpha'\beta'} \left(\Sigma_{s_2} \Sigma_y \Lambda_{l_2} \Lambda_y\right)^{\mu'\xi'}_{\beta'\alpha'}$$

16 diagonal modes

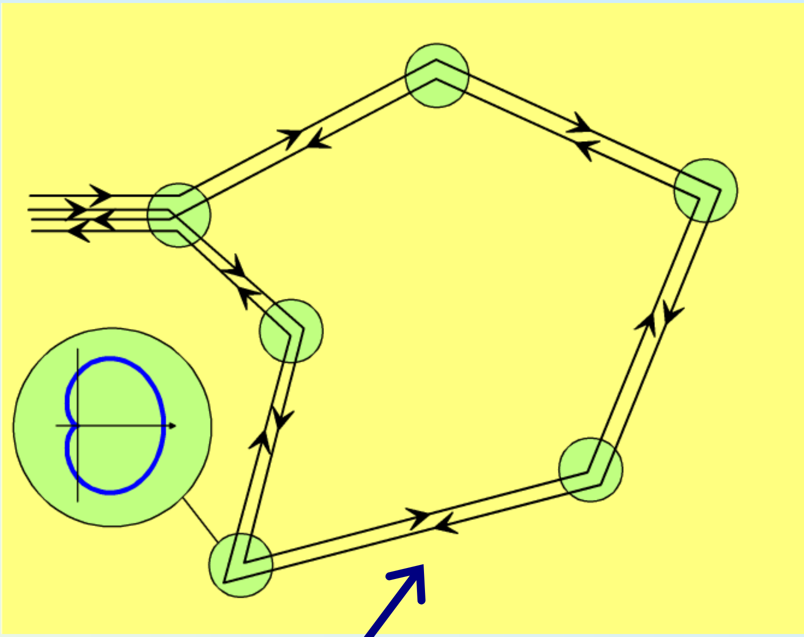
$$C^l_s \equiv C^{ll}_{ss}\,; \quad l = 0, x, y, z; \quad s = 0, x, y, z$$

pseudsospin l_1 = 0,x,y,z

isospin s_1 = 0,x,y,z



Diagonal disorder – weak localisation



plane waves

$$\psi_{\xi,\vec{p}} = \tfrac{1}{\sqrt{2}} e^{i\vec{p}.\vec{r}} \left(e^{-i\frac{\phi}{2}} \left|\uparrow\right\rangle_{\xi} + e^{i\frac{\phi}{2}} \left|\downarrow\right\rangle_{\xi} \right)$$

$$\psi_{\xi,-\vec{p}} = \tfrac{i}{\sqrt{2}} e^{-i\vec{p}.\vec{r}} \left(-e^{-i\frac{\phi}{2}} \left|\uparrow\right\rangle_{\xi} + e^{i\frac{\phi}{2}} \left|\downarrow\right\rangle_{\xi} \right)$$

in terms of isospin up and down:

$$\left|\uparrow\right\rangle_{+1} = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}; \left|\downarrow\right\rangle_{+1} = \begin{pmatrix}0\\1\\0\\0\end{pmatrix}; \left|\uparrow\right\rangle_{-1} = \begin{pmatrix}0\\0\\1\\0\end{pmatrix}; \left|\downarrow\right\rangle_{-1} = \begin{pmatrix}0\\0\\0\\-1\end{pmatrix};$$

plane waves in opposite directions along a ballistic segment:

$$\psi_{\xi,\vec{p}}\psi_{\xi',-\vec{p}} \sim \underbrace{\left|\uparrow\right\rangle_{\xi}\left|\downarrow\right\rangle_{\xi'} - \left|\downarrow\right\rangle_{\xi}\left|\uparrow\right\rangle_{\xi'}}_{\text{isospin singlet } C_0^{\ell}} \underbrace{- e^{-i\phi}\left|\uparrow\right\rangle_{\xi}\left|\uparrow\right\rangle_{\xi'} + e^{i\phi}\left|\downarrow\right\rangle_{\xi}\left|\downarrow\right\rangle_{\xi'}}_{\text{disappear after averaging wrt } \phi}$$

Diagonal disorder – weak localisation

4 isospin singlet modes C_0^l are gapless $\Gamma_0^l = 0$

8 isospin triplet modes C_x^l and C_y^l have a gap $\Gamma_x^l = \Gamma_y^l = \frac{1}{2}\tau_0^{-1}$

4 isospin triplet modes C_z^l have a gap $\Gamma_z^l = \tau_0^{-1}$

Isospin singlets are coupled to triplet modes:

4 by 4 matrix with $C_0^l, C_x^l, C_y^l, C_z^l$ on the diagonal

$$\begin{pmatrix} \frac{1}{2}v^2\tau_0 q^2 + \Gamma_0^l - i\omega & \frac{-i}{2}vq_x & \frac{-i}{2}vq_y & 0 \\ \frac{-i}{2}vq_x & \frac{1}{2}\tau_0^{-1} & 0 & 0 \\ \frac{-i}{2}vq_y & 0 & \frac{1}{2}\tau_0^{-1} & 0 \\ 0 & 0 & 0 & \tau_0^{-1} \end{pmatrix} \overline{C} = 1$$

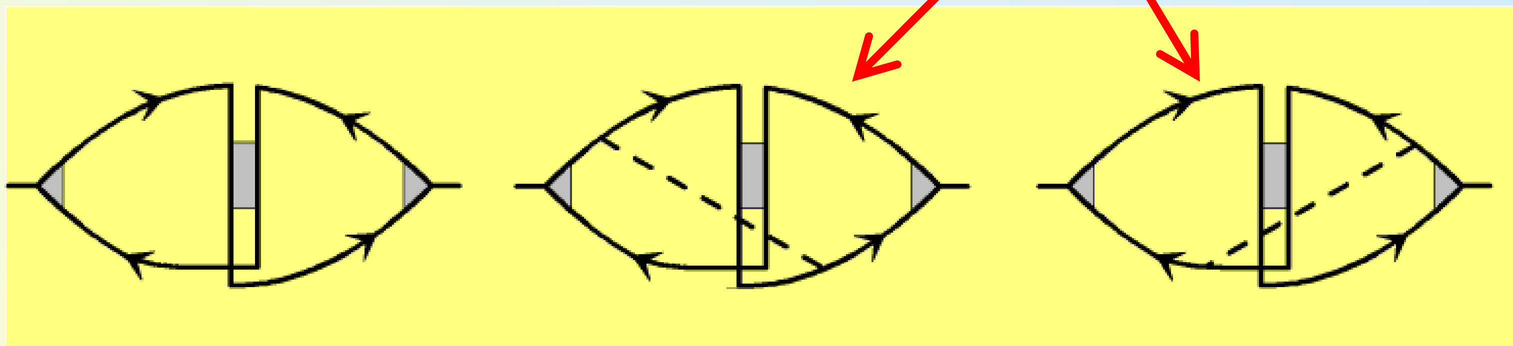
This coupling gives the correct diffusion operator for the gapless modes C_0^l

with $D = \frac{1}{2}v^2\tau_{tr} = v^2\tau_0 : \quad \left(Dq^2 - i\omega + \Gamma_0^l\right)C_0^l = 1$

Diagonal disorder – weak localisation

Relate gapless Cooperons C_0^l to the correction to conductivity

dressing of Hikami boxes leads to a reduction by factor ½



$$\delta g \sim C_0^x + C_0^y + C_0^z - C_0^0$$

For diagonal disorder, isospin singlet modes C_0^l are all gapless $\Gamma_0^l = 0$, leading to weak antilocalisation

Diagonal disorder – weak localisation

What happens to the four gapless modes C_0^l when there is trigonal warping and symmetry breaking disorder?

$$\delta g \sim C_0^x + C_0^y + C_0^z - C_0^0$$

For diagonal disorder, isospin singlet modes C_0^l are all gapless $\Gamma_0^l = 0$, leading to weak antilocalisation

'Hidden' valley symmetry

warping term is invariant with respect to valley transformation Λ_z only $\Gamma_0^0 = \Gamma_0^z = 0; \quad \Gamma_0^x = \Gamma_0^y \neq 0$

$$\hat{H}_1 = v\vec{\Sigma}.\vec{p} - \mu \Sigma_x \left(\vec{\Sigma}.\vec{p}\right) \Lambda_z \Sigma_x \left(\vec{\Sigma}.\vec{p}\right) \Sigma_x$$

$$+ \hat{I}\, u\left(\vec{r}\right) + \sum_{s,l=x,y,z} u_{sl}\left(\vec{r}\right) \Sigma_s \Lambda_l$$

leading terms do not contain valley operators Λ, so they remain invariant with respect to valley transformations $\Gamma_0^0 = \Gamma_0^x = \Gamma_0^y = \Gamma_0^z = 0$

intravalley disorder $\Sigma_s \Lambda_z \Rightarrow \Gamma_0^0 = \Gamma_0^z = 0; \quad \Gamma_0^x = \Gamma_0^y \neq 0$

intervalley disorder $\Sigma_s \Lambda_x \Rightarrow \Gamma_0^0 = \Gamma_0^x = 0; \quad \Gamma_0^y = \Gamma_0^z \neq 0$

$\Sigma_s \Lambda_y \Rightarrow \Gamma_0^0 = \Gamma_0^y = 0; \quad \Gamma_0^x = \Gamma_0^z \neq 0$

Weak localisation

$$\delta g \sim \overbrace{C_0^x + C_0^y}^{\text{same valley}} + \overbrace{C_0^z - C_0^0}^{\text{inter-valley}}$$

acquire gaps due to trigonal warping and symmetry breaking disorder

acquires a gap from inter-valley scattering

truly gapless mode

Expect to observe suppressed weak localisation with an increased amplitude as the degree of inter-valley scattering increases

Weak localisation

same valley inter-valley

$$\delta g(B=0) \sim \frac{e^2}{2\pi^2\hbar}\left\{ \underbrace{2\ln\left(\frac{\tau_\phi/\tau_{tr}}{1+\tau_\phi/\tau_*} \right)}_{\text{same valley}} - \underbrace{\ln\left(1+2\frac{\tau_\phi}{\tau_i}\right)}_{\text{inter-valley}} \right\}$$

trigonal warping and
symmetry breaking disorder

$$\tau_*^{-1} = \Gamma_0^x = \Gamma_0^y$$

inter-valley
scattering

Expect to observe suppressed weak localisation with an increased amplitude as the degree of inter-valley scattering increases

Weak localisation

$$\delta g(B=0) \sim \frac{e^2}{2\pi^2\hbar}\left\{ \overbrace{2\ln\left(\frac{\tau_\phi / \tau_{tr}}{1+\tau_\phi / \tau_*} \right)}^{\text{same valley}} \overbrace{-\ln\left(1+2\frac{\tau_\phi}{\tau_i} \right)}^{\text{inter-valley}} \right\}$$

trigonal warping and
symmetry breaking disorder

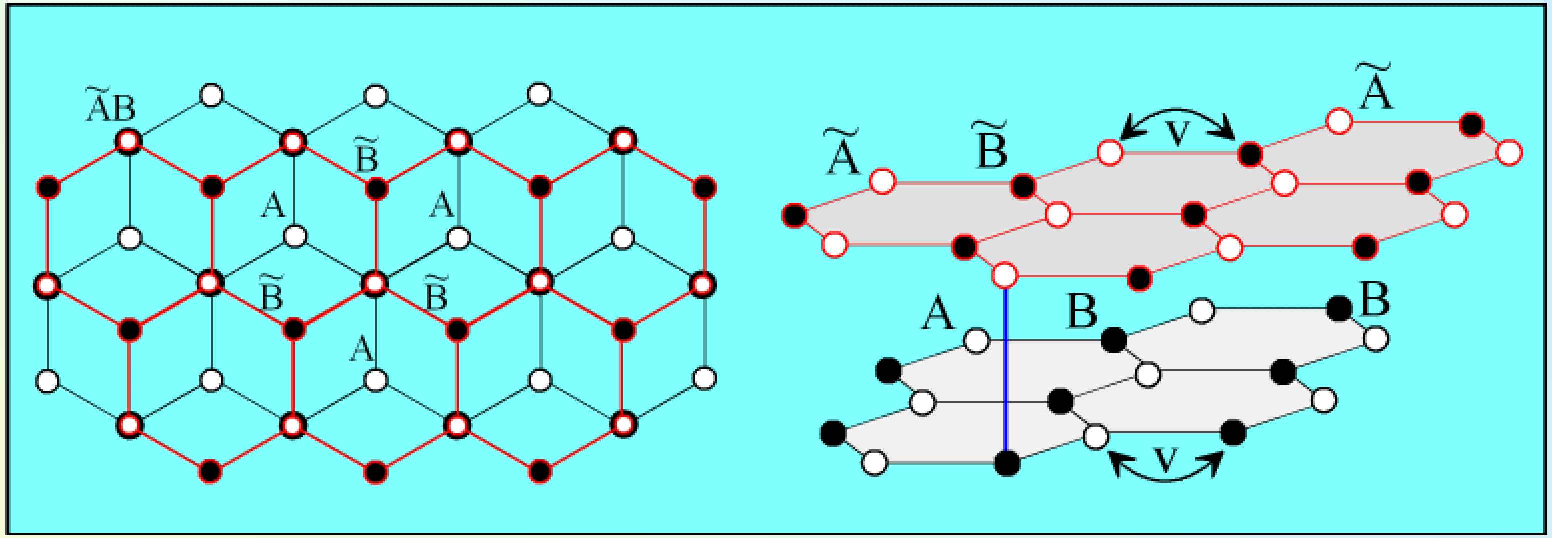
$$\tau_*^{-1} = \Gamma_0^x = \Gamma_0^y$$

inter-valley
scattering

We consider high density $\varepsilon_F\tau$ >> 1. Logarithmic dependence of parameters on energy discussed by Igor Aleiner this morning

[IL Aleiner and KB Efetov, cond-mat/0607200]

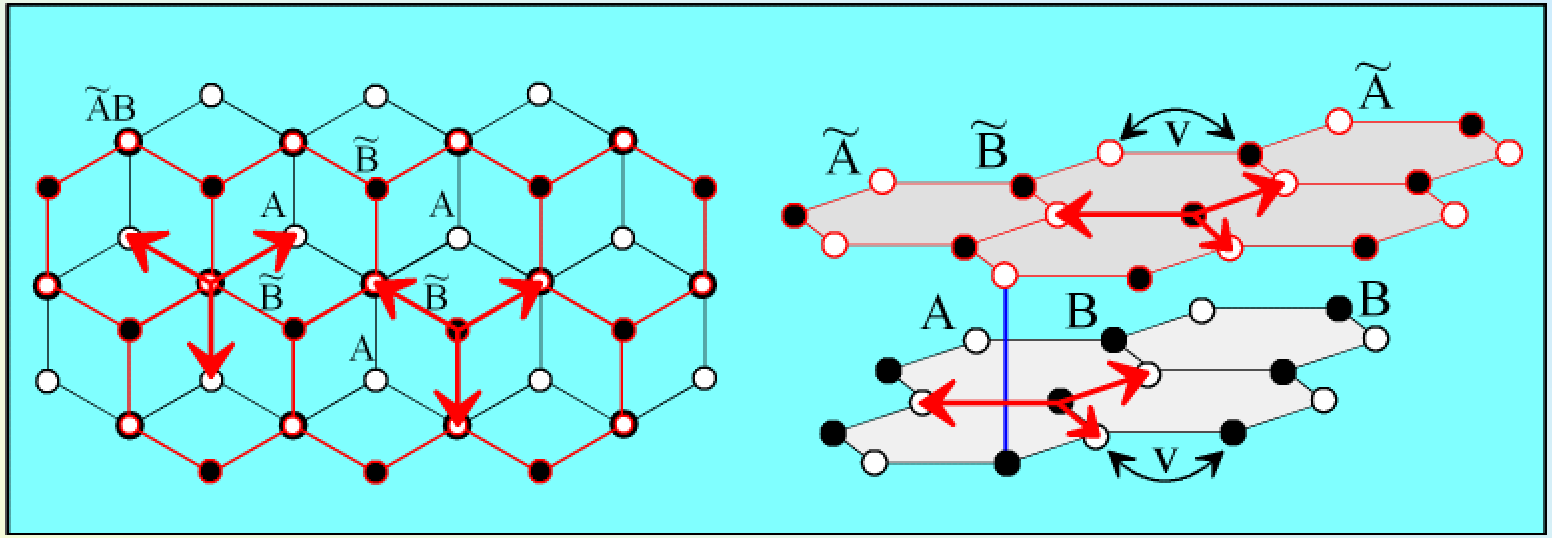
Bilayer [Bernal (AB) stacking]



4 atoms
per unit cell

$$\mathcal{H} = \begin{array}{cc} \begin{matrix} A & \tilde{B} & \tilde{A} & B \end{matrix} & \\ \left(\begin{matrix} \quad & \quad & \quad & \quad \\ & & & \\ & & & \\ & & & \end{matrix} \right) & \begin{matrix} A \\ \tilde{B} \\ \tilde{A} \\ B \end{matrix} \end{array}$$

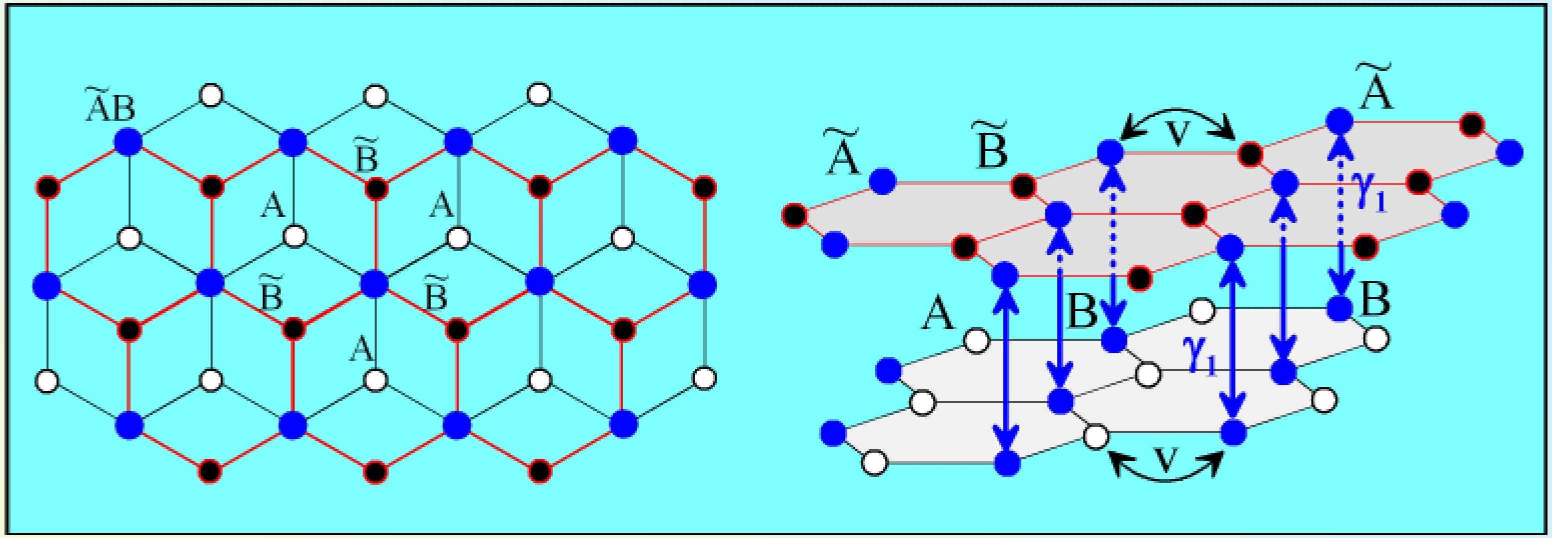
Bilayer [Bernal (AB) stacking]



(B to A) and ($\tilde{B}$ to $\tilde{A}$)
hopping
given by
$\pi^+ = p_x - ip_y$

$$H = \begin{array}{c} \\ \\ \end{array}\begin{array}{cc} \begin{array}{cccc} A & \tilde{B} & \tilde{A} & B \end{array} & \\ \begin{pmatrix} & & & v\pi^+ \\ & & v\pi & \\ & v\pi^+ & & \\ v\pi & & & \end{pmatrix} & \begin{array}{c} A \\ \tilde{B} \\ \tilde{A} \\ B \end{array} \end{array}$$

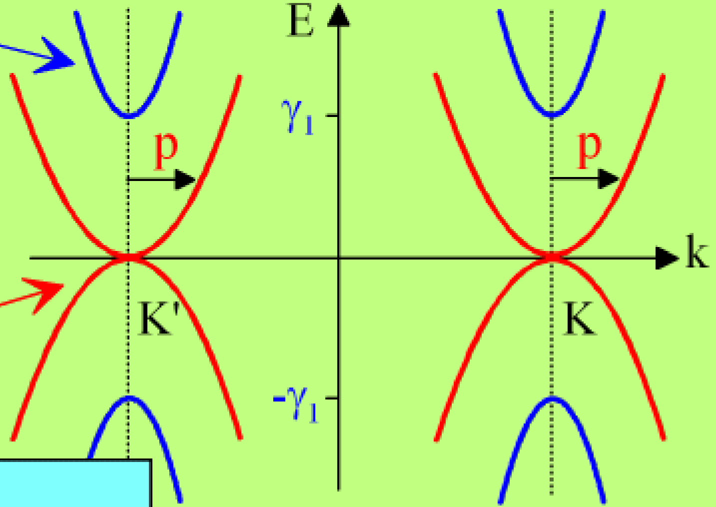
Bilayer [Bernal (AB) stacking]



Bilayer Hamiltonian

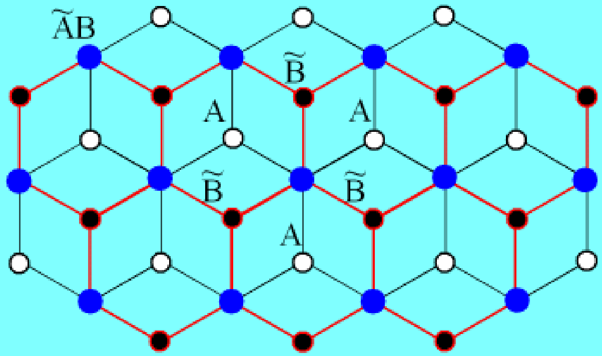
$$
H = \begin{array}{c}
\begin{array}{cccc} A & \tilde{B} & \tilde{A} & B \end{array} \\
\begin{pmatrix}
0 & 0 & 0 & v\pi^{+} \\
0 & 0 & v\pi & 0 \\
0 & v\pi^{+} & 0 & \gamma_1 \\
v\pi & 0 & \gamma_1 & 0
\end{pmatrix}
\end{array}
\begin{array}{c} A \\ \tilde{B} \\ \tilde{A} \\ B \end{array}
$$

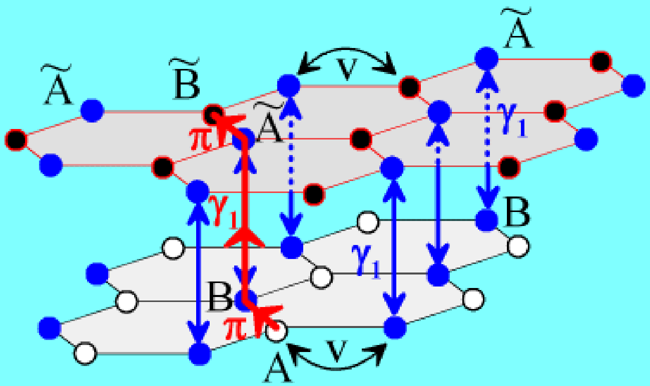
$\tilde{A}B$ orbitals form dimers with energy $|E| \gtrsim \gamma_1$



Quadratic dispersion at low energy:

$$E = \pm \frac{p^2}{2m}$$





E. McCann and V.I. Fal'ko
PRL 96, 086805 (2006)

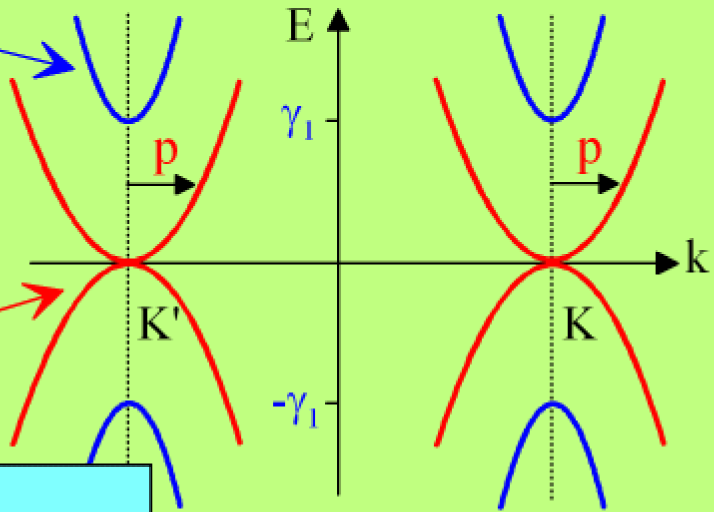
Bilayer Hamiltonian

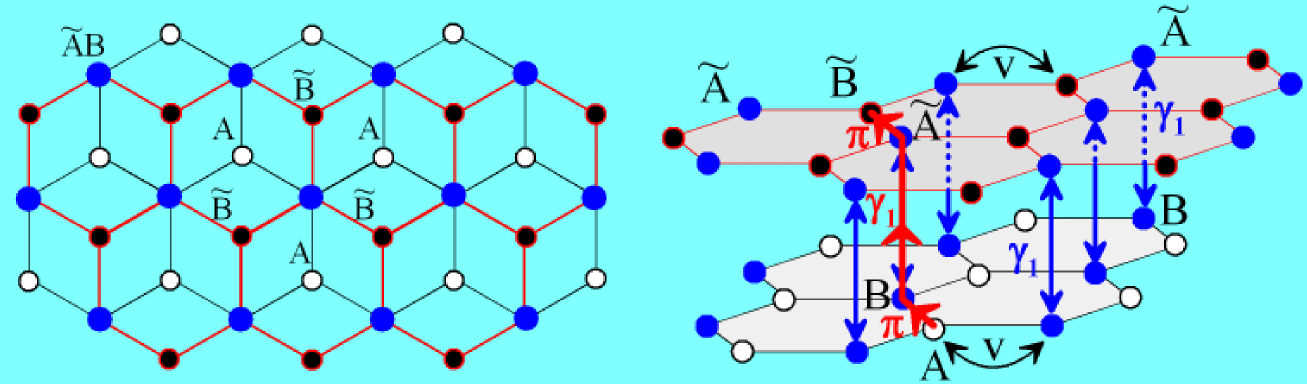
$$H = \begin{array}{c} \\ \end{array}\begin{array}{cc} \begin{array}{cccc} A & \tilde{B} & \tilde{A} & B \end{array} & \\ \begin{pmatrix} 0 & 0 & 0 & v\pi^+ \\ 0 & 0 & v\pi & 0 \\ 0 & v\pi^+ & 0 & \gamma_1 \\ v\pi & 0 & \gamma_1 & 0 \end{pmatrix} & \begin{array}{c} A \\ \tilde{B} \\ \tilde{A} \\ B \end{array} \end{array}$$

$\tilde{A}B$ orbitals form dimers with energy $|E| \gtrsim \gamma_1$

Quadratic dispersion at low energy:

$$E = \pm \frac{p^2}{2m}$$





E. McCann and V.I. Fal'ko
PRL 96, 086805 (2006)

Bilayer Hamiltonian written in a 2 component basis of A and $\tilde{B}$ sites

$$H = \frac{-1}{2m}\begin{pmatrix} 0 & (\pi^+)^2 \\ \pi^2 & 0 \end{pmatrix}$$

A to $\tilde{B}$ hopping

- bottom layer A → B (factor π)
- switch layers via dimer $B\tilde{A}$ (γ_1^{-1})
- top layer $\tilde{A}$ → $\tilde{B}$ (factor π)

$$\pi = p_x + ip_y$$

mass

$m = \gamma_1 / v^2$

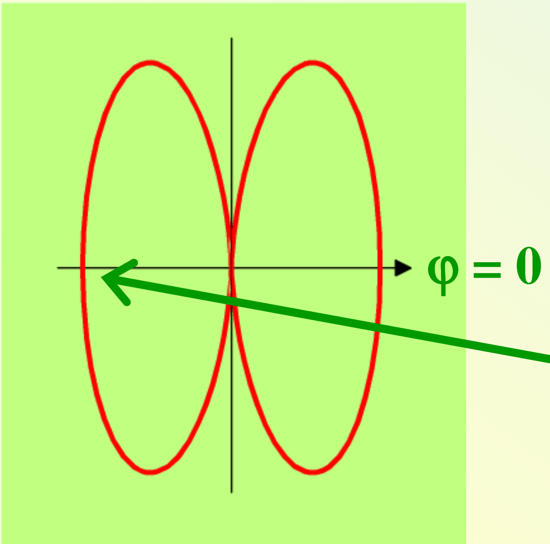
Bilayer graphene

Berry phase 2π quasiparticles

$$H = -\frac{1}{2m}\begin{pmatrix} 0 & (\pi^+)^2 \\ \pi^2 & 0 \end{pmatrix} = -\frac{p^2}{2m}\begin{pmatrix} 0 & e^{-2i\varphi} \\ e^{2i\varphi} & 0 \end{pmatrix}; \qquad E = \frac{p^2}{2m} \quad \Leftrightarrow \quad \psi(\varphi) = \tfrac{1}{\sqrt{2}}\begin{pmatrix} e^{-i\varphi} \\ e^{i\varphi} \end{pmatrix}$$

No absence of backscattering

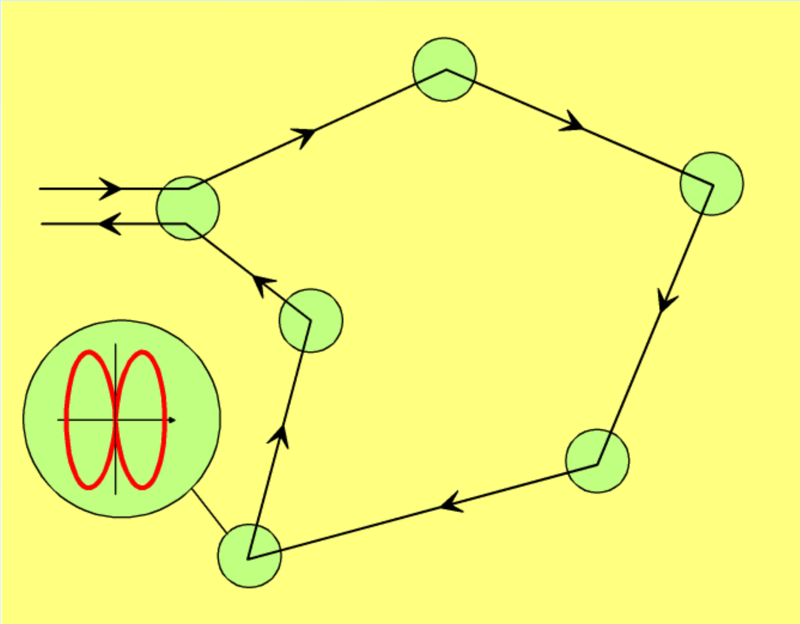
angular scattering probability:



$$\left|\langle \psi(\varphi) | \psi(\varphi = 0) \rangle\right|^2 = \cos^2(\varphi)$$

no suppression of backscattering in a bilayer

Bilayer - Diagonal disorder - Drude conductivity



under isospin conservation,
no suppression of
backscattering in a bilayer

$\tau_{tr} = \tau_0$

Drude conductivity

current operator is momentum-dependent

$$g_{xx} = 4e^2 \upsilon D$$

density of states
per spin
in one valley

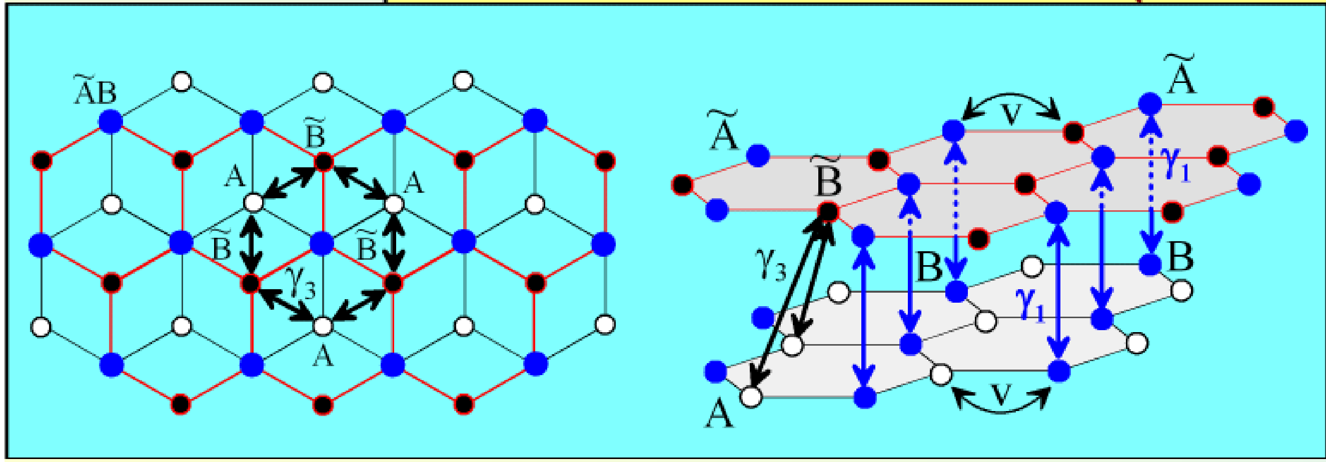
diffusion
coefficient

$$D = \tfrac{1}{2} v^2 \tau_0$$

Trigonal warping

(A to $\tilde{B}$) hopping parameterised by $v_3 = \sqrt{3}\pi a \gamma_3 / h$

$$H = \begin{array}{c} \\ \\ \\ \\ \\ \end{array}\begin{array}{cc} & \begin{array}{cccc} A & \tilde{B} & \tilde{A} & B \end{array} \\ & \begin{pmatrix} 0 & v_3\pi & 0 & v\pi^+ \\ v_3\pi^+ & 0 & v\pi & 0 \\ 0 & v\pi^+ & 0 & \gamma_1 \\ v\pi & 0 & \gamma_1 & 0 \end{pmatrix} \begin{array}{c} A \\ \tilde{B} \\ \tilde{A} \\ B \end{array} \end{array}$$



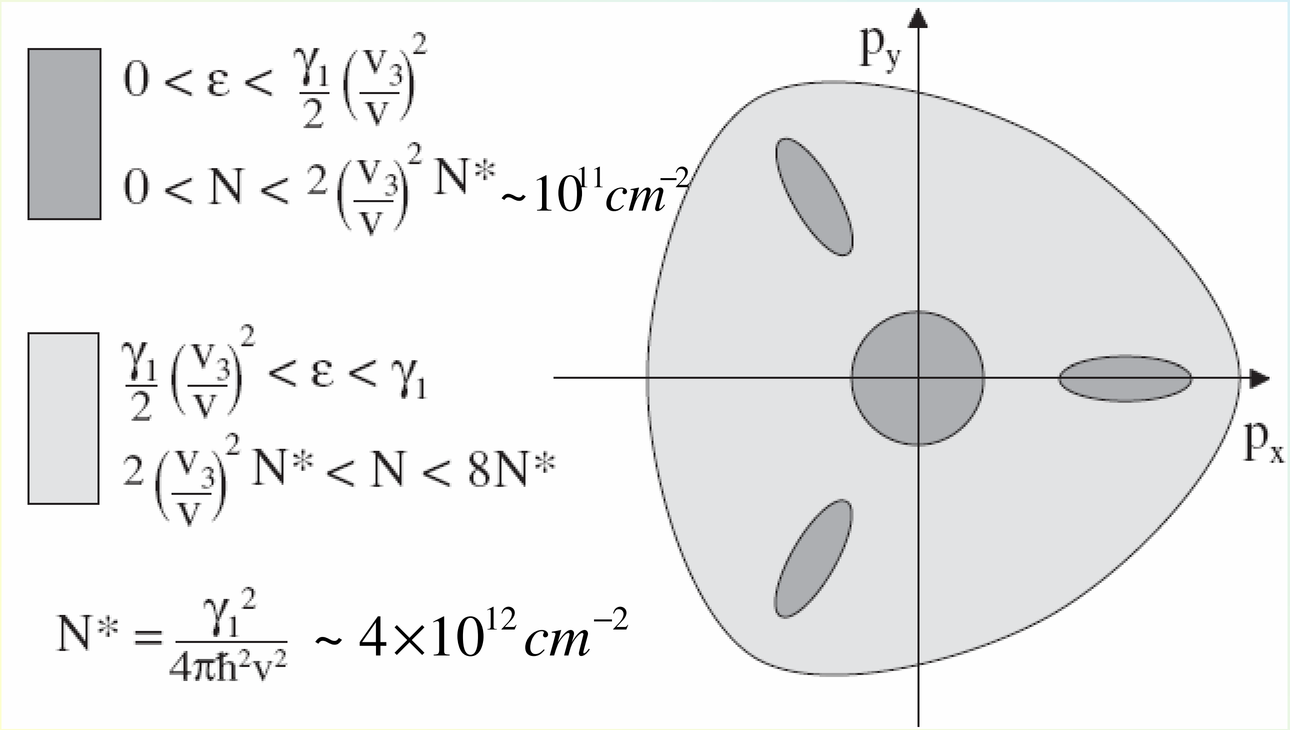
E. McCann and V.I. Fal'ko
PRL 96, 086805 (2006)

$$H_2 = -\frac{1}{2m}\begin{pmatrix} 0 & (\pi^+)^2 \\ \pi^2 & 0 \end{pmatrix} + \xi v_3 \begin{pmatrix} 0 & \pi \\ \pi^+ & 0 \end{pmatrix}$$

$$\begin{pmatrix} A \\ \tilde{B} \end{pmatrix} \xi = +1 \qquad \begin{pmatrix} \tilde{B} \\ A \end{pmatrix} \xi = -1$$

Trigonal warping - bilayer

$$\varepsilon^2 = \left(\frac{p^2}{2m}\right)^2 - \frac{\xi v_3 p^3}{m}\cos 3\phi + v_3^2 p^2$$



$$H_1 = v\begin{pmatrix} 0 & \pi^+ \\ \pi & 0 \end{pmatrix}$$

Berry phase π
suppressed backscattering
weak anti-localisation ?

Berry phase romantics

$$H_2 = -\frac{1}{2m}\begin{pmatrix} 0 & (\pi^+)^2 \\ \pi^2 & 0 \end{pmatrix}$$

Berry phase 2π
weak localisation ?

higher order expansion

$$H_1 = \xi v \begin{pmatrix} 0 & \pi^+ \\ \pi & 0 \end{pmatrix} - \mu \begin{pmatrix} 0 & \pi^2 \\ (\pi^+)^2 & 0 \end{pmatrix} + \hat{V}(\vec{r}) \qquad \begin{pmatrix} A \\ B \\ B \\ A \end{pmatrix} \begin{matrix} \xi=+1 \\ \\ \xi=-1 \end{matrix}$$

‘trigonal warping’:
symmetry of wave vector K is lower than the hexagonal symmetry

valley

$$H_2 = \frac{-1}{2m} \begin{pmatrix} 0 & (\pi^+)^2 \\ \pi^2 & 0 \end{pmatrix} + \xi v_3 \begin{pmatrix} 0 & \pi \\ \pi^+ & 0 \end{pmatrix} + \hat{V}(\vec{r}) \qquad \begin{pmatrix} A \\ \tilde{B} \\ \tilde{B} \\ A \end{pmatrix} \begin{matrix} \xi=+1 \\ \\ \xi=-1 \end{matrix}$$

off-diagonal $A\tilde{B}$
interlayer hopping

Weak localisation correction

$\varepsilon_F \tau >> 1$ High electron (hole) density and remote Coulomb scatterers

~~Berry phase π~~

$$\delta g_1 = - C_{KK'-symm} + C_{KK'-antisymm} \cancel{+ C_{KK} + C_{K'K'}}$$

can only be suppressed by decoherence

may be suppressed by intervalley scattering τ_i due to atomically sharp scatterers or edges

killed by trigonal warping reflecting the asymmetry

$$H(-\vec{p}) \neq H(\vec{p})$$

in each valley

$$\delta g_2 = - C_{KK'-symm} + C_{KK'-antisymm} \cancel{- C_{KK} - C_{K'K'}}$$

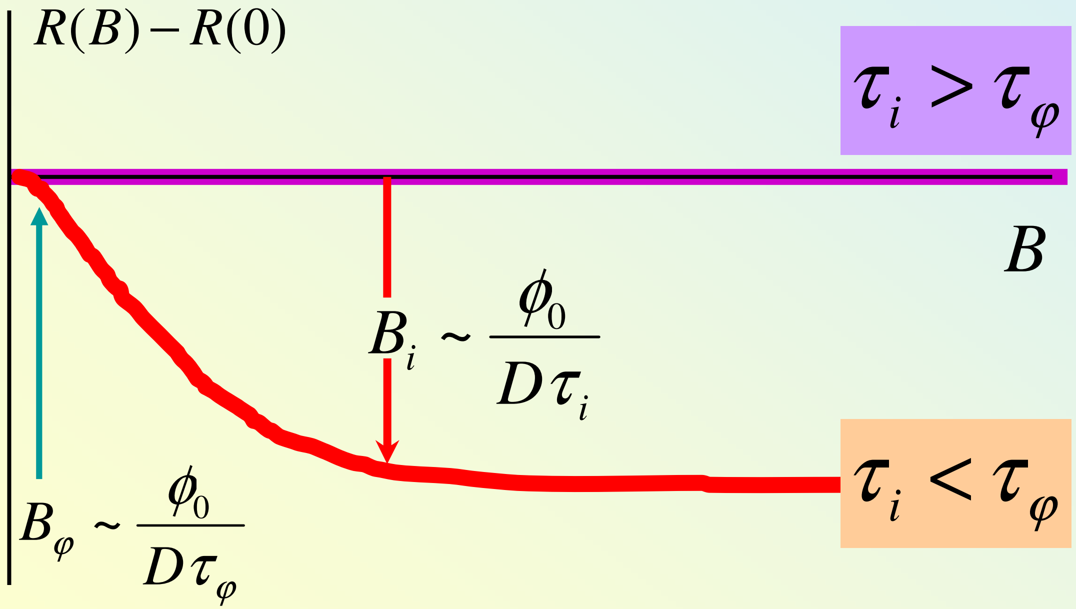
~~Berry phase 2π~~

Weak localisation magnetoresistance

$$\varepsilon_F \tau \gg 1$$

$$\delta g_1 = -\ C_{KK'-symm} \ + \ C_{KK'-antisymm}$$

E. McCann, K. Kechedzhi, V.I. Falko, H. Suzuura, T. Ando, B.L. Altshuler, PRL 97, 146805 (2006)



'slow' inter-valley scattering: neither WL nor WAL magnetoresistance

'fast' inter-valley scattering: usual WL magnetoresistance cut at B_i

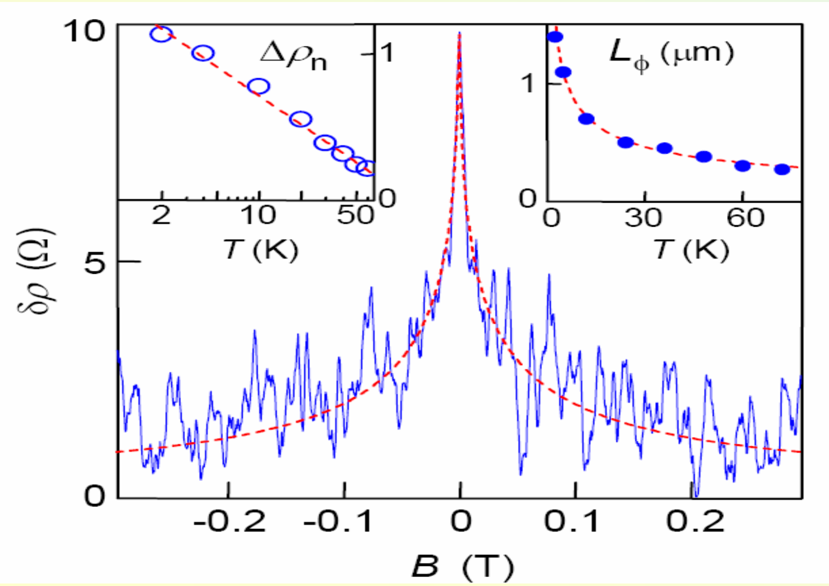
$$\delta g_2 = -\ C_{KK'-symm} \ + \ C_{KK'-antisymm}$$

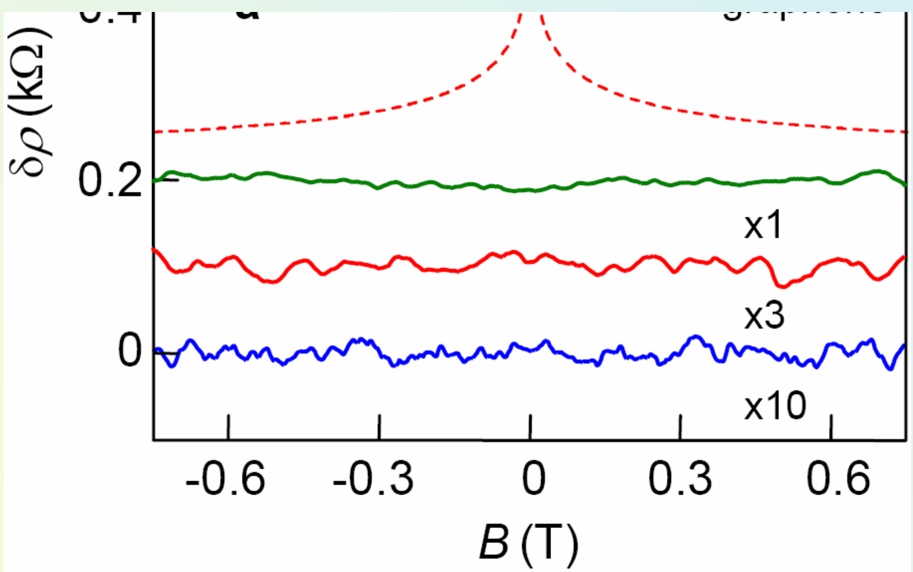
K. Kechedzhi, V.I. Falko, E. McCann, B.L. Altshuler, 2006

Weak localisation magnetoresistance in graphene

"Strong suppression of weak localization in graphene"
SV Morozov, KS Novoselov, MI Katsnelson, F Schedin, LA Ponomarenko, D Jiang, and AK Geim, Phys Rev Lett. 97, 016801 (2006)

Low-field magnetoresistance is ubiquitous in low-dimensional metallic systems with high resistivity and well understood as arising due to quantum interference on self-intersecting diffusive trajectories. We have found that in graphene this weak-localization magnetoresistance is strongly suppressed and, in some cases, completely absent. The unexpected observation is attributed to mesoscopic corrugations of graphene sheets which can cause a dephasing effect similar to that of a random magnetic field.

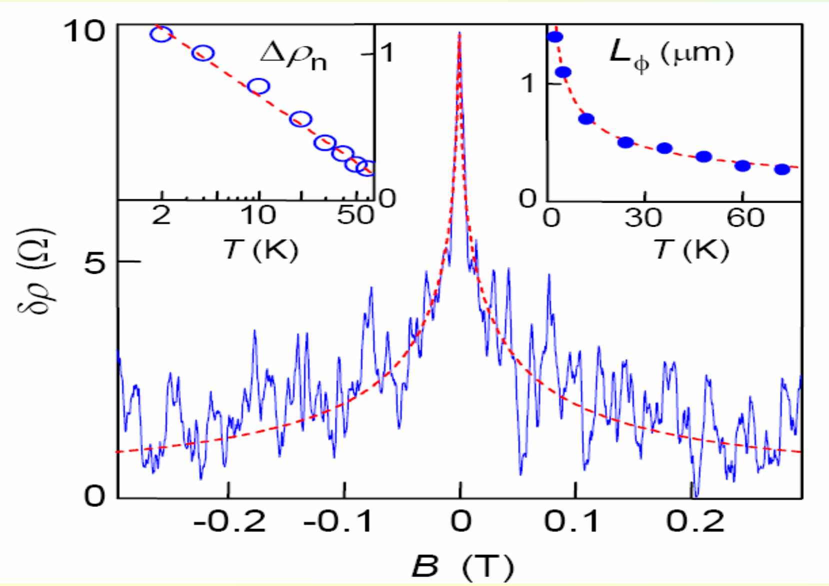




Weak localisation magnetoresistance in graphene

"Strong suppression of weak localization in graphene"
SV Morozov, KS Novoselov, MI Katsnelson, F Schedin, LA Ponomarenko, D Jiang, and AK Geim, Phys Rev Lett. 97, 016801 (2006)

Curve shown for density $n \approx 3\times10^{12}\,cm^{-2}$
and mean free path $l_{tr} \approx 80nm$



My estimates for parameters:

Fermi energy	$\varepsilon_F \approx 200meV$
Scattering time	$\tau_0 \approx 0.04\,ps$
Perturbative parameter	$\varepsilon_F\tau_0/\hbar \approx 12$
Warping time	$\tau_w \approx 1\,ps$
Inelastic decoherence	$\tau_\phi \approx 50\,ps$

$$\tau_0^{-1} >> \tau_w^{-1} >> \tau_\phi^{-1}$$

Weak localisation magnetoresistance in graphene

"Strong suppression of weak localization in graphene"
SV Morozov, KS Novoselov, MI Katsnelson, F Schedin, LA Ponomarenko, D Jiang, and AK Geim,
Phys Rev Lett. 97, 016801 (2006)

Note added in proofs.—Most recently, to improve the quality of our graphene samples, we attempted to eliminate the mesoscopic ripples discussed in this Letter. To this end, we have changed our microfabrication procedure [1] by depositing flakes on the freshly cleaned SiO_2 surface (within 1 h). This technological change resulted in samples with generally higher mobility (of about 15 000 $cm^2/V\,s$) and no ripples visible in AFM. Moreover, such structures exhibited the full, unsuppressed WL peak. The experimental curves look very similar to the one shown in Fig. 3 but with a much larger negative MR peak so that no additional fitting parameter is required to explain its amplitude. This proves that the WL amplitude (but not its sign) is sensitive to fabrication procedures and further supports the inferred importance of ripples in the suppression of WL in graphene.

Summary

Crossovers between
weak localisation/anti-localisation magnetoresistance
in graphene monolayers and bilayers

Trigonal warping suppresses the effect of chirality
[weak antilocalisation in a monolayer]
while intervalley scattering tends to restore
conventional negative magnetoresistance

E. McCann, K. Kechedzhi, V.I. Fal'ko, H. Suzuura, T. Ando, B.L. Altshuler,
Phys Rev Lett. **97**, 146805 (2006)

K. Kechedzhi, V.I. Fal'ko, E. McCann, B.L. Altshuler (2006)