Decoherence in superconducting nanocircuits

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Noise sources in SC qubits

\[ \mathcal{H} = E_C \left( \hat{N} - N_g \right)^2 - E_J \cos(\delta/2) \cos(\theta) \]

- Flux/E\textsubscript{J} noise

- Two-port design: Quantronium, Saclay 2002 (courtesy D. Ésteve)

**charge noise (1/f)**

\( \longleftrightarrow \) switching impurities close to the device

*Paladino, Faoro, Falci, Fazio, PRL 2002*

- **THE** problem in high Q charge based qubits
- Affects two-qubit operations in spin-qubits

![Graph showing switching probability over time between pulses]
Noise characterization

- Noise due to *charged bistable impurities* → RTN

- Distribution of *charged bistable impurities* with switching rates leads to 1/f noise

  *non Gaussian, non Markovian*

1/f noise measurements:
- Uncertainty on all high-frequency properties, e.g. $\gamma_M$ vs $\Omega$

1/f noise measurements: uncertainty on all high-frequency properties, eg. $\gamma_M$ vs $\Omega$
white noise, ohmic noise????

Saclay qubit Ithier et al., subm PRB 04

Charge qubit exps: Astafiev et al., PRL 04

- Variety of experimental features material & device dependent
- Slow noise components make unstable the calibration of the device (env. with memory) → signal decay strongly depends on protocol
Environment 1: understanding the nature of noise sources

\[ \mathcal{H} = -\frac{\epsilon}{2} \sigma_z - \frac{\Delta}{2} \sigma_x - \frac{1}{2} \sigma_z \hat{X} + \mathcal{H}_R \]

Coupling with continuous variables (bosons)

\[ \hat{X} = \sum_\alpha \lambda_\alpha (a_\alpha^\dagger + a_\alpha) \quad \mathcal{H}_R = \sum_\alpha \omega_\alpha a_\alpha^\dagger a_\alpha \]

- electromagnetic environment, sum of many microscopic variables

Coupling with impurities or switching noise sources

\[ \hat{X} = \sum_\alpha v_\alpha d_\alpha^\dagger d_\alpha \quad \text{intrinsic discrete nature} \]

- eg. Fano impurities, Kondo like traps, defects in Junction oxides, flux noise

Goal: more and more accurate microscopic (semi-phenomenological)
characterization of the nature of the environment

Solid-state coherent nanodevices as detectors
Environment 2: handling the effects of noise sources

Solid-state coherent nanodevices as processors

Devices may have substantial coupling to the environment but often they work in regimes of limited sensitivity to its details

Why?

➢ In practice:

  Limited control of protocols $\rightarrow$ details of the environment blurred
  (e.g. when inhomogeneous broadening dominates)
  Interest in protocols which effectively decouple the environment

➢ In principle: typical situation for Quantum Information

  several qubits nearly decoupled from the environment
  short time dynamics $\sim$ 10000 operation
  time-dependent control

Goal: reasonable approximations including systematically only relevant information about noise, focusing on the effects of the environment on the controlled dynamics rather than on the specific nature of the noise sources.

Applications to multiqubit devices
Main effects of the environment

\[
\mathcal{H} = -\frac{\epsilon}{2} \sigma_z - \frac{\Delta}{2} \sigma_x - \frac{1}{2} v \tau(t) \sigma_z
\]

Visibility of the induced splitting

\[
g = \frac{\Omega' - \Omega}{\gamma}
\]

\[T_2 \propto \frac{1}{\gamma}\]

\[g > 1: \text{2 narrow lines at } \Omega \text{ and } \Omega' \]

\[\text{Linewidth } \gamma \rightarrow 0 \]

\[\text{Beats } \rightarrow \text{bad qubit} \]

\[\text{Good detector} \]

\[\text{Inhomogeneous broadening} \]

\[\text{Non-exponential decay} \]

\[\text{Linewidth } v \]

\[S(\omega) = \frac{v^2}{2} \left(1 - \frac{v^2}{\gamma^2 + \omega^2}\right) \frac{\gamma}{\gamma^2 + \omega^2}\]

\[T_1^{-1} = \frac{1}{2} \sin^2 \theta \times S(\Omega)\]

\[T_2^{-1} = \frac{1}{2} T_1^{-1} + \frac{1}{2} \cos^2 \theta \times S(0)\]

**g < 1 (golden rule)**

Nearly decoupled qubit:

- Exponential decay
- Single narrow line at \(\Omega\)
Main effects of the environment

\[ \mathcal{H} = -\frac{\epsilon}{2} \sigma_z - \frac{\Delta}{2} \sigma_x - \frac{1}{2} v \tau(t) \sigma_z \]

\[ \tau(t) \]

\[ \Omega \quad \Omega' \]

\[ \tau = 0 \quad \tau = 1 \]

visibility of the induced splitting

\[ g = \frac{\Omega' - \Omega}{\gamma} \]

sensitivity depends on the operating point

\[ \Omega' - \Omega \]

\[ \gamma \]

\[ g > 1 \text{ Strongly coupled} \]

\[ g < 1 \text{ Nearly decoupled} \]
Three classes of noise : I

Classification: according to the effect rather than to the nature of noise

Quantum noise
1. Spontaneous decay + ...
2. Depends on $S(\omega)$, not on details
3. Weakly coupled and short-time correlated
4. Markovian Master Equation
5. Electromagnetic fluctuations, fast impurities, meter off, crosstalk, ...

$$\mathcal{H} = -\frac{1}{2} \vec{\Omega} \cdot \vec{\sigma} - \frac{1}{2} \sigma_z \hat{E} + \mathcal{H}_E$$

Master Equation
$$\partial_t \rho^Q(t) = \hat{L}[\rho^Q(0)] \quad \rho^Q(t) = Tr_f [(W^Q + i(t))] = \mathcal{E}_t[\rho^Q(0)]$$

Relaxation
$$T_1^{-1} = \frac{1}{2} \sin^2 \theta \ S(\Omega)$$

Dephasing
$$T_2^{-1} = \frac{1}{2} T_1^{-1} + \frac{1}{2} \cos^2 \theta \ S(0)$$
Three classes of noise: II

Strongly coupled noise
1. Uncontrolled “chemical shift” + …
2. Sensitive to details of protocol. Needed information beyond $S(\omega)$
3. Possibly sample specific
4. **Enlarge Hilbert space of the system** including few environmental degrees of freedom
Enlarging system’s Hilbert space

E. Paladino et al. PRL 2002

Quantum model
Bauernschmitt & Nazarov (1993)

\[ \mathcal{H} = -\frac{1}{2} \tilde{\Omega} \sigma - \frac{\nu}{2} \sigma_z d^\dagger d + \epsilon_c d^\dagger d + \sum_k [T_k c_k^\dagger d + T_k^* d^\dagger c_k] + \sum_k \epsilon_k c_k^\dagger c_k \]

Fano impurity included in the system

“strong coupling” technique \(\rightarrow\) theory from adiabatic to quantum noise
Enlarging system’s Hilbert space

\[ \mathcal{H} = -\frac{1}{2}\bar{\Omega}\bar{\sigma} - \frac{\nu}{2}\sigma_z d^\dagger d + \sum_k \left[ T_k c_k^\dagger d + T_k^* d^\dagger c_k \right] + \sum_k \varepsilon_k c_k^\dagger c_k \]

System

4 state system

\[ |\textit{qubit}\rangle \otimes |\sim \text{incoher. impurity}\rangle \]

Master eq. for $4 \times 4$ reduced density matrix

\[ \partial_t \rho^Q + S(t) = \hat{\mathcal{L}}_Q + S[\rho^Q + S(0)] \]

Trace over the impurity to get the qubit density matrix

\[ \rho^Q(t) = Tr_S[\rho^Q + S(t)] \]

Cf. also Marquard et al. PRB 2002
Enlarging system’s Hilbert space

& exact diagonalisation of the $8 \times 8$ Redfield tensor
Three classes of noise: III

**Adiabatic noise**

1. Inhomogeneous broadening + …
2. Sensitive to details of protocol. Beyond $S(\omega)$?
3. Long time-correlated

4. **Adiabatic approximation**
5. Low-frequency part of 1/f noise.
Adiabatic noise (Cooper-pair–box based nanodevices)

\[ \mathcal{H}_{BOX} = E_C (\hat{q} - q_x)^2 - E_J \cos \hat{\phi} \]

\[ [\hat{\phi}, \hat{q}] = i \]

Cooper pair box

charge fluctuactions \[ q_x \to q_x + \delta q_x \]

splittings \[ \Omega(q_x, X) = \Omega_0(q_x) + \delta \Omega(q_x, X) \]

Environment as a classical stochastic drive

\[ \mathcal{H} = \mathcal{H}_{BOX} + q \, X(t) \quad X(t) \to -2E_c \delta q_x(t) \]
Adiabatic noise (Cooper-pair–box based nanodevices)

\[ \mathcal{H} = \mathcal{H}_{\text{BOX}} + q X(t) \]

Environment as a classical stochastic drive

\[ \gamma_M \ll \Omega \rightarrow t \ll T_1 \]

Adiabatic approximation

\[ |\psi(t)\rangle = \sum_m |m(X_t)\rangle\langle m(X_0)|0\rangle e^{-i \int_0^t ds E_m[X(s)]} \]

Average over realizations

\[ \rho(t) = \int \mathcal{D}X(t) P[X(t)] \rho(t|X(s)) \]

\[ \rho(t) = \int \mathcal{D}X(t) P[X(t)] \sum_{mn} \hat{R}_{mn}(X_0, X_t) e^{-i \int_0^t ds \Omega_{mn}(s)} \]

\[ P[X(s)] = F[X(s)] p[X(s)] \]

Filter (protocol)

Joint probability distribution

\[ p[X(s)] = \lim_{n \to \infty} p_{n+1}(X_{nt}; \ldots; X_{1t_1}; X_00) \]
Adiabatic noise: inhomogeneous broadening

Falci, D’Arrigo, Mastellone, Paladino, PRL 2005, & cond-mat/0407484

Phase fluctuations accumulate in time \[\rightarrow \text{retain longitudinal noise} \quad \rho_{mn}(t) = \rho_{mn}(0)e^{-i\Phi_{mn}(t)}\]

\[-i\Phi_{mn}(t) = \ln\int \mathcal{D}X(t)P[X(t)]e^{-i\int_0^t ds\Omega_{mn}[X(s)]}\]

Static Path Approximation (SPA) \[-i\Phi_{mn}(t) = \ln\int dX_0 P[X_0]e^{-i\Omega_{mn}(X_0)t}\]

Large \(N_f\) central limit theorem \[\rightarrow \mathcal{P}(X_0)\text{ gaussian distributed}\]

Variance from number of switching impurities during \(t_{\text{meas}} = 1/\gamma^*\)

\[
\sigma_X^2 = \frac{\langle v^2 \rangle}{4} N_{fl} = 16E_C A \ln \frac{\gamma M}{\gamma^*} \rightarrow \int_{\gamma^*}^{\gamma M} \frac{d\omega}{\pi} S(\omega) \quad S(\omega) \rightarrow 1/\omega
\]

for 1/f noise
Static Path Approximation (SPA)

\[ -i \Phi_{mn}(t) = \ln \int dX_0 P[X_0] e^{-i \Omega_{mn}(X_0) t} \]

Steepest descent \sim quadratic expansion (lowest eigenstates)

\[ \Omega(q_x, X) \approx \Omega_0(q_x) + c(q_x) X + \frac{1}{2} s^2(q_x) \frac{X^2}{\Omega_0(q_x)} \]

\[ -i \Phi_{01}(t) = -i \Omega_0 - \frac{1}{2} \ln (1 + i s^2 \frac{\sigma^2_X}{\Omega_0} t) - \frac{1}{2} \frac{(c \sigma_X t / \Omega_0)^2}{1 + i s^2 \frac{\sigma^2_X}{\Omega_0} t / \Omega_0^2} \]
low-frequency noise characterization in:

1) Time domain:  short-time dynamics
dephasing time

2) Frequency domain
Adiabatic noise in the Quantronium

Falci, D’Arrigo, Mastellone, Paladino, PRL 2005, & cond-mat/0407484

Cf. Diagrammatic approach: Shnirman-Makhlin, PRL 2004
Cf. Ornstein-Uhlenbeck processes: Rabenstein-Averin 2004

\[ \rho_{10}(t) = \rho_{10}(0) e^{-i\Omega_0 t} \left( 1 + i \frac{\sigma_X^2 t}{\Delta} \right)^{-\frac{1}{2}} \propto t^{-1/2} \]

For simple protocols relevant effects due to static impurities (inhom. broadening)

Data courtesy Gregoire Ithier (Saclay) 2004
Low frequency noise characterization: $T_2^*$

D’Arrigo, Falci, Mastellone, Paladino, proc. MS+S2006

Extract the dephasing time from $\text{Im}[\Phi_{01}(T_2^*)] = -1$

Charge qubit

\[ \frac{(\sqrt{2})}{\sigma^*} \]

\( q_x \)

(a)

courtesy of J. Nakamura (MS+S2006)
Low frequency noise characterization: $T_2^*$

Extract the dephasing time from $\text{Im}[\Phi_{01}(T_2^*)] = -1$

Charge qubit

$\frac{\sqrt{2}}{\sigma^*}$

\[ c = 1 \quad T_2^{*-1} \approx \frac{\sigma_X |c|}{\sqrt{2}} \]

\[ c = 0 \quad T_2^{*-1} \approx \frac{\sigma_X^2}{\sqrt{2}\Omega_0} \]

Master equation $T_2^{*-1} \approx c^2 S(0)$ where $\sigma_X^2 \leftrightarrow S(0)$
Low frequency noise characterization: $T_2^*$

$c(q_x) = (q_{11} - q_{00}) \leftrightarrow$ charge sensitivity of the device

$s^2(q_x) = 2\{2q_{10}^2 - \sum_{m \geq 2} \left[ \frac{q_{1m}^2 \Omega_0}{E_m - E_1} - \frac{q_{0m}^2 \Omega_0}{E_m - E_0} \right] \}$

$\leftrightarrow$ quantum capacitance CPB $C_Q^k \equiv (C_q/e)^2 \partial^2 E_k/\partial q_x^2$

$\Omega(q_x, X) \approx \Omega_0(q_x) + c(q_x) X + \frac{1}{2} s^2(q_x) \frac{X^2}{\Omega_0(q_x)}$

$T_2^* \text{ vs } c(q_x) : \text{ low-frequency noise characterization}$

BUT charge fluctuations $\rightarrow$ leakage from Hilbert space spanned by the lowest eigenstates at the optimal point
Low-frequency noise characterization: Lineshapes

\[ \rho_{10}(t) = \rho_{10}(0) e^{-\Gamma t} \int dX \, p(X) \, e^{-i \int_0^t ds \Omega[X(s)]} \]

**Quadratic splitting** about the optimal point

\[ \Omega(X) = \Omega_0 + a X + \frac{X^2}{2\Delta} \]

Close to the optimal point: sharp asymmetric lineshape

\[ |a| < \sqrt{2\sigma_X/\Delta} \]

\[ a = 2E_C(q_x - 1/2)/E_J. \]

\[ \tilde{\rho}_{10}(\omega) \approx \rho_{10}(0) \, 2 \sqrt{2\pi} \, \Delta \, \frac{\Theta(\omega - \Delta)}{\sqrt{\omega - \Delta}} \, p(0) \]

**Regularized by exponential tail**

\[ \Gamma = \frac{1}{2T_1} \]

\[ \tilde{\rho}_{10}(\omega) = \rho_{10}(0) \, 2 \sqrt{2\pi}\Delta \, \frac{z}{z \sigma_X} \, e^{-\frac{(z\Delta)^2}{2\sigma_X^2}} \left[ 1 + \text{Erf} \left( \frac{iz\Delta}{\sqrt{2}\sigma_X} \right) \right] \]

\[ z = \frac{2}{\Delta} \, (\omega - \Delta - i\Gamma) \]

insensitive to details of fluctuations

D’Arrigo, Falci, Mastellone, Paladino, proc. MS+S2006

\[ q_x \]

\[ q_x = 0.4 \]

\[ \omega/\Delta \]
Low-frequency noise characterization: Lineshapes

\[ \rho_{10}(t) = \rho_{10}(0) \ e^{-\Gamma t} \int dX \ p(X) \ e^{-i \int_{0}^{t} ds \ \Omega[X(s)]} \]

\[ \Omega(X) = \Omega_0 + \alpha X + \frac{X^2}{2\Delta} \]

\[ \alpha = 2E_C(q_x - 1/2)/E_f. \]

Non optimal points: \[ |\alpha| > \sqrt{2\sigma_X/\Delta} \]

broadened line peaked at \[ \omega = \Omega_0 \]

containing information on \[ p(X) \]

\[ \tilde{\rho}_{10}(\omega) \approx \rho_{10}(0) \ \frac{\sqrt{2\pi} \ \Delta}{\sqrt{\omega - \Delta}} \ p(|\alpha|\Delta - \sqrt{2(\omega - \Delta)}) \quad \omega > \Delta \]
Three classes of noise

\[ \ln S \]

\[ \ln \omega \]

\[ \Omega \]

Adiabatic noise
1. Inhomogeneous broadening + …
2. Adiabatic approximation
3. Low-frequency part of 1/f

Strongly coupled noise
1. Uncontrolled chemical shift + …
2. Enlarge Hilbert space of the system
3. Not weakly coupled impurities

Quantum noise
1. Spontaneous decay + …
2. Markovian Master equation

Classification according to the effect rather than to the nature of noise

Each class has its specific approximation scheme which does not work (or it is impractical) for other classes of noise
Multistage elimination

\[ H = -\frac{\varepsilon}{2}\sigma_z - \frac{\Delta}{2}\sigma_x - \frac{1}{2}\sigma_z \hat{X} + H_R \]

Split

\[ -\frac{1}{2}\sigma_z \hat{X}_S + H_{RS} - \frac{1}{2}\sigma_z X(t) - \frac{1}{2}\sigma_z \hat{X}_f + H_{R_f} \]

In general

\[ \rho^Q(t) = \text{Tr}_S \{ \int \mathcal{D}X(t) P[X(t)] \text{Tr}_f [ W^{Q+S+f}(t|X(t)) ] \} \]
Adiabatic + one extra fluctuator

\[ \rho^Q(t) = e^{-\Gamma t} \text{Tr}_S\left\{ \int dX p(X) \rho^{Q+S}(t|X(t)) \right\} \]

Features washed out by inhomogeneous broadening away from optimal point

- (Asymmetric) features of the extra fluctuator appear close to the optimal point
- Does not modify the lineshape at optimal point

Extra strongly coupled fluctuators
- Limit flexible control
- Pose reliability problems in networks

*Paladino, Mastellone, D’Arrigo, Falci, cond-mat/0407484*
*c.f. Non gaussian effects in Bergli, Galperin, Altshuler PRB 2006*
1. **n-joint** probability of the **sum of many independent** stochastic processes via **product** of the **n-point** generating functionals of each variables

2. An individual fluctuator is markovian \(\rightarrow\) **n-point** generating functional via **(2-point) conditional probability**

3. If needed gaussian approximation via retaining second cumulant

4. Systematic approach \(\sim\) derivative expansion
Systematic path-integral

Falci, D'Arrigo, Mastellone, Paladino, PRL 2005

\[ P[X_t, t ; X_{tM-1}, t_{M-1} ; \cdots ; X_{t_1}, t_1 ; X_0, 0] \rightarrow P[X(t)] \]

\[ \Delta t \rightarrow 0 \]

**Static path**  \( P[X_0, 0] \)

**1-st correction**  \( P[X_t, t ; X_0, 0] \)

**2-nd correction**  \( P[X_t, t ; X_{t_1}, t_1 ; X_0, 0] \)

More and more accurate sampling of the process “Derivative” expansion

**First correction: adiabatic noise during evolution in FID or Ramsey**

\[ i\Phi(t) = \frac{1}{2} \ln \left[ 1 + \frac{\sigma_X^2 [1 - \pi(t)] t}{\Omega} \right] + \frac{1}{2} \ln \left[ 1 + i \frac{\sigma_X^2 \pi(t) t}{3 \Omega} \right] \]

\[ \pi(t) = \frac{1}{2 \sigma^2} \int_0^\infty \frac{d\omega}{\pi} S(\omega) \left( 1 - \cos \omega t \right) \]

**First correction: adiabatic noise during evolution with Feedback and Echo**

\[ \Gamma_F(t) \approx -\frac{1}{4} \ln \left[ 1 + \left( \frac{4s^2 \sigma^2 t}{3\Omega} \pi_2(t) \right)^2 \right] \]

\[ \Gamma_1(t) \approx -\frac{1}{4} \ln \left[ 1 + \left( \frac{2s^2 \sigma^2 t}{2\Omega} \pi_2(t) \right)^2 \right] \]

\[ \pi_2(t) = \pi(t) \left[ 1 - \pi(t) \right] \]
Feedback control and nongaussian effects

\[ \gamma_M = 10 \text{ MHz with Recalibration} \]
\[ \gamma_M = 1 \text{ MHz with Recalibration} \]

Repeated measurements

Experiment

Slower decay, starts as \( \approx \exp(-\Gamma^3 t^3) \) for discrete environments

Protocol more sensitive to details of the environment
1/f adiabatic + 1/f quantum

Environment of slow + fast impurities, $\gamma_M = 10 \Delta$

No separation of time scales $T_1 \approx T_2$

Theory: two-stage elimination

\[ \hat{X} \to X + \hat{\chi} \]

\[ \rho^Q(t) = \int dX \, p(X) \, \rho^Q_f(t|X) \]

\[ \langle \sigma_y(t) \rangle = \langle \sigma_y(0) \rangle \, e^{-t/(2T_1)} \left( 1 + [i\Delta + T_1^{-1}] \frac{\sigma_X^2 t}{\Delta^2} \right)^{-1/2} \]

Falci, D’Arrigo, Mastellone, Paladino, PRL 2005
Applications to more complex systems

Quantum bits coupled to resonators
- Selection rules between entangled doublet → adiabatic approximation is exact
- System tunable to a working point where fluctuations are

\[ \Omega(X) = \Omega_0 + aX^2 + \beta X^4 \]

extra protection against fluctuations due to interaction.

Driven multistate systems
- Adiabatic fluctuations may induce level crossings
- Limitations in the implementation of quantum optics protocols as STIRAP

Acknowledgements: IST-SQUBIT2, EuroSQUIP,