Landau-Zener Transitions with quantum noise

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Participants:
Outline

• Introduction and motivation
• Landau-Zener problem for 2-level crossing
• Fast classical noise in 2-level systems
• Noise and regular transitions work together
• Quantum noise and its characterization
• Transitions due to quantum noise in the LZ system
• Production of molecules from atomic Fermi-gas at Feshbach resonance
• Conclusions
Introduction

LZ theory

Diabatic levels

Adiabatic levels

Avoided level crossing (Wigner-Neumann theorem)

Schrödinger equations

\[ i\dot{a}_1 = E_1(t)a_1 + \Delta a_2 \]
\[ i\dot{a}_2 = \Delta^* a_1 + E_2(t)a_2 \]

\[ E_2(t) - E_1(t) = \Omega(t); \quad \hbar = 1 \]

\[ \Omega(t) = \dot{\Omega}t \]
Adiabatic levels:

\[ E_{\pm} = \frac{E_1 + E_2}{2} \pm \sqrt{\left( \frac{E_1 - E_2}{2} \right)^2 + |\Delta|^2} \]

\[ E_2 = -E_1 = \dot{\Omega} t / 2 \]

Center-of mass energy = 0

LZ parameter:

\[ \gamma = \frac{\Delta}{\hbar \sqrt{\dot{\Omega}}} \quad \dot{\Omega} = \frac{g \mu_B \dot{B}_z}{\hbar} \]

\[ \gamma \neq 1 \]

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LZ transition matrix

\[
U = \begin{pmatrix}
\alpha & \beta \\
-\beta^* & \alpha^*
\end{pmatrix}
\]

| \alpha |^2 + | \beta |^2 = 1

Amplitude to stay at the same d-level
\[
\alpha = e^{-\pi \gamma^2}
\]

Amplitude of transition
\[
\beta = -\frac{\sqrt{2\pi} \exp \left( -\frac{\pi \gamma^2}{2} + i \frac{\pi}{4} \right)}{\gamma \Gamma \left( -i \gamma^2 \right)}
\]

LZ transition time:
\[
\tau_{LZ} \approx m a x \left( \frac{\Delta}{\dot{\Omega}} \right)^{1/2} \]

Condition of validity:
\[
\tau_{LZ} \leq \tau_{sat} = \left| \dot{\Omega} / \ddot{\Omega} \right|
\]

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Dresden, 2006
Nanomagnets: Brief description

$S = 10 : \text{Mn}_{12}$, $\text{Fe}_8$. $S = 1/2 : \text{V}_{15}$. 

$S_{\text{total}} = 8S_2 - 4S_1 = 10$
Spin reversal in nanomagnets

Controllable switch between states for quantum computing:

The noise introduces mistakes to the switch work.

Transverse noise

Longitudinal noise creates decoherence
Landau-Zener tunneling in noisy environment


Classical fast noise in 2-level system

\[ \mathbf{b}_{tot} = \mathbf{b}_{reg} + \eta; \quad \mathbf{b}_{reg} = \hat{z} \hat{\Omega} t + \hat{x} \Delta \]

\( \eta(t) \) -- Gaussian noise

\[ \langle \eta_i(t) \eta_k(t') \rangle = f_{ik} \left( \frac{t - t'}{\tau_n} \right) \]

Noise is fast if

\[ \tau_n \ll \dot{\Omega}^{-1/2} \]
Density matrix:  \[ \hat{\rho}(t) = \frac{1}{2} I + g(t) \cdot s \]

\( g(t) \) — *Bloch vector*. It obeys Bloch equation:

\[ \dot{g} = -b_{tot} \times g \]

\[ g_z = \frac{1}{2} (n_1 - n_2) \equiv \frac{1}{2} (n_\uparrow - n_\downarrow) \]  Difference of populations

\[ g_\pm = g_x \pm ig_y \]  Coherence amplitude

**Integral of motion:**  \[ g^2 = const \]
Transitions produced by noise

\[ \Omega(t) = \dot{\Omega} t \]

It produces transitions until

\[ \Omega(t) = \dot{\Omega} t \leq \frac{1}{\tau_n} \]

Accumulation time:

\[ \tau_{acc} = \frac{1}{\dot{\Omega} \tau_n} \]

\[ \langle g_z(t) \rangle \] is slowly varying

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Transition is produced by a spectral component of noise whose frequency equal to its instantaneous value in the LZ 2-level system.

\[ \dot{n}_1 = - \left\langle \eta_{\Omega(t)}^* \eta_{\Omega(t)} \right\rangle n_1 \]

Transition probability measures the spectrum of noise.

Transition probability for infinite time

\[ P_{1 \to 1} = \exp \left( - \frac{2\pi \left\langle |\eta|^2 \right\rangle}{\hbar^2 \dot{\Omega}} \right) \]

Fermi golden rule is exact for gaussian fast noise!

Regular and random field act together

Separation of times: noise is essential only beyond $\tau_{LZ}$

$$P_{1 \rightarrow 2} = \frac{1}{2} \left[ 1 - e^{-\frac{2\pi \langle \eta^2 \rangle}{\hbar^2 \Omega}} \left( 2e^{-\frac{2\pi \Delta^2}{\hbar^2 \Omega}} - 1 \right) \right]$$
It is possible to get $P$ larger than $\frac{1}{2}$ at faster sweep rate or stopping the process at some specific field.

\[ J = \frac{2\pi\langle\eta^2\rangle}{\dot{\omega}} \]

Plot of transition probability vs. inverse frequency rate.

$\beta = \dot{\Omega}$
Graph representation

\[ G(t, t') = \exp \left( \frac{i\dot{\Omega}(t^2 - t'^2)}{2} \right) \]

Chronological time order

4 times are close
What was omitted: Debye-Waller factor $W = \left\langle \exp \left[ -i \int_{t'}^{t} \eta_z (t'') dt'' \right] \right\rangle$

$W \approx 1, \text{ if } \left\langle \eta_z \right\rangle^2 \tau_n^2 \geq 1$

Diagonal noise leads to decoherence for a long time

$\left\langle g_\pm (+\infty) \right\rangle = 0$

Decoherence time: $\tau_{dec} = \frac{1}{\left\langle \eta_z \right\rangle^2 \tau_n}$
Quantum noise and its characterization

\[ \langle \eta(t)\eta(t') \rangle \neq \langle \eta(t')\eta(t) \rangle \]

Model of noise: phonons

\[ H_{\text{int}} = u (a_1^\dagger a_2 + a_2^\dagger a_1) ; \]

\[ u = \eta + \eta^\dagger ; \quad \eta = \frac{1}{\sqrt{V}} \sum_k g_k b_k \]

\[ H_n = \sum_k \omega_k b_k^\dagger b_k \]

\[ H_2 = \frac{\Omega(t)}{2} (a_1^\dagger a_1 - a_2^\dagger a_2) ; \quad \Omega(t) = \dot{\Omega} t \]
\[ \langle \eta_\omega \eta_\omega \rangle = N_\omega \int d\mathbf{k} \delta(\omega - \omega_k) |g_k|^2 ; \quad N_\omega = \left( e^{\omega/T} - 1 \right)^{-1} \]

\[ \langle \eta_\omega \eta_\omega^\dagger \rangle = (N_\omega + 1) \int d\mathbf{k} \delta(\omega - \omega_k) |g_k|^2 = e^{\omega/T} \langle \eta_\omega \eta_\omega \rangle \]

\[ \langle \eta^\dagger(t) \eta(t') \rangle = \int \frac{d\omega}{2\pi} \langle \eta_\omega^\dagger \eta_\omega \rangle e^{-i\omega(t-t')} \]

Different time scales for induced and spontaneous transitions

\[ \tau_{ni} \square T^{-1} \quad \tau_{ns} \square \omega_D^{-1} \]

Noise is fast if \[ T, \omega_D \square \sqrt{\Omega} \]
Bloch equation for the fast quantum noise

\[
\frac{ds_z}{dt} = \text{sign}(t) \left\langle \left[ \eta_{|\Omega|}, \eta_{|\Omega|}^\dagger \right] \right\rangle - \left( \left\langle \eta_{|\Omega|}^\dagger \eta_{|\Omega|} \right\rangle + \left\langle \eta_{|\Omega|} \eta_{|\Omega|}^\dagger \right\rangle \right) s_z
\]

\[\Omega = \Omega(t) = \dot{\Omega} t\]

Solution

\[
s_z(t) = s_z(t_0) \exp \left( -\int_{t_0}^{t} \left( \left\langle \eta_{|\Omega(t')|}^\dagger \eta_{|\Omega(t')|} \right\rangle + \left\langle \eta_{|\Omega(t')|} \eta_{|\Omega(t')|}^\dagger \right\rangle \right) dt' \right) + \int_{t_0}^{t} dt' \text{sign}(t') \left\langle \left[ \eta_{|\Omega(t')|}, \eta_{|\Omega(t')|}^\dagger \right] \right\rangle \exp \left( -\int_{t'}^{t} \left( \left\langle \eta_{|\Omega(t'')|}^\dagger \eta_{|\Omega(t'')|} \right\rangle + \left\langle \eta_{|\Omega(t'')|} \eta_{|\Omega(t'')|}^\dagger \right\rangle \right) dt'' \right)
\]

\(s_z\) turns into zero in the limit of strong noise, but not exponentially
Essential graphs

Golden rule picture
Saturation of frequency

\[ \tau_{acc} = \left( \dot{\omega} \tau_n \right)^{-1} \tau_s \]

Long time limit

\[ s_{z\infty} = -\frac{\left[ \eta_{\Omega_{\infty}}, \eta_{\Omega_{\infty}}^\dagger \right]}{\left\langle \eta_{\Omega_{\infty}} \eta_{\Omega_{\infty}}^\dagger + \eta_{\Omega_{\infty}}^\dagger \eta_{\Omega_{\infty}} \right\rangle} \]

Noise in thermal equilibrium

\[ \langle \eta_{\Omega} \eta_{\Omega}^\dagger \rangle = e^{T} \langle \eta_{\Omega}^\dagger \eta_{\Omega} \rangle \]

\[ s_{z\infty} = \tanh \frac{\hbar \Omega_{\infty}}{2T} \]
Creation of ultracold molecules from a Fermi gas of atoms

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Figure 4. Absorption images of the quantum gas using a Stern–Gerlach technique. We start with ultracold fermionic atoms in the \( m_F = -5/2 \) and \( m_F = -9/2 \) states of \( ^{40}\text{K} \). A magnetic field ramp through the Feshbach resonance causes 50% atom loss, owing to adiabatic conversion of atoms to diatomic molecules. To directly detect these bosonic molecules we apply a r.f. photodissociation pulse; the dissociated molecules then appear in the \( m_F = -7/2 \) and \( m_F = -9/2 \) atom states. The shaded bar indicates the optical depth.
Conversion of an Atomic Fermi Gas to a Long-Lived Molecular Bose Gas

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We have completed an adiabatic passage bound, $N = 38$, molecules remain adiabatic sweep. DOE 10.1103/Phys

FIG. 3. Dependence of atom loss on inverse sweep rate. The field is ramped linearly from high to low field. The solid circles represent the average of 6-10 measurements and the error bars are the standard deviations of these measurements. The solid line is an exponential fit, giving a decay constant of 1.3 ms/G.
Feshbach Resonance driven by sweeping magnetic field

Hamiltonian: \[ H = H_0 + V \]

\[ H_0 = \sum_p \left[ (\varepsilon_p - \mu - h(t)) (a_p^\dagger a_p + b_p^\dagger b_p) \right] + \sum_q \left( E_q - 2\mu \right) c_q^\dagger c_q \]

\[ h(t) = \dot{h}t \]

\[ V = \frac{g}{\sqrt{V}} \sum_{p,q} \left( a_p b_q c_{p+q}^\dagger + h.c. \right); \quad g \propto \varepsilon_{hf} \sqrt{a_m^3} \]
Suggestions of LZ probability for the molecule production

\[
\frac{N_{mol}(t)}{N_a(0)} = \alpha P_{LZ}(t, \gamma)
\]

\[
\gamma^2 = \frac{g^2 n}{\hbar^2 \dot{\Omega}} \frac{\varepsilon_{hf}^2}{\hbar^2 \dot{\Omega}} n a_m^3
\]

\[
\alpha = ? - \text{combinatorial factor. } \alpha = 1/2
\]
Perturbation theory for molecular production


Keldysh technique for time-dependent field

\[
\begin{align*}
\mathcal{A}_{\alpha\beta}(p; t, t') &= -i\left\langle T_c \left( a_{\alpha}(p, t)a^{\dagger}_{\beta}(p, t') \right) \right\rangle \\
\mathcal{B}_{\alpha\beta}(p; t, t') &= -i\left\langle T_c \left( b_{\alpha}(p, t)b^{\dagger}_{\beta}(p, t') \right) \right\rangle \\
\mathcal{C}_{\alpha\beta}(p; t, t') &= -i\left\langle T_c \left( c_{\alpha}(p, t)c^{\dagger}_{\beta}(p, t') \right) \right\rangle 
\end{align*}
\]

\[\alpha, \beta = \pm\]

Number of molecules

\[
N_m(t) = i \sum_{p \in \text{Fermi sphere}} C_{\alpha\beta}(p; t, t)
\]
Interaction representation

\[ A_{+,+}^{(0)}(p,t,t') = i\theta(\varepsilon_F - \varepsilon_p) \exp \left[ \frac{i}{\hbar} \int_t^{t'} \left( \varepsilon_p - \mu - h(t) \right) dt \right] \]

\[ A_{-,+}^{(0)}(p,t,t') = -i\theta(\varepsilon_p - \varepsilon_F) \exp \left[ \frac{i}{\hbar} \int_t^{t'} \left( \varepsilon_p - \mu - h(t) \right) dt \right] \]

\[ B_{\alpha,\beta}(p,t,t') = A_{\alpha,\beta}(p,t,t') \]

\[ C_{+,+}^{(0)}(p,t,t') = 0; \quad C_{-,+}^{(0)}(p,t,t') = \exp \left[ \frac{i}{\hbar} \left( \varepsilon_c(p) - 2\mu \right) (t' - t) \right] \]

\[ G_{+,+}(p,t,t') = \theta(t - t')G_{-,+}(p,t,t') + \theta(t' - t)G_{+,+}(p,t,t') \]

\[ G_{-,+}(p,t,t') = \theta(t - t')G_{+,+}(p,t,t') + \theta(t' - t)G_{-,+}(p,t,t') \]

Vertices:

\[ \frac{g}{\sqrt{V}} \]
Graphs

Second order:

Fourth order:
Results

\[ \frac{N_m(t = \infty)}{N_a(t = -\infty)} = \Gamma - \frac{88}{105} \Gamma^2 + 0(\Gamma^4) \]

\[ \Gamma = \frac{g^2 n_a}{\hbar^2 \Omega} \quad \Omega = \frac{g_S \mu_B B}{\hbar} = \frac{h(t)}{\hbar} \]

In first two orders of perturbation theory

\[ \frac{N_m(t = \infty)}{N_a(t = -\infty)} = f(\Gamma) \]

Compare to effective LZ theory:

\[ \frac{N_m(t = \infty)}{N_a(t = -\infty)} = \Gamma - \frac{1}{2} \Gamma^2 + 0(\Gamma^4) \]

Collective effects suppress the transition probability

During the transition process atoms can perform an exchange.

Renormalizability?
Conclusions

• LZ transition proceeds during time interval $\tau_{LZ} = \Delta / \dot{\Omega}$

• Fast transverse noise produces transitions during the time interval $\tau_{acc} = (\dot{\Omega} \tau_n)^{-1}$. Condition $\tau_n \parallel \tau_{acc}$ is assumed

• Transitions at any moment of time are due to the spectral component of noise which is in the resonance with the current frequency of the LZ system

• Classical noise tends to establish equal population of levels. Strong quantum noise also leads to the equal population on the timescale between $\tau_{acc}$ and $\tau_{sat}$ and equilibrates the LZ system on longer time scale.

• Transformation of the Fermi gas of cooled alkali atoms into molecules driven by sweeping magnetic field is a collective process which can not be described by the LZ formula. Collective effects suppress the transitions.

• The decoherence time due to longitudinal noise is $\tau_{dec} = \frac{1}{\langle \eta_z^2 \rangle} \tau_n \parallel \tau_n$.
Transitions of spin $S>1/2$ in the sweeping magnetic field

\[ H_s = -bS; \quad b = g\mu_B B \]

\[ b = \left( b_x, 0, \dot{b}_z t \right) \]

Correlations in the noisy LZ transitions

Response to a weak pulse signal

\[ \langle \delta g_\alpha (t) \rangle = \int_{-\infty}^{t} \langle g_\alpha (t) g_\beta (t') \rangle \delta h_\beta (t') dt' \]

\[ \langle g_\alpha (t) g_\beta (t') \rangle = K_{\alpha \beta} (t, t') \]

Solution of the Bethe-Salpeter equations

Solution of the Bloch equations

\[ \dot{g}_z = \frac{1}{2i}(\eta_+ g_+ - \eta_- g_-) \]
\[ \dot{g}_\pm = \mp i\dot{\Omega} g_\pm \pm i\eta_\pm g_z \]

Integral equation for \( g_z \)

\[ \dot{g}_z = -\frac{1}{2} \int_{-\infty}^{t} \left[ \frac{i\dot{\Omega}(t^2-t'^2)}{e^{2\frac{t-t'}{2}}} \eta_+(t)\eta_-(t') + c.c. \right] g_z(t')dt' \]

Complete initial decoherence

Averaging procedure:
\[ \langle \eta_+(t)\eta_-(t')g_z(t') \rangle = \langle \eta_+(t)\eta_-(t') \rangle \langle g_z(t') \rangle \]

Precision:
\[ \tau_n / \tau_{acc} = \dot{\Omega}\tau^2 \]