# Dynamic response of 1D fermions

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# Outline

- introduction
- solvable model 1 (free fermions)
- solvable model 2 (Calogero-Sutherland)
- perturbation theory
- beyond perturbation theory

M.P., PRL **97**, 036404 (2006)
M.P. *et al*., PRL **97**, 196405 (2006)

# Introduction

**Landau's Fermi liquid theory (1956):**

excitations of strongly interacting system of fermions ⟷ excitations of free Fermi gas

→ a liquid of weakly interacting **quasiparticles**

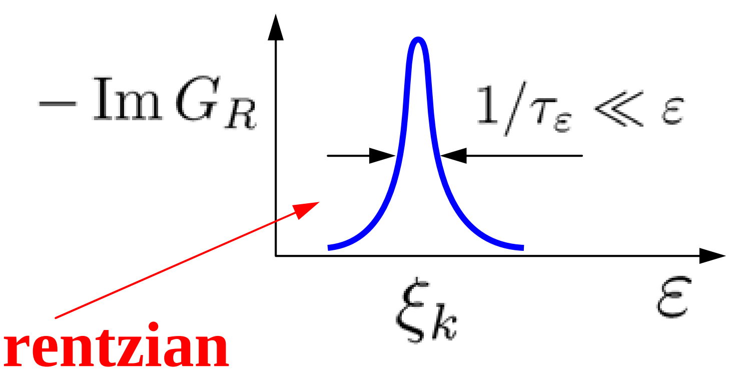
**how well the quasiparticles are defined?**

$$G_R(\varepsilon, k) = \left\langle -i\theta(t)\{\psi(x,t), \psi^\dagger(0,0)\} \right\rangle_{\varepsilon,k}$$

$$= \left[\varepsilon - \xi_k - i/\tau_\varepsilon\right]^{-1}$$

$$\frac{1}{\tau_\varepsilon} \sim r_s^2 \frac{\varepsilon^2}{\epsilon_F} \qquad r_s = e^2/\hbar v_F$$

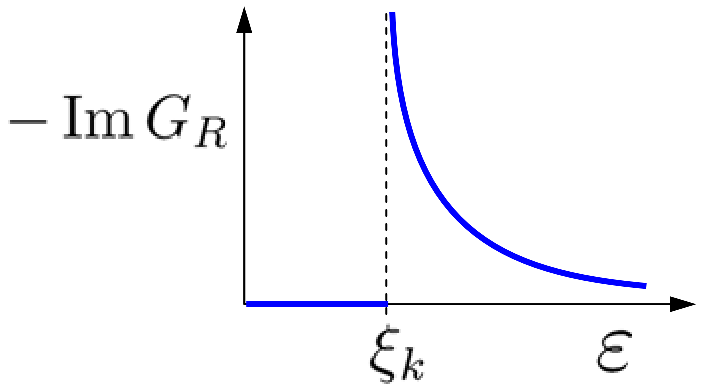
spectral function



# Failure of perturbation theory in 1D

$$-\operatorname{Im} G_R(\varepsilon,k) \propto \lambda^2 \frac{\theta(\varepsilon-\xi_k)}{\varepsilon-\xi_k}\left\{1+\lambda^2 \ln\left(\frac{\varepsilon+\xi_k}{\varepsilon-\xi_k}\right)+\dots\right\}$$

$$\approx \lambda^2 \frac{\theta(\varepsilon-\xi_k)}{\varepsilon-\xi_k}\left[\frac{\varepsilon+\xi_k}{\varepsilon-\xi_k}\right]^{\lambda^2}$$



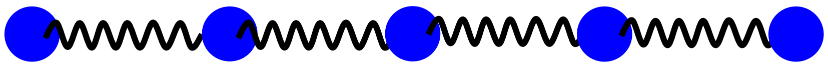

exact for **linear** spectrum $\xi_k = vk$

**Dzyaloshinskii, Larkin (1973)**

**FL's particles/holes are not 'good' quasiparticles in 1D**

# Elementary excitations in 1D

**1D**: only collective excitations



- waves of density (sound waves) - **bosons**

proper quantity is now density-density cor. function

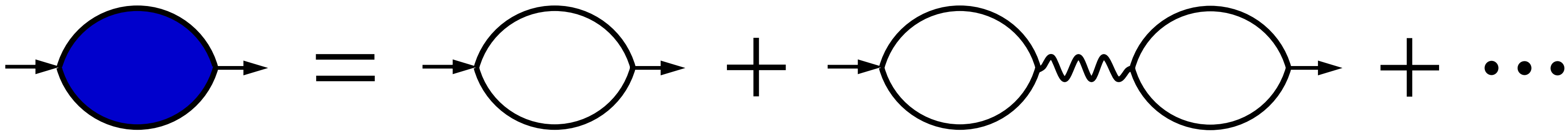
$$\chi(q,\omega) = \left\langle -i\theta(t)\left[\rho(x,t),\rho^{\dagger}(0,0)\right]\right\rangle_{q,\omega}$$

dynamic structure factor:

$$S(q,\omega) = \int dx\, dt\, e^{i(\omega t - qx)}\langle\rho(x,t)\rho(0,0)\rangle \;=\; 2\,\mathrm{Im}\chi(q,\omega)$$

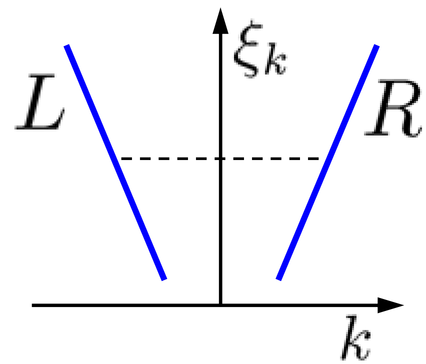
at $T=0$ (FDT)

# Interacting 1D fermions: RPA

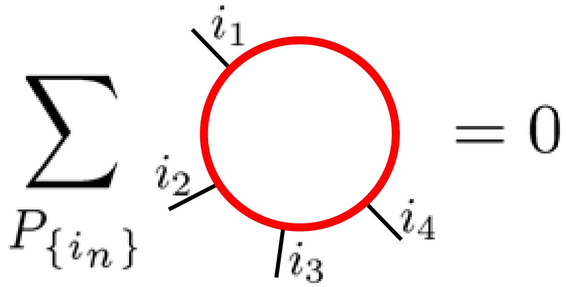


$$\Rightarrow \quad S^{\text{RPA}} \propto q\delta(\omega - vq)$$

exact for **linear** spectrum $\xi_k = \pm vk$




**Dzyaloshinskii & Larkin (1973)**




**Do bosons make ‘better’ quasiparticles in 1D than quasiparticles/holes are in 3D Fermi liquid?**

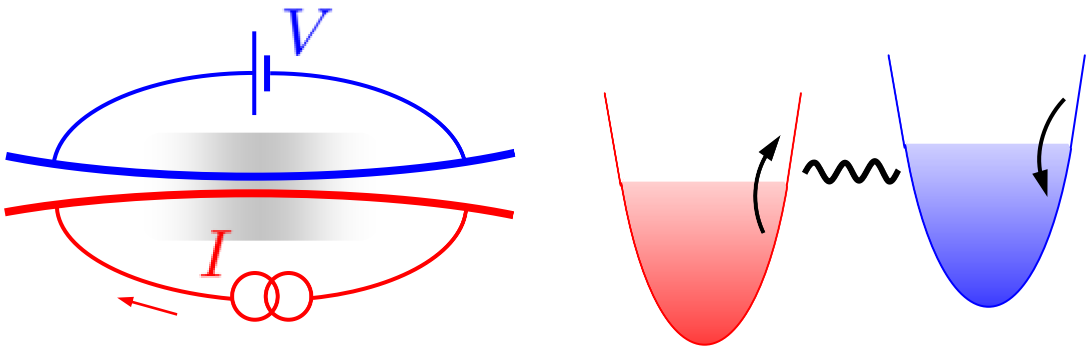
# Motivation

**Challenge:**

**how to account for interaction and nonlinearity of spectrum simultaneously?**

**Applications**

**Coulomb drag**




$$R_{\rm drag} = V_2/I_1$$

$$\propto T^{-1}\int_0^\infty d\omega\, dq\, q^2 V_{12}^2(q) e^{-\omega/T} S_1(q,\omega) S_2(q,\omega)$$

PRL **91**, 126805 (2003)

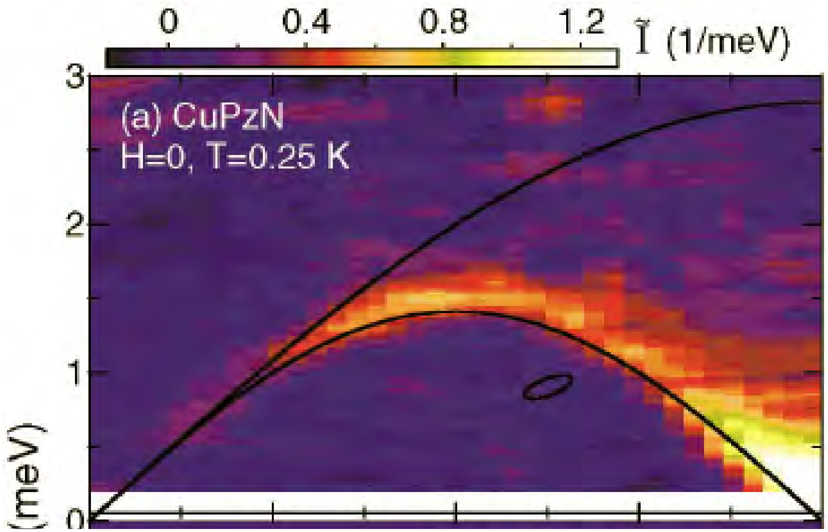
# Motivation

**Challenge:**

**how to account for interaction and nonlinearity of spectrum simultaneously?**

**Applications**

**Inelastic neutron scattering off antiferromagnetic spin chains**




(maps to structure factor of 1D spinless fermions)

# Free electrons

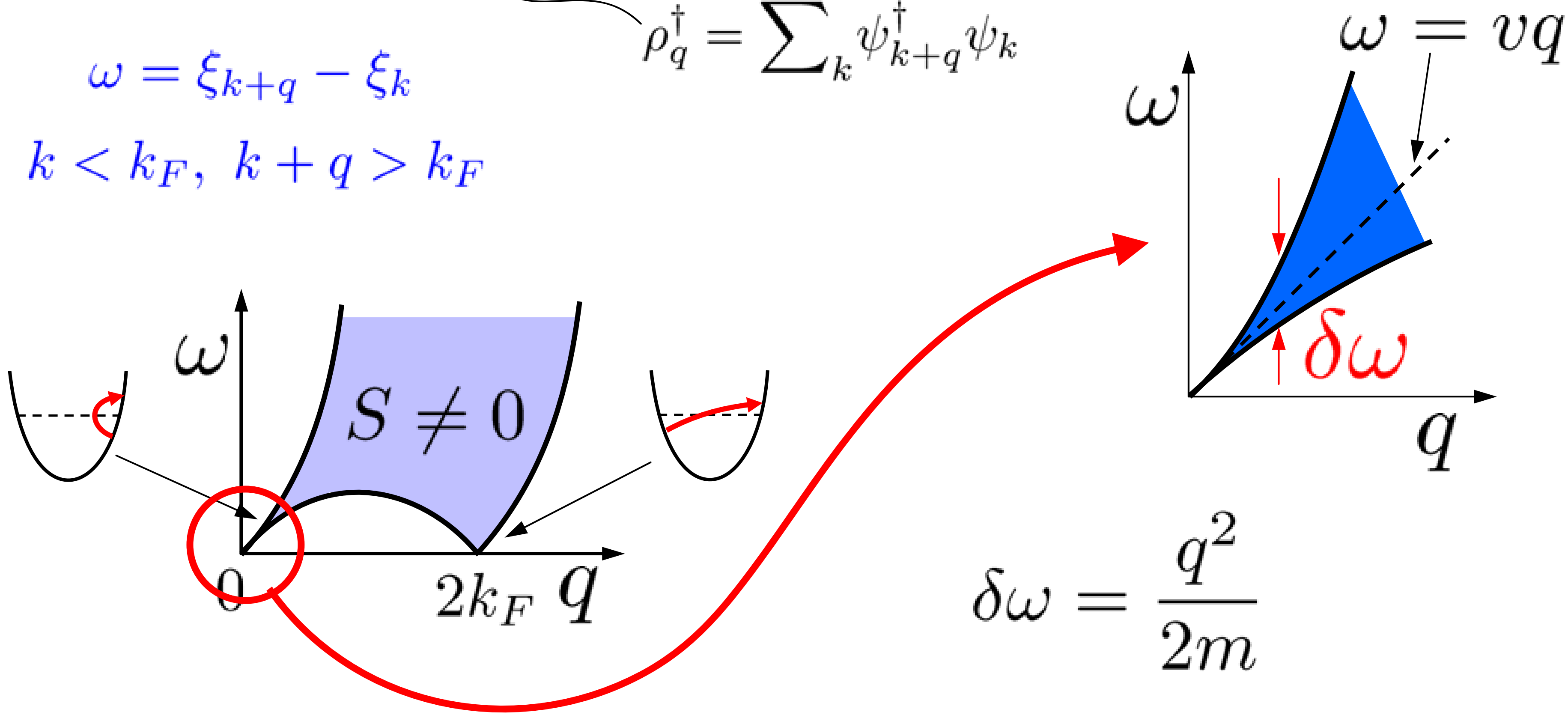
Lehmann (Golden rule – like) representation

$$S(q,\omega) \propto \sum_f \langle f|\rho_q^\dagger|0\rangle^2 \delta(\omega - \epsilon_f + \epsilon_0)$$

$$\rho_q^\dagger = \sum_k \psi_{k+q}^\dagger \psi_k$$

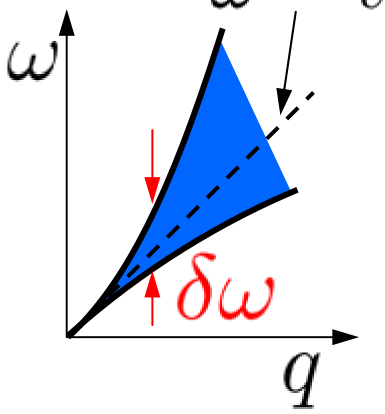
$$\omega = \xi_{k+q} - \xi_k$$

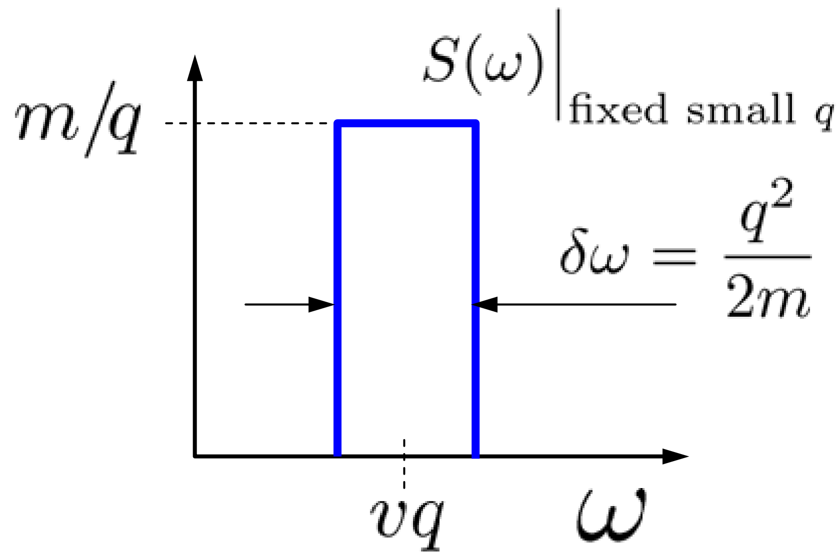
$$k < k_F,\ k+q > k_F$$




$$\delta\omega = \frac{q^2}{2m}$$

# Free electrons




$$\delta\omega = q^2/2m \sim \omega^2/\epsilon_F \implies$$

- **the peak is narrow – good news!**

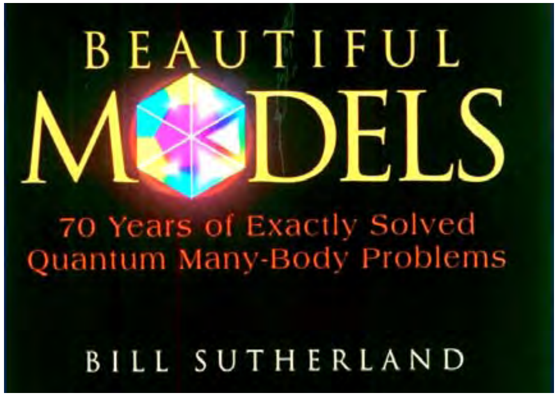
**but…**

- **it is not a Lorentzian**
- $\delta\omega \propto 1/m$ **(non-perturbative in curvature)**

# Calogero-Sutherland model

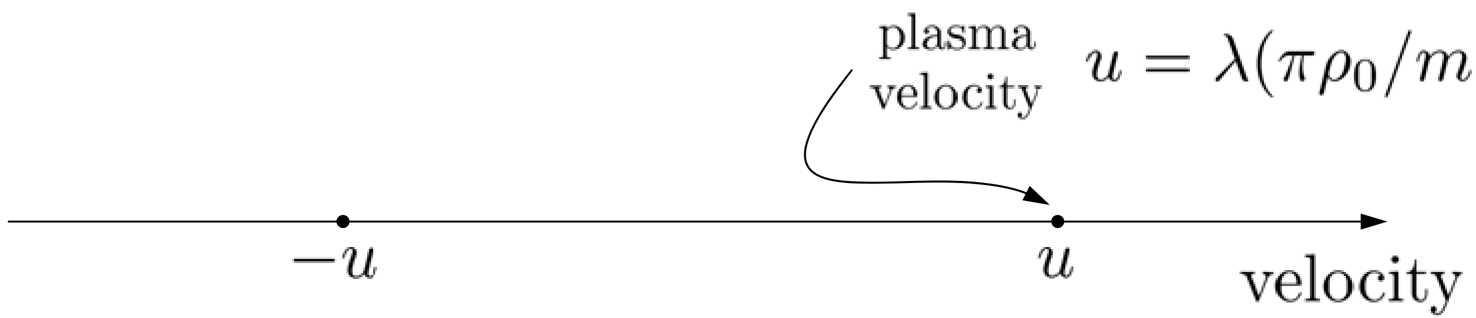
$$H = -\sum_i \frac{1}{2m}\frac{\partial^2}{\partial x_i^2} + \sum_{i<j} V(x_i - x_j)$$

$$V(x) = \frac{\lambda(\lambda-1)}{m}\frac{1}{x^2}$$




**elementary excitations** $\begin{cases} \textbf{quasiparticles } |v| > u \\ \textbf{quasiholes } |\bar{v}| < u \end{cases}$

$$u = \lambda(\pi\rho_0/m)$$



# Calogero-Sutherland model

excited state with $\{v_i, \bar{v}_j\}$:

$$P = \sum_i m v_i - \sum_j \bar{m}\bar{v}_j \qquad \bar{m} = m/\lambda$$

$$E = \sum_i \frac{1}{2} m(v_i^2 - u^2) + \sum_j \frac{1}{2}\bar{m}(u^2 - \bar{v}_j^2)$$

$|R\rangle = \rho^\dagger_{q>0}|0\rangle$ ⟶ all quasiparticles are **right-moving**

$v_i > u$

$\rho^\dagger_q = \sum_k \psi^\dagger_{k+q}\psi_k$

(valid for $\forall\lambda$)

not the case generically!

# Calogero-Sutherland model

excited state with $\{v_i, \bar{v}_j\}$: $\quad |R\rangle = \rho^\dagger_{q>0}|0\rangle$

$$P = \sum_i m v_i - \sum_j \bar{m}\bar{v}_j$$

$$E = \sum_i \frac{1}{2} m (v_i^2 - u^2) + \sum_j \frac{1}{2} \bar{m} (u^2 - \bar{v}_j^2)$$

momentum and energy conservation:

$$q = P_{|R\rangle}, \quad \omega = E_{|R\rangle}$$

$\Rightarrow$ bounds on $\omega$ for a given $q$:

$$S \neq 0 \text{ for } \omega_- < \omega < \omega_+$$

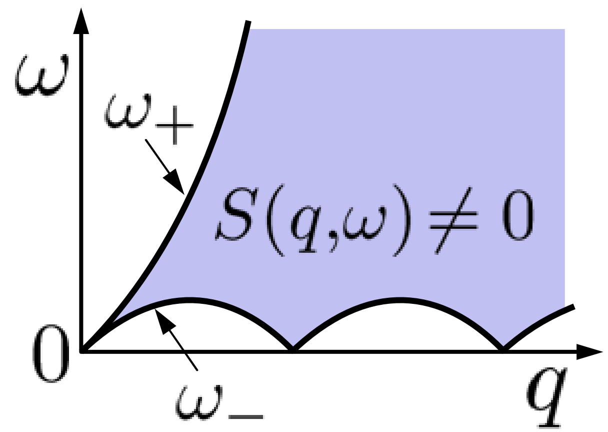
(like for free fermions)

$$\omega_+ = uq + q^2/2m$$

$$\omega_- = uq - \lambda q^2/2m$$

# Calogero-Sutherland model

$$\omega_+ = uq + q^2/2m$$
$$\omega_- = uq - \lambda q^2/2m$$




structure factor

$$S(q,\omega) = q^2 \int \prod_{i,j} dv_i d\bar{v}_j \, \boldsymbol{F}(\ldots) \, \delta(q - P_{|R\rangle}) \, \delta(\omega - E_{|R\rangle})$$

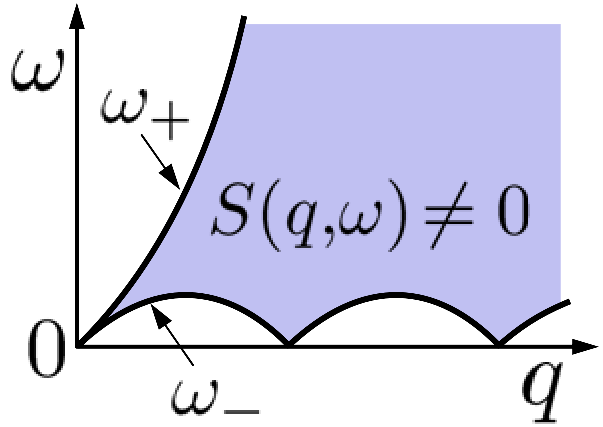
form-factor $\boldsymbol{F} = \langle \{v_i, \bar{v}_j\} | \rho_q^\dagger | 0 \rangle^2$

– known for CSM: Simons *et al.* (93), Haldane (93), Ha (94)

# Calogero-Sutherland model

$$\omega_+ = uq + q^2/2m$$

$$\omega_- = uq - \lambda q^2/2m$$



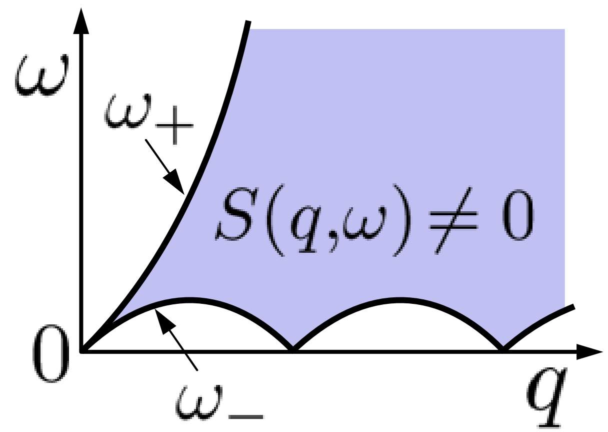

at $\omega \to \omega_\pm$ almost all momentum and energy are carried by a **single** quasiparticle/hole **!**


(for $\omega \to \omega_+$)

**- this observation allows the evaluation of the integral**

# Calogero-Sutherland model

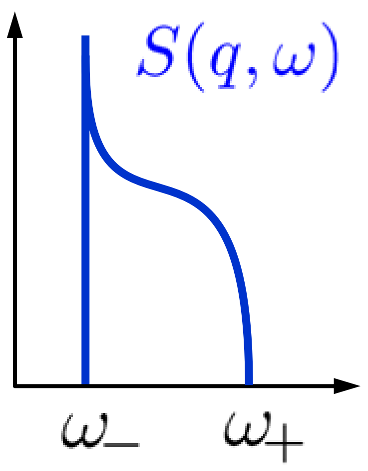
$$\omega_+ = uq + q^2/2m$$
$$\omega_- = uq - \lambda q^2/2m$$




**power-law singularities** at $\omega \to \omega_\pm$:

$$\frac{S(q,\omega)}{S_0} \propto \left[\frac{\omega-\omega_-}{\delta\omega}\right]^{1/\lambda-1}, \quad \omega \to \omega_-$$

$$\frac{S(q,\omega)}{S_0} \propto \left[\frac{\omega_+-\omega}{\delta\omega}\right]^{\lambda-1}, \quad \omega \to \omega_+$$

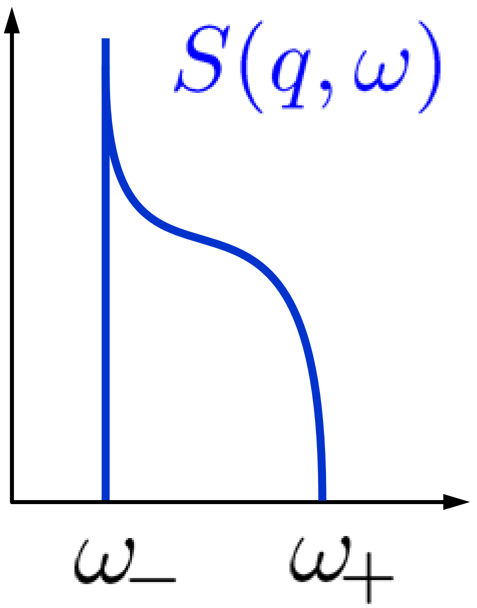



see PRL **97**, 036404 (2006)

# Calogero-Sutherland model

weak interaction: $|\lambda - 1| \ll 1$

**the exponents** $\left\{ \begin{array}{c} 1/\lambda - 1 \\ \lambda - 1 \end{array} \right. \longrightarrow \mp(\lambda - 1)$




$$\Longrightarrow \quad \delta S/S_0 \sim \left[\frac{\omega_+ - \omega}{\omega - \omega_-}\right]^{\lambda - 1}$$

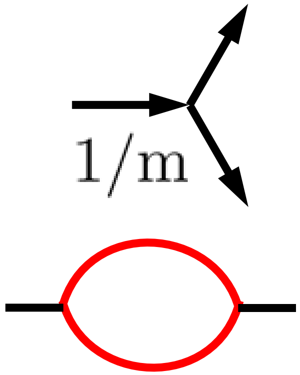
(exact for $\omega \to \omega_\pm$)

**the exponents are 1st order in the interaction strength**

# Perturbation theory: bosons

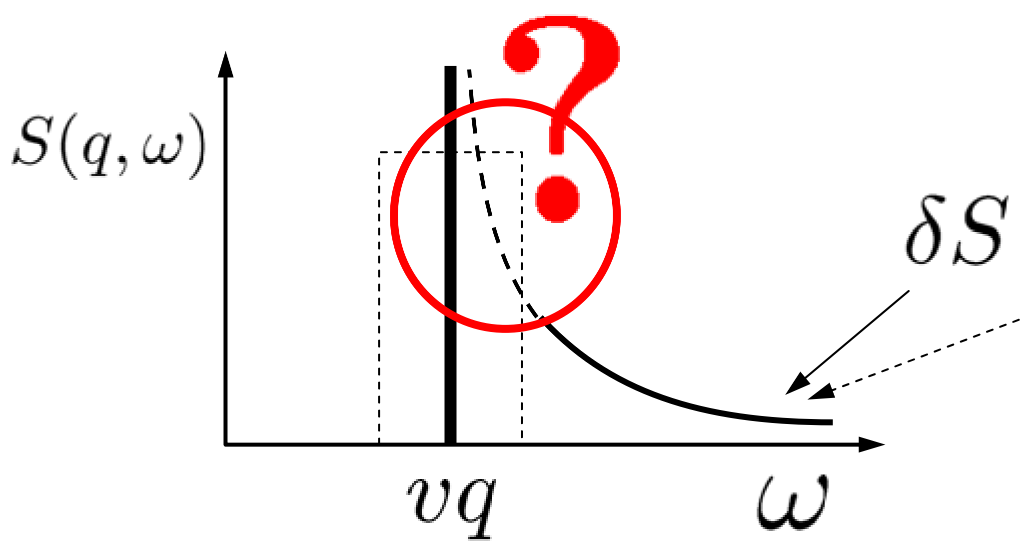
**bosonization**:

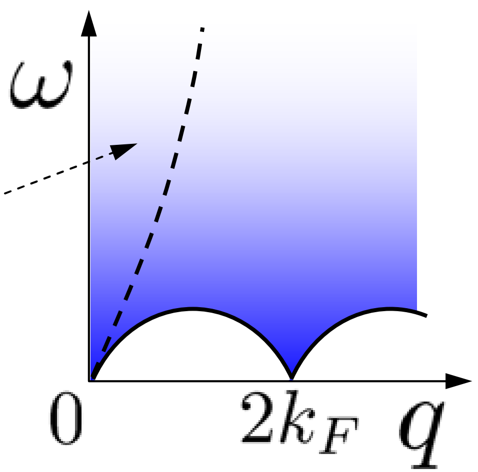
$$\psi_{R/L} \to \sqrt{k_0}\, e^{\pm i\varphi_{R/L}}$$



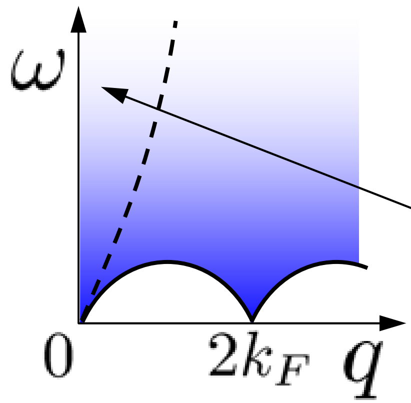

nonlinearity as a perturbation: $\delta H \propto \frac{1}{m}\int dx \sum_\alpha (\partial_x \varphi_\alpha)^3$

$$\Rightarrow \quad \delta S \propto \frac{(q^2/m)^2}{\omega^2 - (vq)^2}\,\theta(\omega - vq) \quad \text{- diverges on the shell}$$



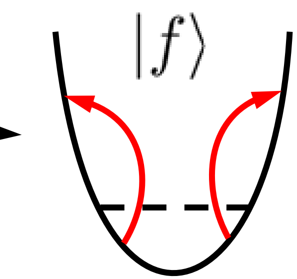



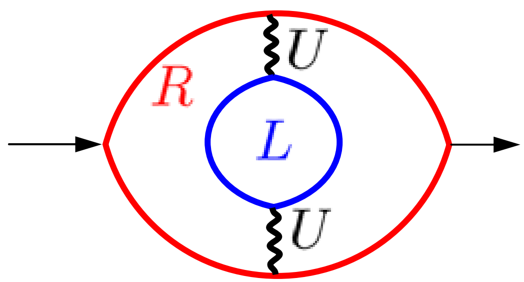
# Perturbation theory: fermions

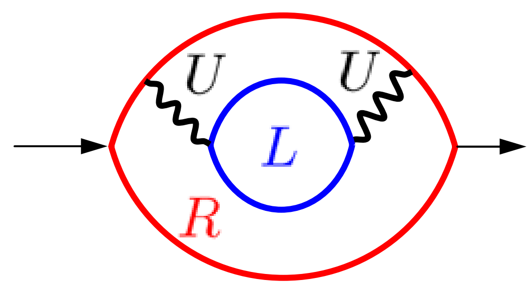



$$S(q,\omega) \propto \sum_f \langle f|\rho_q^\dagger|0\rangle^2 \delta(\omega - \epsilon_f + \epsilon_0)$$

large $\omega$, small $q$



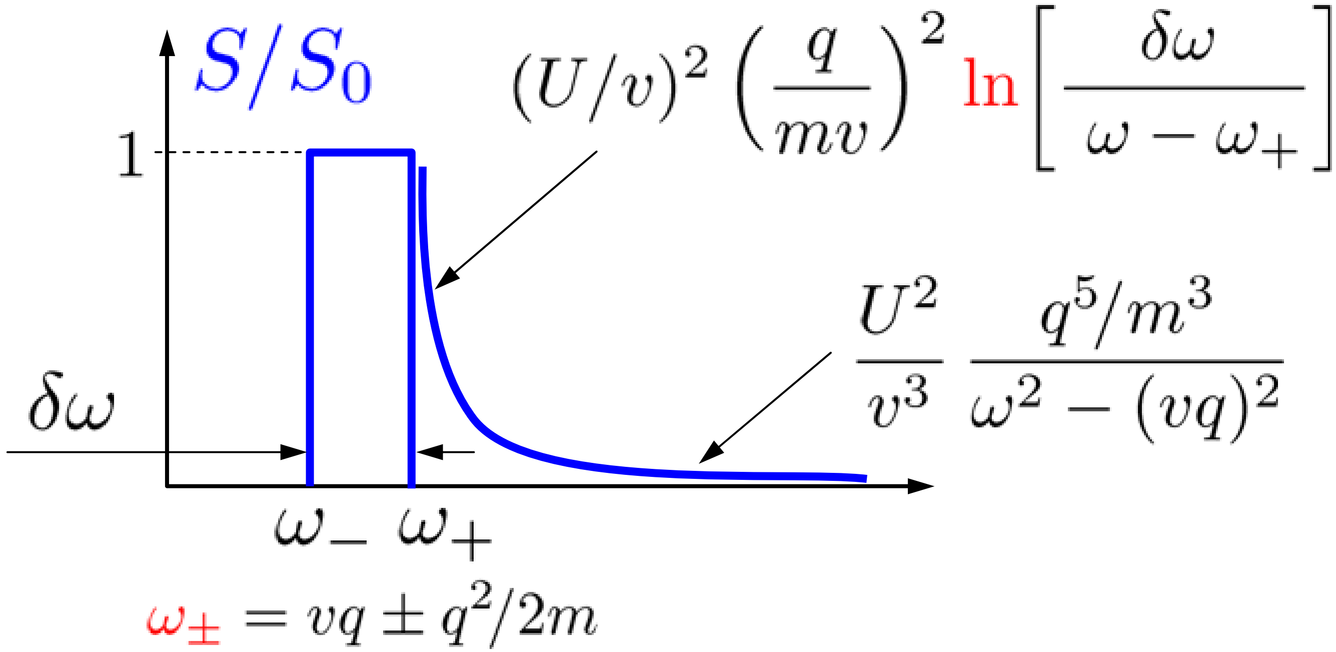




$$S/S_0 \sim (U/v)^2 \left(\frac{q}{mv}\right)^2 \ln\left[\frac{\delta\omega}{\omega - \omega_+}\right]$$

$$0 < \omega - \omega_+ \ll \delta\omega$$

$$S_0 = m/q$$

# Perturbation theory: $\omega > \omega_+$

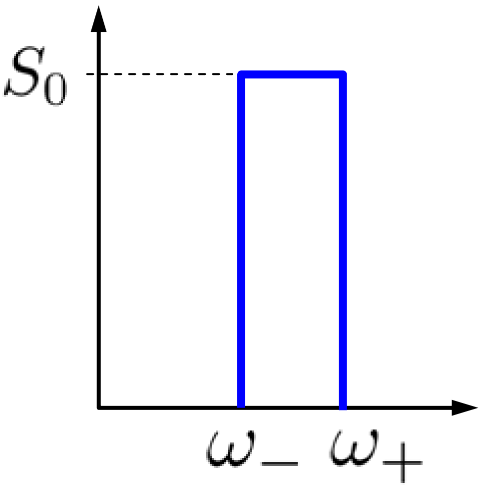



$S/S_0 \sim 1$ when $\omega - \omega_+ \sim$ exponentially small

main spectral weight of $S(q,\omega)$ is confined to

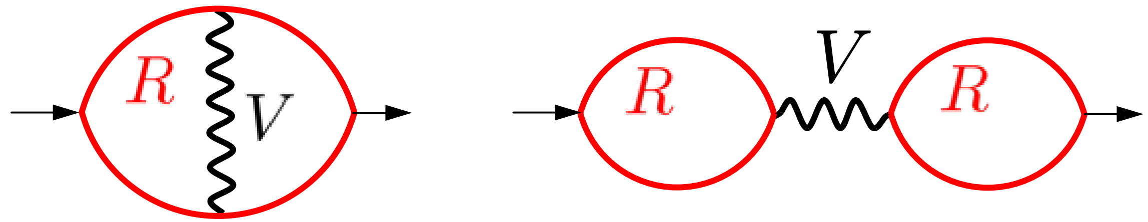
$$\omega_- < \omega < \omega_+$$

# Perturbation theory: $\omega_- < \omega < \omega_+$




$\omega_\pm = vq \pm q^2/2m$

leading corrections (in $V/v \ll 1$ and $q/mv \ll 1$)




$$\delta S \propto (V_0 - V_q)\,\mathrm{Im}\chi_0(q,\omega)\,\mathrm{Re}\chi_0(q,\omega)$$

$$\Rightarrow \delta S/S_0 \propto \frac{m}{q}(V_0 - V_q)\,\ln\left[\frac{\omega_+ - \omega}{\omega - \omega_-}\right]$$

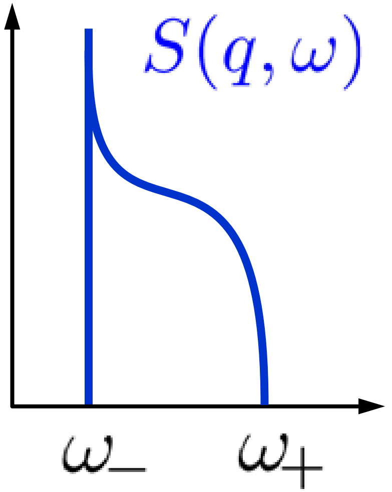
$> 0$ for $\omega \to \omega_-$

$< 0$ for $\omega \to \omega_+$

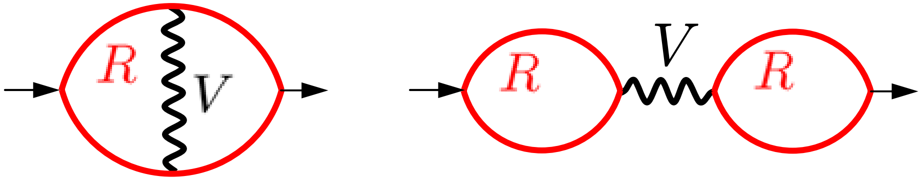
# Back to Calogero-Sutherland model

recall that

$$\delta S/S_0 \sim \left[\frac{\omega_+ - \omega}{\omega - \omega_-}\right]^{\lambda-1}$$




(exact for $\omega \to \omega_\pm$ and $|\lambda - 1| \ll 1$)



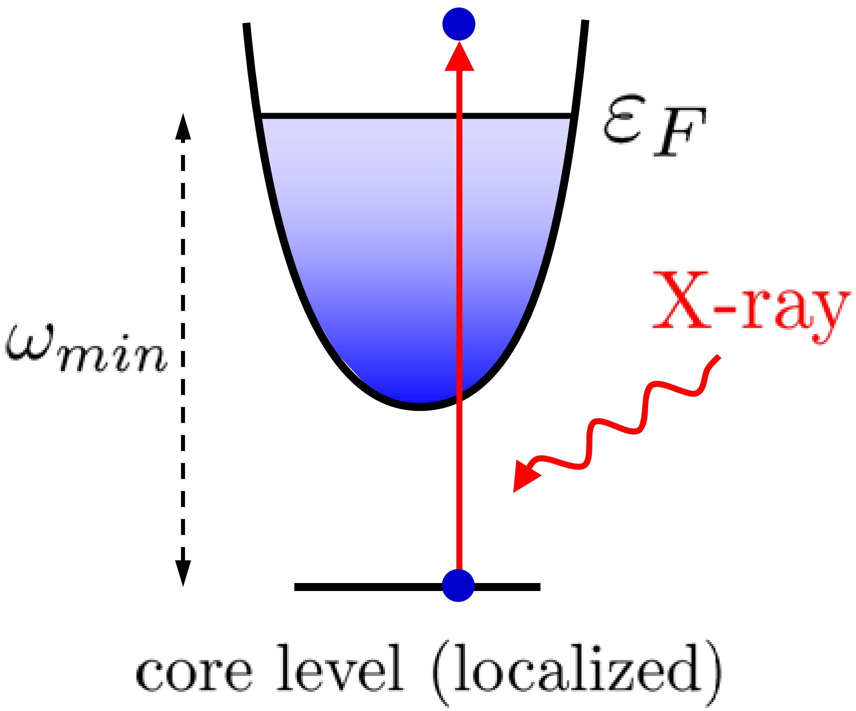

$$\delta S/S_0 = \mu \ln(\ldots) + \ldots$$

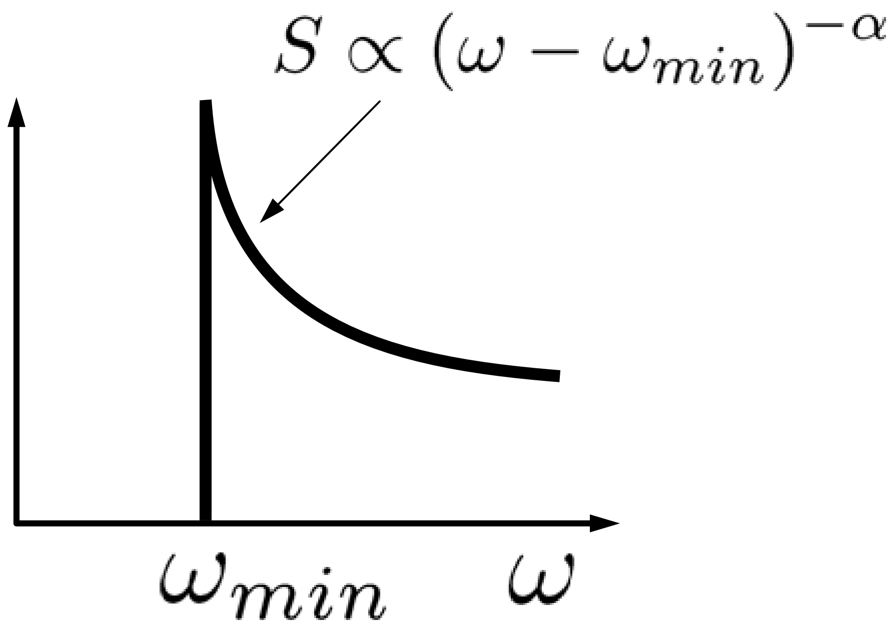
for $V(x) = \dfrac{\lambda(\lambda-1)}{mx^2}$

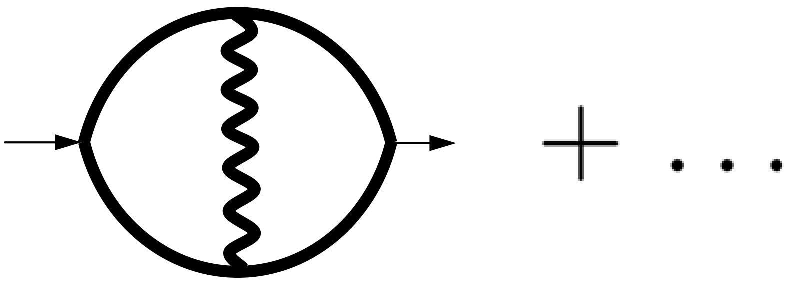
one finds $\mu(q) = \dfrac{m}{\pi q}(V_0 - V_q) = \lambda(\lambda - 1) \approx \lambda - 1$

bingo!

# Fermi edge singularity in metals

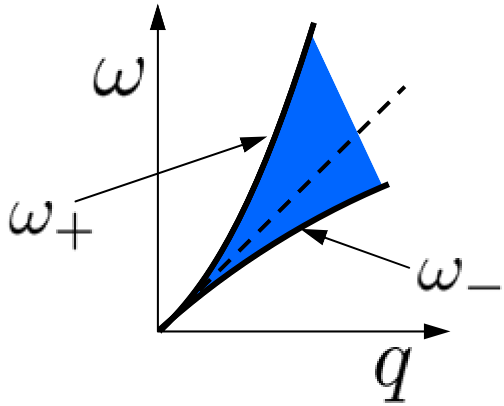


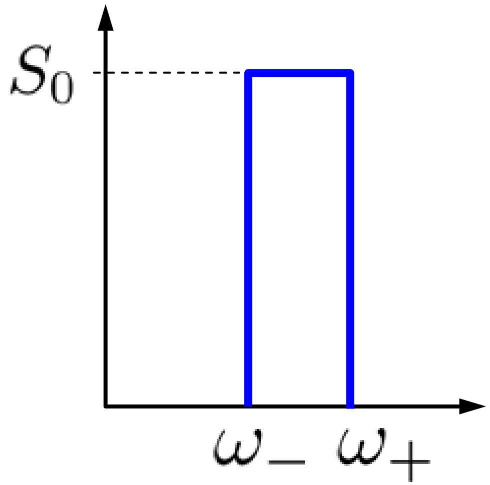



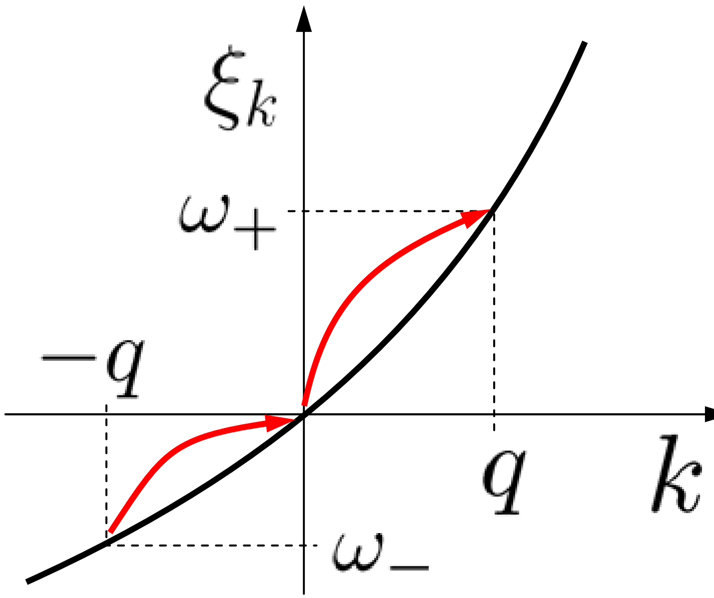



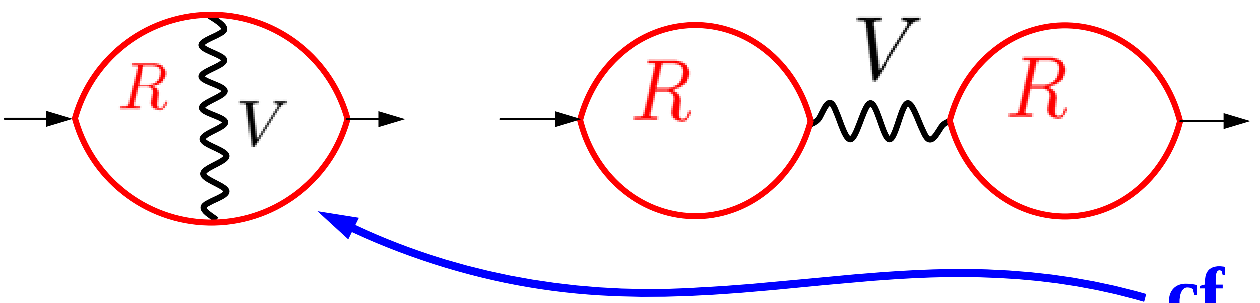

**Mahan (1967)**
**Nozieres & De Dominicis (1969)**

# Back to structure factor



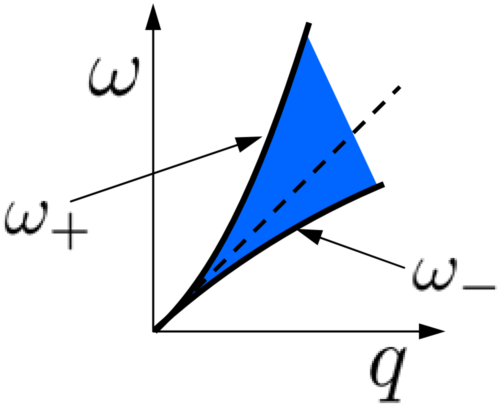



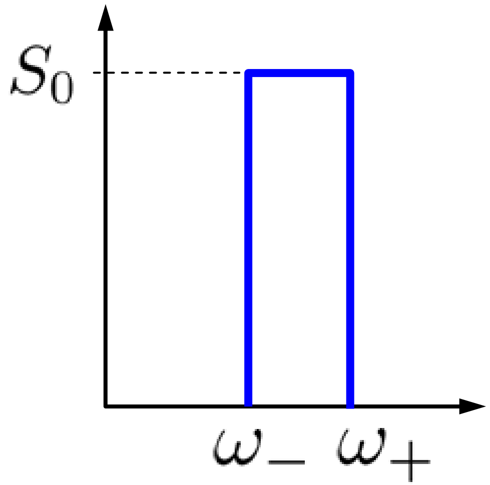



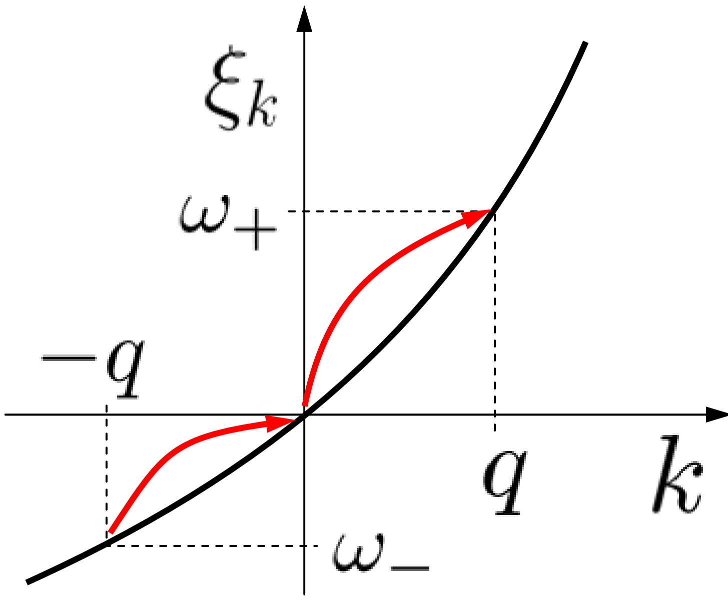

**cf. edge singularity**

$$\delta S/S_0 = \mu(q) \ln \left[ \frac{\omega_+ - \omega}{\omega - \omega_-} \right] \qquad \mu(q) = \frac{m}{\pi q}(V_0 - V_q)$$

# Back to structure factor

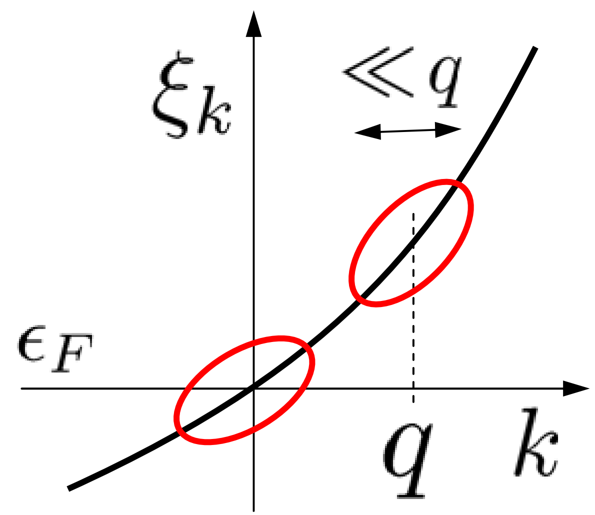


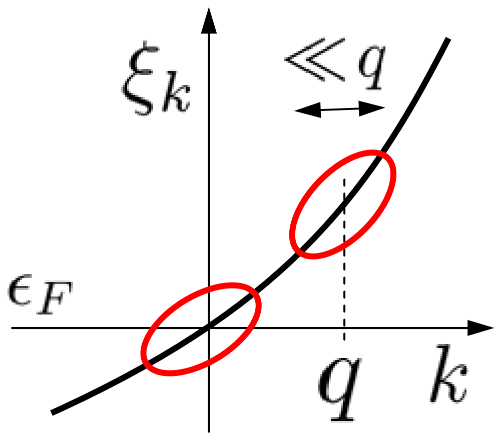

log-singularities at $\omega \to \omega_\pm$

**important states:**

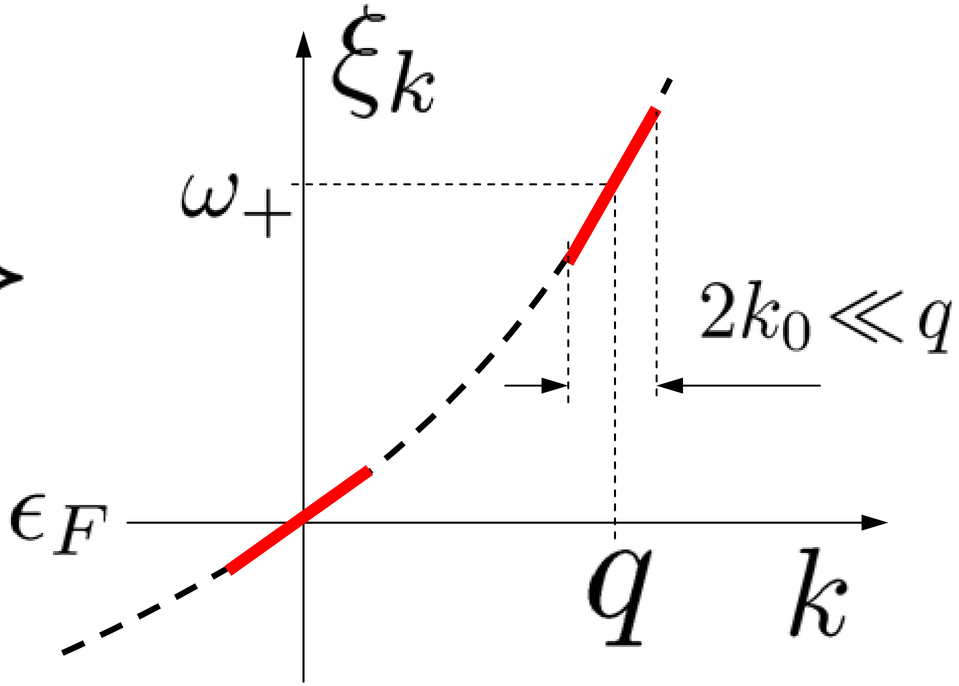


# Beyond perturbation theory

**important states:**

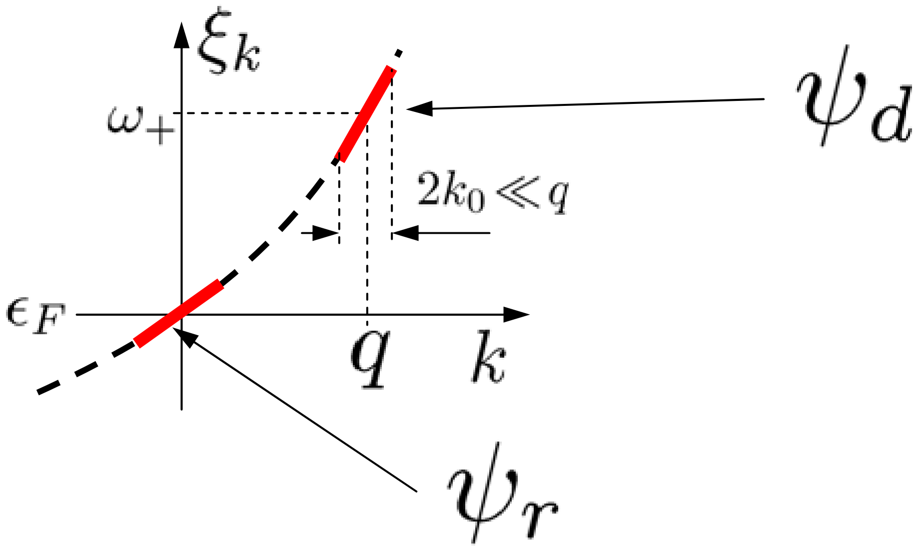



$\Longrightarrow$




**the idea:** **project all other states out!**

# Beyond perturbation theory




$$\rho_q^\dagger \to \psi_d^\dagger \psi_r$$

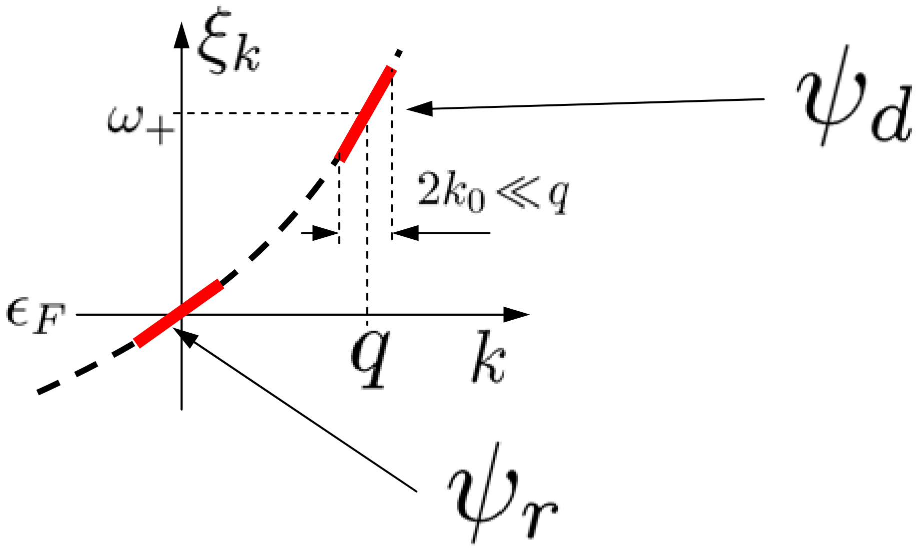
(creates an exciton)

$$v_d = v + q/m$$

**effective two-band model:**

$$H_+ = \int dx\, \psi_r^\dagger(-iv\partial_x)\psi_r + \int dx\, \psi_d^\dagger(\omega_+ - iv_d\partial_x)\psi_d$$
$$+ (V_0 - V_q)\int dx\, \rho_d(x)\rho_r(x).$$

# Beyond perturbation theory

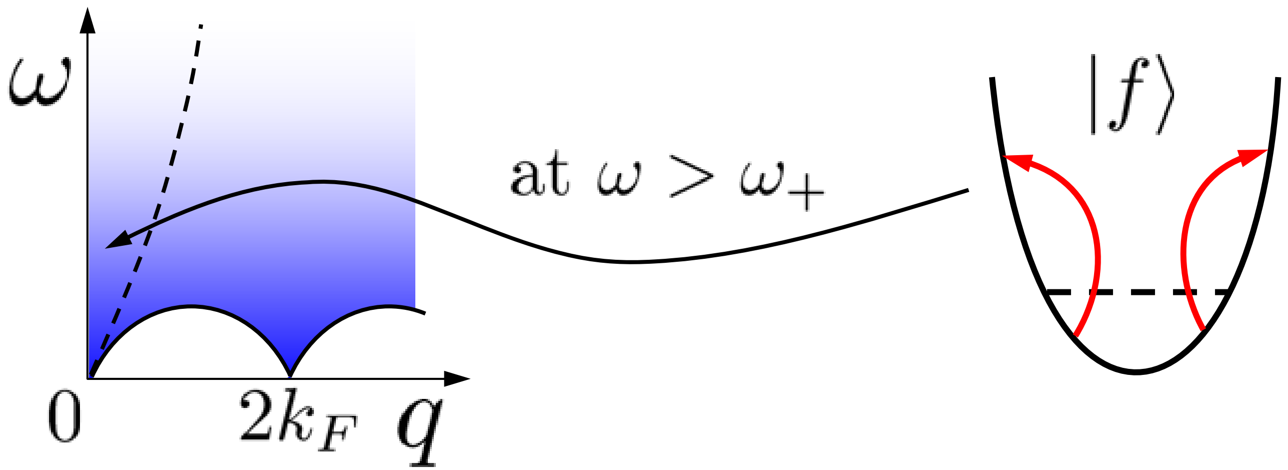



log. accuracy

$$\Rightarrow \quad \delta S/S_0 = \left[\frac{\omega_+ - \omega}{(k_0/q)\delta\omega}\right]^{\mu} \qquad 0 < \omega_+ - \omega \ll \delta\omega$$

$$\mu(q) = \frac{V_0 - V_q}{\pi(v_d - v)} = \frac{m}{\pi q}(V_0 - V_q)$$

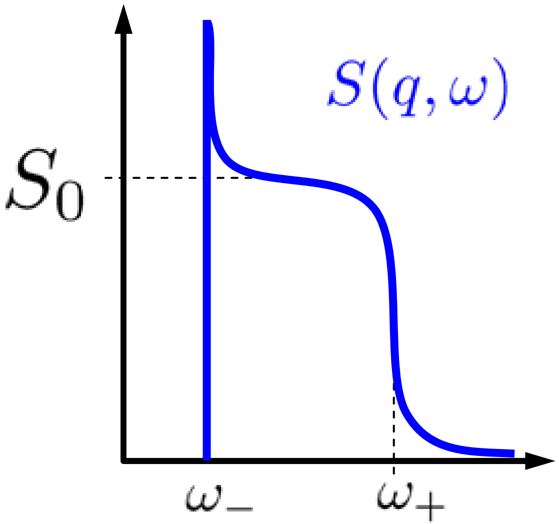
# Including the left-movers…




perturbation theory $\rightarrow$ $S/S_0 \sim (U/v)^2 \left(\frac{q}{mv}\right)^2 \ln\left[\frac{\delta\omega}{\omega - \omega_+}\right]$

$R - L$ interaction

# Including the left-movers (results)




$$S/S_0 = \left[\frac{\delta\omega}{\omega - \omega_-}\right]^{\mu}, \quad \omega \to \omega_- + 0$$

$$\mu(q) = \frac{m}{\pi q}(V_0 - V_q)$$

$$S/S_0 = \begin{cases} \left[\frac{\omega_+ - \omega}{\delta\omega}\right]^{\mu} + \frac{\nu}{\mu}, & \omega < \omega_+ \\ \frac{\nu}{\mu}\left(1 - \left[\frac{\omega - \omega_+}{\delta\omega}\right]^{\mu}\right), & \omega > \omega_+ \end{cases}$$

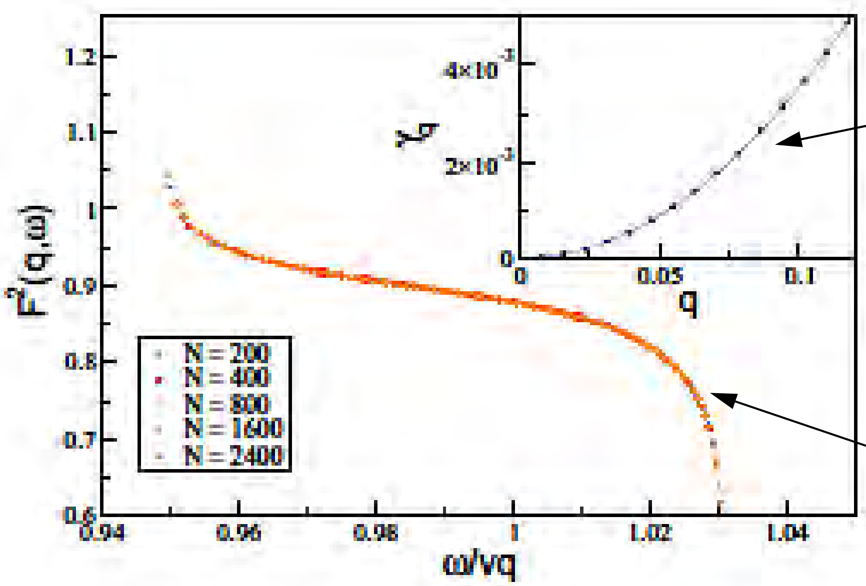
$$\nu(q) = \left(\frac{q}{4mv}\right)^2 \left(\frac{U_0}{2\pi v}\right)^2 \ll \mu(q)$$

see PRL **97**, 196405 (2006)

# AFM spin chain - numerics

**The dynamical spin structure factor for the anisotropic spin-1/2 Heisenberg chain**

R. G. Pereira,[1] J. Sirker,[1] J.-S. Caux,[2] R. Hagemans,[2] J. M. Maillet,[3] S. R. White,[4] and I. Affleck[1]



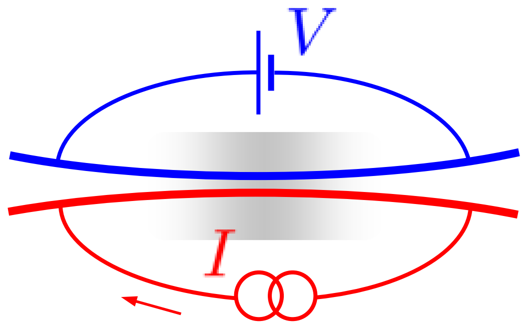

$\delta\omega(q)$

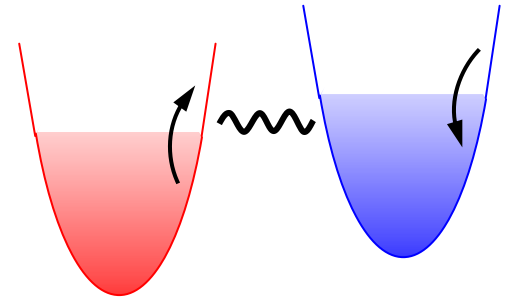
arXiv:cond-mat/0603681

$\propto S(q,\omega)$

FIG. 1: Form factors squared for the single particle-hole type excitations and different $N$ at $\Delta = 0.25$, $s = -0.1$ and $q = 2\pi/25$. The inset shows the scaling of the width $\gamma_q$. The points are obtained by an extrapolation $N \to \infty$ of the numerical data. The solid line is the prediction (6), $\gamma_q = 0.356\, q^2$.

# Coulomb drag



$$R_{\text{drag}} = V_2/I_1$$

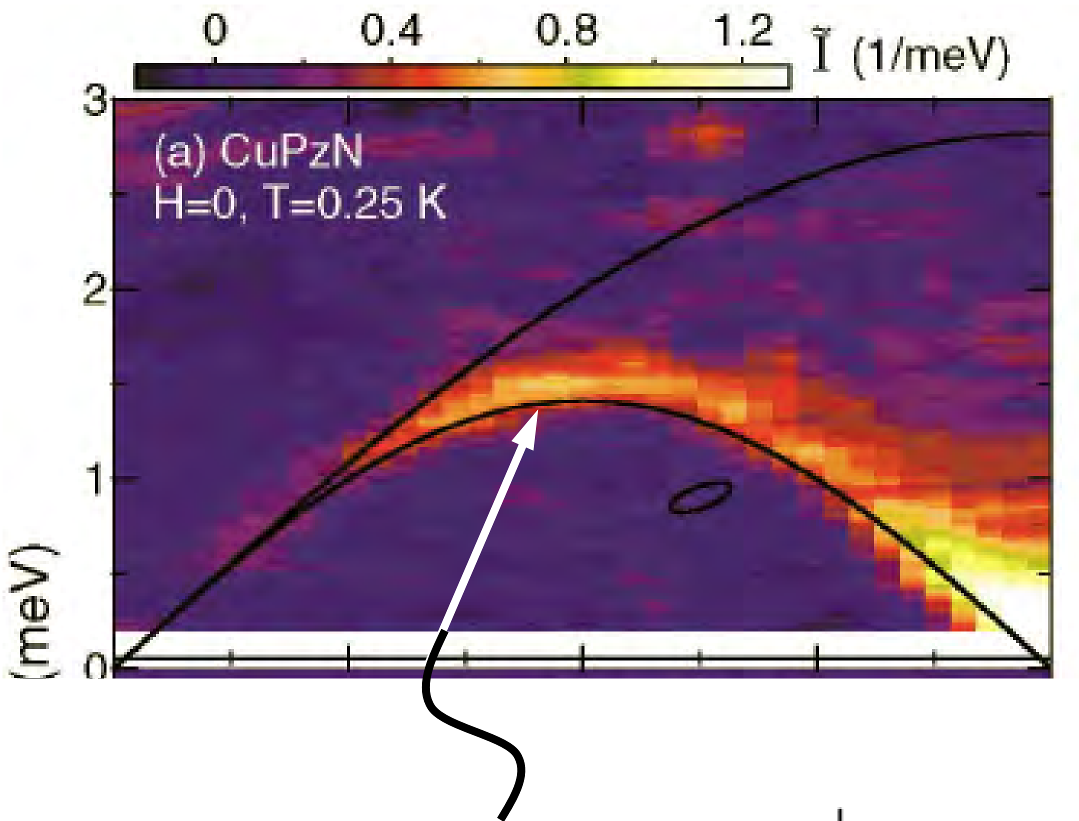
$$\propto T^{-1}\int_0^\infty d\omega\, dq\, q^2 e^{-\omega/T} S_1(q,\omega) S_2(q,\omega)$$

$$\mu(q) = \frac{m}{\pi q}(V_0 - V_q) \propto q \xrightarrow[q \to 0]{} 0$$

$\Rightarrow$ no effect on $R_{\text{drag}}$: $R_{\text{drag}} \propto T^2$ at low $T$

PRL **91**, 126805 (2003)

# AFM spin chain

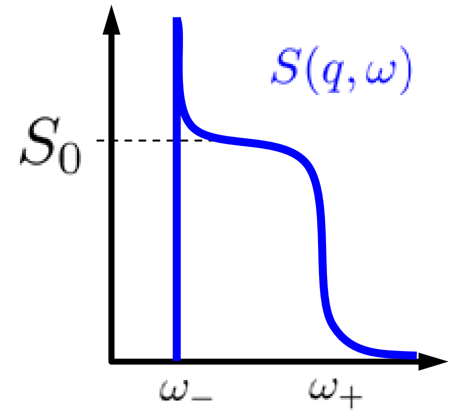



$$\max S(\omega, q)\Big|_{\text{fixed } q} \propto T^{-\alpha(q)}$$

# Conclusions

- **a challenge:**

simultaneous account for
**interactions & nonlinear dispersion**
in a system of 1D fermions




✓ dynamic structure factor (density-density cor. function)

single-particle cor. function

fermions with spin

PRL **97**, 036404 (2006)
PRL **97**, 196405 (2006)
**more is coming soon…**