

Quantum chemistry approaches for electronic structure II: single- and many-electron bound-continuum transitions

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Single-electron transition: Photoionization

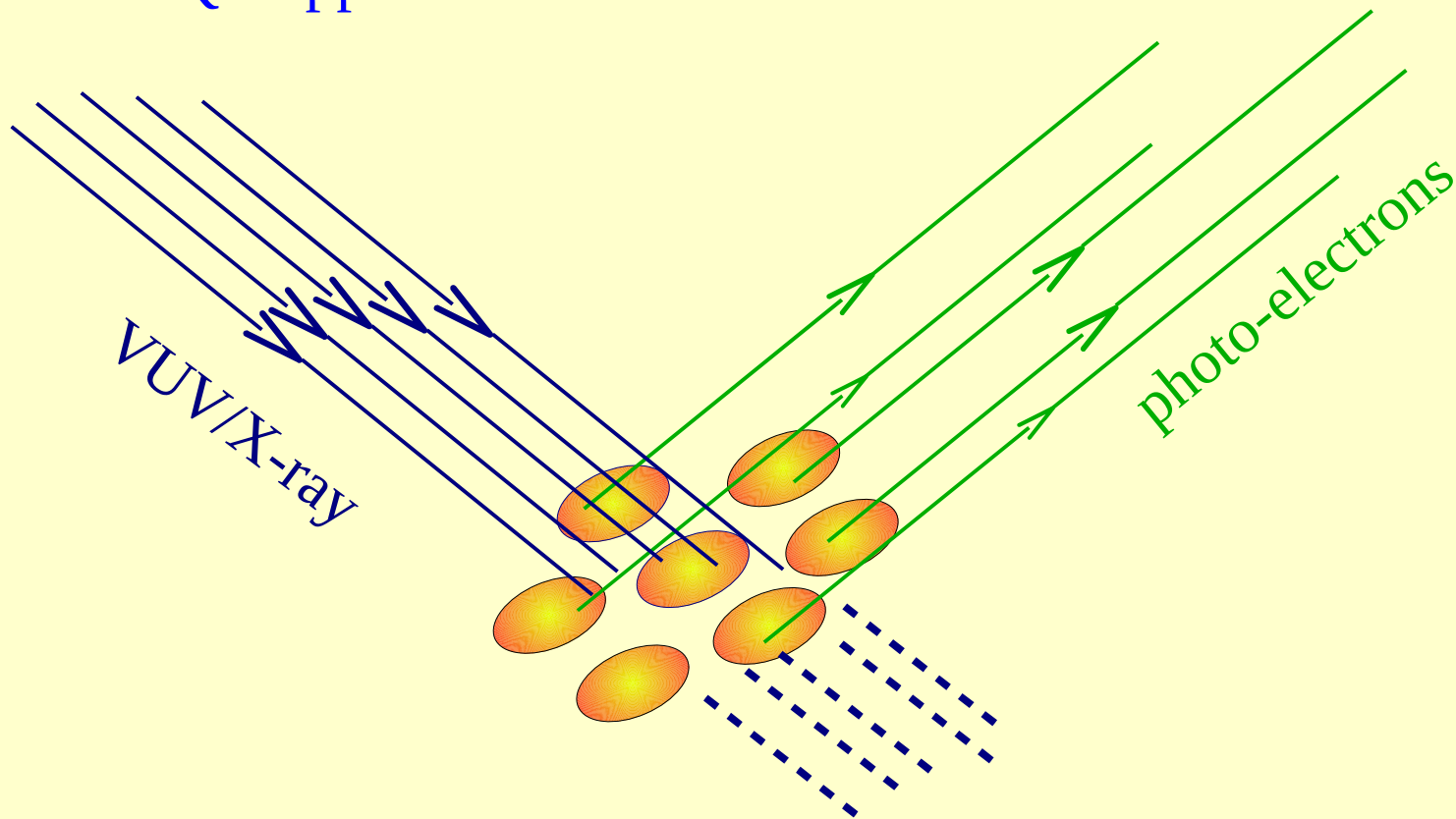
$$\sigma_E \propto E \left| \langle \Phi | \sum_i \vec{r}_i | \chi_E \rangle \right|^2$$

A bound (typically the ground) state
 $\langle \Phi | \Phi \rangle = 1$

A continuum state
 $\langle \chi_E | \chi_{E'} \rangle = \delta(E' - E)$

✓ Standard QC applies

✗ Standard QC does not apply



Ab initio approach: ADC

The purpose: *ab initio* calculation of Φ, χ wave functions in $\sigma_E \propto E |\langle \Phi | \sum \vec{r}_i | \chi_E \rangle|^2$

The method: Algebraic Diagrammatic Construction (**ADC**)
in the Intermediate State Representation (**ISR**)

$$\Psi = \sum_{ia} Y_i^a \Psi_i^a + \sum_{ijab} Y_{ij}^{ab} \Psi_{ij}^{ab} + \sum_{ijkabc} Y_{ijk}^{abc} \Psi_{ijk}^{abc} + \dots$$

CI-like expansion

	1h1p	2h2p	3h3p
	single	double	triple
	excitations	excitations	excitations

$$\Psi_i^a = c_a^\dagger c_i \Psi_0, \quad \Psi_{ij}^{ab} = c_a^\dagger c_b^\dagger c_i c_j \Psi_0, \quad \dots, \quad \text{where } \Psi_0 = \Phi_0^{\text{HF}} + \underbrace{\Psi_0^{(1)} + \Psi_0^{(2)} + \dots}_{\text{MBPT for The ground state}}$$

ADC: Schirmer and Cederbaum (since 1980s)

ISR: Schirmer and Trofimov (since 1990s)

MBPT for
The ground state

ADC-ISR for photoionization: Averbukh, Cederbaum, and co-workers (2009)

The problems

☹ Problem No1: $\langle \chi_E | \chi_{E'} \rangle = \delta(E'-E)$ is not fulfilled in an \mathcal{L}^2 *basis*, instead we have $\langle \chi_j | \chi_{j'} \rangle = \delta_{j,j'}$

☹ Problem No2: the energies ε_j obtained in an \mathcal{L}^2 *basis* are discrete $\Rightarrow \sigma_E$ can not be obtained at an arbitrary E

Substitution of χ_ε obtained in \mathcal{L}^2 *basis* into $A = |\langle \Phi | \hat{A} | \chi_E \rangle|^2$ would give wrong dimensions for the physical quantity A



A solution: Stieltjes-Chebyscheff moment theory

Spectral moments of \hat{H} with $\hat{A}|\Phi\rangle$ can be calculated using either true or discretized continuum functions:

$$M_n = \langle \Phi | \hat{A}^\dagger \hat{H}^n \hat{A} | \Phi \rangle = \int dE \langle \Phi | \hat{A}^\dagger \hat{H}^n | \chi_E \rangle \langle \chi_E | \hat{A} | \Phi \rangle = \int dE E^n |\langle \Phi | \hat{A} | \chi_E \rangle|^2$$

$$M_n = \langle \Phi | \hat{A}^\dagger \hat{H}^n \hat{A} | \Phi \rangle = \sum_j \langle \Phi | \hat{A}^\dagger \hat{H}^n | \chi_j \rangle \langle \chi_j | \hat{A} | \Phi \rangle = \sum_j E_j^n |\langle \Phi | \hat{A} | \chi_j \rangle|^2$$

Convergence problems \Rightarrow negative n

One can use the techniques of moment theory to obtain the best approximation for $f(E) = |\langle \Phi | \hat{A} | \chi_E \rangle|^2$ using a finite number of M_n 's

P. W. Langhoff (1973)

- ☹ The *apparent* price: full spectrum of $\{\chi_j\}$ is required to calculate the M_n 's
Not feasible for large systems!

Overcoming the full diagonalization bottleneck: Stieljes imaging with Lanczos pseudospectrum

Krylov states: $\varphi_k = \hat{H}^k |\Phi\rangle, k = 0, \dots, N$

Gram-Schmidt orthonormalization \rightarrow **Lanczos basis** $\{\psi_j\}, j = 0, \dots, N$

Representation of \hat{H} in **Lanczos basis**: $\hat{H}^{(N)} = \sum_{j,k} |\psi_j\rangle \langle \psi_j| \hat{H} |\psi_k\rangle \langle \psi_k|$

Approximation [Meyer & Pal, 1989]:

$$F(\hat{H}) \approx F(\hat{H}^{(N)})$$

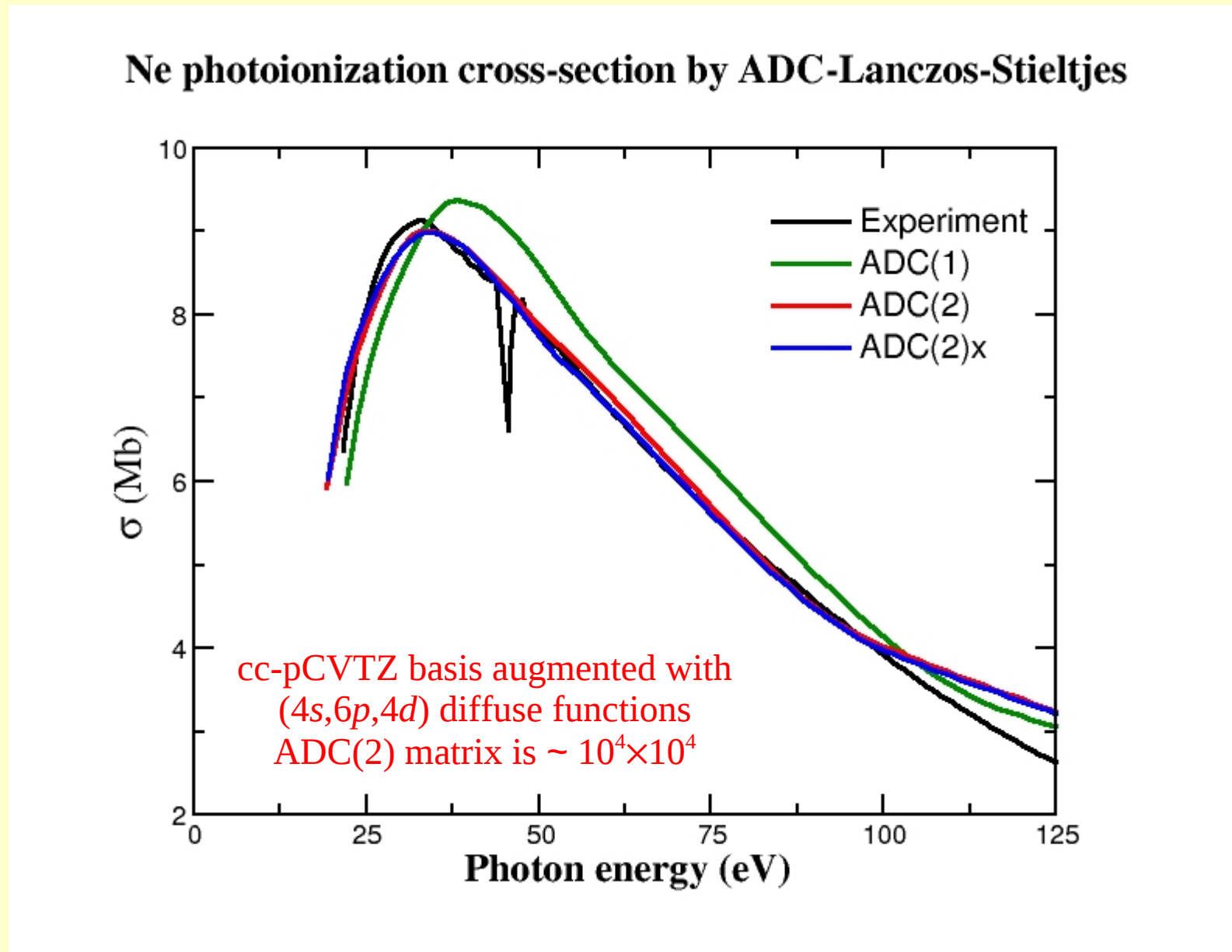
(exact for $\langle \Phi | F(\hat{H}) | \Phi \rangle$, if $F(x) = x^n, n = 0, \dots, 2N$)

Application to Stieltjes imaging:

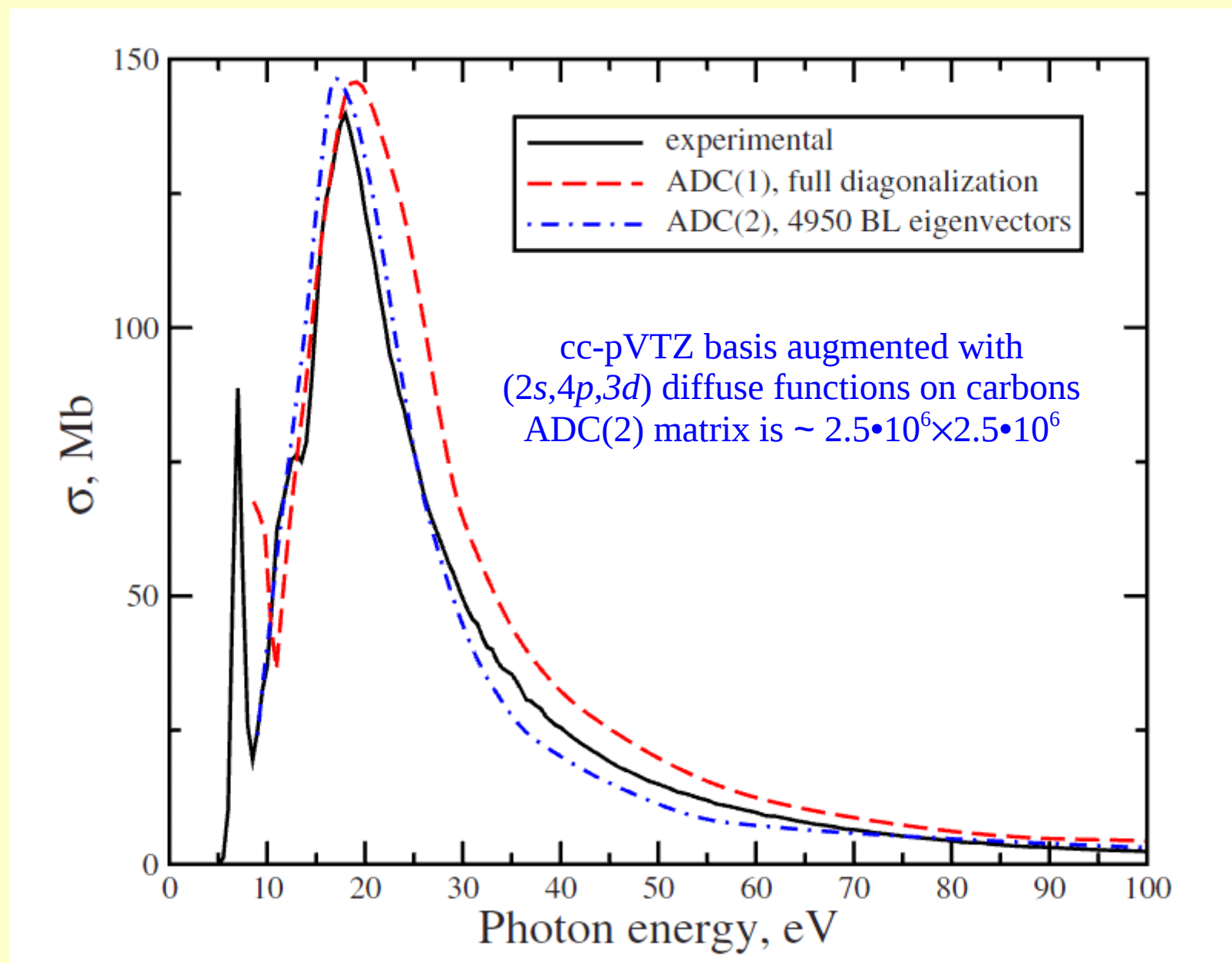
$$M_n \approx \sum_{j=0}^N (E_j^{(N)})^n |\langle \Phi | \hat{A} | \chi_j \rangle|^2,$$

where $|\chi_j\rangle$ are obtained by **Lanczos diagonalization** after N iterations

Testing the use of Lanczos pseudospectrum in Stieltjes imaging



Application to photoionization cross-section: Benzene

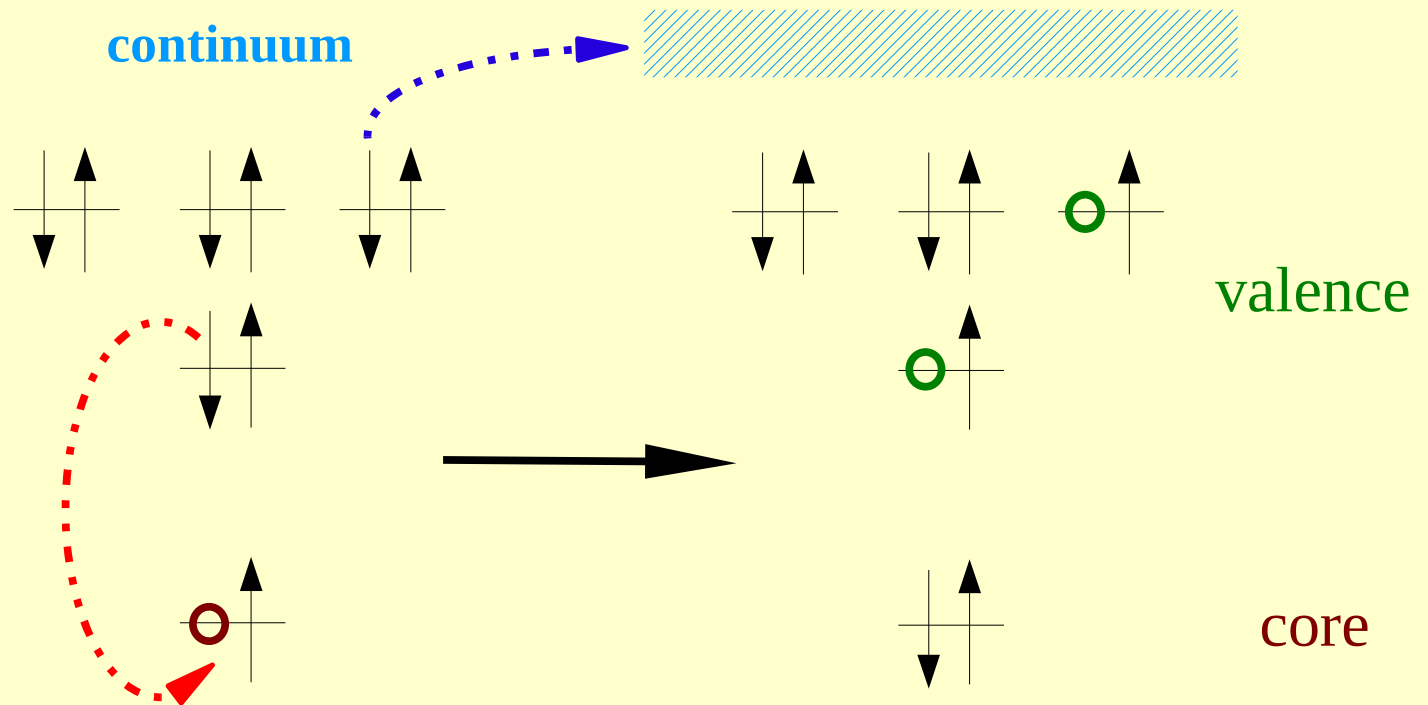


The basic two-electron process: Auger effect



Pierre Auger

“*Sur L’effet Photoélectrique Composé*” (1925)



Photoelectrons: $E_k = \hbar\omega - E(A^+)$

Auger electrons: $E_k = E(A^+) - E(A^{++})$

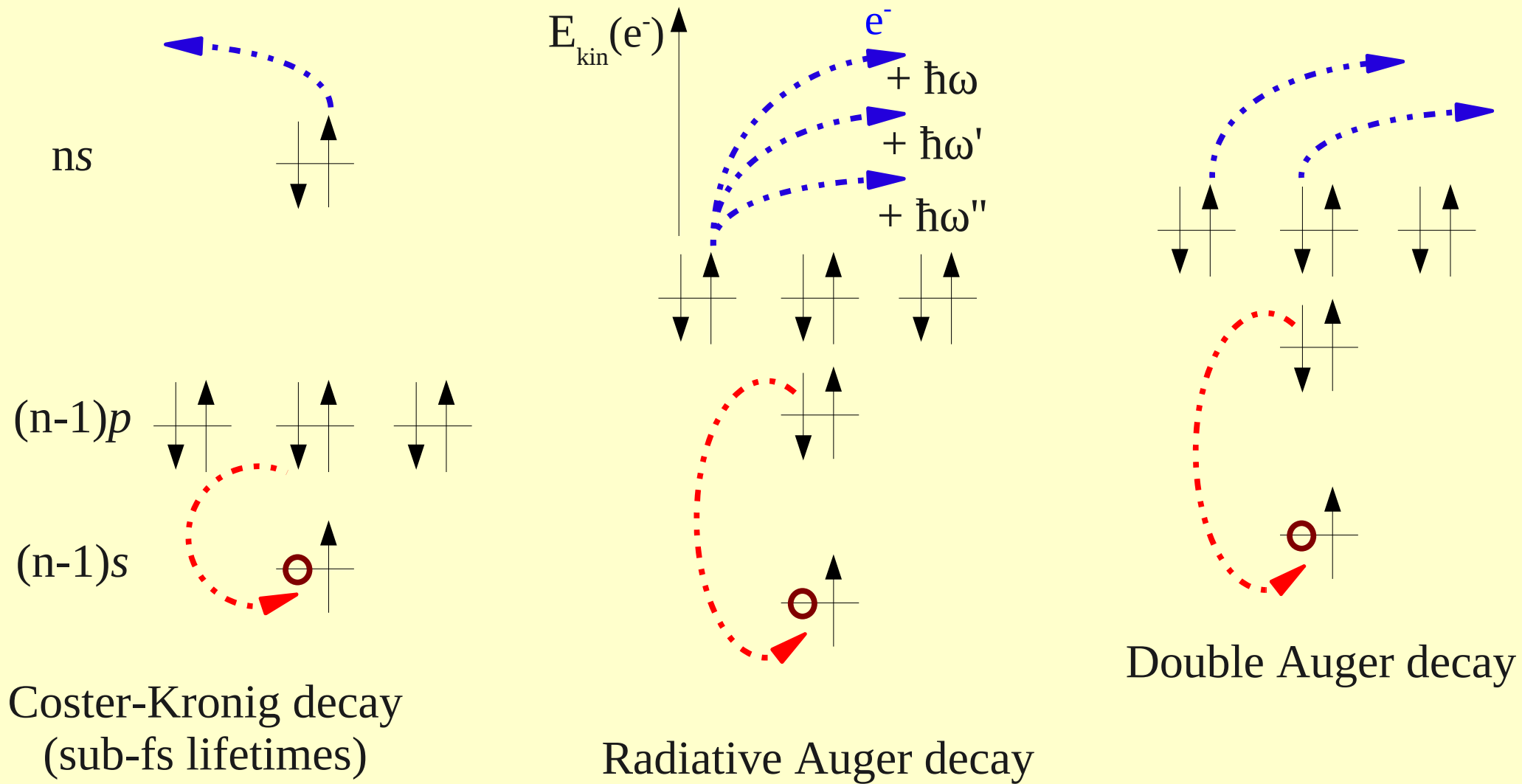
$$\Gamma_{\text{Auger}} = 2\pi \left| \left\langle \psi_{v1}(r_1) \psi_{v2}(r_2) \left| e^2/r_{12} \right| \psi_{\text{core}}(r_1) \psi_{\text{continuum}}(r_2) \right\rangle \right|^2$$

“*Über strahlunglose Quantensprünge*” (1927)



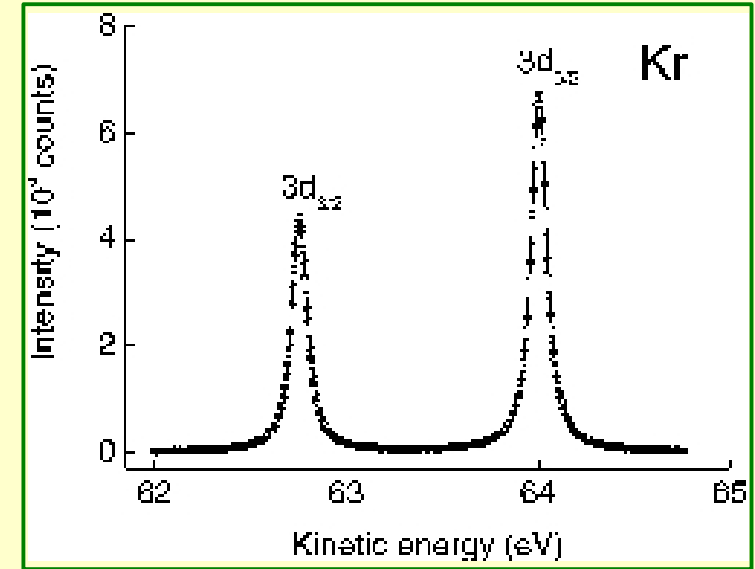
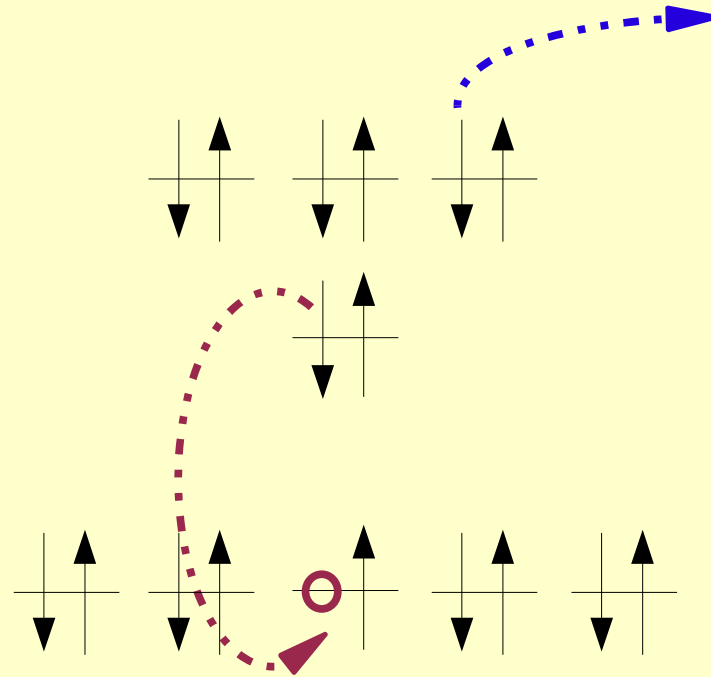
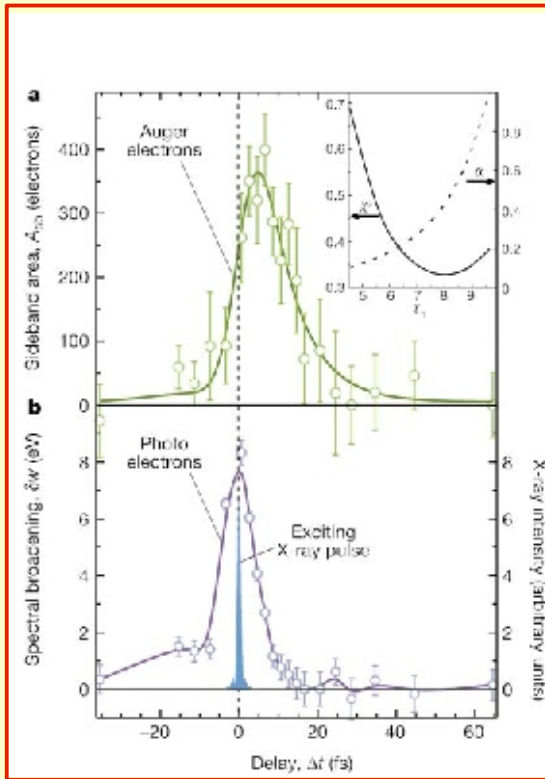
Gregor Wentzel

More Auger processes



Motivation: Attosecond science

Time-resolved vs. energy-resolved Auger decay in $(3d^{-1}) \text{Kr}^+$



[Drescher *et al.*, Nature **419**, 803 (2002)]

[Jurvansuu *et al.*, PRA **64**, 012503 (2001)]

$$\tau \approx 7.9 \text{ fs}$$

$$\Gamma \approx 88 \text{ meV}$$

Is it always straightforward to draw such a correspondence

???

Molecular Auger decay: The effect of nuclear motion

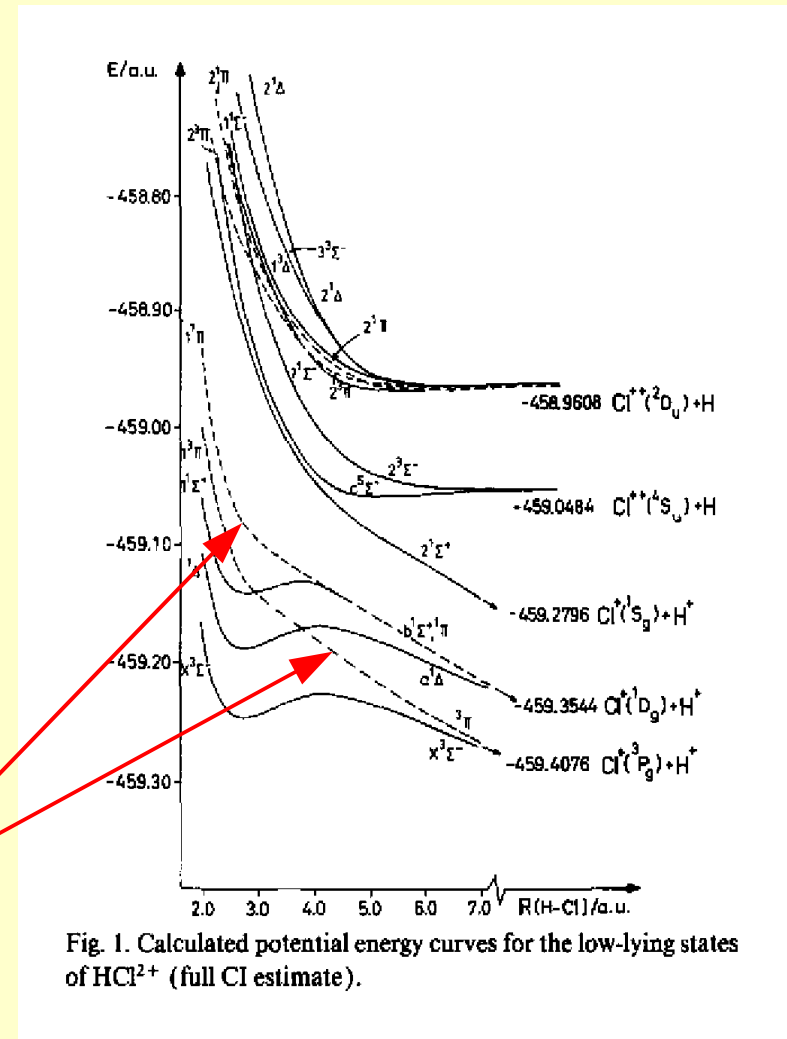
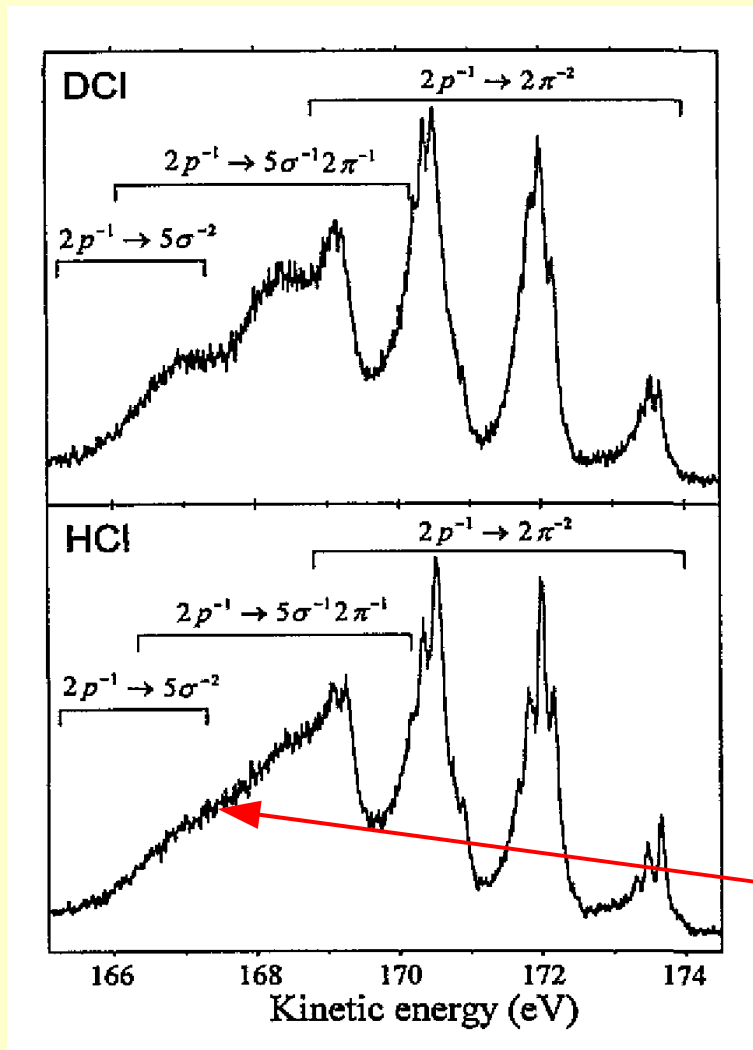
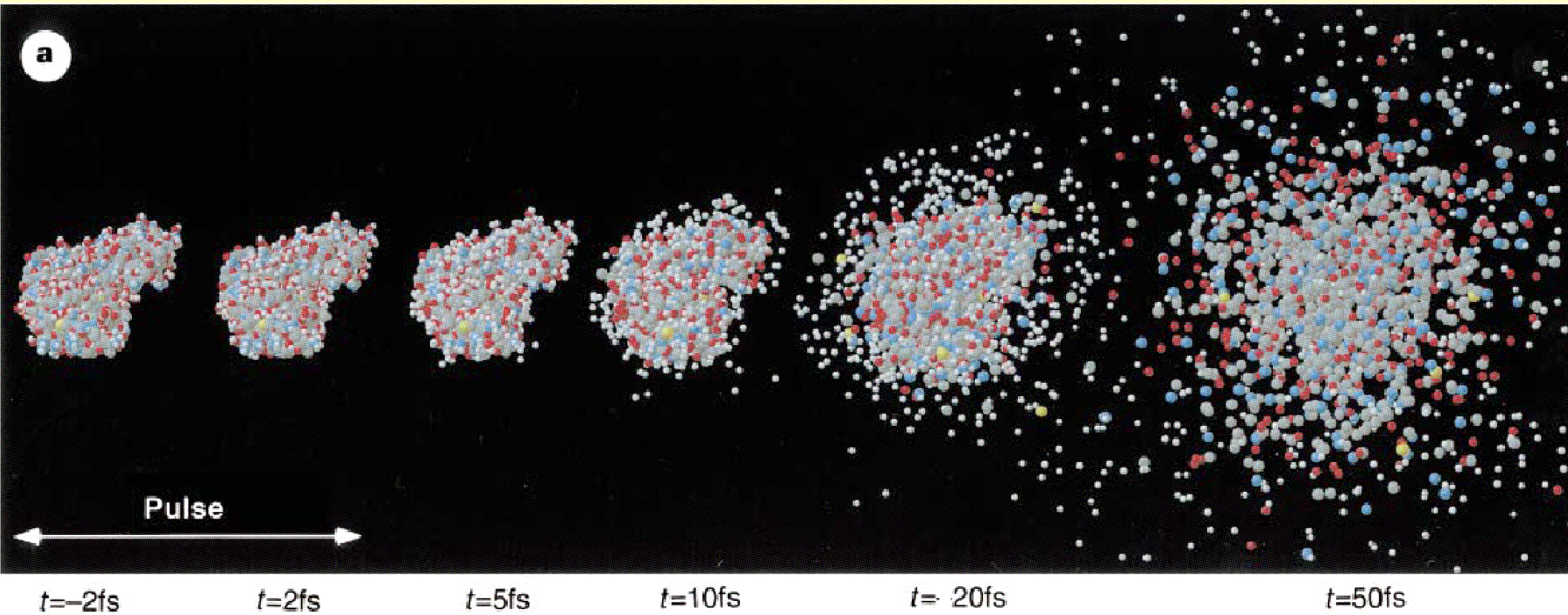


Fig. 1. Calculated potential energy curves for the low-lying states of HCl^{2+} (full CI estimate).

[Aksela *et al.*, JPB 28, 4259 (1995)] [Banichevich *et al.*, CP 121, 351 (1988)]

Auger transitions to dissociative states “produce very wide ... lineshapes, ... which rules out the possibility of their detailed study”.

Motivation: XFEL science

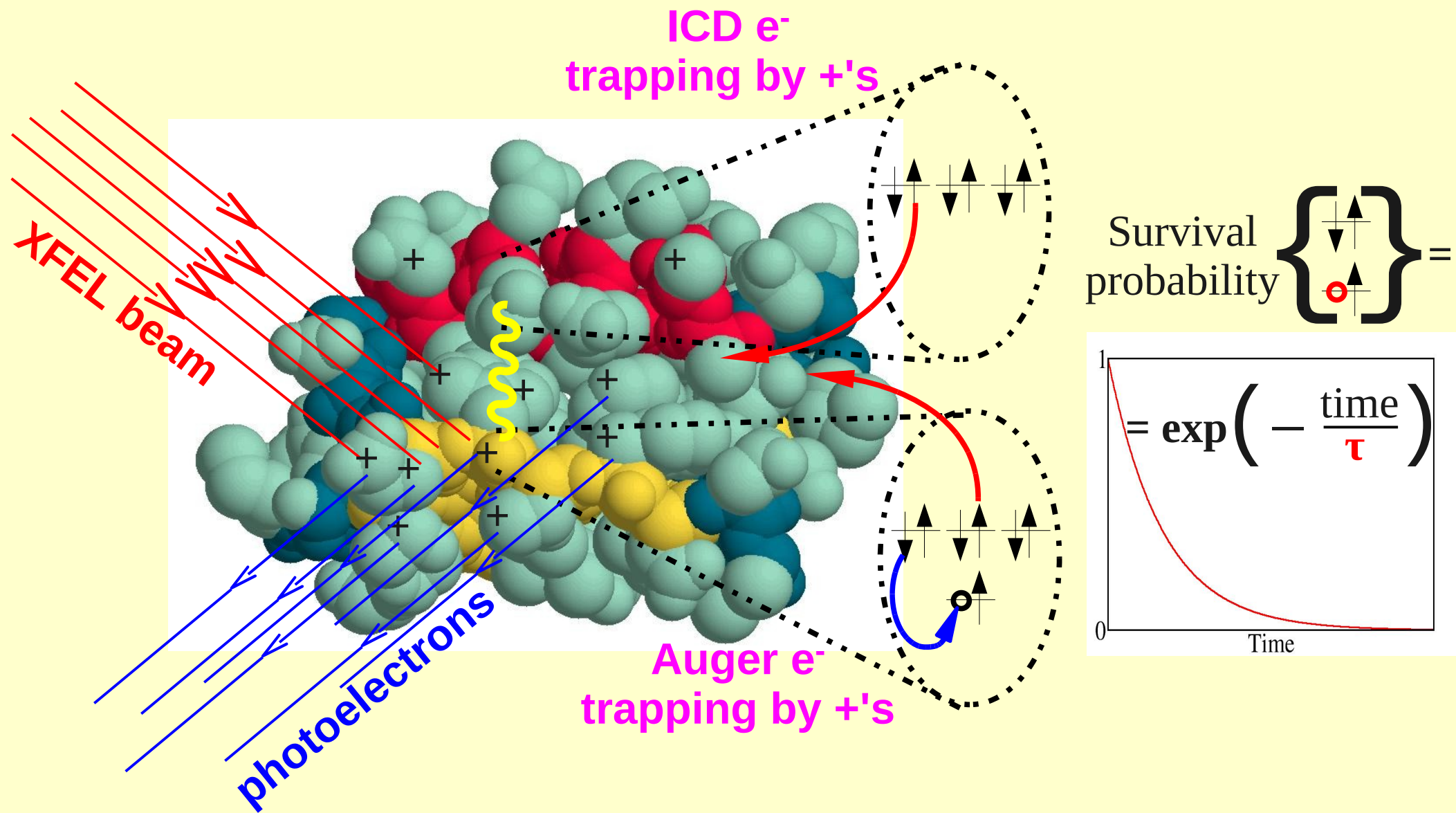


“*Potential for biomolecular imaging with femtosecond X-ray pulses*”,
R. Neutze, R. Wouts, D. van der Spoel, E. Weckert & J. Hajdu, *Nature* **406**, 752 (2000).

Obtain a diffraction image of a single molecule using a short pulse of high-intensity X-rays before the molecule is destroyed by radiation damage

“**Radiation damage**” - Coulombic explosion due to accumulation of a large number of positive charges

Macromolecule-XFEL interaction: Schematic picture



State of the art radiation damage simulations rely on the isolated-species lifetimes (τ) for the intra-atomic processes and disregard the inter-atomic processes completely...

Open questions

- ★ How different is the ***time scale*** of a core hole decay in a charged environment from the one in a singly ionized species?
- ★ Does the core hole dynamics in a highly charged environment follow the familiar ***exponential pattern***?
- ★ Can one ***control*** the time scale of the electronic decay by shaping the XFEL pulse or by using an additional laser source?
- ★ Can we follow the electronic decay dynamics in ***molecules and clusters*** directly using the ***attosecond streaking*** technique?
- ★ What ***new electronic decay processes*** are possible in multiply charged and/or laser-driven systems?

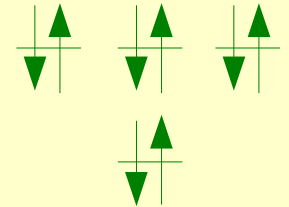
Simple quantum chemical theory for singly ionized states: Koopmans theorem



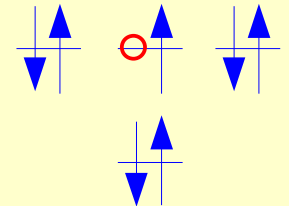
Tjalling C. Koopmans (Nobel prize winner in *economics*, 1975)

If both the ground state of the neutral and the eigenstate of the cation are approximated by single HF configurations:

$$\Psi_0^{(N)} = \Phi_0^{\text{HF}}$$



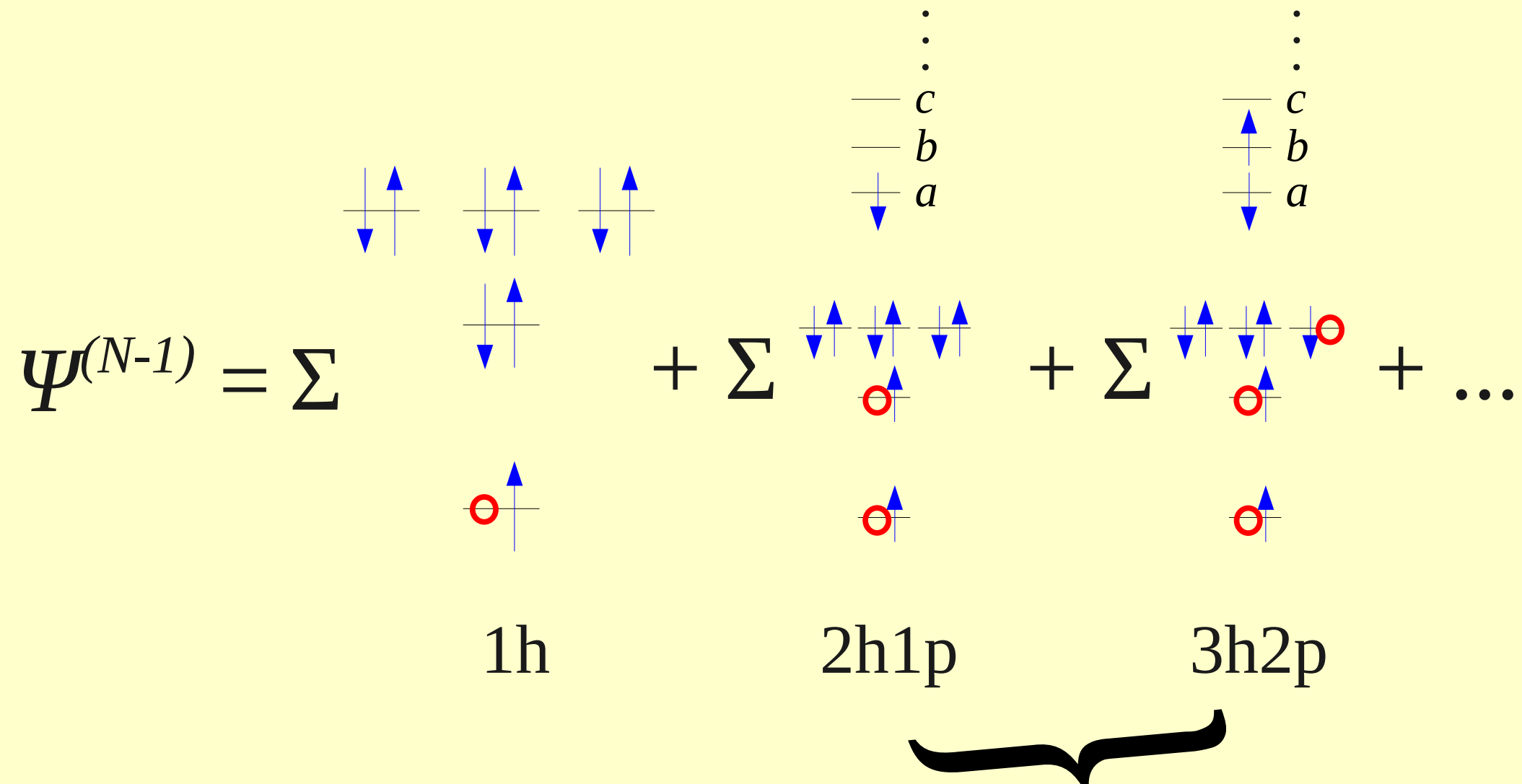
$$\Psi_0^{(N-1)} = \hat{a}_i \Phi_0^{\text{HF}}$$



then the corresponding ionization potential is given by the HF orbital energy:

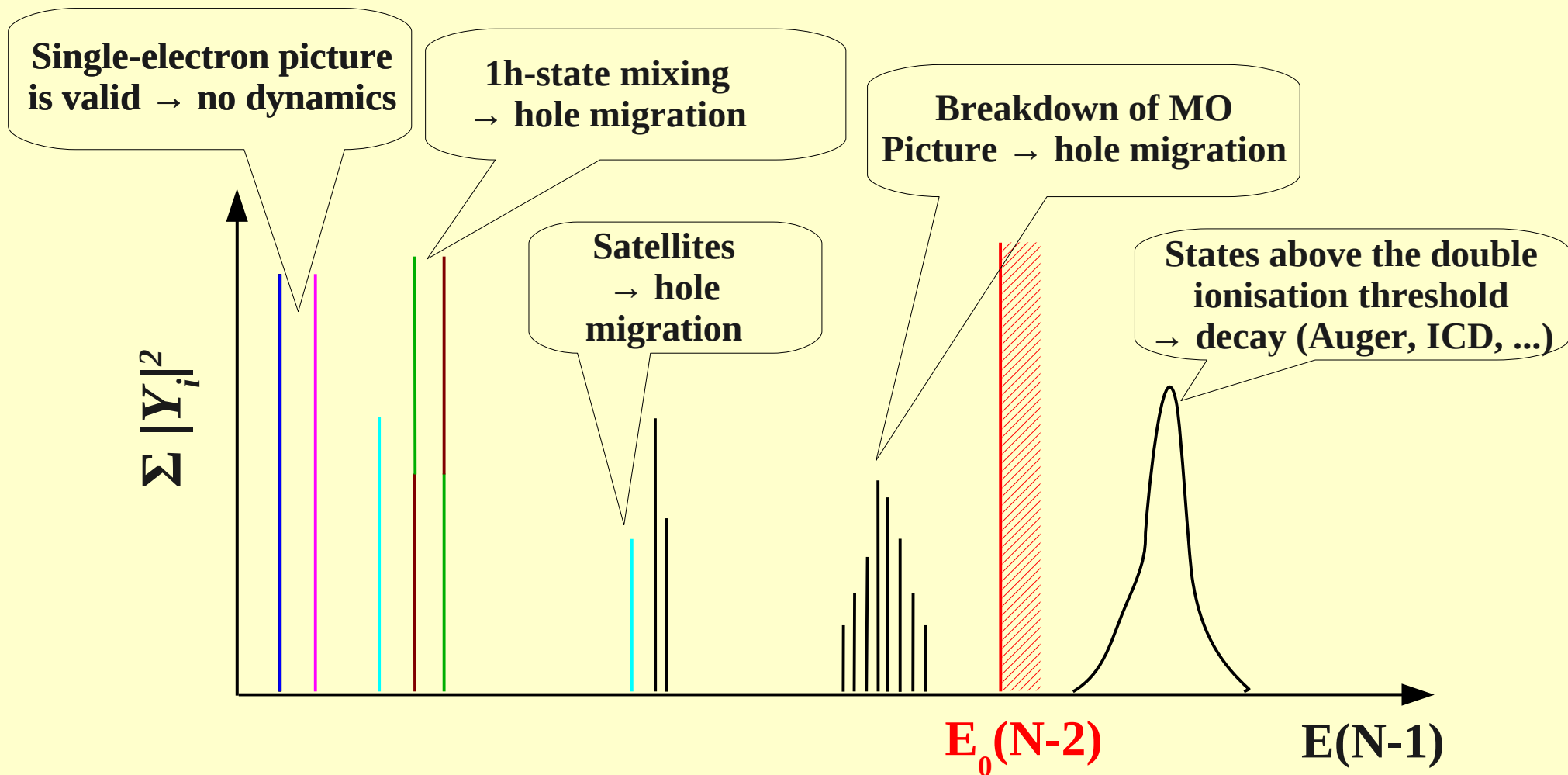
$$\text{IP} = E_i^{(N-1)} - E_0^{(N)} = -\epsilon_i$$

The real world: CI for singly ionised states



What is the physical effect of this contribution?

Effects of the CI: From spectral patterns to physical phenomena



$$\Psi^{(N-1)} = \sum_i Y_i \Psi_i + \sum_{ij} Y_{ij}^a \Psi_{ij}^a + \sum_{ijk} Y_{ijk}^{ab} \Psi_{ijk}^{ab} + \dots$$

1h
2h1p
3h2p

Quantitative approach to Γ_{Auger} : Fano theory of resonances

$$\Psi_E = a(E) \Phi + \int b(E, \varepsilon) \chi_\varepsilon d\varepsilon$$

exact wave function at energy E in the continuum

bound-like part

continuum part

Assumption: no continuum-continuum interaction

$$\langle \chi_{\varepsilon'} | \hat{H} - E | \chi_\varepsilon \rangle = (\varepsilon - E) \delta(\varepsilon' - \varepsilon)$$

No orthogonality requirement (unlike in Feshbach theory): $\langle \Phi | \chi_\varepsilon \rangle \neq 0$

Solving $(\hat{H} - E)\Psi_E = 0$ gives $|a(E)|^2 = \frac{\Gamma/2\pi}{(E - E_r)^2 + \Gamma^2/4}$

Lorentzian width \Rightarrow decay rate

$$\Gamma = 2\pi \left| \langle \Phi | \hat{H} - E_r | \chi_{E_r} \rangle \right|^2$$

Fano-ADC approach: Configuration selection scheme

$$\Gamma = 2\pi \left| \langle \Phi | \hat{H} - E_r | \chi_{E_r} \rangle \right|^2$$

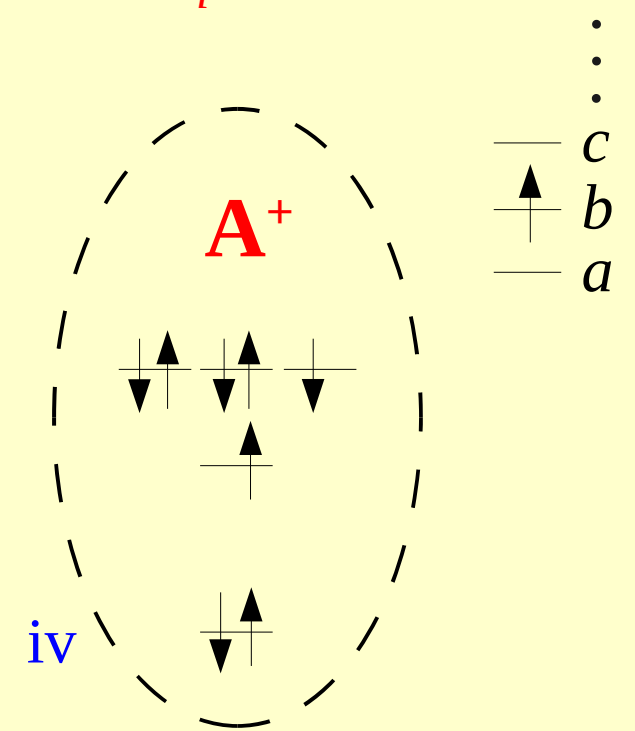
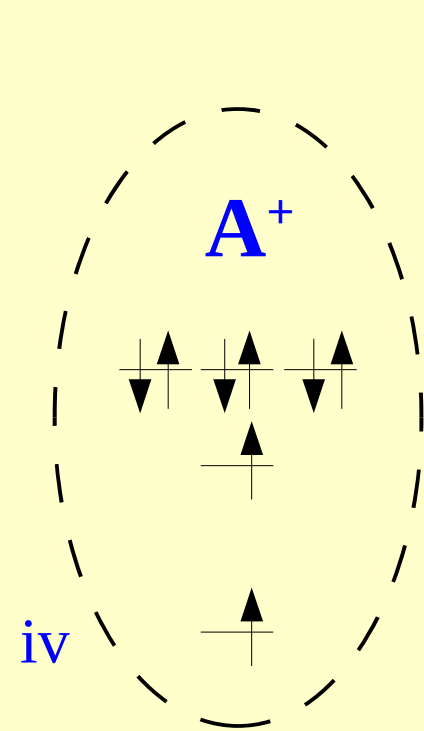
$$\Phi = \sum_i Y_i \Psi_i + \sum_{ija} Y_{ij}^a \Psi_{ij}^a$$

$E_{ij}(A^{2+}) > E_{iv}(A^+)$

$$E_r \approx \langle \Phi | \hat{H} | \Phi \rangle$$

$$\chi_n = \sum_i Y_i \Psi_i + \sum_{ija} Y_{ij}^a \Psi_{ij}^a$$

$E_{ij}(A^{2+}) < E_{iv}(A^+)$
 $\sum_i |Y_i|^2 \ll 1$



Testing the theory on the ADC(2)x level

Ne ($2s^{-1}np$) autoionization

TABLE I. Experimental and theoretical decay widths Γ for the autoionizing $2s^{-1}np$ ($n=3,4,5$) states of Ne.

	$n=3$	$n=4$	$n=5$
$\Gamma_{\text{expt}}^{\text{a}}$ (meV)	13 (± 2)	4.5 (± 1.5)	2 (± 1)
Γ_{TDA} (meV)	30.48	9.31	5.59
$\Gamma_{\text{ADC}(2)}$ (meV)	8.93	2.86	1.72
$\Gamma_{\text{ADC}(2e)}$ (meV)	11.46	3.78	1.94
$\Gamma_{\text{IDLDA}}^{\text{b}}$ (meV)	13.90	3.86	1.62
$\Gamma_{\text{RM}}^{\text{c}}$ (meV)	34.9	6.65	2.47

^aData taken from Ref. 13.

^bData taken from Ref. 17.

^cData taken from Ref. 16.

Ar ($3s^{-1}np$) autoionization

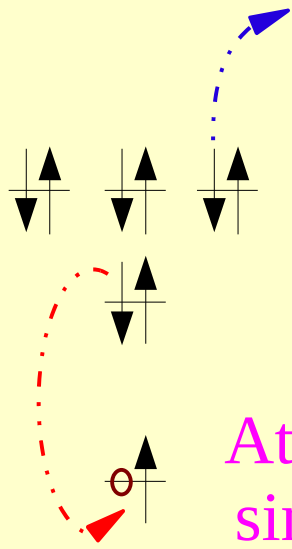
TABLE II. Experimental and theoretical decay widths Γ for the autoionizing $3s^{-1}np$ ($n=4,5,6$) states of Ar.

	$n=4$	$n=5$	$n=6$
$\Gamma_{\text{expt}}^{\text{a}}$ (meV)	76 (± 5)	25 (± 7)	16 (± 7)
Γ_{TDA} (meV)	50.61	13.52	5.59
$\Gamma_{\text{ADC}(2)}$ (meV)	61.5	18.42	8.05
$\Gamma_{\text{ADC}(2e)}$ (meV)	67.76	25.85	12.14
$\Gamma_{\text{IDLDA}}^{\text{b}}$ (meV)	183.4	42.8	18.2

^aData taken from Ref. 15.

^bData taken from Ref. 17.

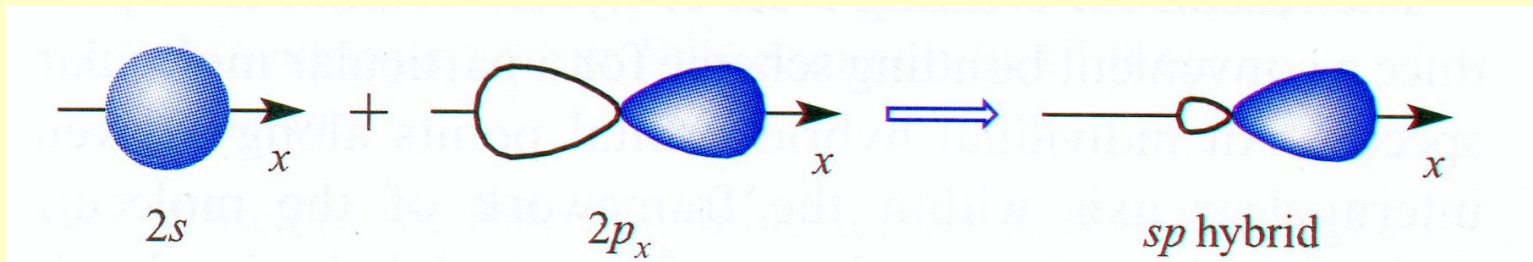
Auger effect in multiply charged systems: Effect of a single neighboring charge



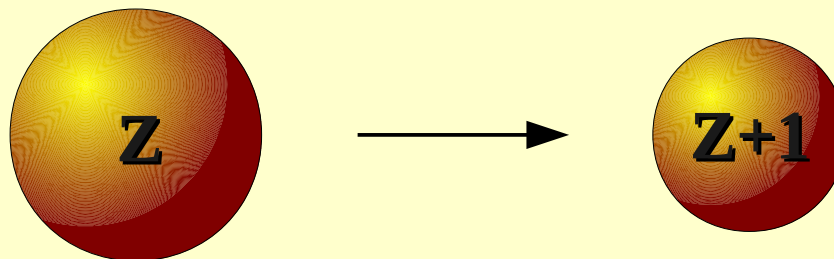
Wentzel's ansatz:

$$\Gamma_{\text{Auger}} = 2\pi \left| \langle \psi_{v1}(r_1) \psi_{v2}(r_2) | e^2/r_{12} | \psi_{\text{core}}(r_1) \psi_{\text{continuum}}(r_2) \rangle \right|^2$$

At large distances, a single charge causes *orbital mixing*, similar to hybridization in the theory of chemical bond:

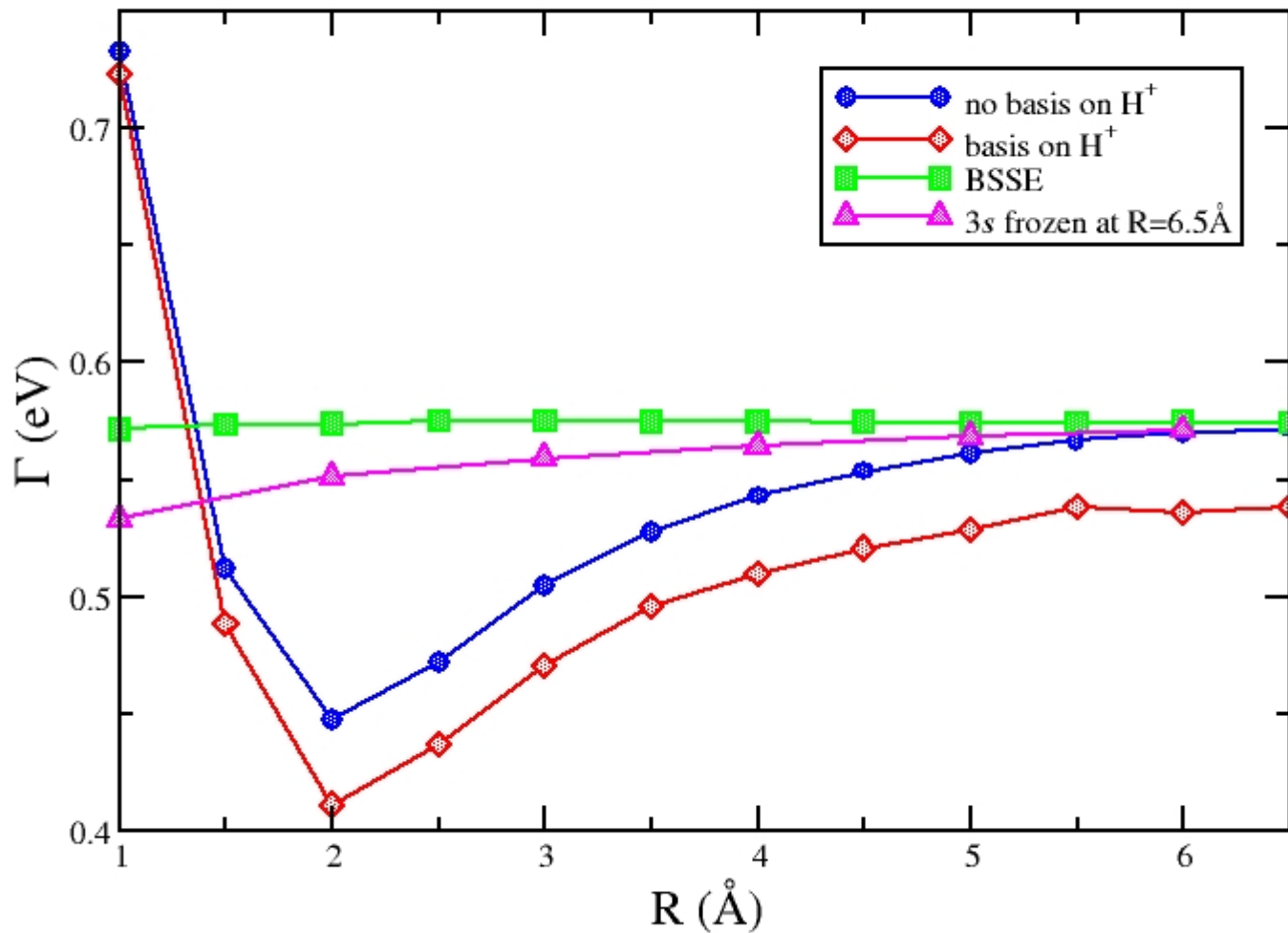


At small distances, the charge penetrates the electron cloud causing orbital contraction:



Auger decay in the field of a charge: $2s^{-1}\text{Mg}^+ \dots \text{H}^+$

$\text{Mg-H}^+ 2s^{-1}$ total width



The leading decay channel is $2p_z^{-1}3s^{-1}$ singlet

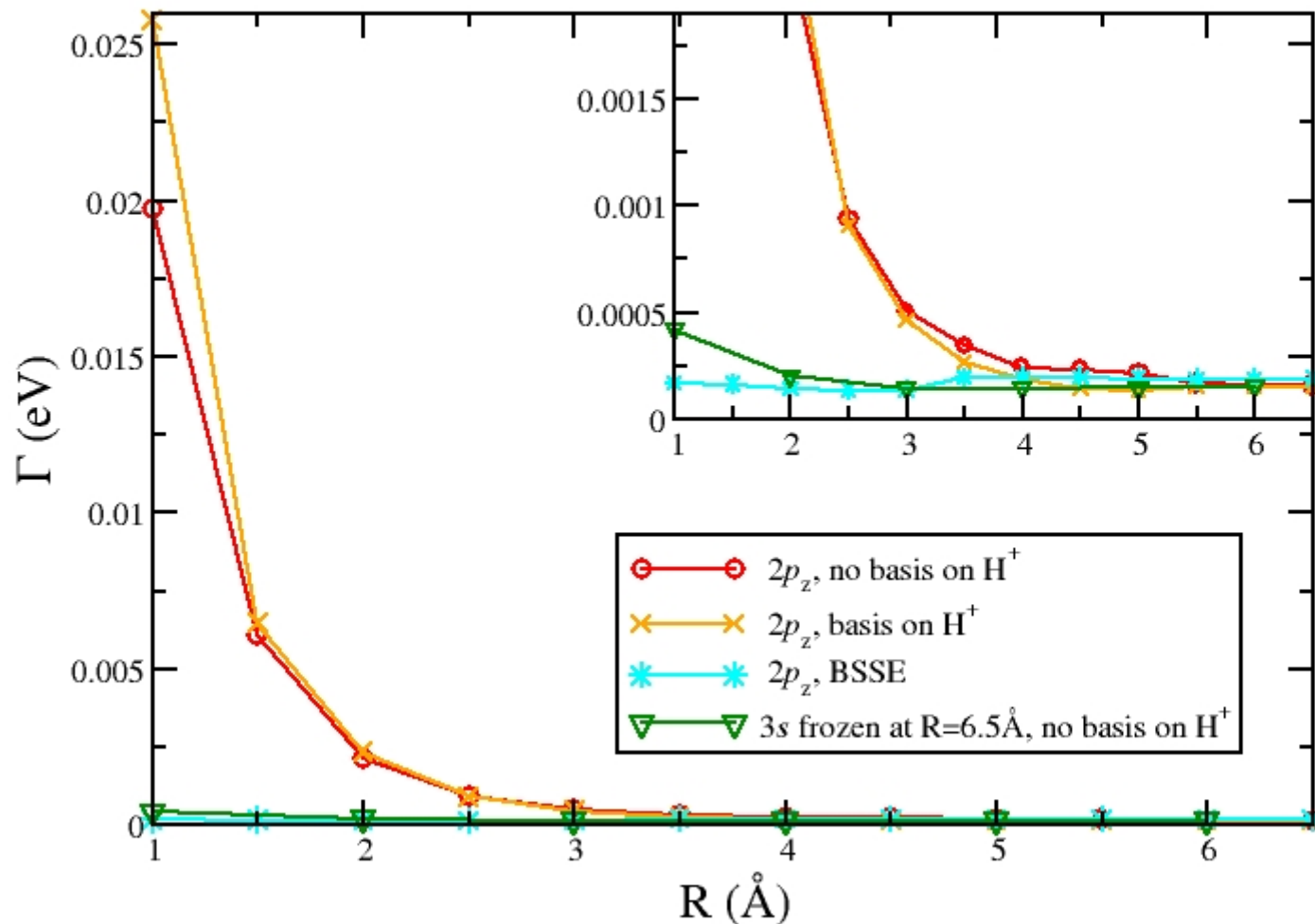
Calculation is done using Fano-ADC(2)x-Stieltjes method
[Averbukh & Cederbaum JCP **123**, 204107 (2005)]

The outermost (3s) orbital determines the Auger rate

[Averbukh, Saalman & Rost, Phys. Rev. A **85**, 063405 (2012)]

Auger decay in the field of a charge: $2p^{-1}\text{Mg}^+ \dots \text{H}^+$

$\text{Mg-H}^+ 2p_z^{-1}$ width



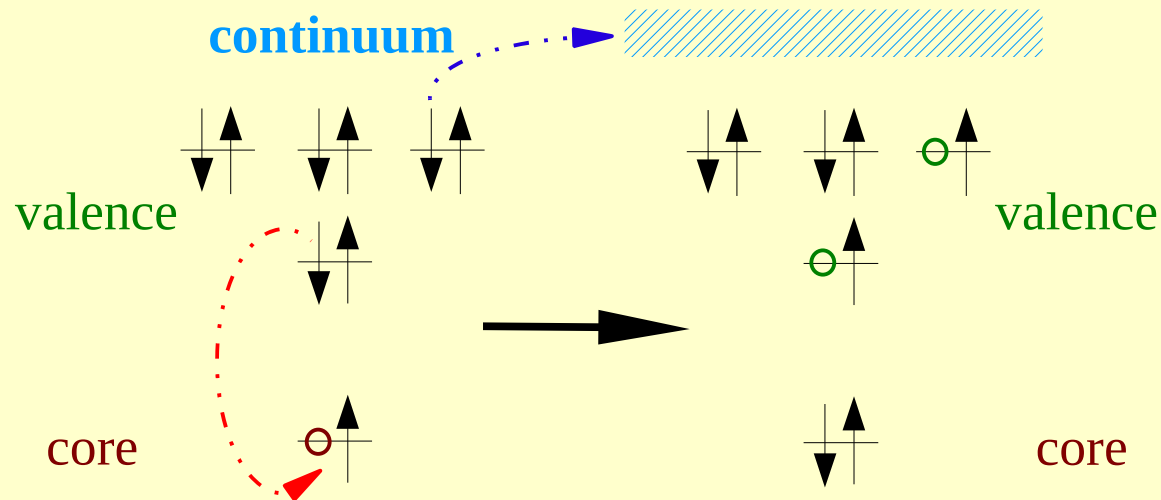
The single open decay channel is $3s^{-2}$ singlet

Calculation is done using Fano-ADC(2)x-Stieltjes method
[Averbukh & Cederbaum JCP **123**, 204107 (2005)]

The outermost ($3s$) orbital determines the Auger rate

[Averbukh, Saalman & Rost, Phys. Rev. A **85**, 063405 (2012)]

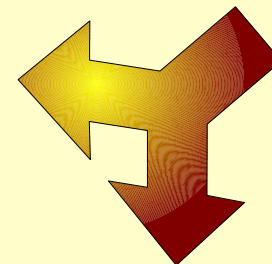
Auger decay in the field of a single charge: a more detailed interpretation



$$\Gamma_{\text{Auger}} = 2\pi |V|^2, \quad V = \langle \psi_{v1}(r_1) \psi_{v2}(r_2) | e^2/r_{12} | \psi_{\text{core}}(r_1) \psi_{\text{continuum}}(r_2) \rangle$$

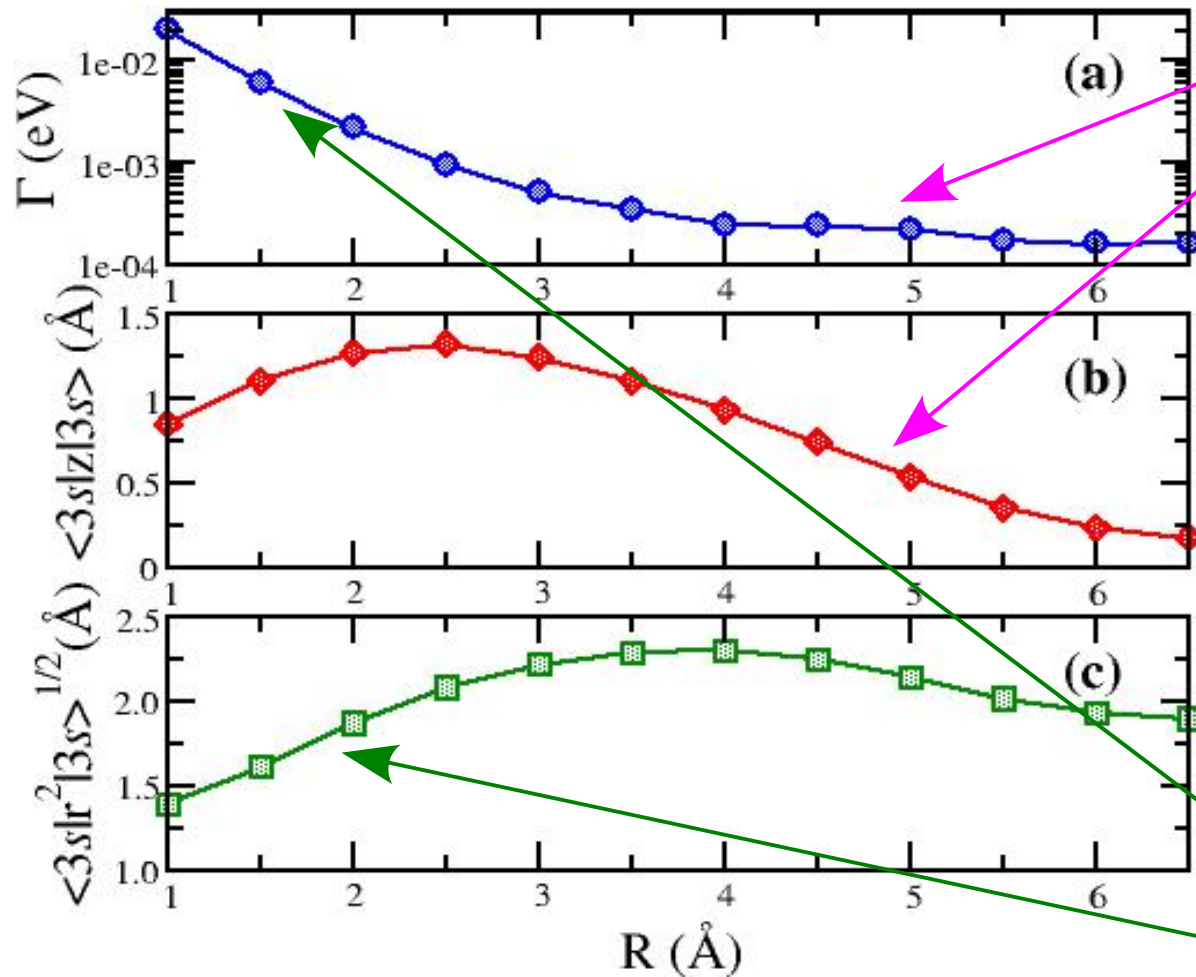
$$V = \sum_{l,m} \frac{4\pi}{(2l+1)} \langle \psi_{v1}(r_1) | r_1^l Y_{lm}^*(\mathbf{1}) | \psi_{\text{core}}(r_1) \rangle \langle \psi_{v2}(r_2) | Y_{lm}(\mathbf{2}) / r_2^{l+1} | \psi_{\text{continuum}}(r_2) \rangle$$

Contraction of ψ_{v2}
leads to higher Auger rate



Transitions described by lower-order
multipole transition moments lead to higher Auger rate

Auger decay in the field of a single charge: a more detailed interpretation

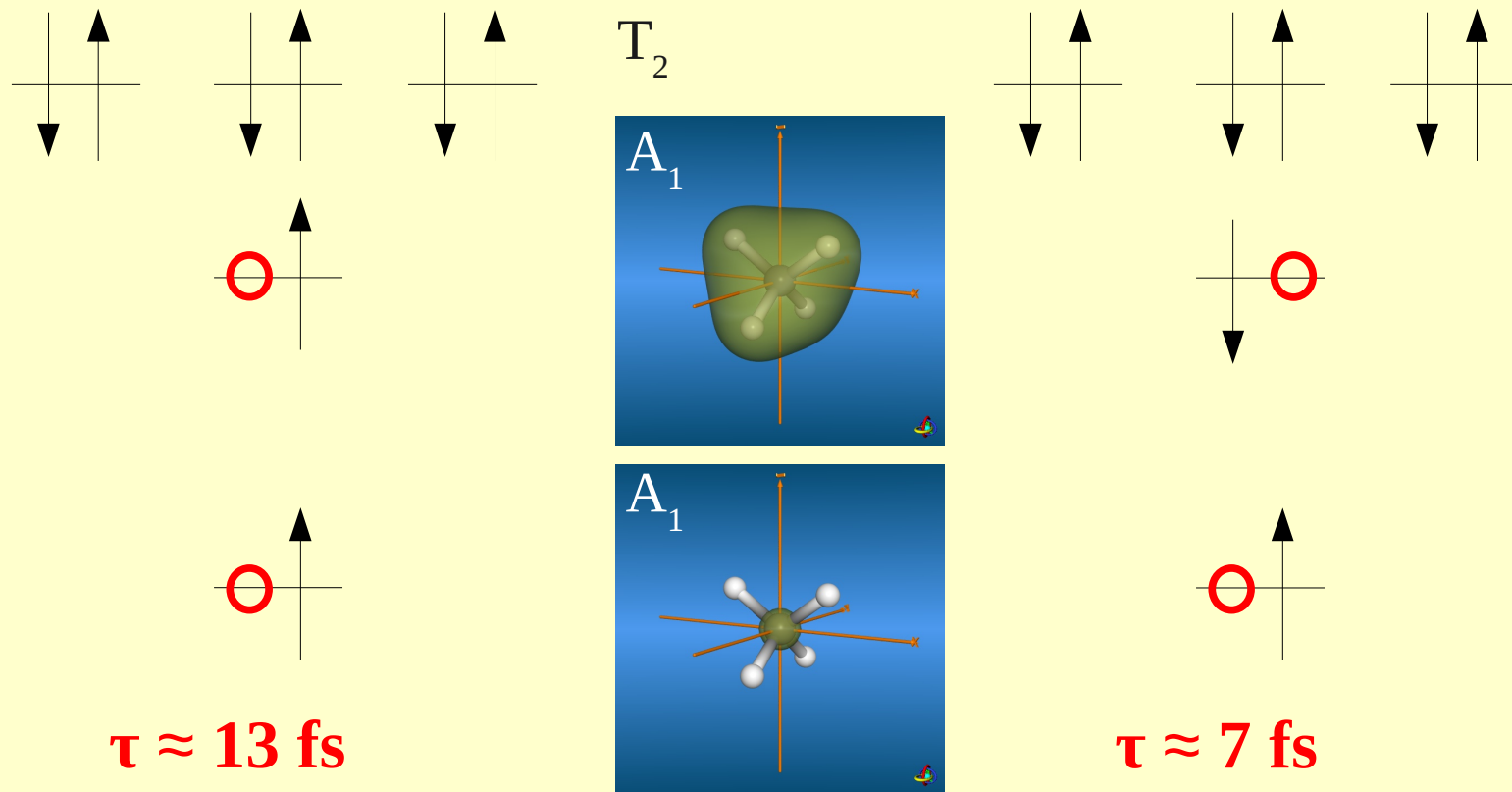


3s-3p hybridization
turns the dipole-dipole
transition into
monopole-monopole

Contraction of 3s
orbital further
increases the
Auger rate

Auger decay rate modified by an on-site charge: The spin effect

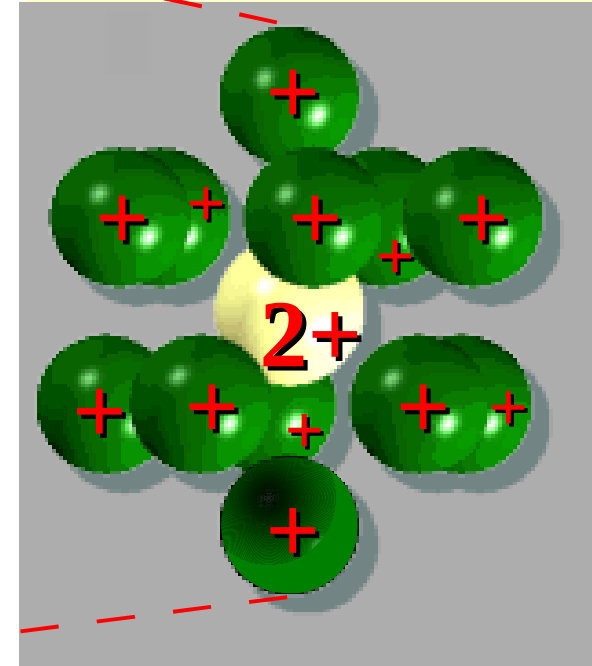
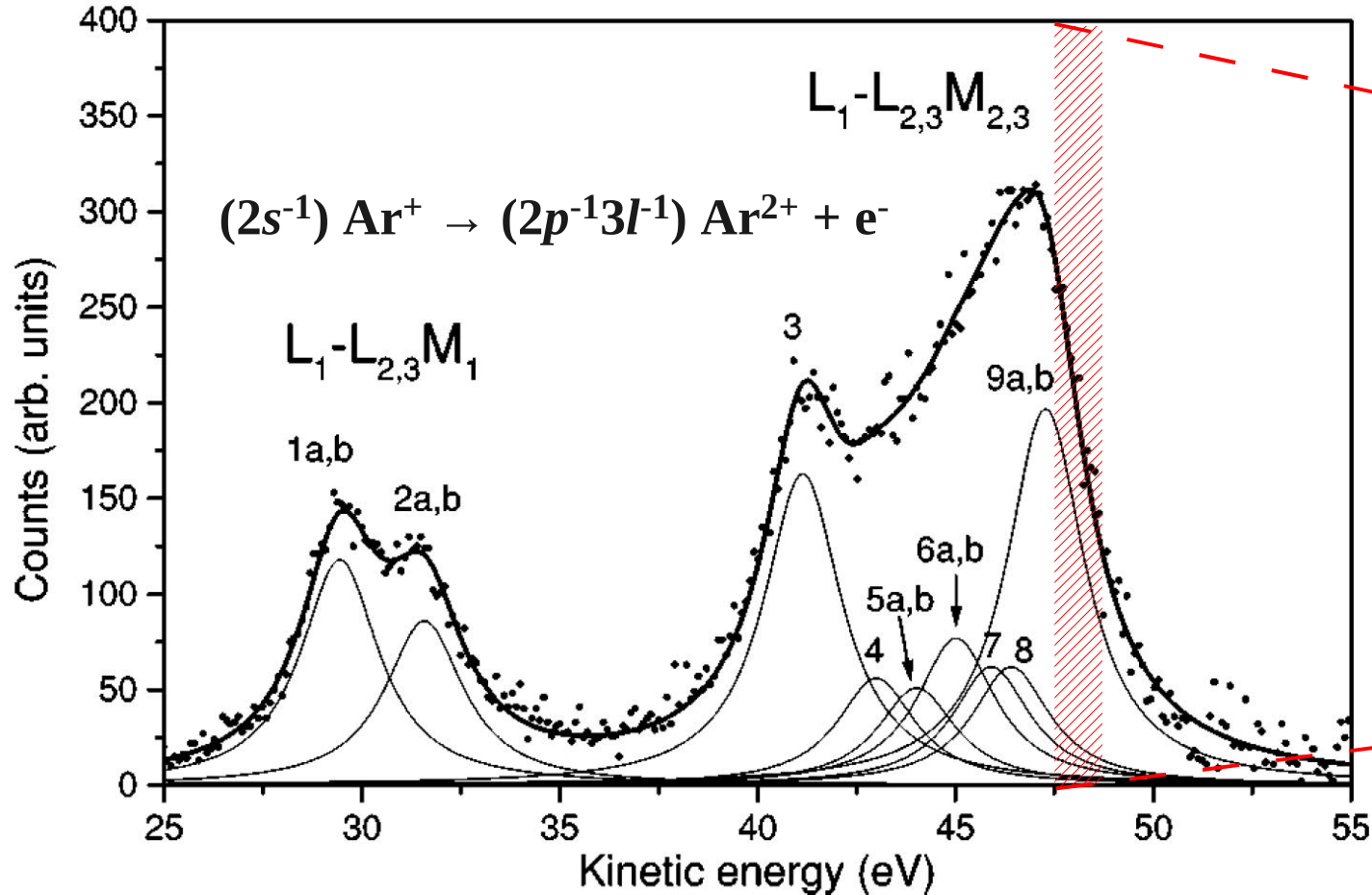
1s Auger in $1s^{-1}2s^{-1}$ CH₄



A similar effect is predicted for NH₃ and H₂O molecules

[Averbukh and Kolorenč, J. Chem. Phys. **135**, 134314 (2011)]

Adding more charges: trapping of secondary electrons



[Kylli *et al.*, PRA 59, 4071 (1999)]

12 charges around $(2s^{-1}) Ar^+$ in Ar^{13+} close the Coster-Kronig channels!

Exponential decay without a true continuum?

Bixon & Jortner (1968): Exponential decay without a true continuum can take place only for a finite time

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15 JANUARY 1968

Intramolecular Radiationless Transitions

MORDECHAI BIXON AND JOSHUA JORTNER

Department of Chemistry, Tel-Aviv University, Tel-Aviv, Israel

(Received 7 August 1967)

$$\begin{pmatrix} E_s & v & v & \cdots \\ & \cdot & & \\ v & \cdot & & 0 \\ & & \cdot & \\ v & & E_i & \\ \cdot & & & \cdot \\ \cdot & & & \\ \cdot & 0 & & \cdot \end{pmatrix} \begin{pmatrix} a_n \\ \cdot \\ \cdot \\ \cdot \\ b_i^n \\ \cdot \\ \cdot \\ \cdot \end{pmatrix} = E_n \begin{pmatrix} a_n \\ \cdot \\ \cdot \\ \cdot \\ b_i^n \\ \cdot \\ \cdot \\ \cdot \end{pmatrix}$$

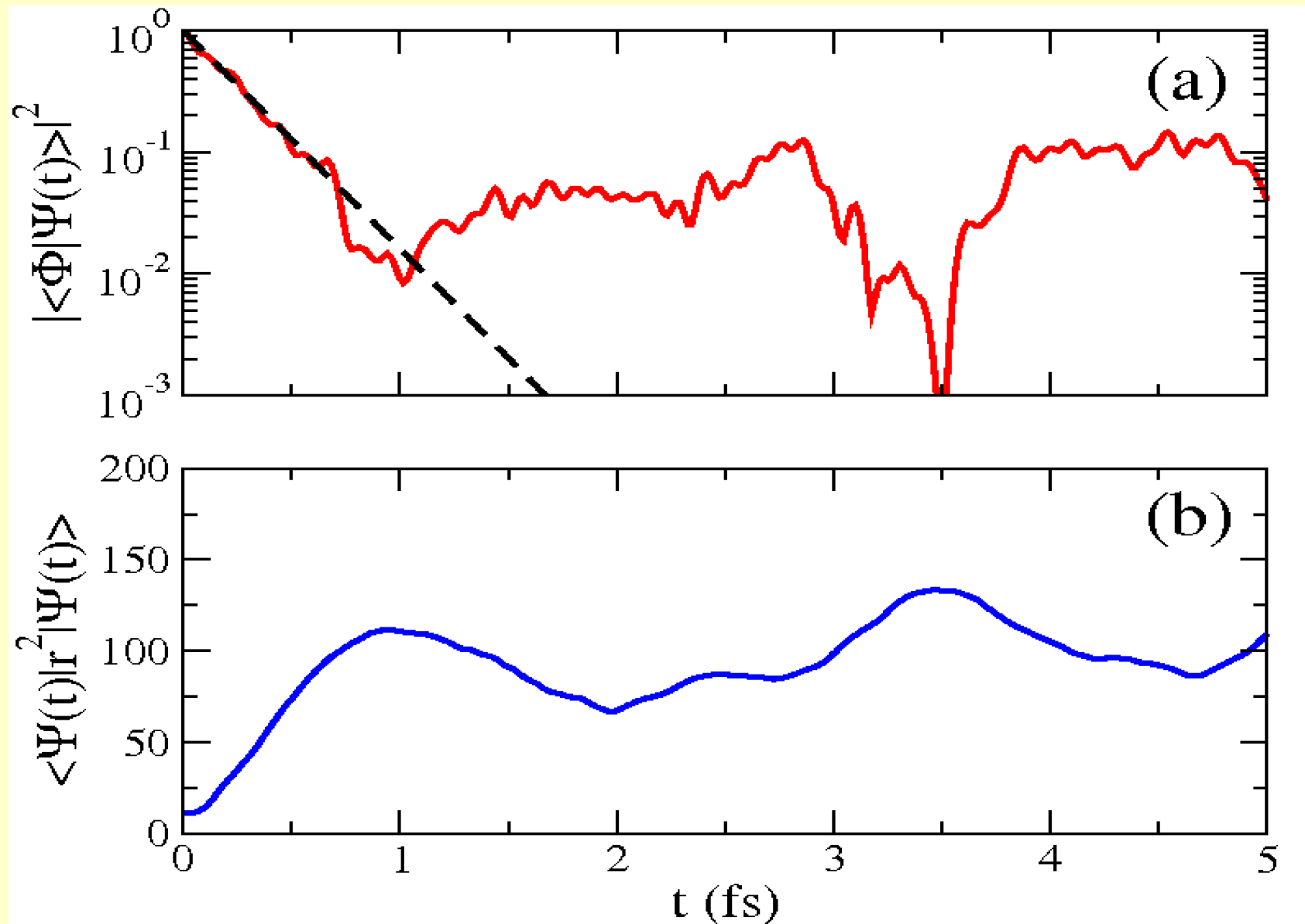
Equidistant, $E_{i+1} - E_i = \epsilon$,
“quasicontinuum” levels

Uniform bound-quasicontinuum
coupling, v

Exponential decay takes
place for $t < \hbar/\epsilon$

$$E_i = E_s - \alpha + i\epsilon, \quad i = 0, \pm 1, \pm 2, \dots$$

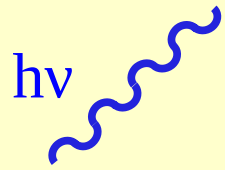
Results for a model $(2s^{-1})\text{Ar}^+(\text{H}^+)_{12}$ cluster



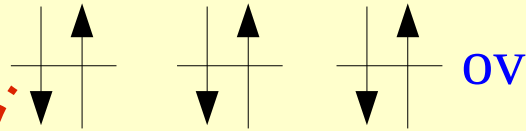
Quasi-bound state Φ_0 is propagated under Fano-type Hamiltonian

[Averbukh, Saalman & Rost, PRL **104**, 233002 (2010)]

From core to inner valence holes: ICD



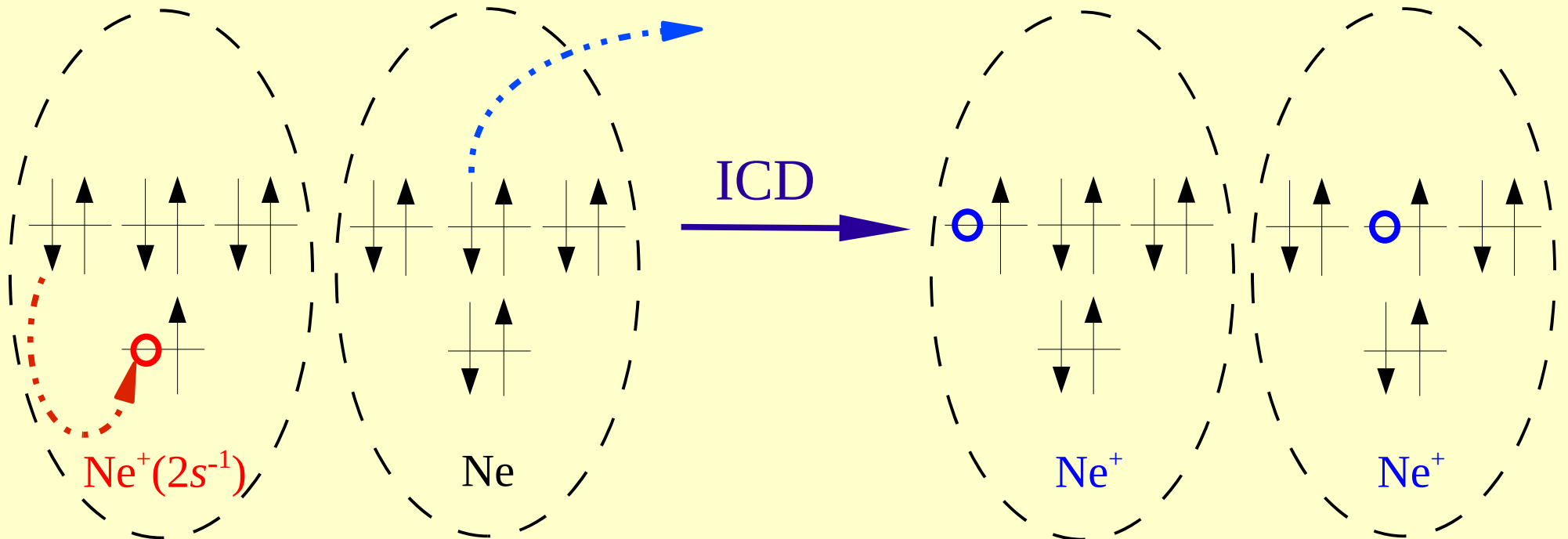
★ Inner valence ionization of **isolated species** leads to the slow (ns) radiative decay



$$E[\text{Ne}^+ (2s^{-1})] < E[\text{Ne}^{2+}]$$

★ **BUT...** the situation is dramatically different in a **cluster!** [Cederbaum, Zobeley & Tarantelli, PRL **79**, 4778 (1997)]

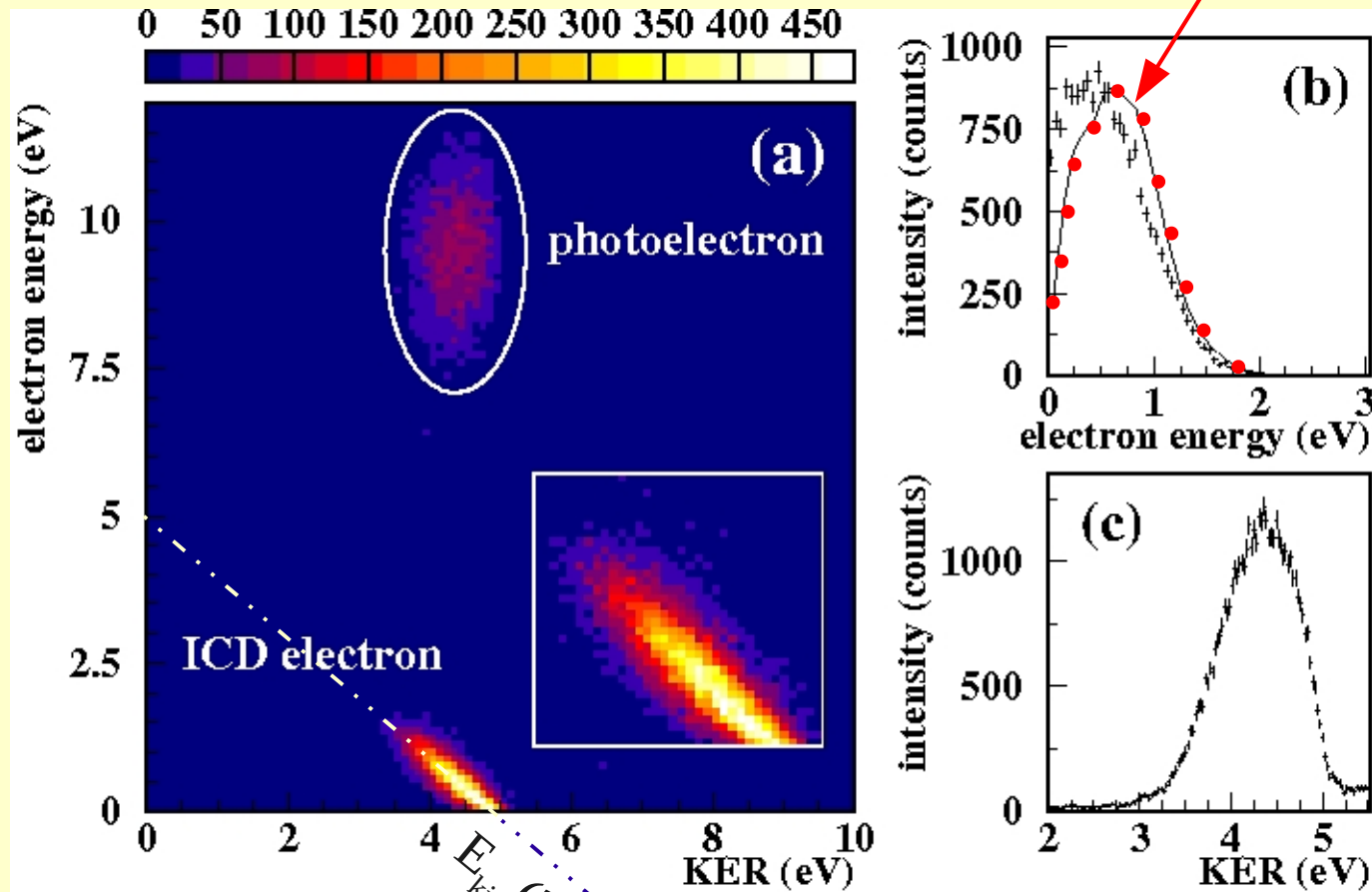
$$E[\text{Ne}^+ (2s^{-1})] > E[\text{Ne}_2^{2+}] \text{ leads to: } \text{NeNe}^+ (2s^{-1}) \rightarrow \text{Ne}^+ (E_{\text{kin}}) + \text{Ne}^+ (E_{\text{kin}}) + e^-(E_{\text{kin}})$$



ICD in Ne₂: The experiment

Cold Target Recoil Ion Momentum Spectroscopy
(COLTRIMS) experiment of Dörner's group,
PRL **93**, 163401 (2004)

Theoretical calculation of Scheit, Averbukh *et al.*,
JCP **121**, 8393 (2004)

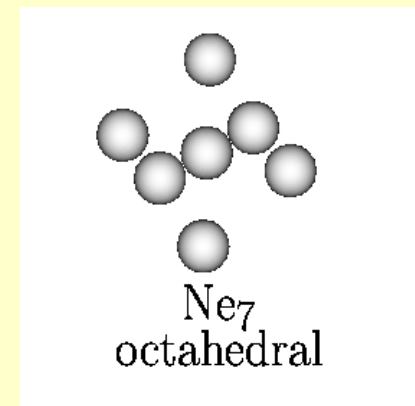


ICD: A general phenomenon

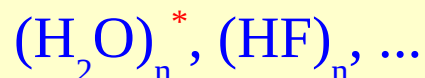
★ Van der Waals clusters



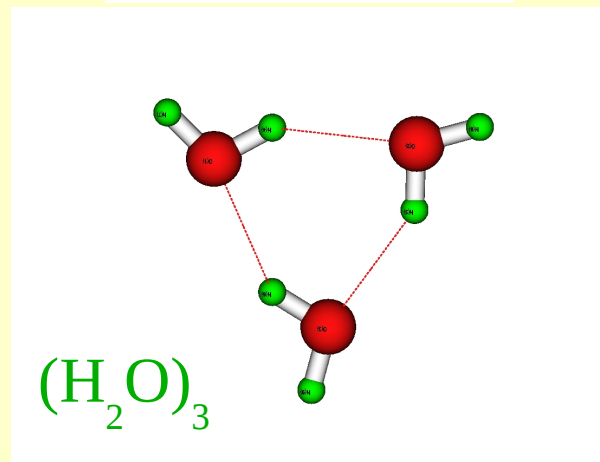
* - confirmed experimentally: Hergenbahn and coworkers, PRL **90**, 203401 (2003); Björneholm, Svensson and coworkers, PRL **93**, 173401 (2004); Dörner and coworkers, PRL **93**, 163401 (2004).



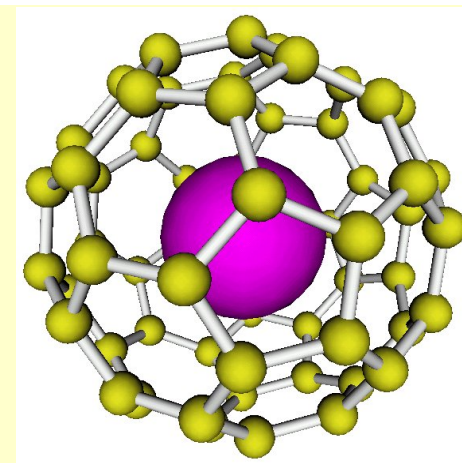
★ Hydrogen bonded clusters



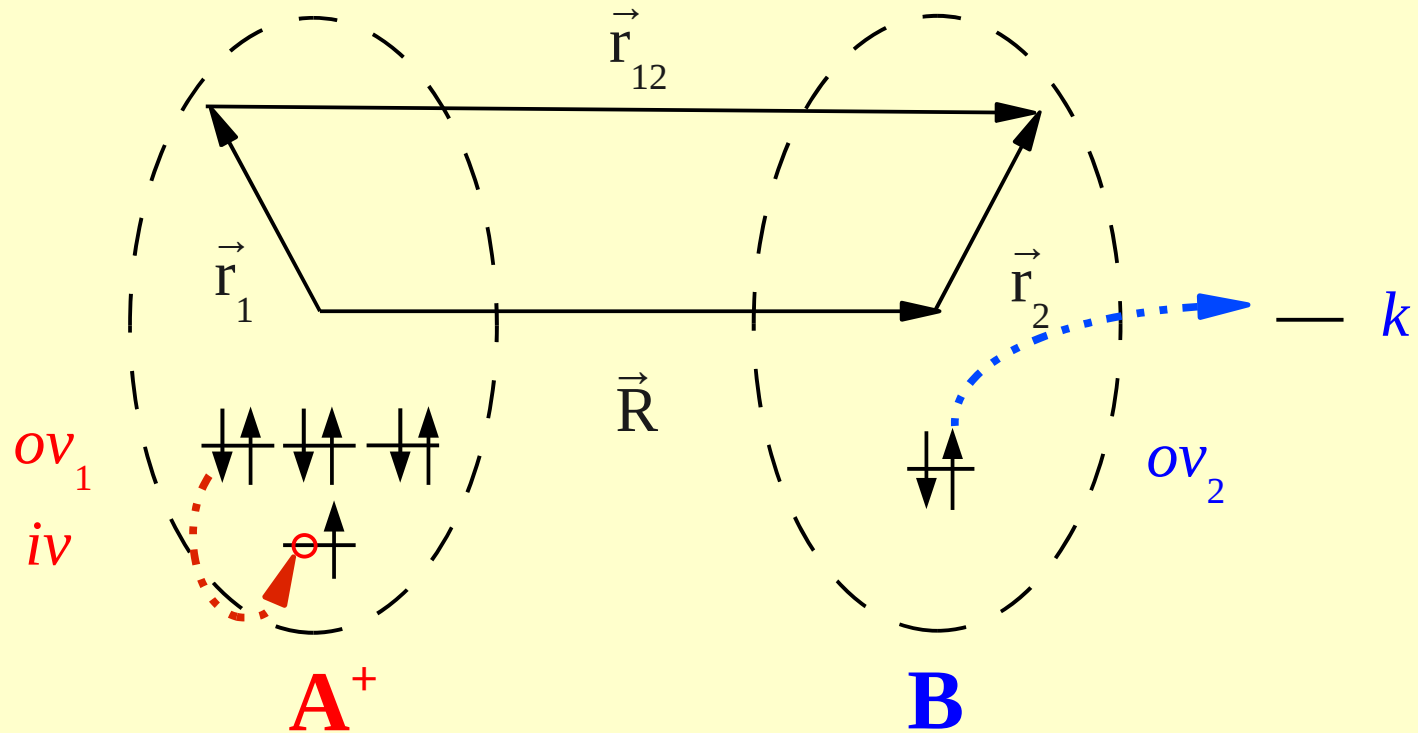
* - confirmed experimentally: Dörner and co-workers, Nature Physics **6**, 139 (2010), Hergenbahn and co-workers, Nature Physics **6**, 143 (2010)



★ Endohedral fullerenes



ICD: Virtual photon transfer mechanism

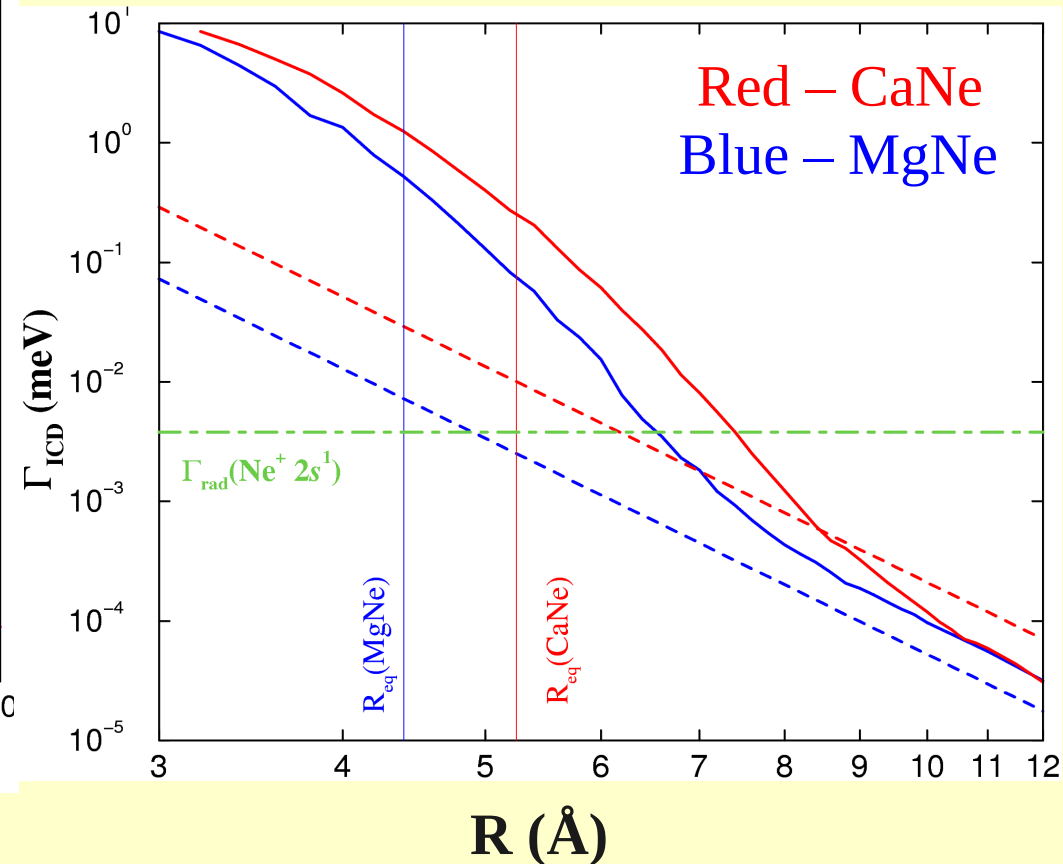
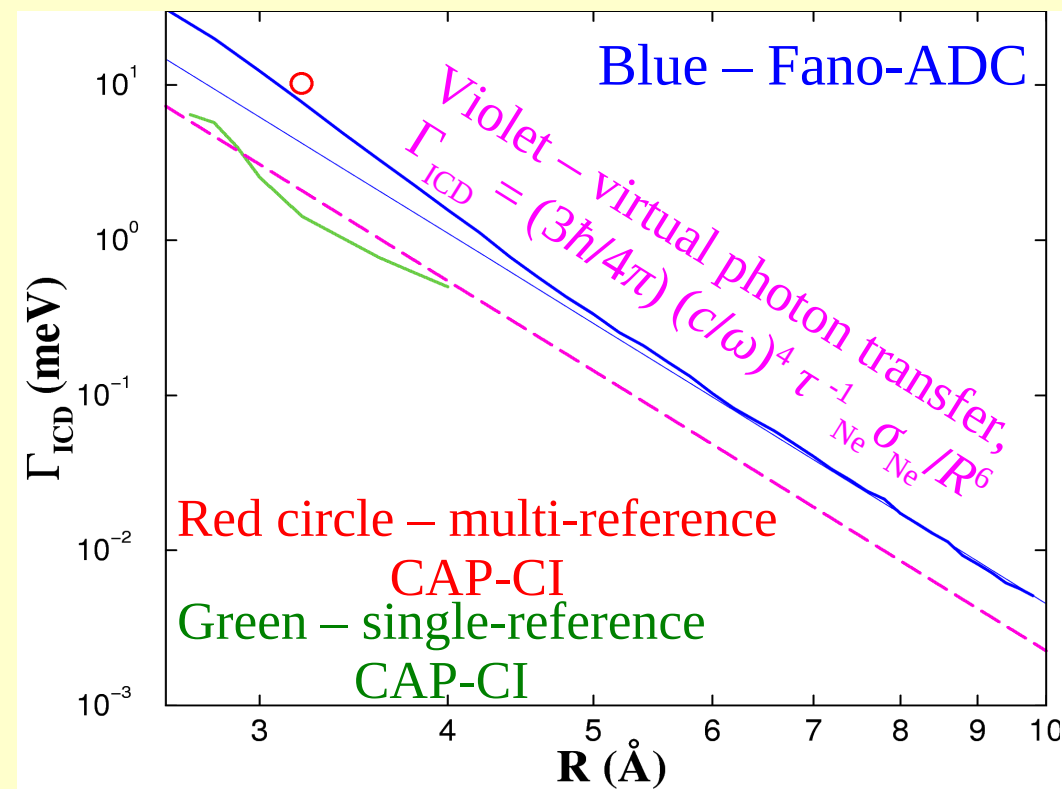


The dipole-dipole contribution to:

$$\Gamma_{\text{Auger}} = 2\pi \left| \langle ov_1(r_1) ov_2(r_2) | e^2/r_{12} | iv(r_1) k(r_2) \rangle \right|^2$$

gives: $\Gamma = (3\hbar/4\pi) (c/\omega)^4 \tau_{A^+}^{-1} \sigma_B / R^6$ $\tau_{A^+}^{-1}$ is the radiative lifetime of A^+
 σ_B is the total ionization cross-section of B

ICD: The overlap enhancement effect

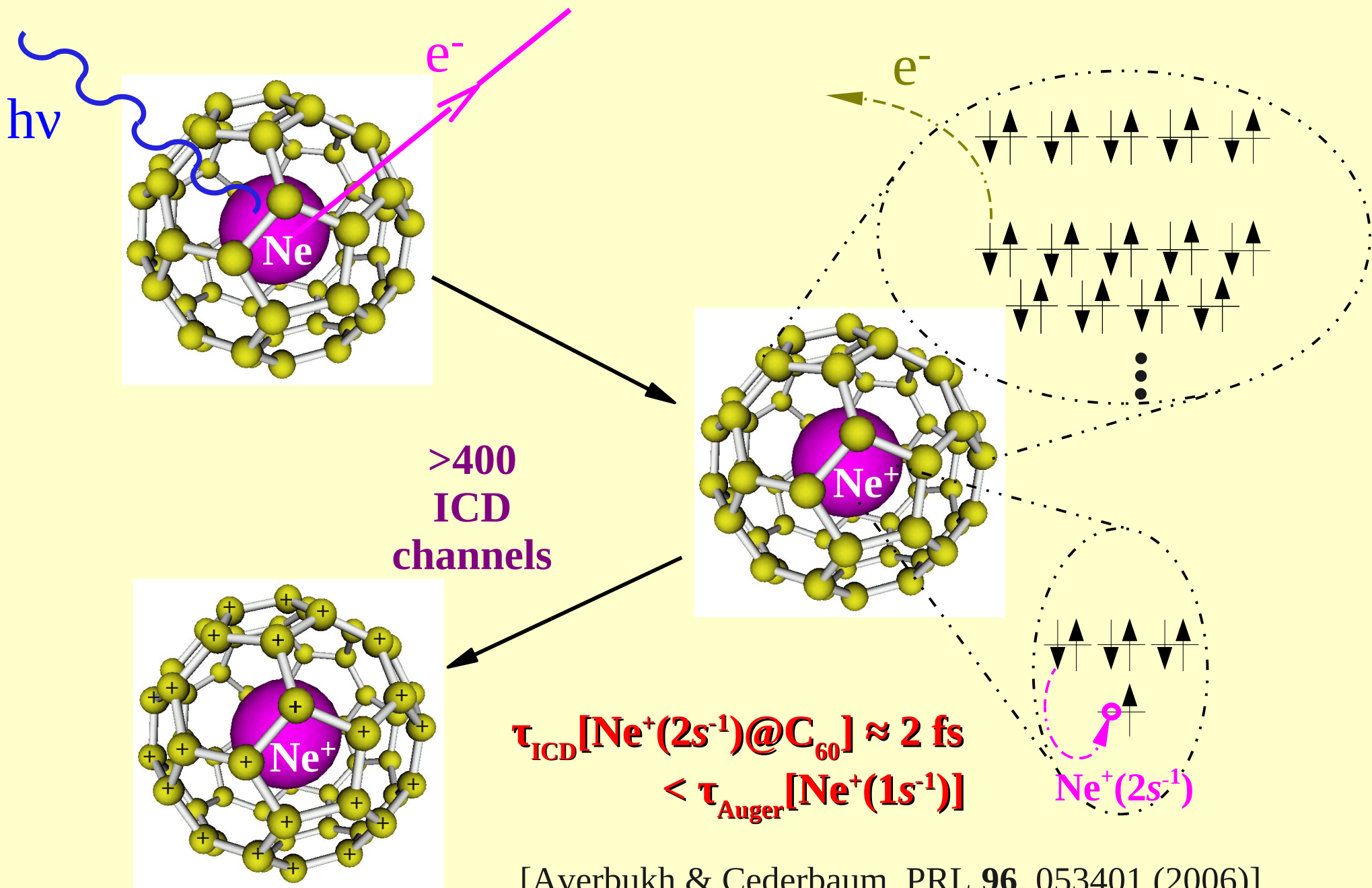


★ While the virtual photon transfer model is qualitatively correct for rare gas clusters, it can fail badly for other systems. Overlap enhancement in MgNe, CaNe diatoms reaches two orders of magnitude!

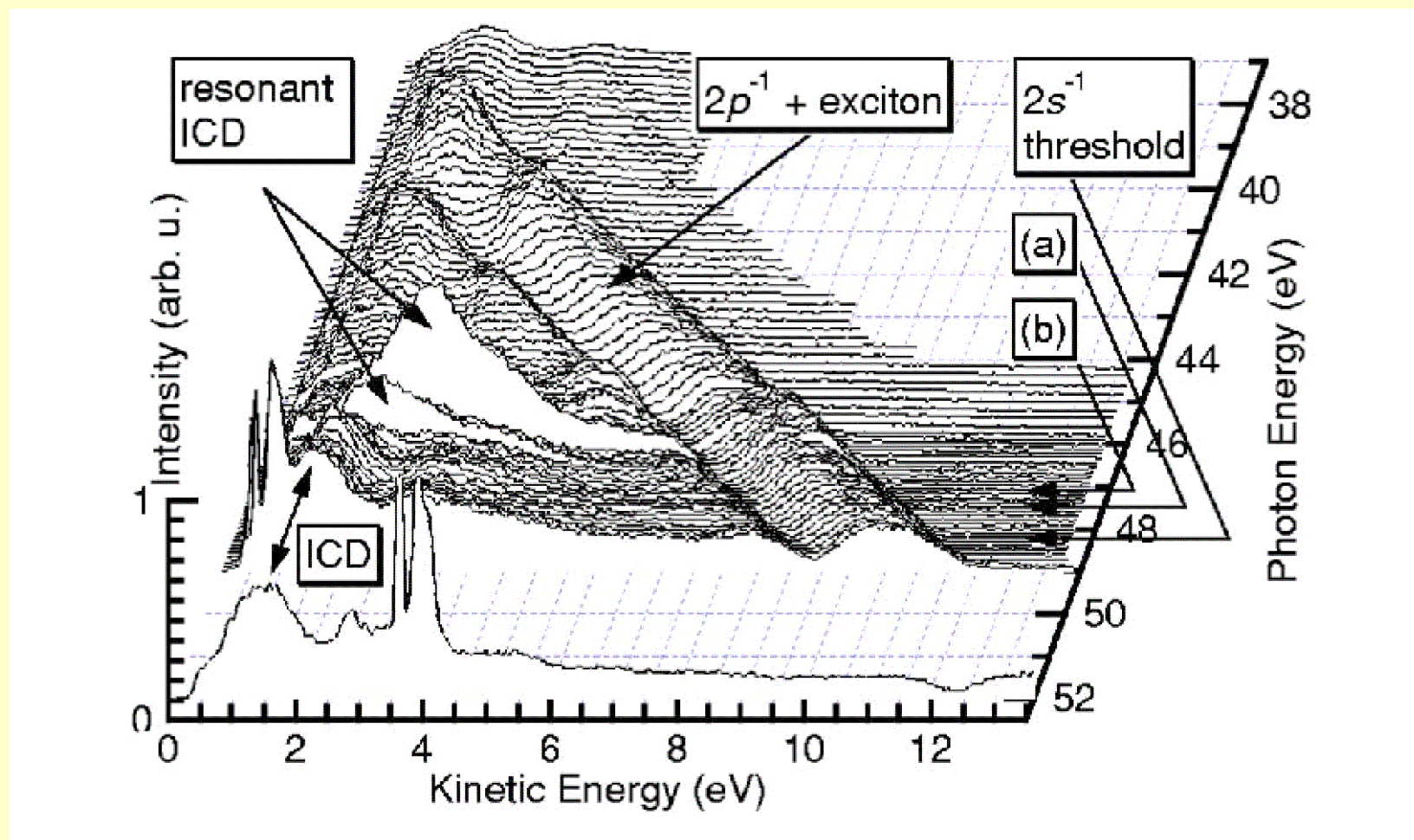
[Averbukh, Müller & Cederbaum, PRL **93**, 263002 (2004)]

[Averbukh & Cederbaum, JCP **123**, 204107 (2005)]

ICD in Endohedral Fullerene Complexes

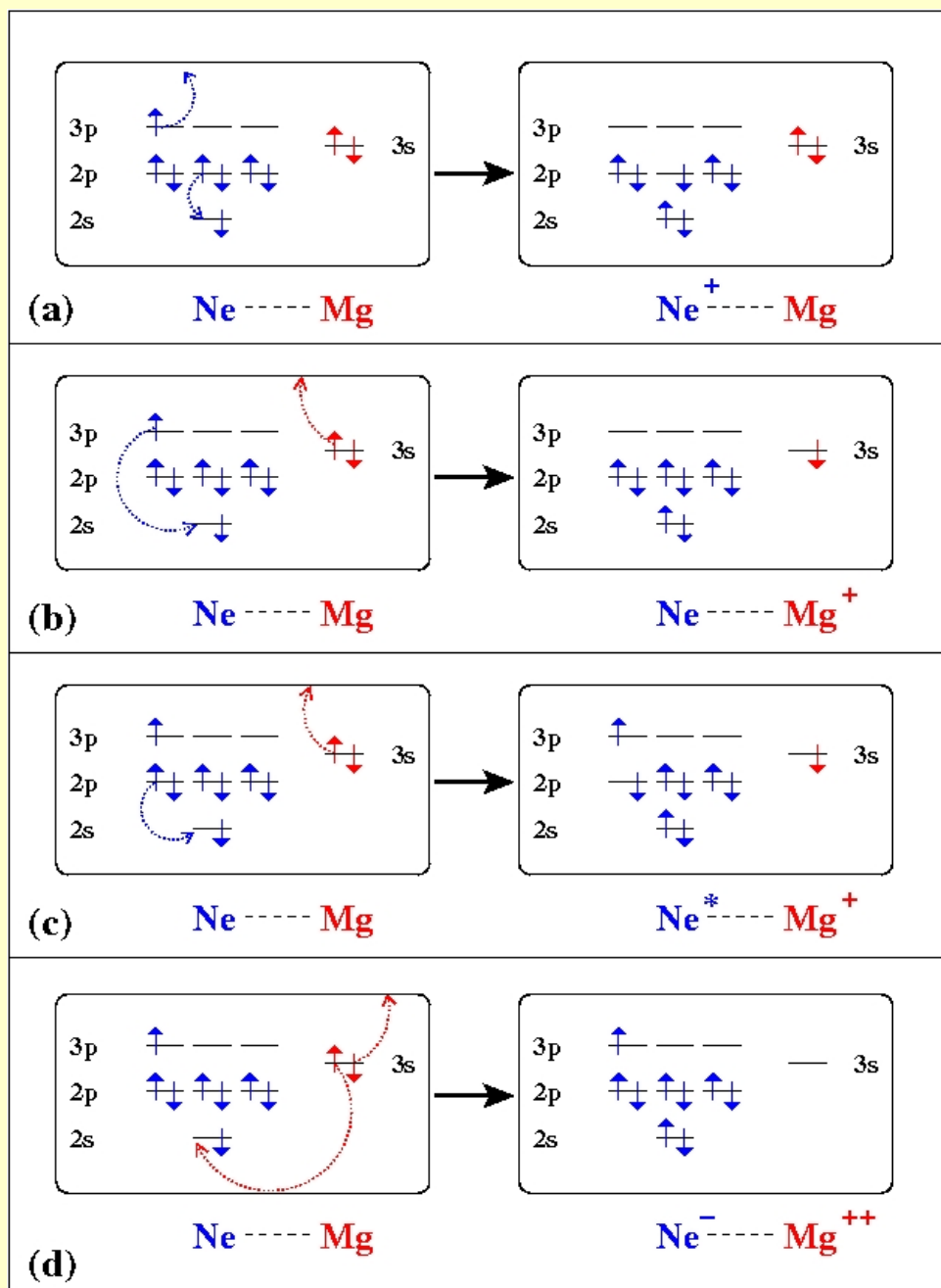


Interatomic decay of *excited* clusters: RICD



Photon energy independent feature below Ne 2s threshold
→ *spectator* resonant ICD (sRICD) –
interatomic analog of spectator resonant Auger decay

Resonant ICD in MgNe: possible decay pathways



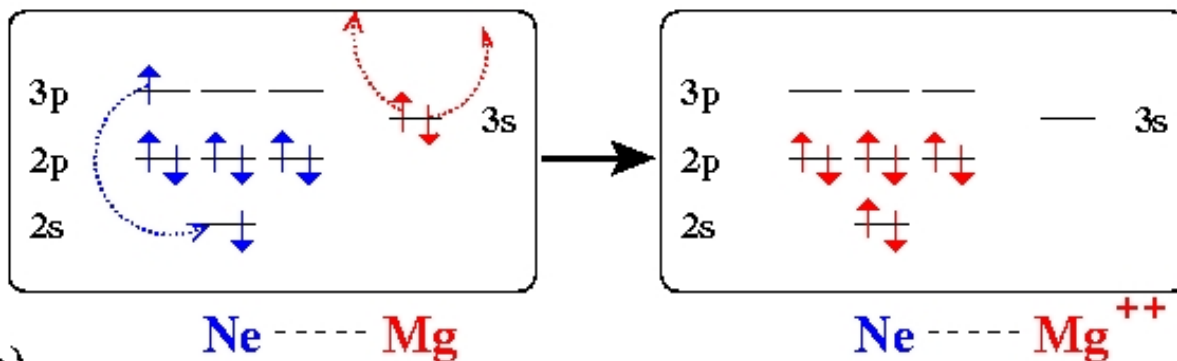
Autoionization (AI)

Participator resonant ICD (*p*RICD)

Spectator resonant ICD (*s*RICD)

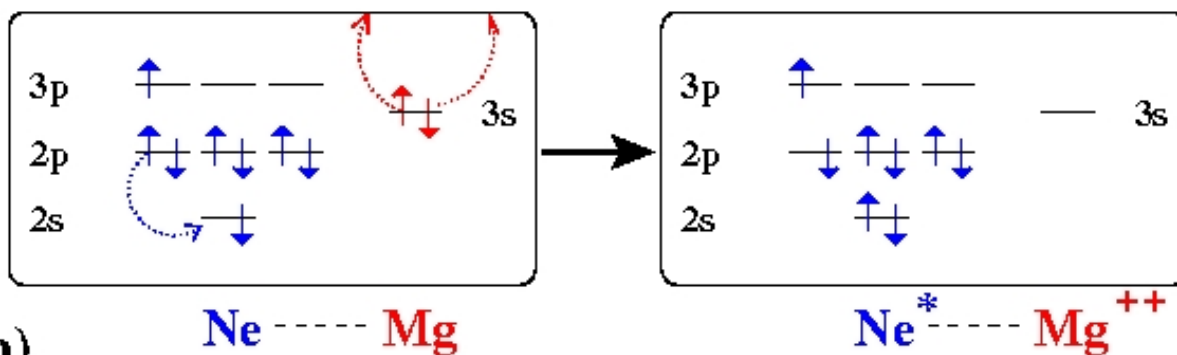
ETMD? - Not feasible!

Resonant ICD in MgNe: still more decay pathways



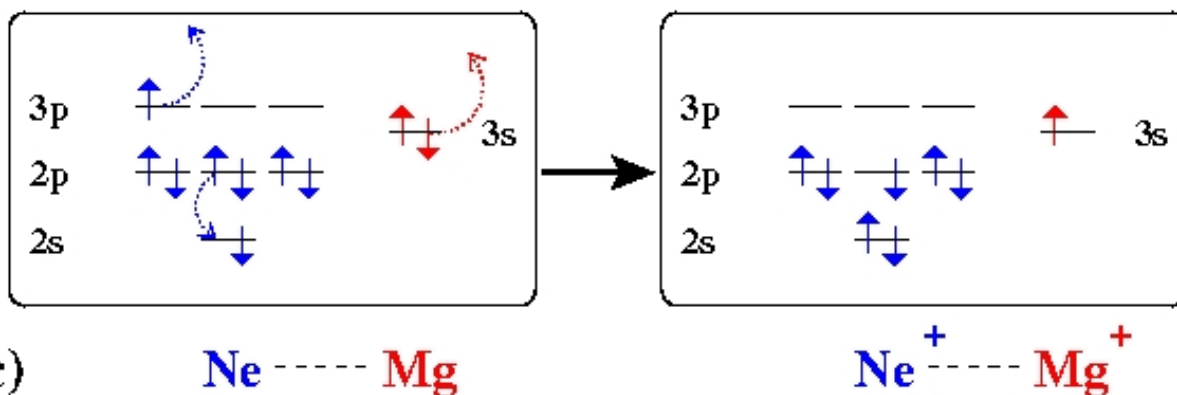
(a)

Participator double resonant ICD (pDRICD)



(b)

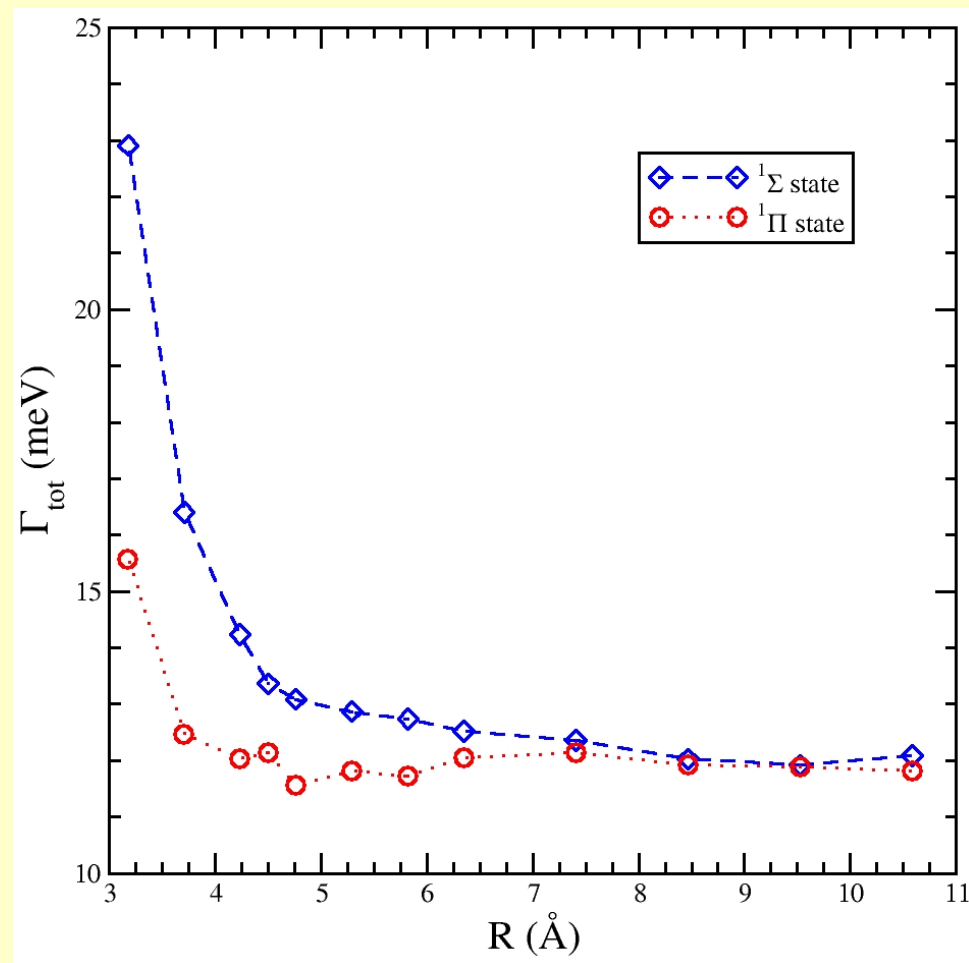
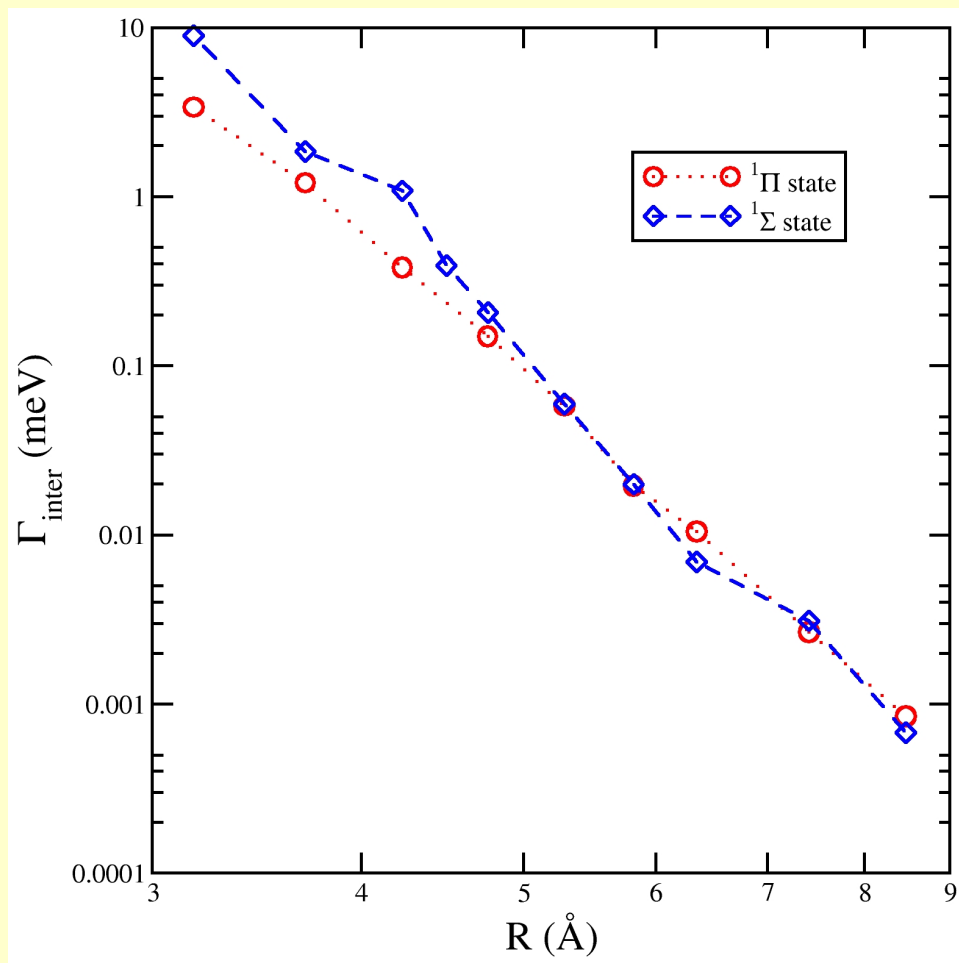
Spectator double resonant ICD (sDRICD)



(c)

AI - RICD hybrid

Resonant ICD in MgNe: the Fano-ADC rates

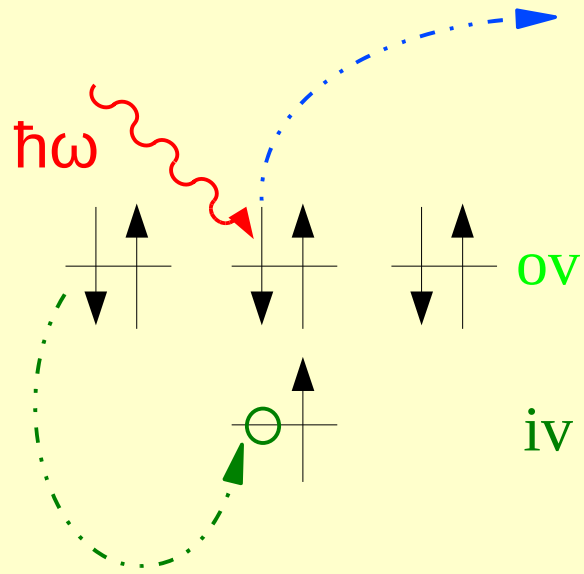


★ The spectator RICD process is much stronger than the participator decay

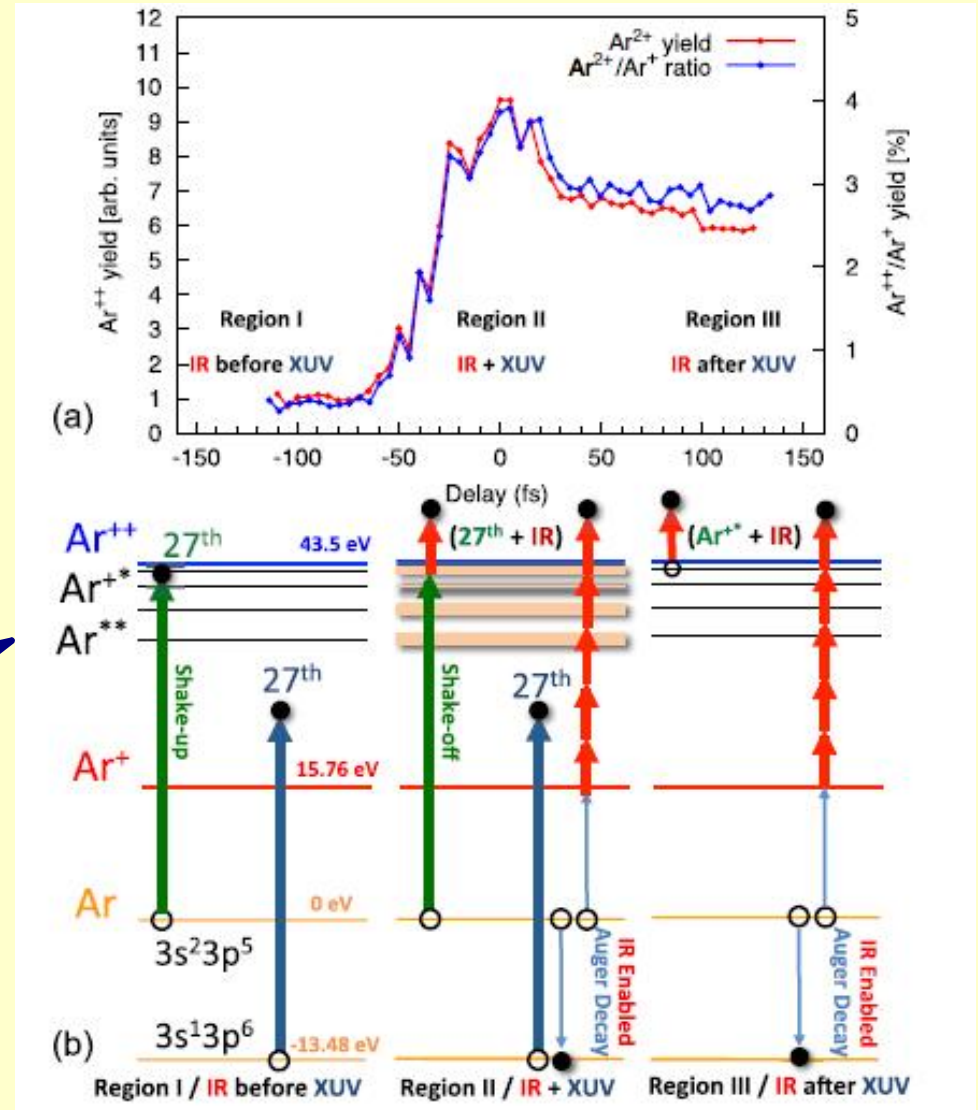
★ At small R 's, RICD and AI are comparably fast

More on the inner valence holes: Laser-enabled Auger decay

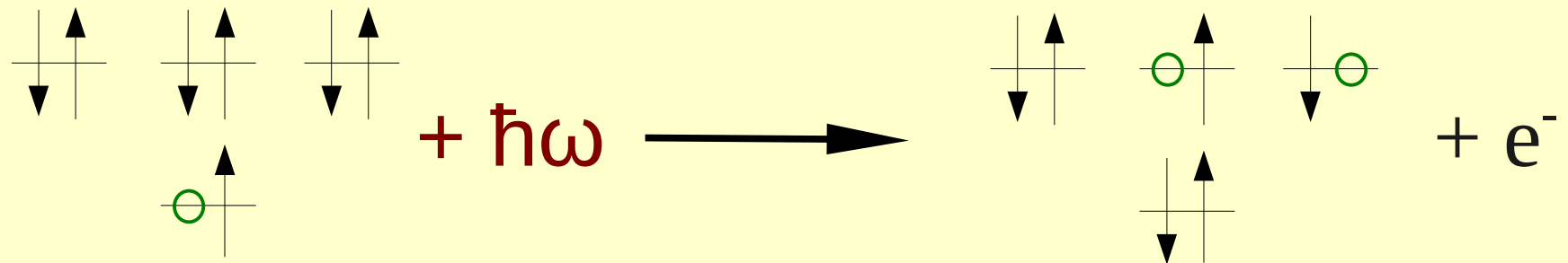
An idea inspired by
radiative Auger:



Very recent experimental realization
in the multiphoton regime



Single-photon laser-enabled Auger decay...

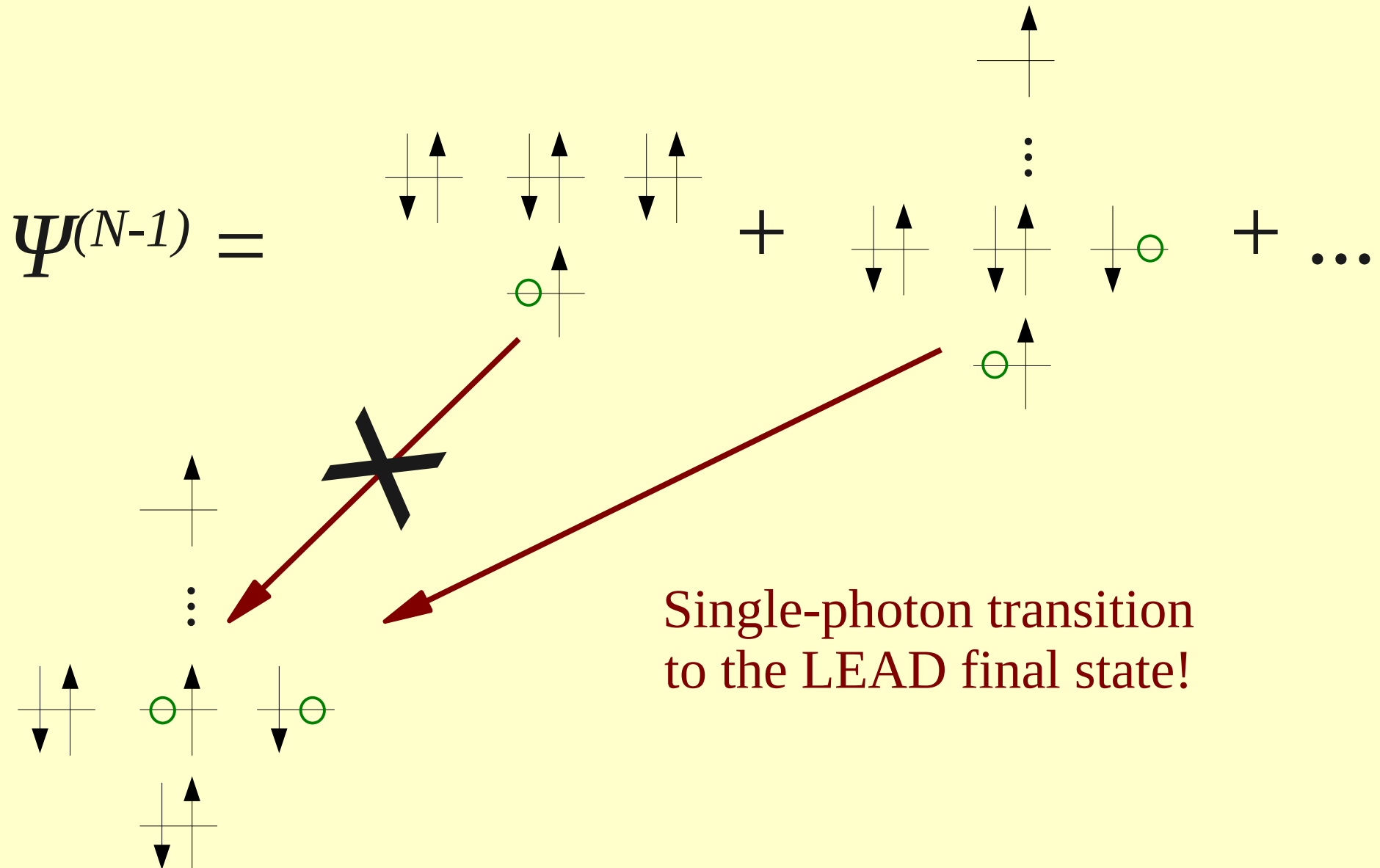


... is forbidden in the single Slater determinant approximation!

$$\langle \text{Slater Determinant}_1 | \sum_j \vec{r}_j | \text{Slater Determinant}_2 \rangle = 0$$

The diagram shows the matrix element of the dipole operator $\sum_j \vec{r}_j$ between two Slater determinants. The left determinant has three occupied orbitals and one partially occupied orbital (green circle). The right determinant has the same three occupied orbitals but the partially occupied orbital is now fully occupied (up arrow). The matrix element is shown to be zero, indicating that the transition is forbidden in the single Slater determinant approximation.

Single-photon LEAD: Measure of CI in the inner-valence-ionized states



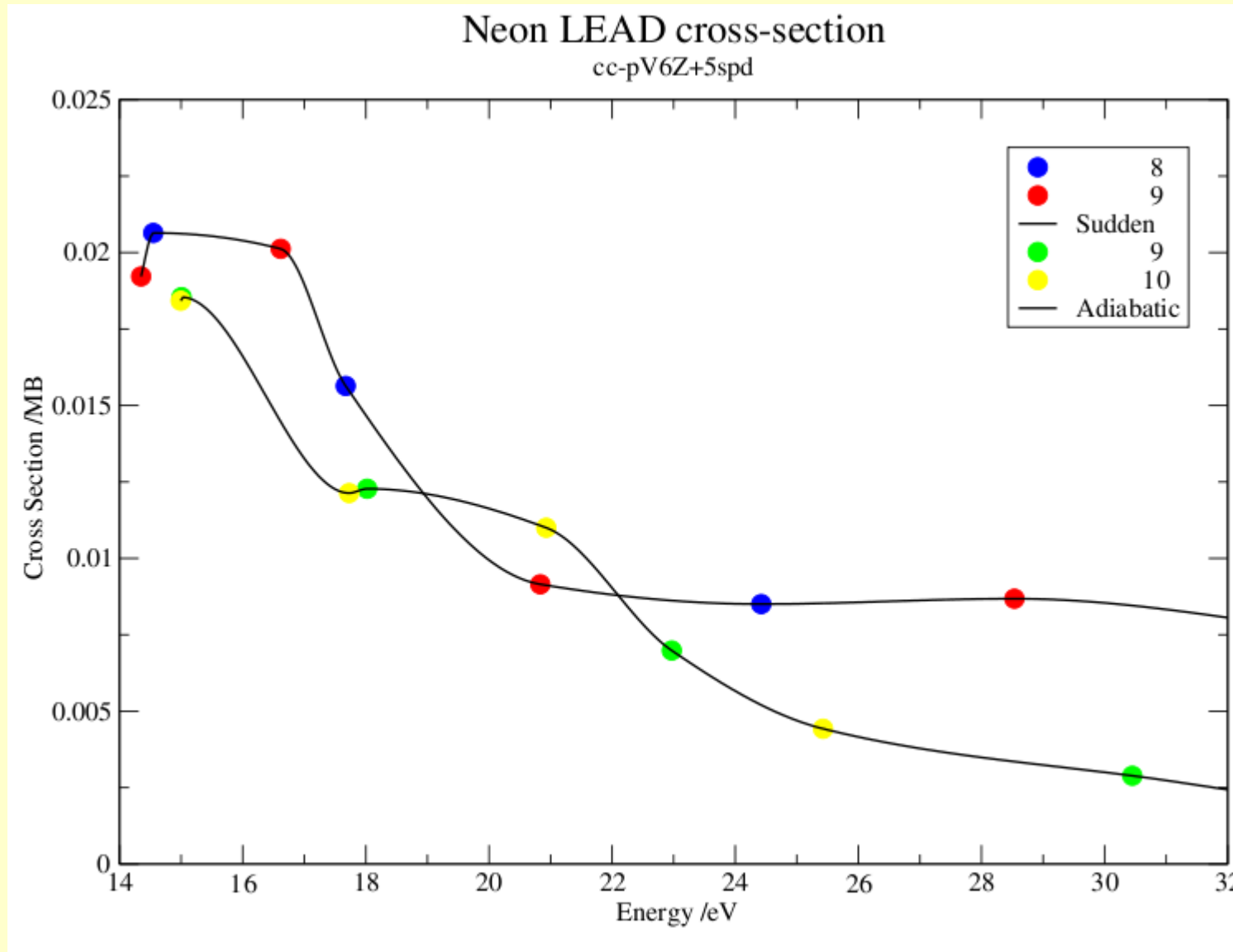
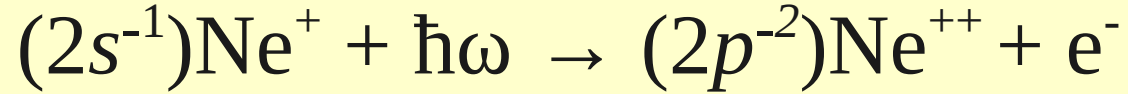
The initial state of LEAD: Sudden ionization?

$$\Psi_0^{(N)} = C_1 \begin{array}{c} \text{---} \\ | \\ \text{---} \\ | \\ \uparrow \\ | \\ \downarrow \end{array} + C_2 \begin{array}{c} \text{---} \\ | \\ \uparrow \\ | \\ \text{---} \\ | \\ \downarrow \end{array} + C_3 \begin{array}{c} \text{---} \\ | \\ \downarrow \\ | \\ \uparrow \\ | \\ \text{---} \end{array}$$

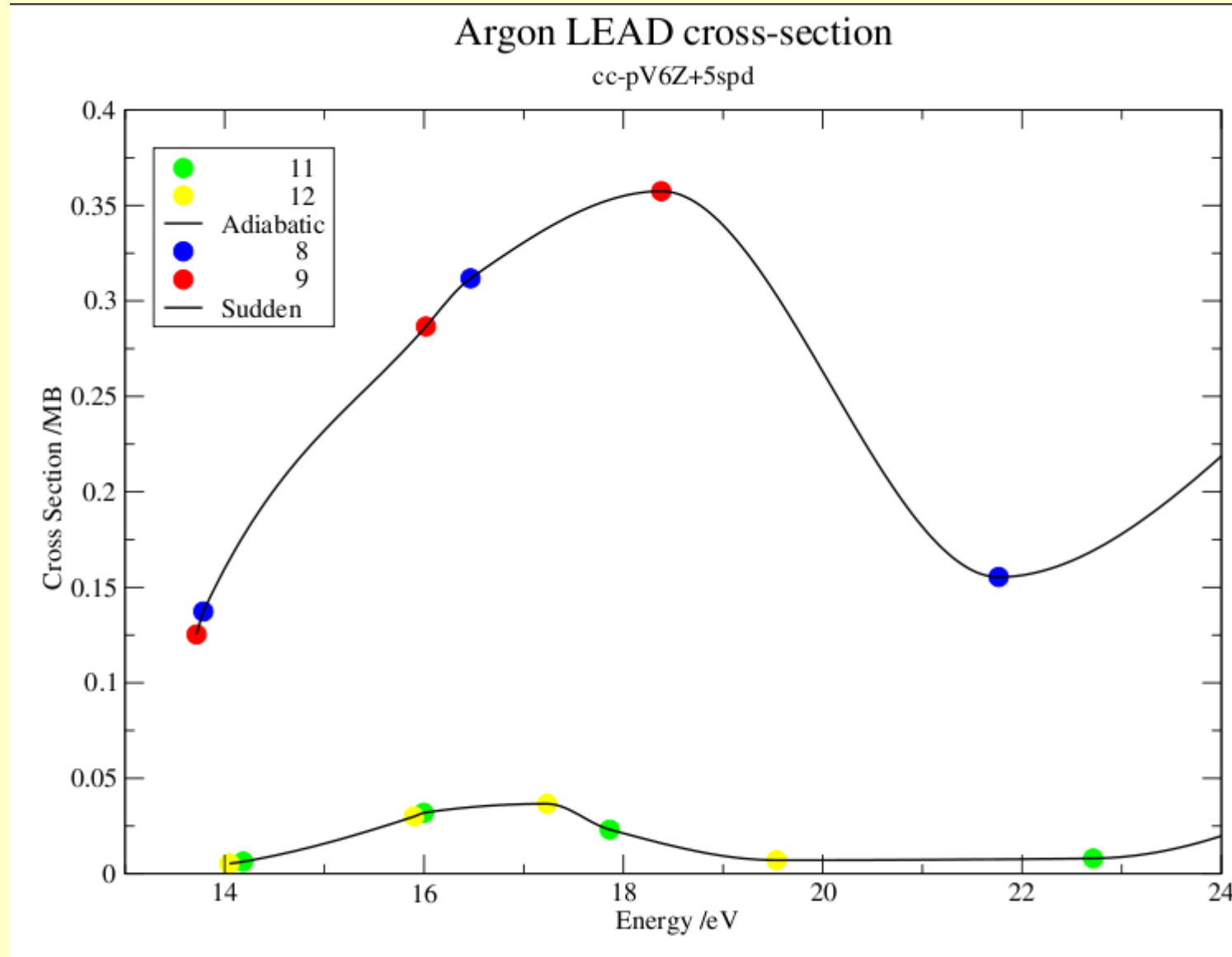
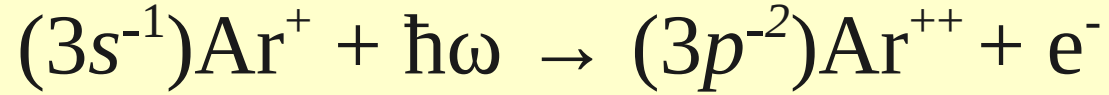
$$\hat{a}_{1\beta} \Psi_0^{(N)} = C'_1 \begin{array}{c} \text{---} \\ | \\ \uparrow \\ | \\ \text{---} \end{array} + C'_2 \begin{array}{c} \text{---} \\ | \\ \uparrow \\ | \\ \text{---} \end{array} \neq \Psi_j^{(N-1)}$$

Sudden annihilation of an electron produces a non-stationary state of the ion!

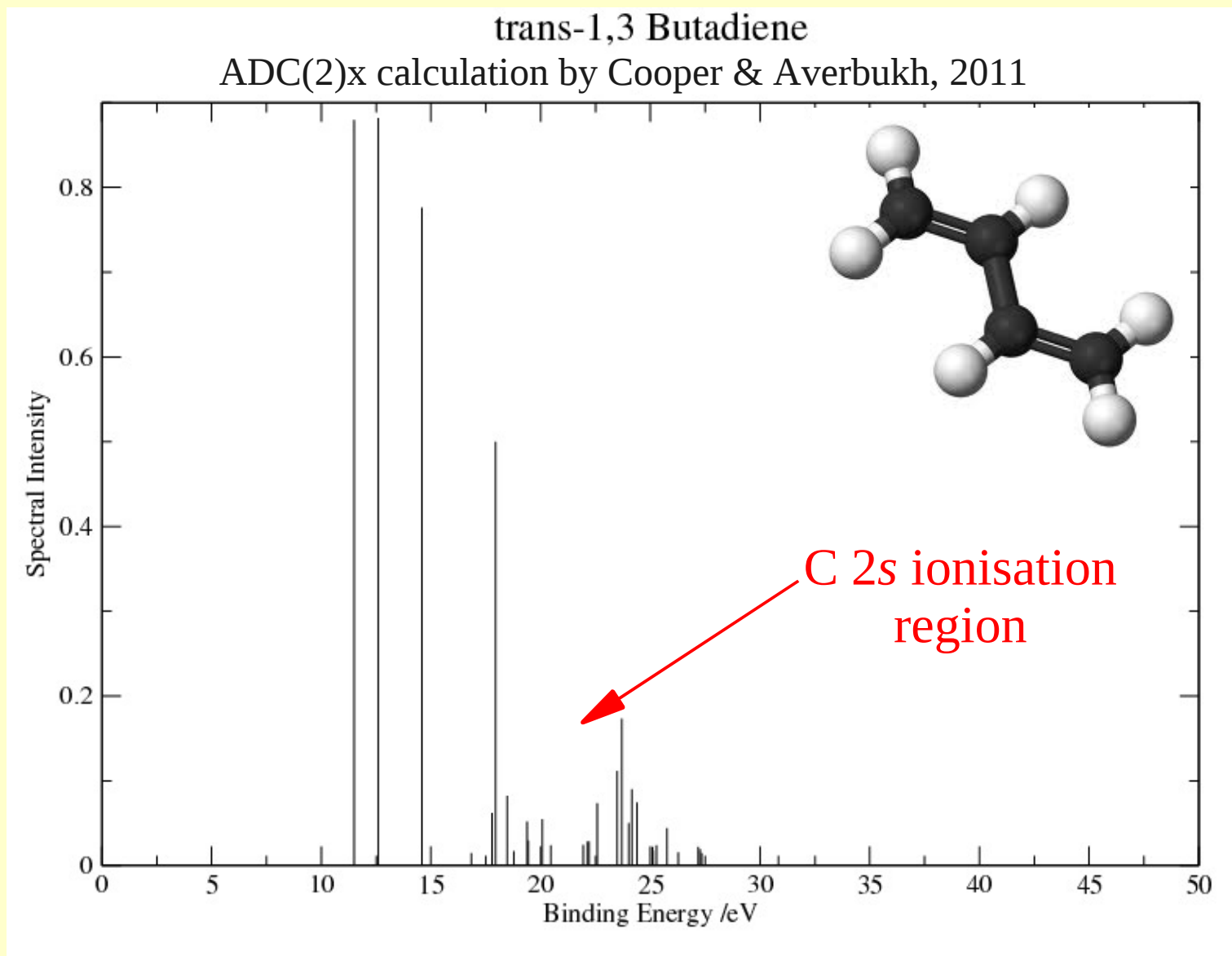
Single-photon LEAD: ADC(2)x-Stieltjes results



Single-photon LEAD: ADC(2)x-Stieltjes results



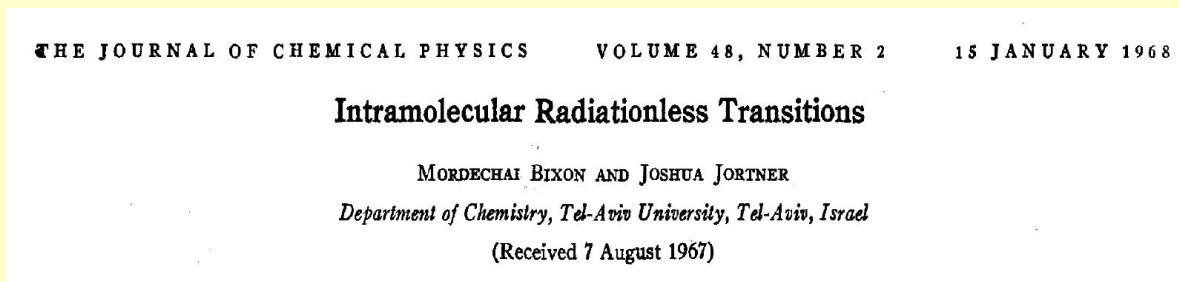
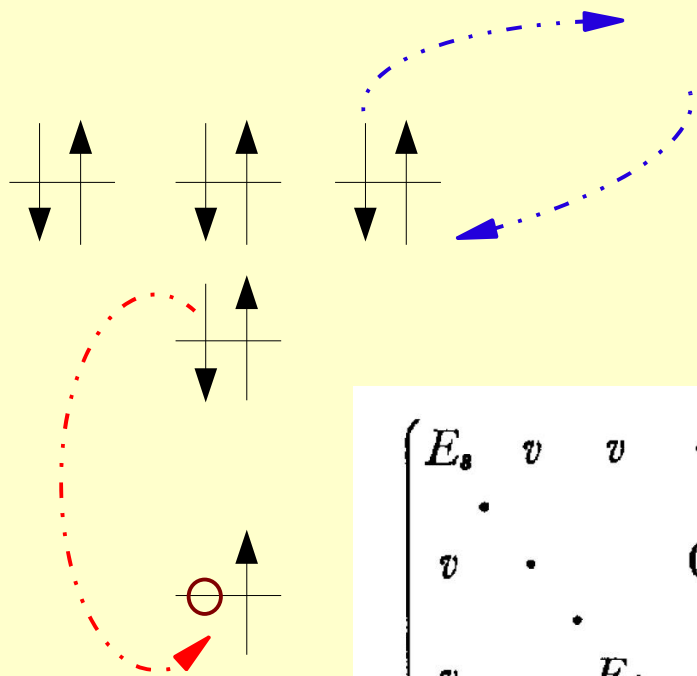
Inner valence ionisation in molecules: Breakdown of MO picture



[Cederbaum, Domcke, Schirmer & von Niessen, Adv. Quantum Chem. **65**, 115 (1986)]

Breakdown of MO picture: an Auger transition that did not happen...

Bixon-Jortner type instead of Fano-Feshbach type situation



$$\begin{pmatrix} E_s & v & v & \cdots \\ & \cdot & & \\ v & & & 0 \\ & & \cdot & \\ v & & E_i & \\ \cdot & & & \cdot \\ \cdot & & & \\ \cdot & 0 & & \cdot \end{pmatrix} \begin{pmatrix} a_n \\ \cdot \\ \cdot \\ \cdot \\ b_i^n \\ \cdot \\ \cdot \\ \cdot \end{pmatrix} = E_n \begin{pmatrix} a_n \\ \cdot \\ \cdot \\ \cdot \\ b_i^n \\ \cdot \\ \cdot \\ \cdot \end{pmatrix}$$

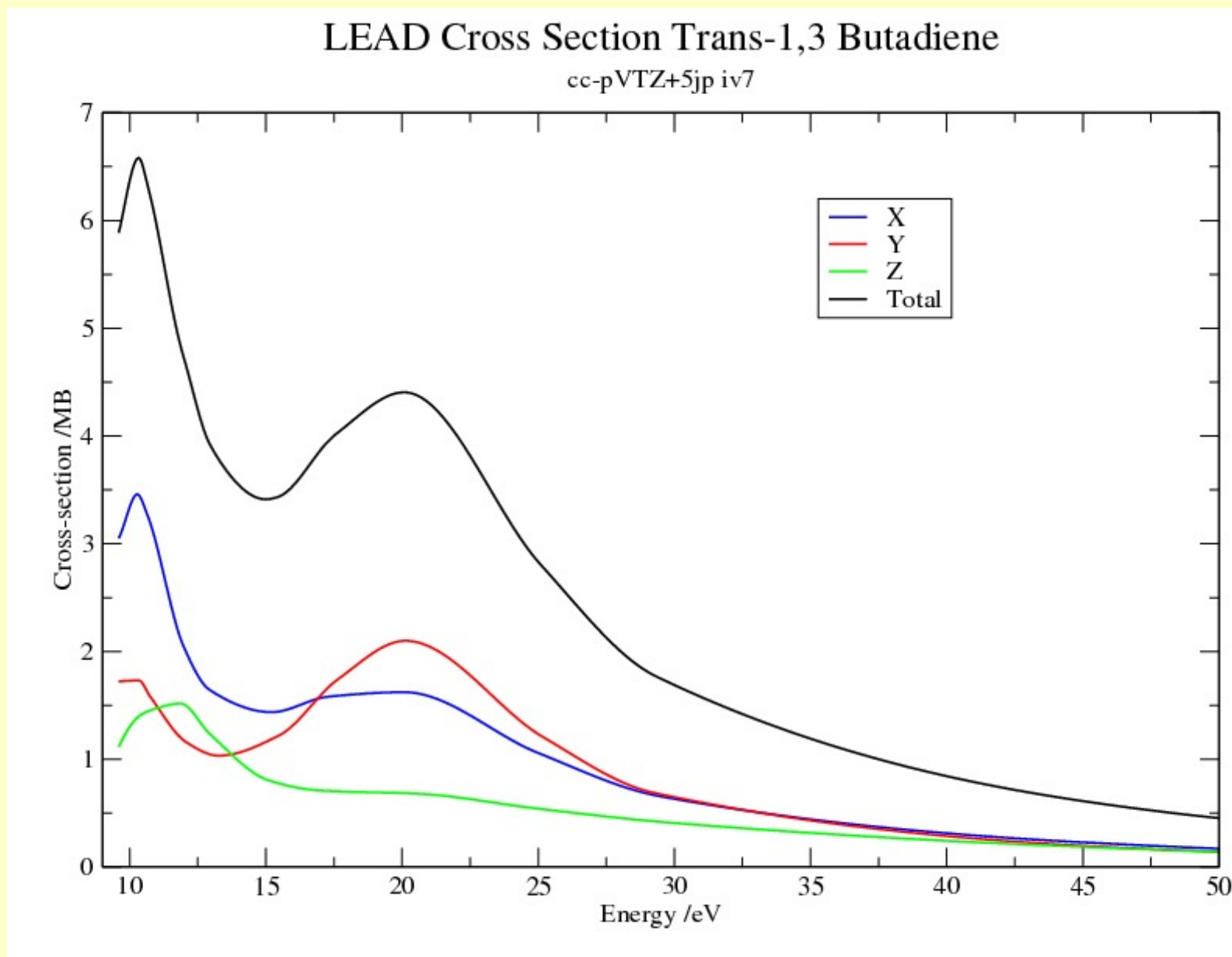
Equidistant, $E_{i+1} - E_i = \epsilon$,
“quasicontinuum” levels

Uniform bound-quasicontinuum
coupling, v

Exponential decay takes
place for $t < \hbar/\epsilon$

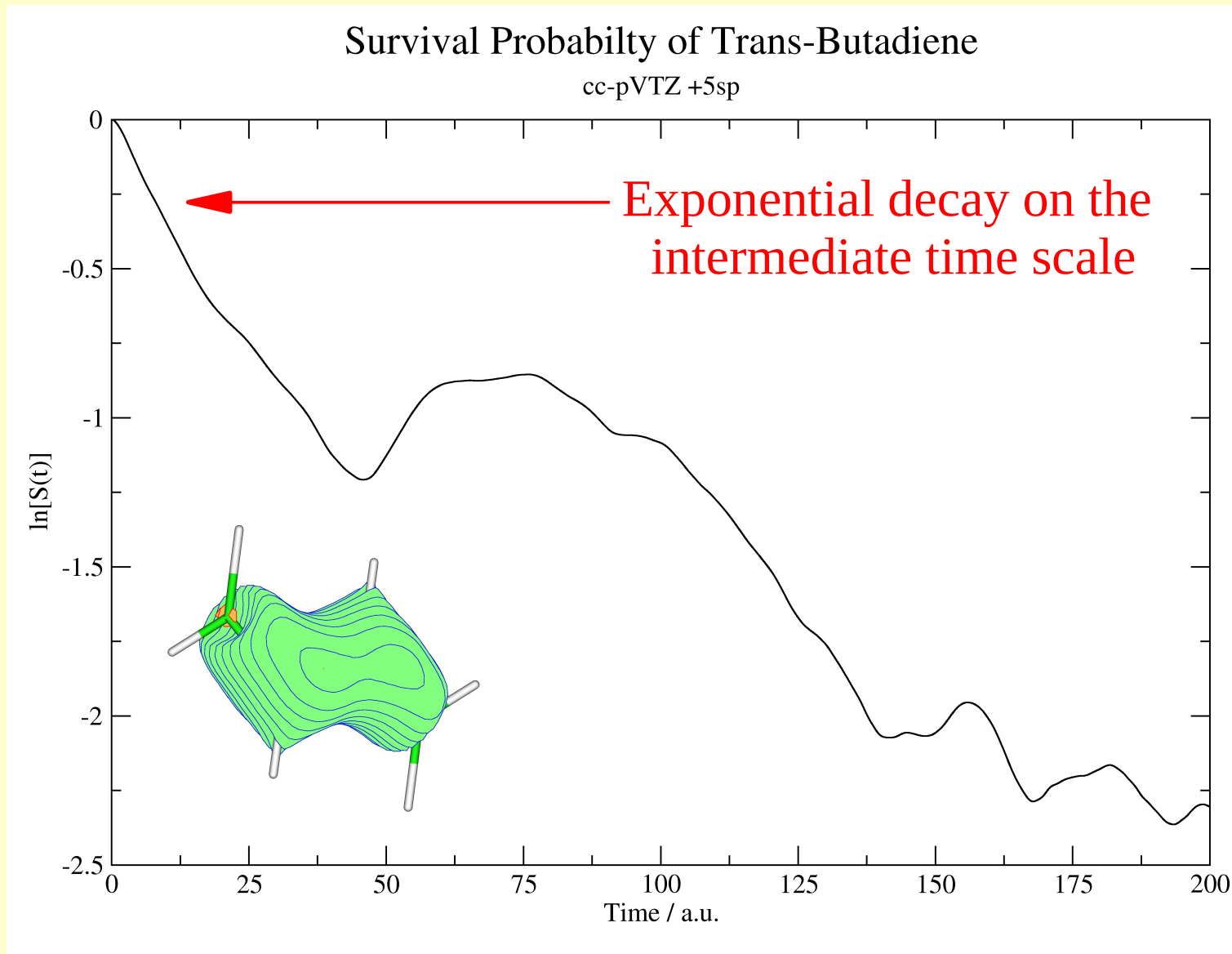
$$E_i = E_s - \alpha + i\epsilon, \quad i = 0, \pm 1, \pm 2, \dots$$

Single-photon LEAD & MO picture breakdown: Strong enhancement relative to atomic case



Cooper & Averbukh, 2012

Breakdown of MO picture: The time-dependent picture



Rate of quasi-exponential decay without time propagation

True exponential decay widths: $\Gamma = 2\pi |\langle \Phi | \hat{H} - E_r | \chi_{E_r} \rangle|^2$
can be obtained using discretized continuum and Stieltjes imaging.

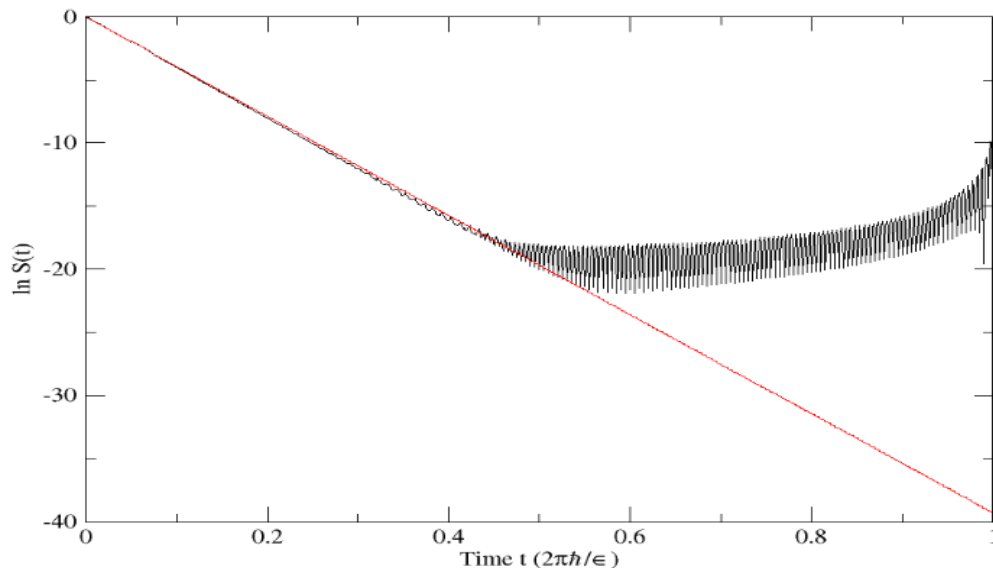
Why not to apply the Stieltjes imaging procedure to truly discrete final states?

This is a mapping of the real discrete-level system onto imaginary system with a discrete state coupled to a continuum.

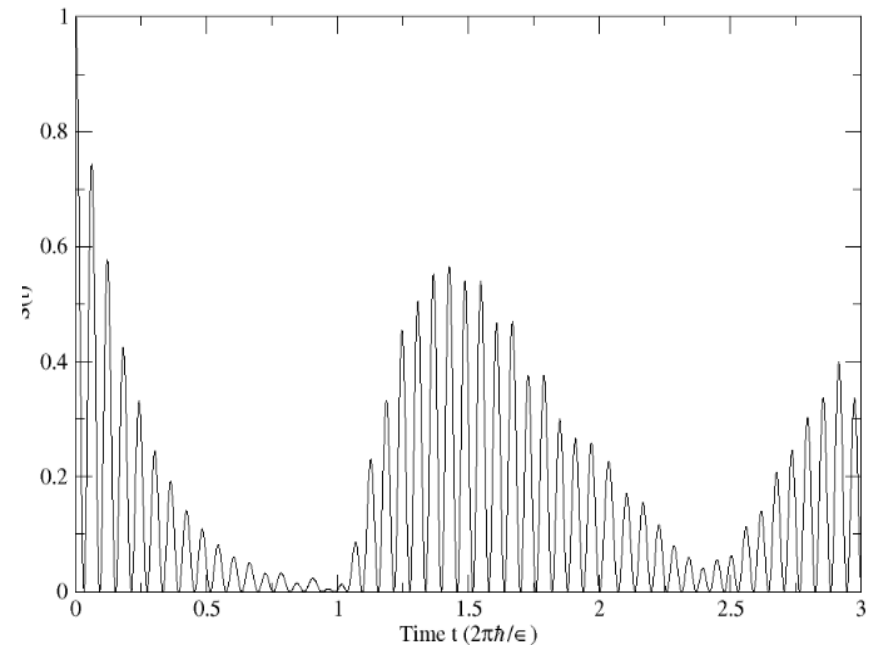
Will it work...?

Rate of quasi-exponential decay without time propagation

... it will!



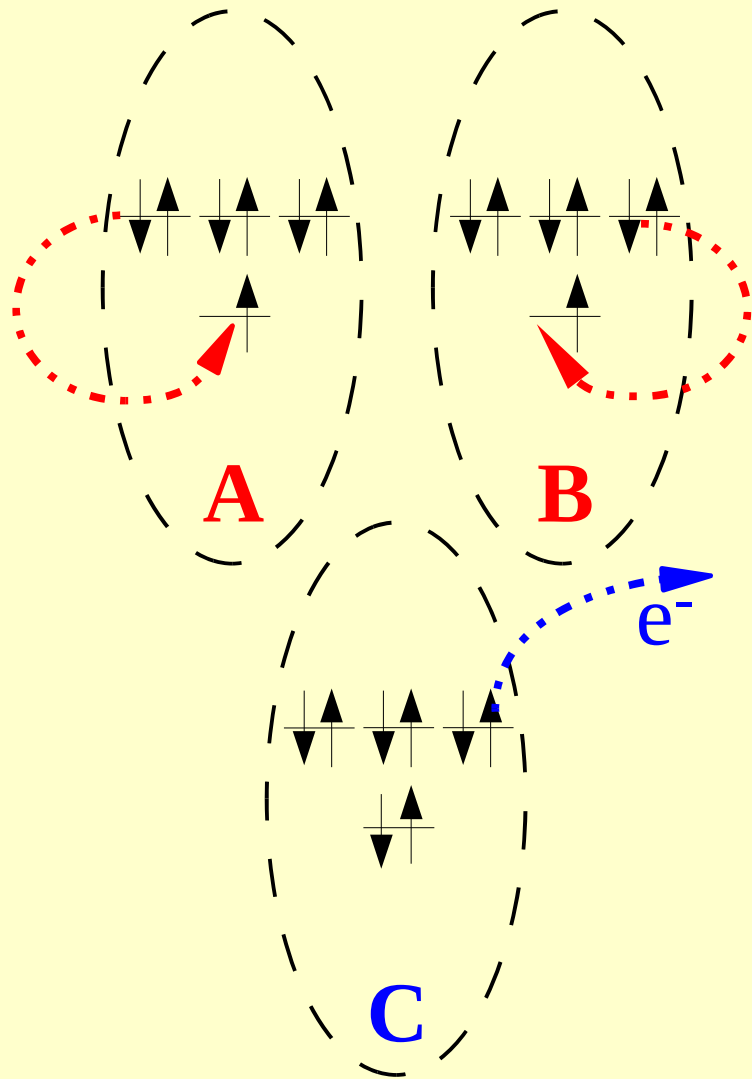
... it won't!



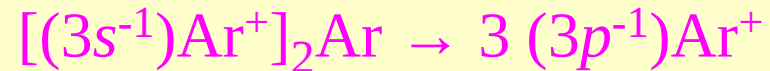
if there IS exponential decay
(Bixon-Jortner model).

if there IS NO exponential decay
(Morokuma-Freed model).

Three-electron transitions: Collective decay



Example:
 $(3s^{-1})\text{Ar}^+\text{Ar} \not\rightarrow$
 but



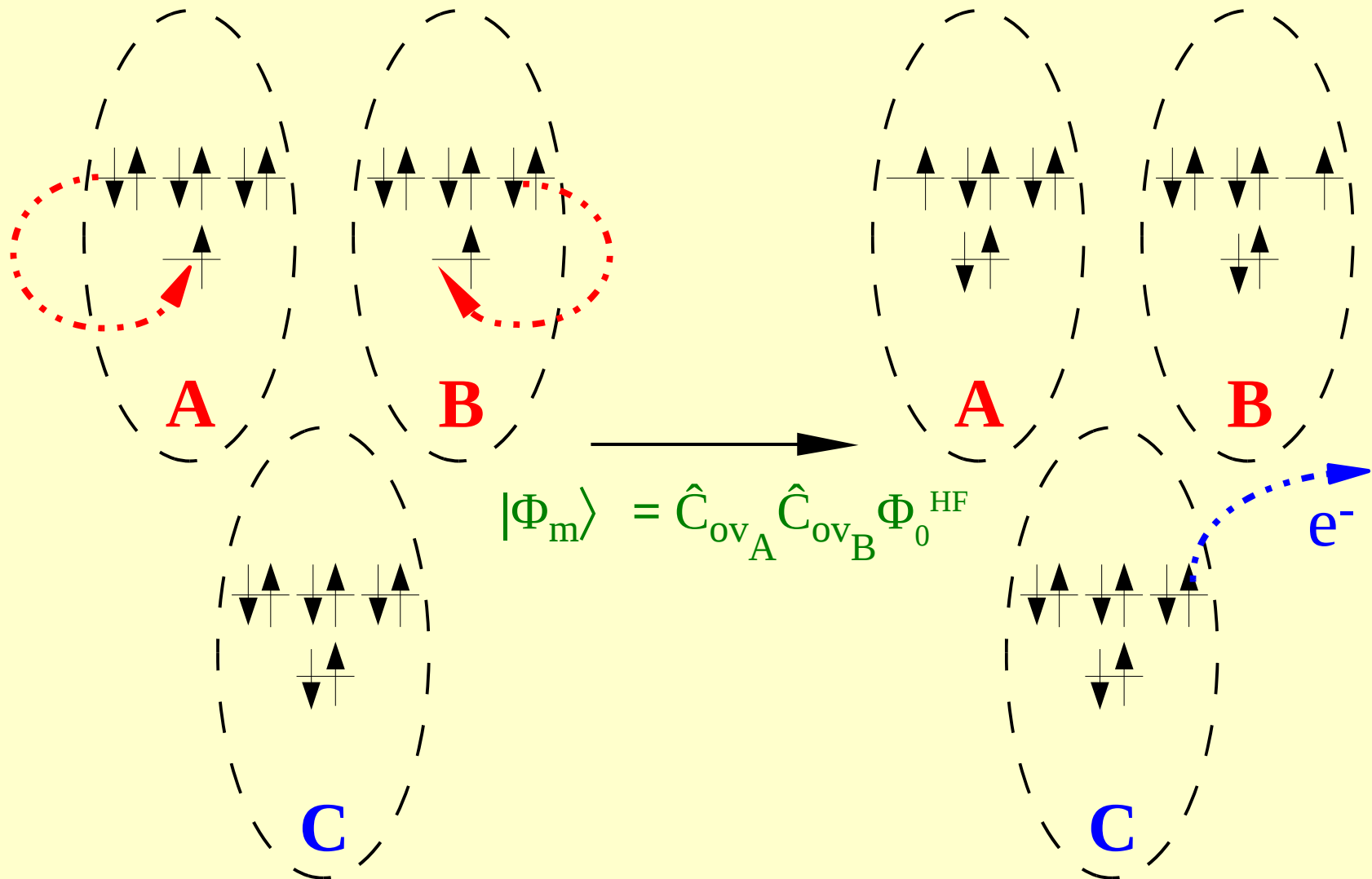
Two-virtual-photon transfer, an equivalent of a two-photon ionization process

Description of the collective decay requires second-order perturbation theory:

$$\Gamma = 2\pi \left| \sum_{\mathbf{m}} \frac{\langle \Phi_{\text{fin}} | V | \Phi_{\mathbf{m}} \rangle \langle \Phi_{\mathbf{m}} | V | \Phi_{\text{in}} \rangle}{E_{\text{in}} - E_{\mathbf{m}}} \right|^2 \delta(E_{\text{fin}} - E_{\text{in}})$$

an intermediate state $|\Phi_{\mathbf{m}}\rangle$ defines a two-step decay pathway contributing to Γ

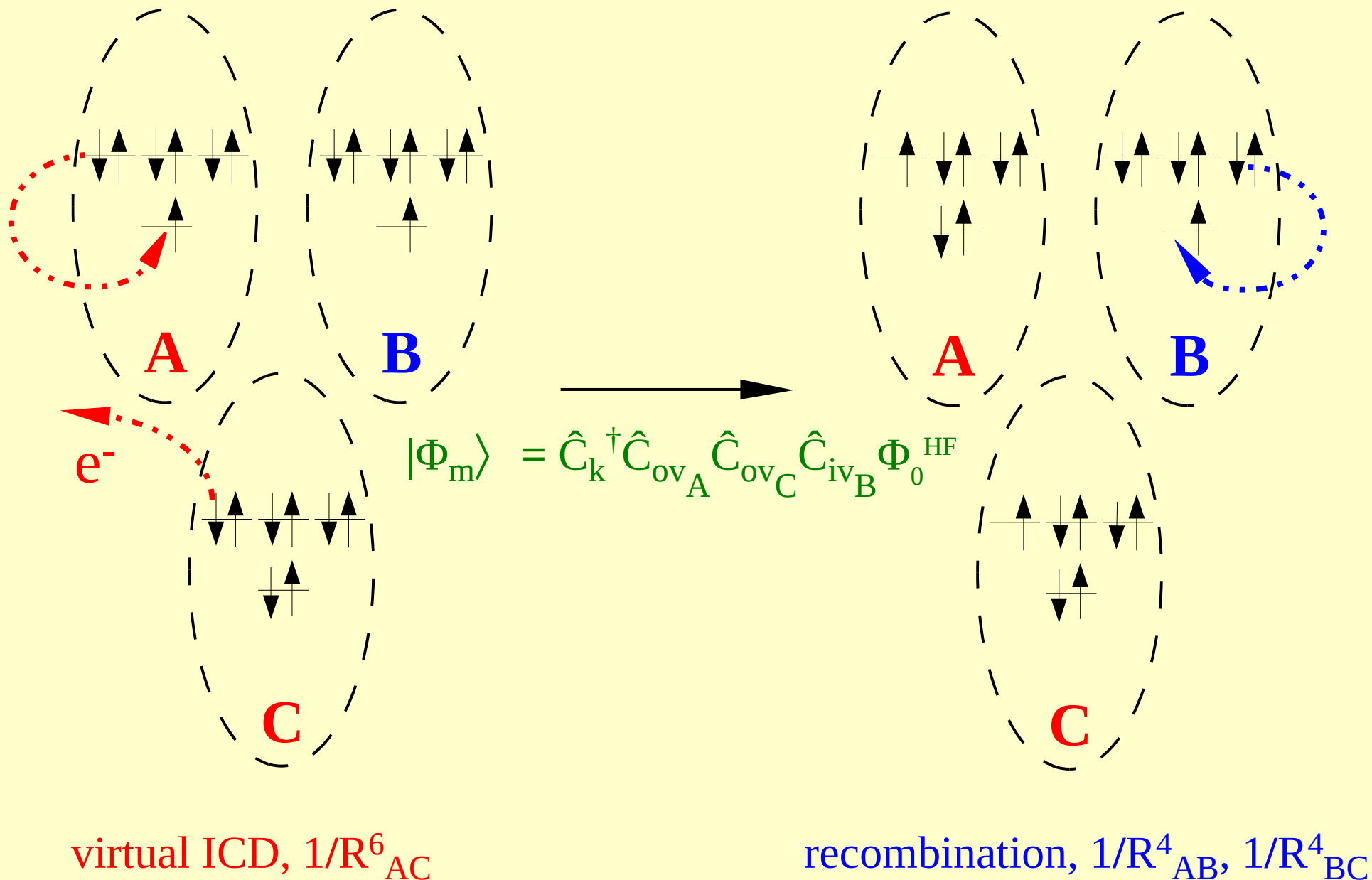
Examples of collective decay pathways



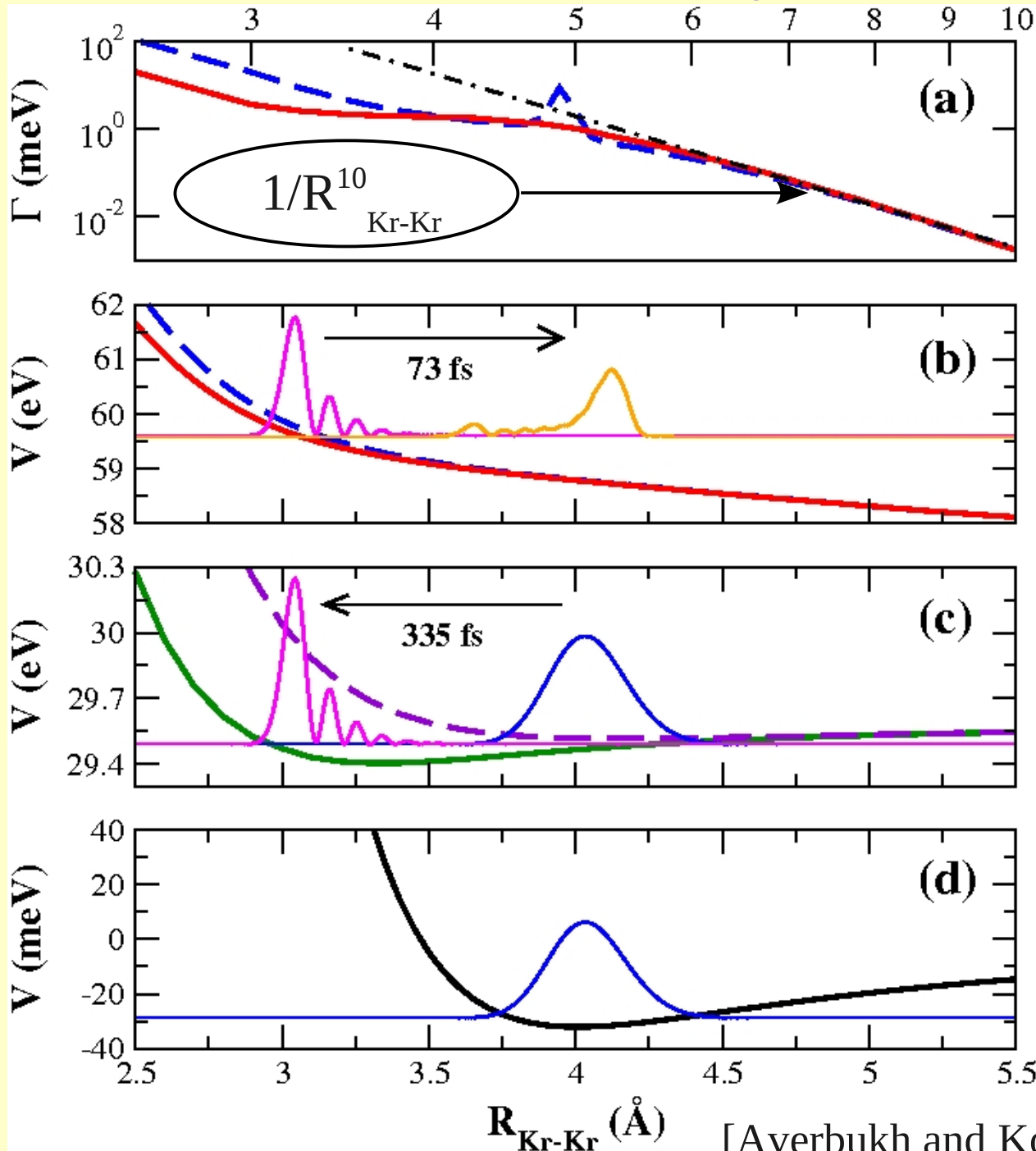
virtual recombination, $1/R_{AB}^6$

ionization, $1/R_{AC}^4, 1/R_{BC}^4$

Examples of collective decay pathways

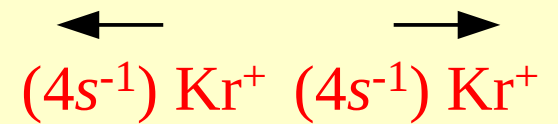


Collective decay in $[(4s^{-1}) \text{Kr}^+]_2 \text{Ar}$

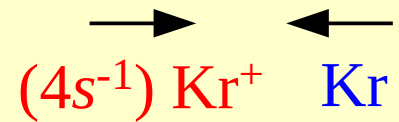


$\tau(4\text{\AA}) \approx 300 \text{ fs}$

30%-65% yields for the collective decay



Ar



Ar

Kr Kr

Ar

Summary

- ★ Quantum chemistry is not only for bound state energies and properties – with some tricks, it can be used for bound-continuum transitions.
- ★ But there is no magic, of course, and using \mathcal{L}^2 functions does impose very strict limits on what you can do, that is not much beyond total cross-sections and decay width...
- ★ So, future belongs to hybrid computational approaches, where quantum chemistry machinery is used for bound states and some kind of one- or (better!) two-electron true continuum states are constructed.

The team

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Dr. B. Cooper – laser-enabled Auger decay

Mr. J. Leeuwenburgh – streaking and HHG spectroscopy of Auger decay

Mr. M. Ruberti – photoionization by B-spline ADC

Perugia:

Prof. F. Tarantelli

MPI PKS, Dresden:

Prof. J.-M. Rost

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TC Heidelberg:

Prof. L. S. Cederbaum

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Dr. K. Gokhberg

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