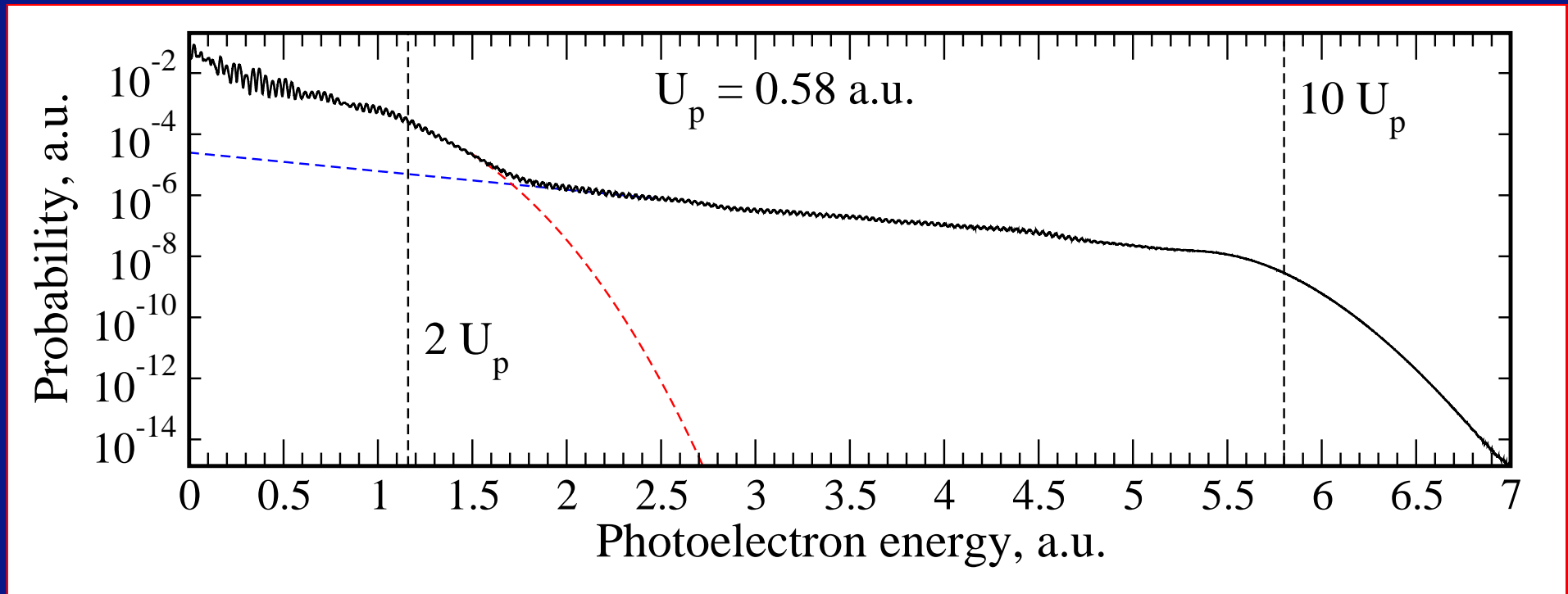


## Example electron spectrum (ATI)



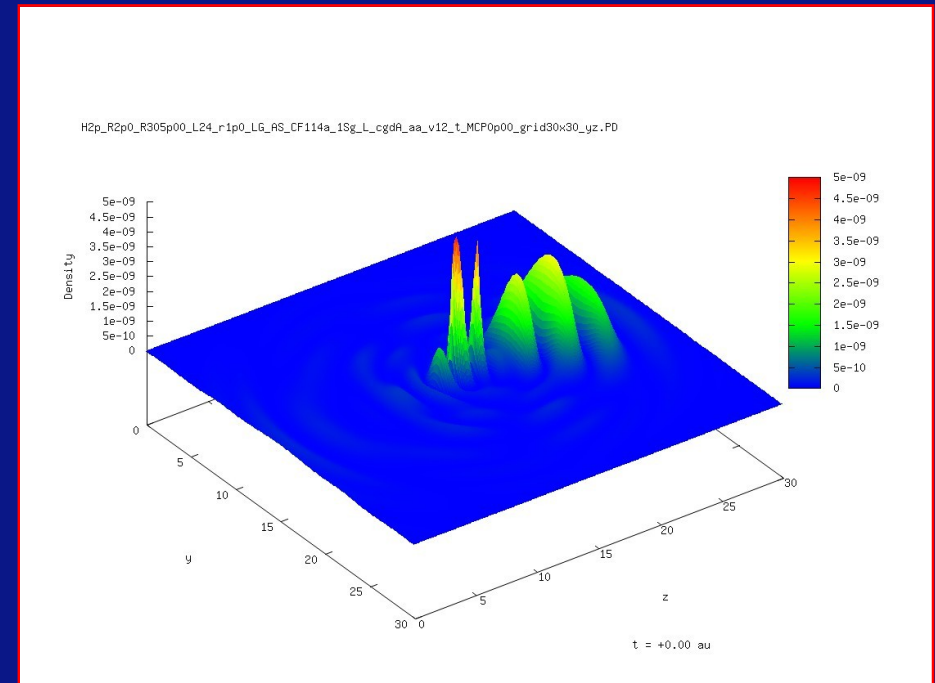
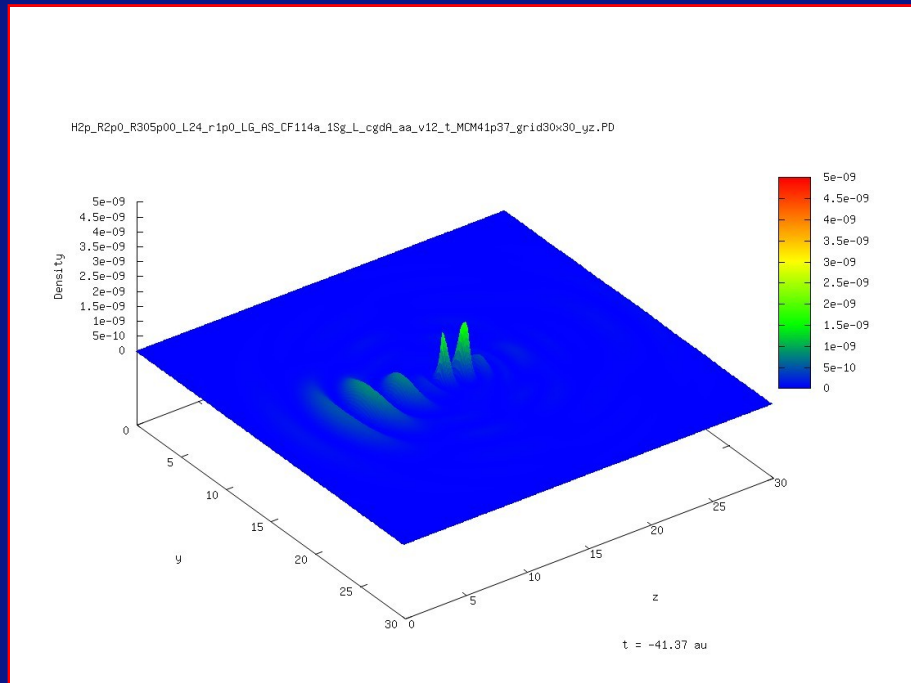
Hydrogen atom

Laser parameters: 1300 nm; 6 cycles;  $\cos^2$ ;  $I_{\max} = 10^{14}$  W/cm<sup>2</sup>.

**Direct electrons:** 0 to about 2 times the ponderomotive energy  
 $U_p = I/(4\omega^2)$ .

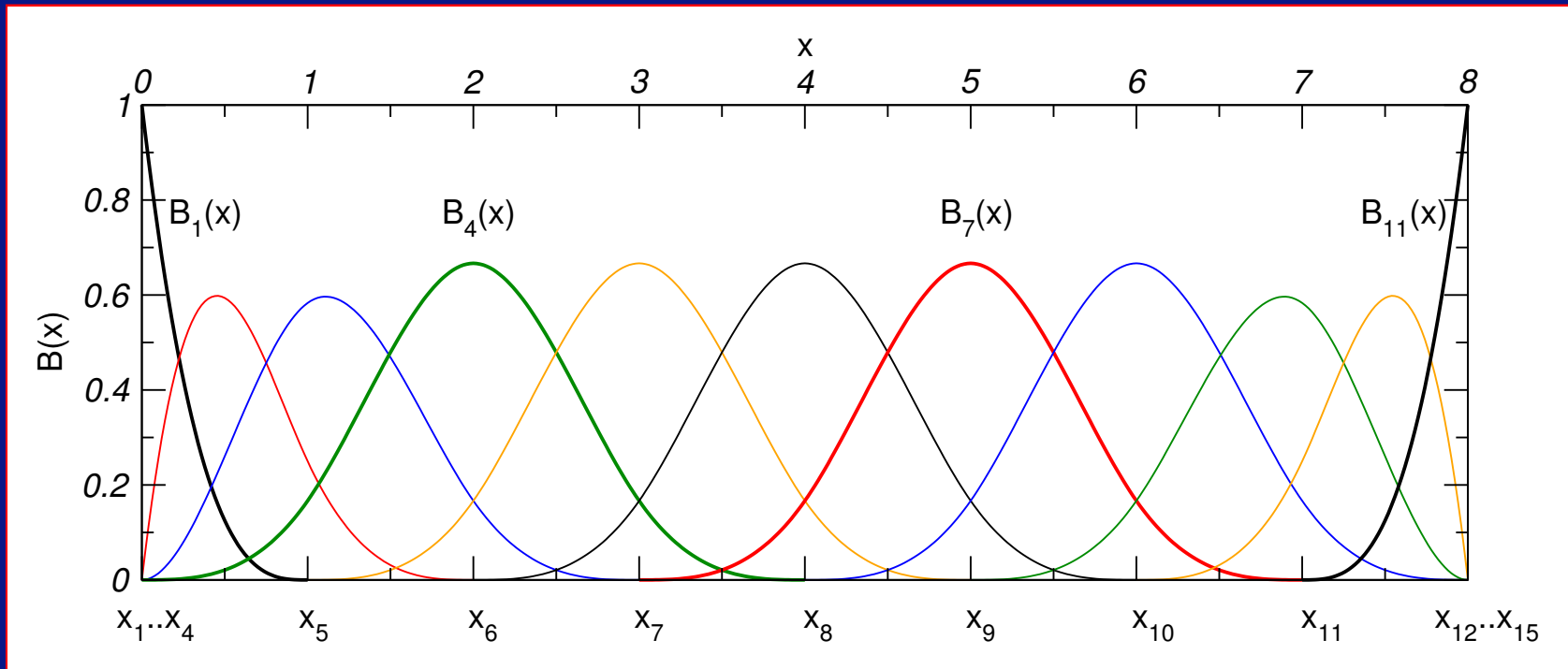
**Rescattered electrons:** dominate spectrum beyond  $2 U_p$ .

# Example electronic wavepacket ( $H_2^+$ )



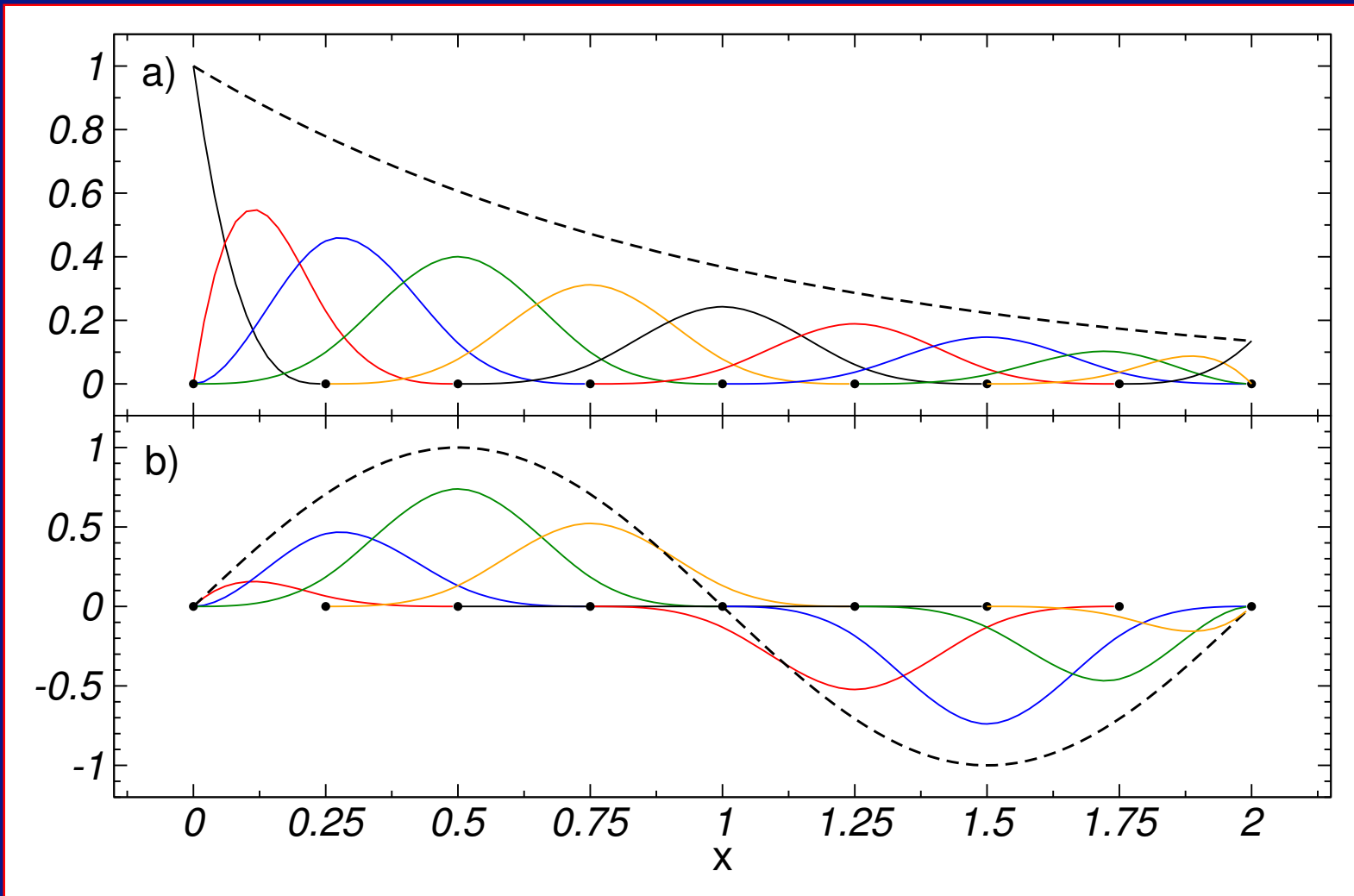
Electronic wavepacket at two different times within a 2-cycle laser pulse.  
(Only the continuum part is shown.)

## B-spline properties (I)



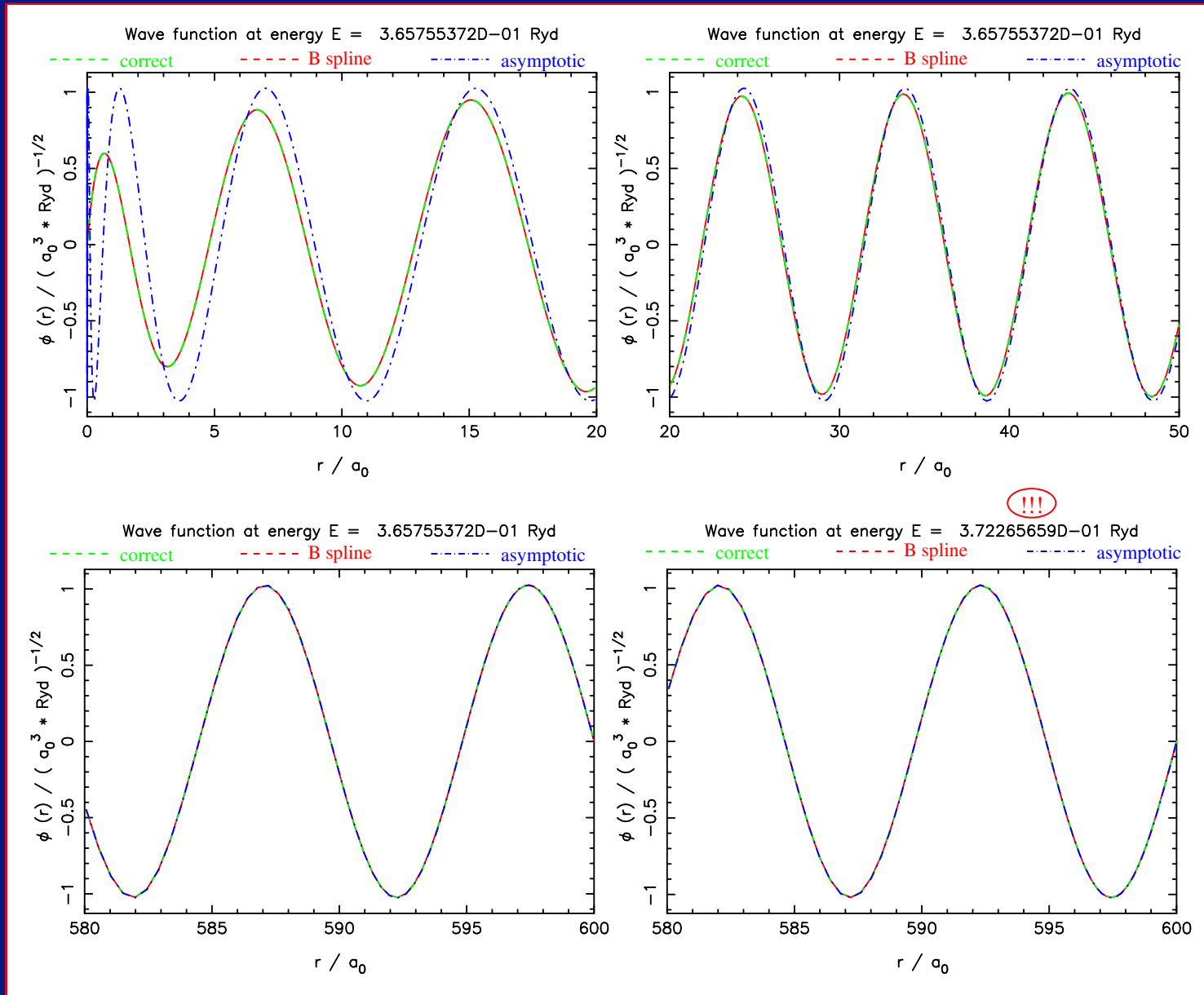
All 11  $B$  splines of order  $k = 4$  for knot sequence  $\{t_i\} = \{0, 0, 0, 0, 1, 2, 3, 4, 5, 6, 7, 8, 8, 8, 8\}$ .

## *B*-spline properties (II)

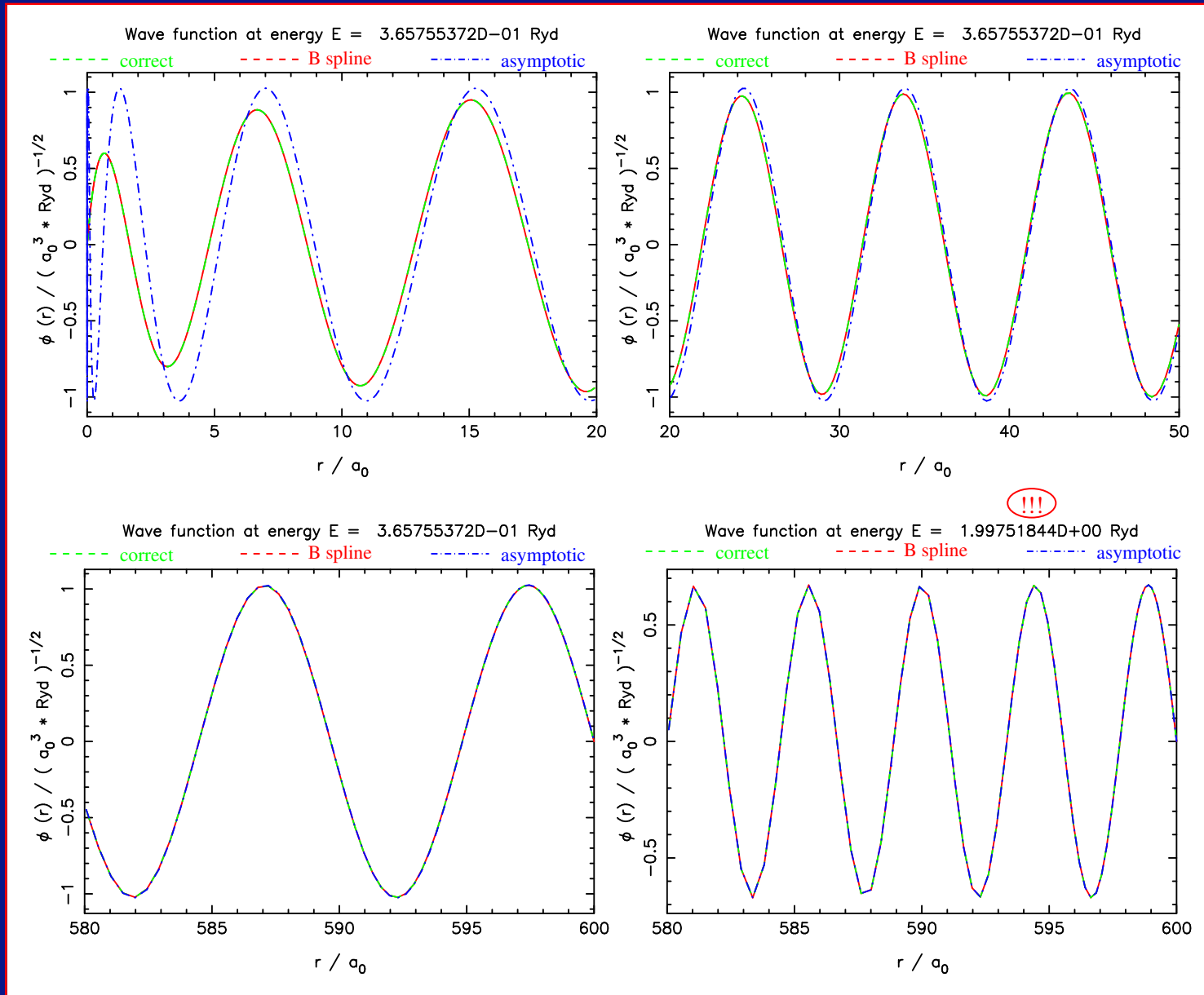


Fitting (a)  $e^{-x}$  or (b)  $\sin(\pi x)$  with *B* splines (order  $k = 4$  and  $s = 8$  knot points).

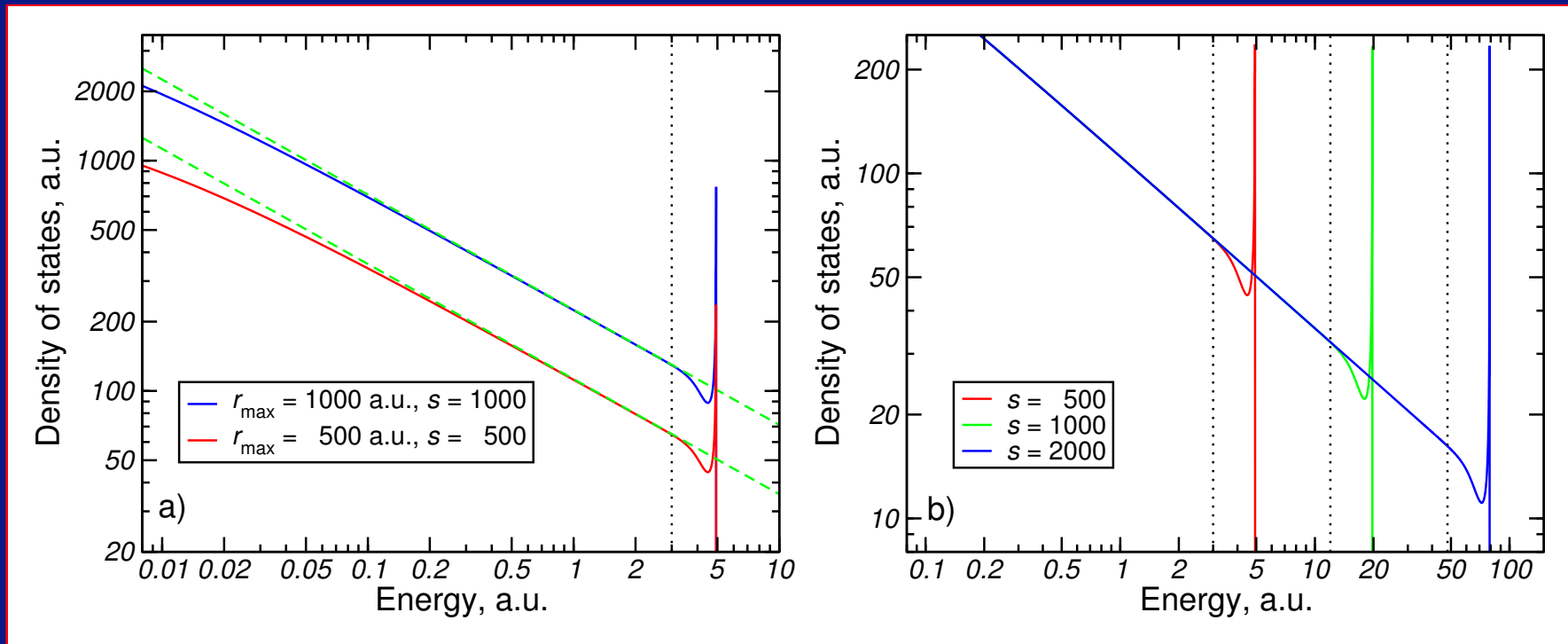
# Example: continuum wavefunctions for H atom (I)



# Example: continuum wavefunctions for H atom (II)



# Box discretization with $B$ splines



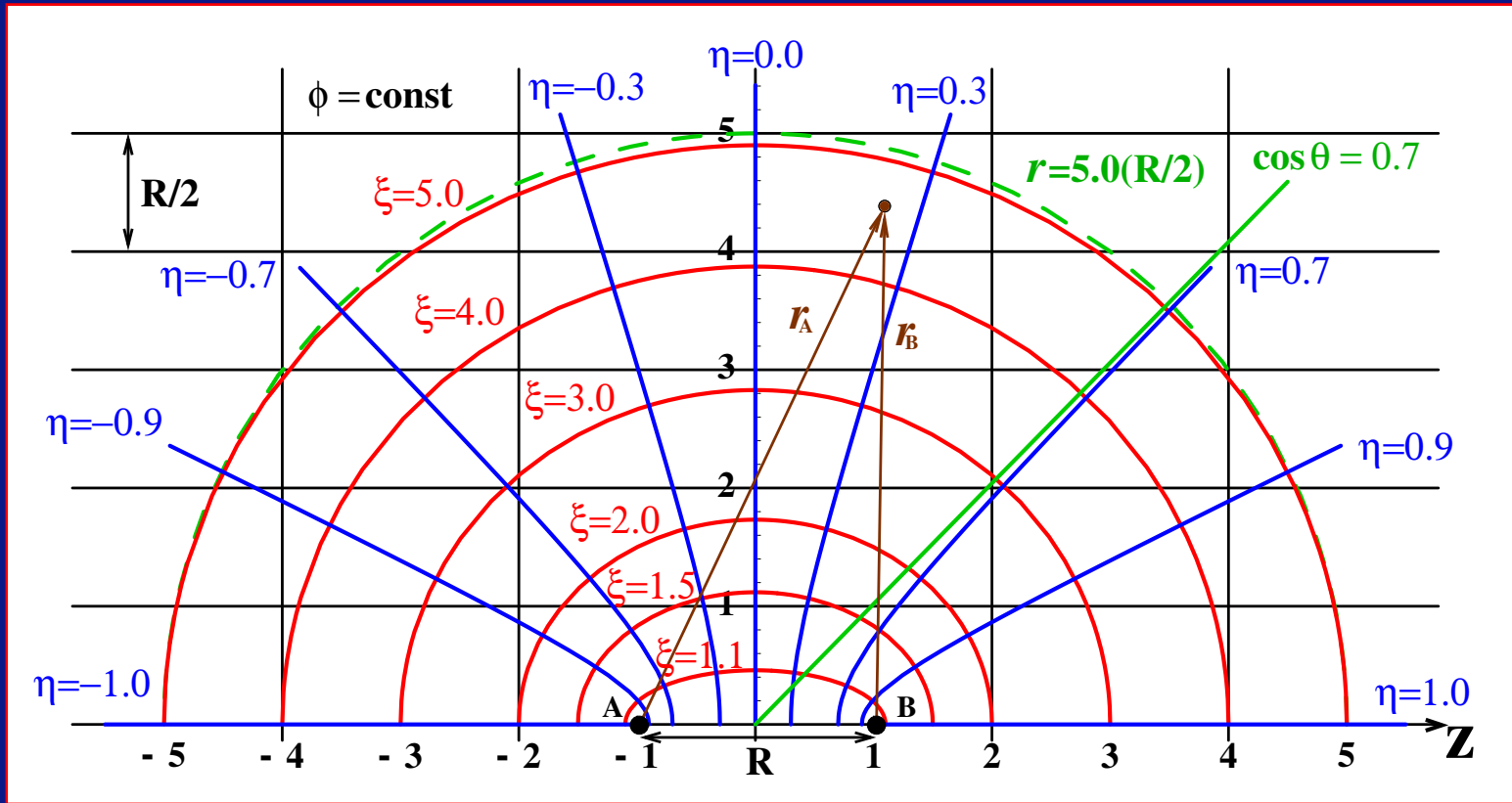
Size of the (radial) “box”:  $r_{\max}$

Number of (radial) knot points:  $s$

(a): same knot spacing;

(b): variable knot spacing ( $r_{\max} = 500 a_0$ ).

# Prolate spheroidal coordinates (for diatomics)



$$\xi \in [1, \infty)$$

$$\eta \in [-1, 1]$$

$$\phi \in [0, 2\pi)$$

$$\xi = \frac{r_A + r_B}{R}$$

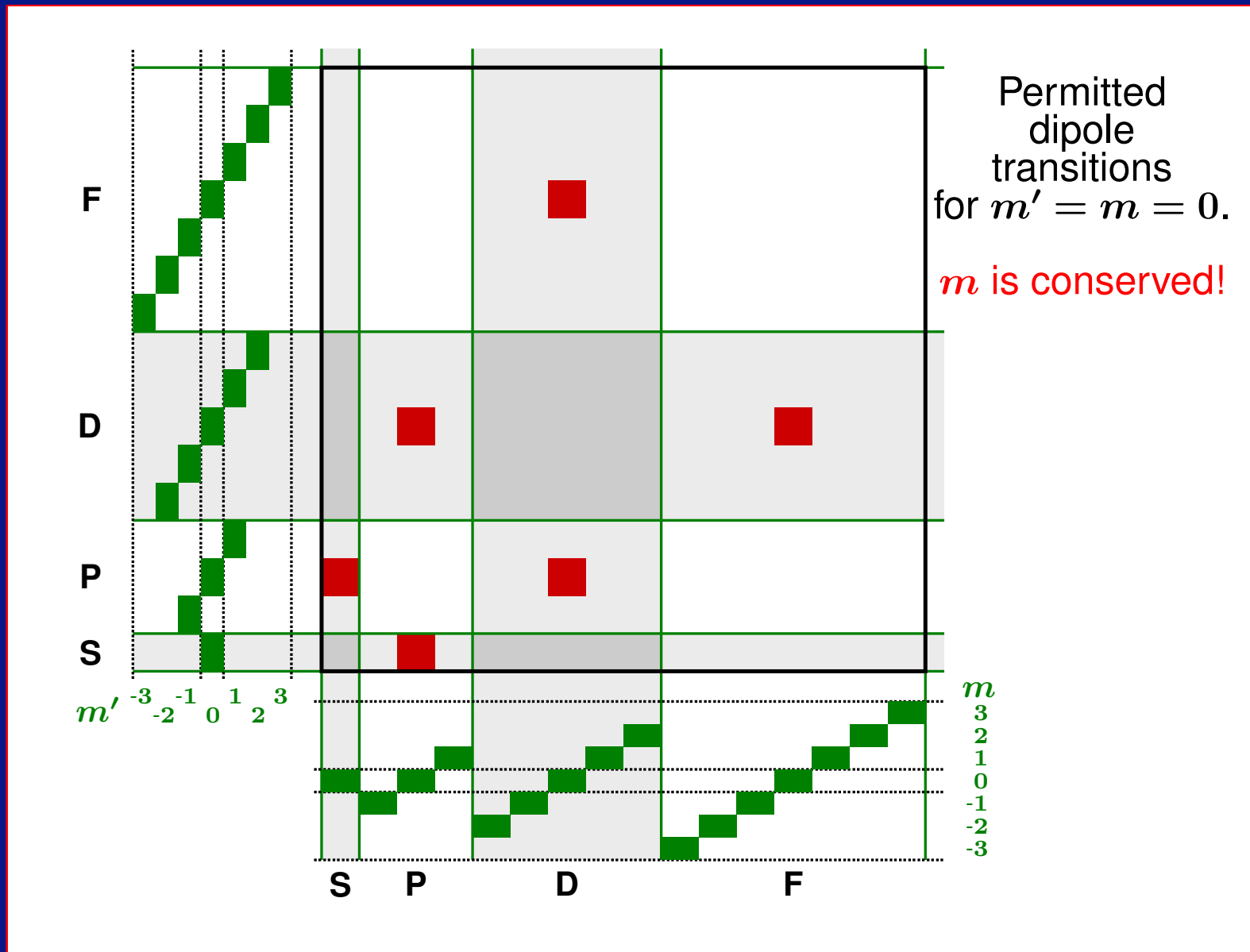
$$\eta = \frac{r_A - r_B}{R}$$

$$r = \frac{R}{2} \sqrt{\xi^2 + \eta^2 - 1} \quad \xrightarrow{\xi \gg 1} \quad \frac{R}{2} \xi$$

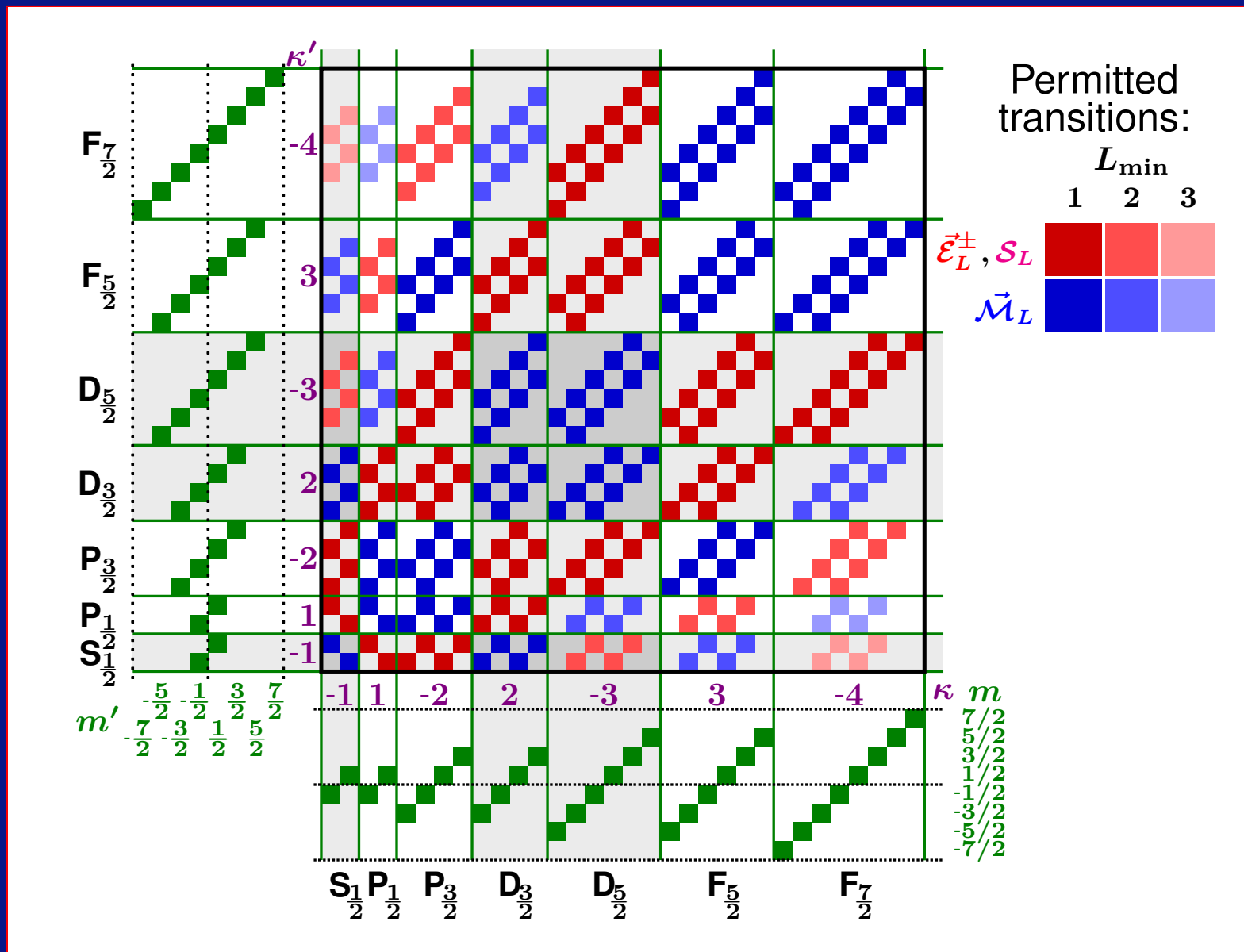
$$\cos \theta = \frac{\xi \eta}{\sqrt{\xi^2 + \eta^2 - 1}} \quad \xrightarrow{\xi \gg 1} \quad \eta$$



# Transitions within non-relativistic dipole approximation



# Transitions within relativistic beyond-dipole treatment



# Normalization of continuum states

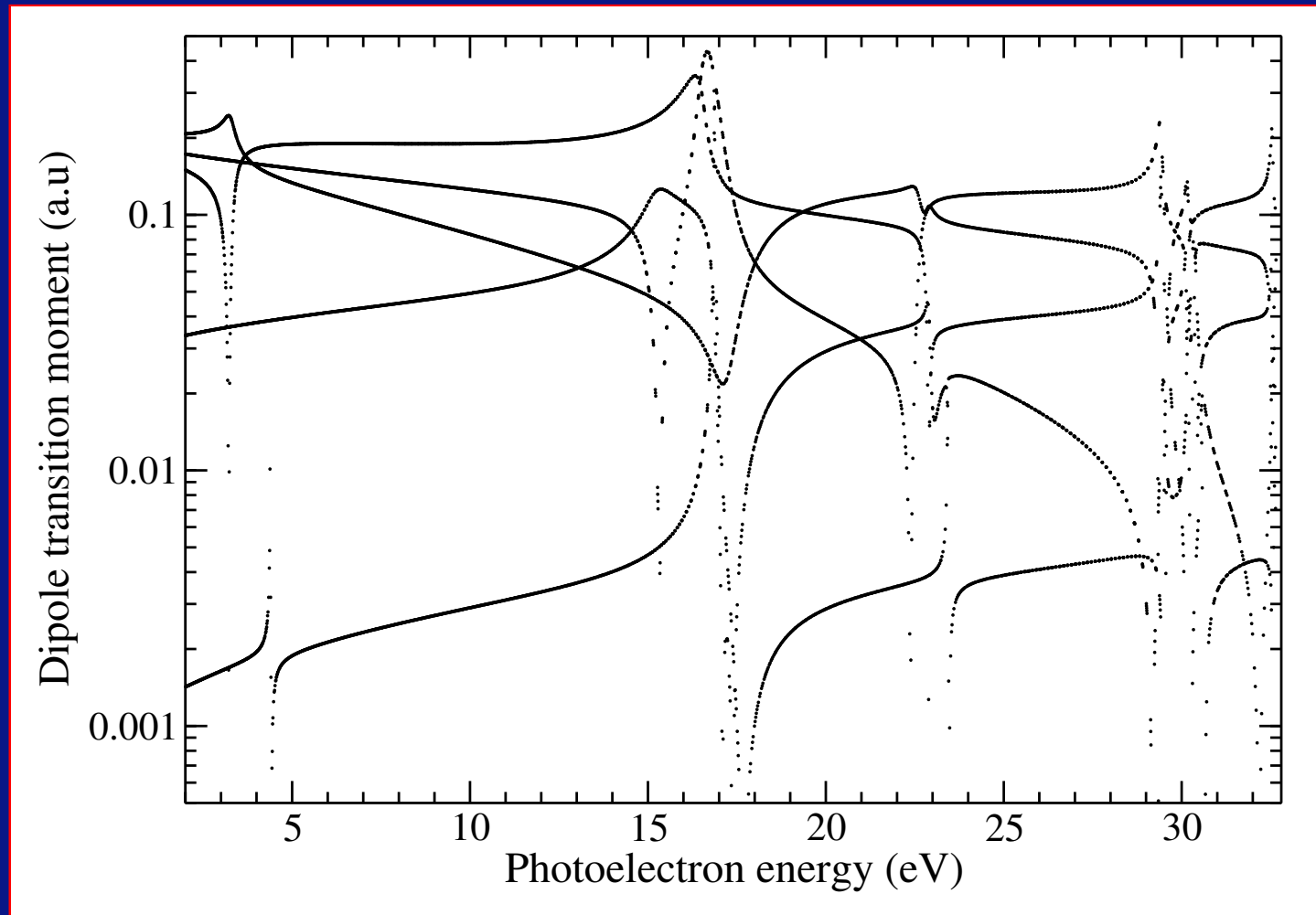
## Uncoupled continuum states:

- separable potential ( $H_2^+$ ): channels are separately obtained,
- normalization (of originally) box-discretized states via density of states or asymptotic behaviour.

## Coupled continuum states:

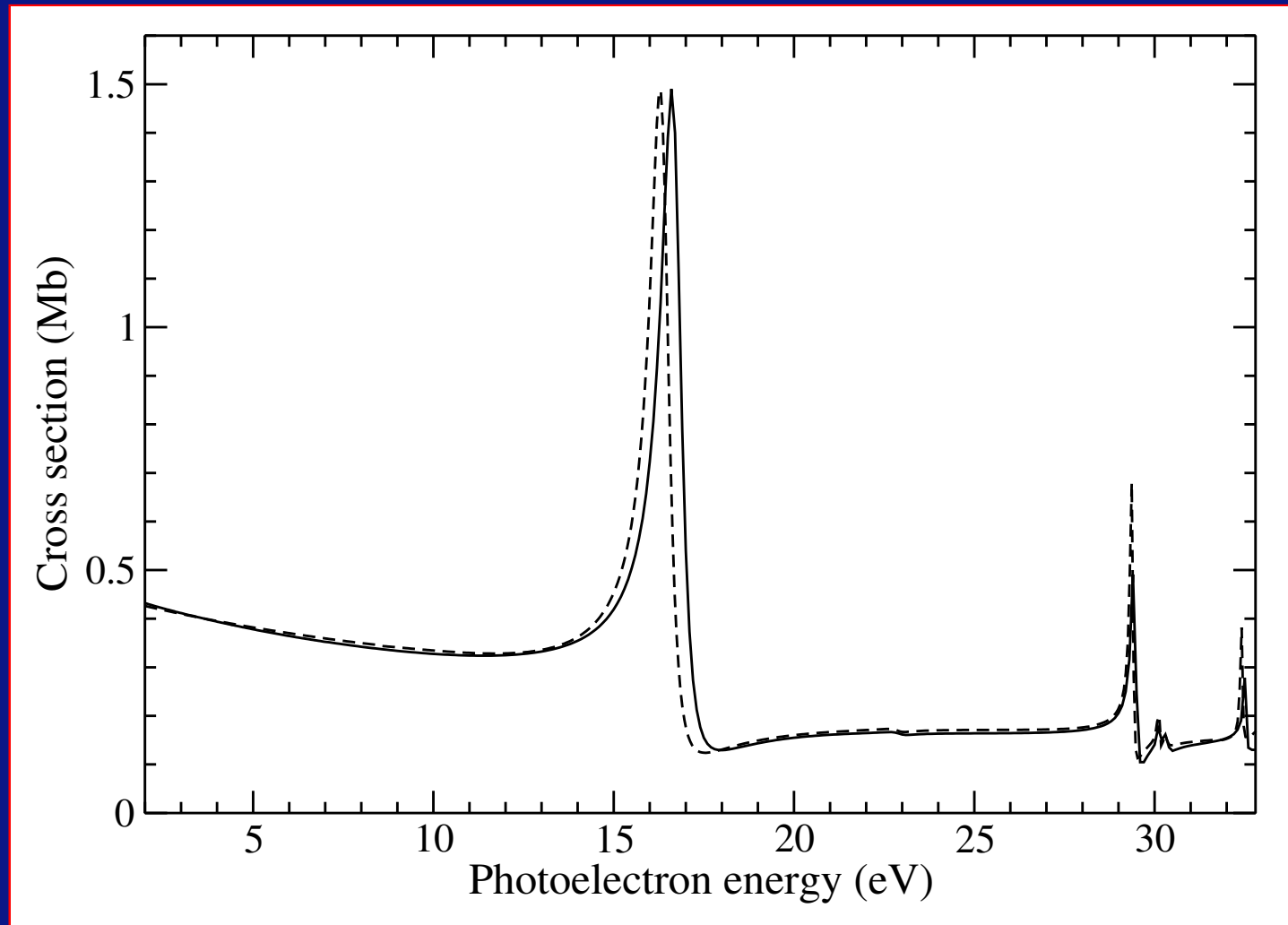
- non-separable potential (like  $Na_2^+$ ) or two-electron case,
- **Note:** this differs from atoms (and larger molecules) where the electron-electron interaction does not break a symmetry!
- analysis of leading configurations (two-electron case): [Apalategui & Saenz, *J. Phys. B* **35**, 1909 (2002)].
- asymptotic analysis in terms of linear combinations of spherical harmonics (more robust): [Vanne & Saenz, *J. Phys. B* **37**, 4101 (2004)].

# Continuum transition moments for HeH<sup>+</sup>



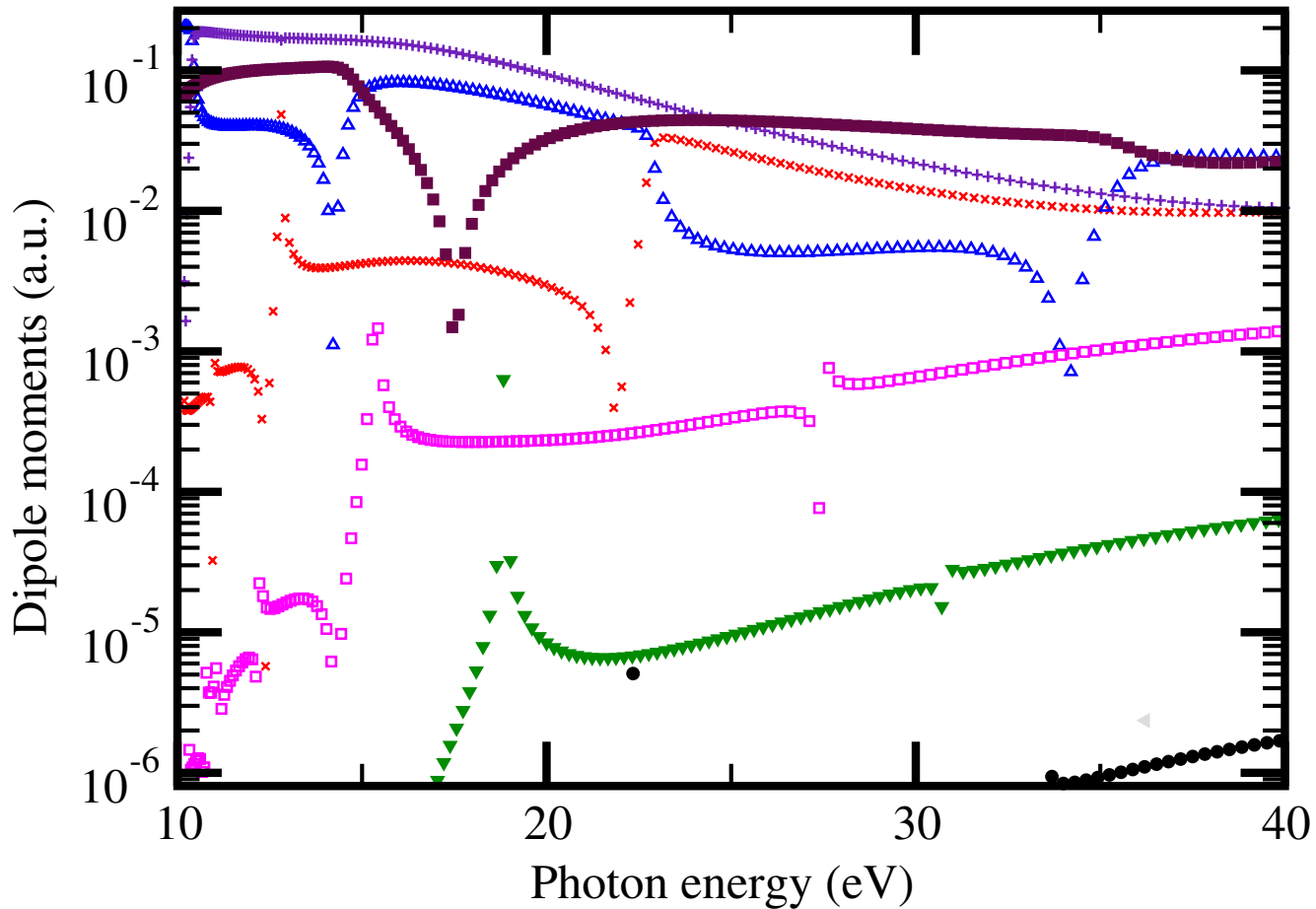
$$X^1\Sigma \rightarrow ^1\Sigma (R = 1.45 a_0)$$

# Partial photoionization cross-section for $\text{HeH}^+$



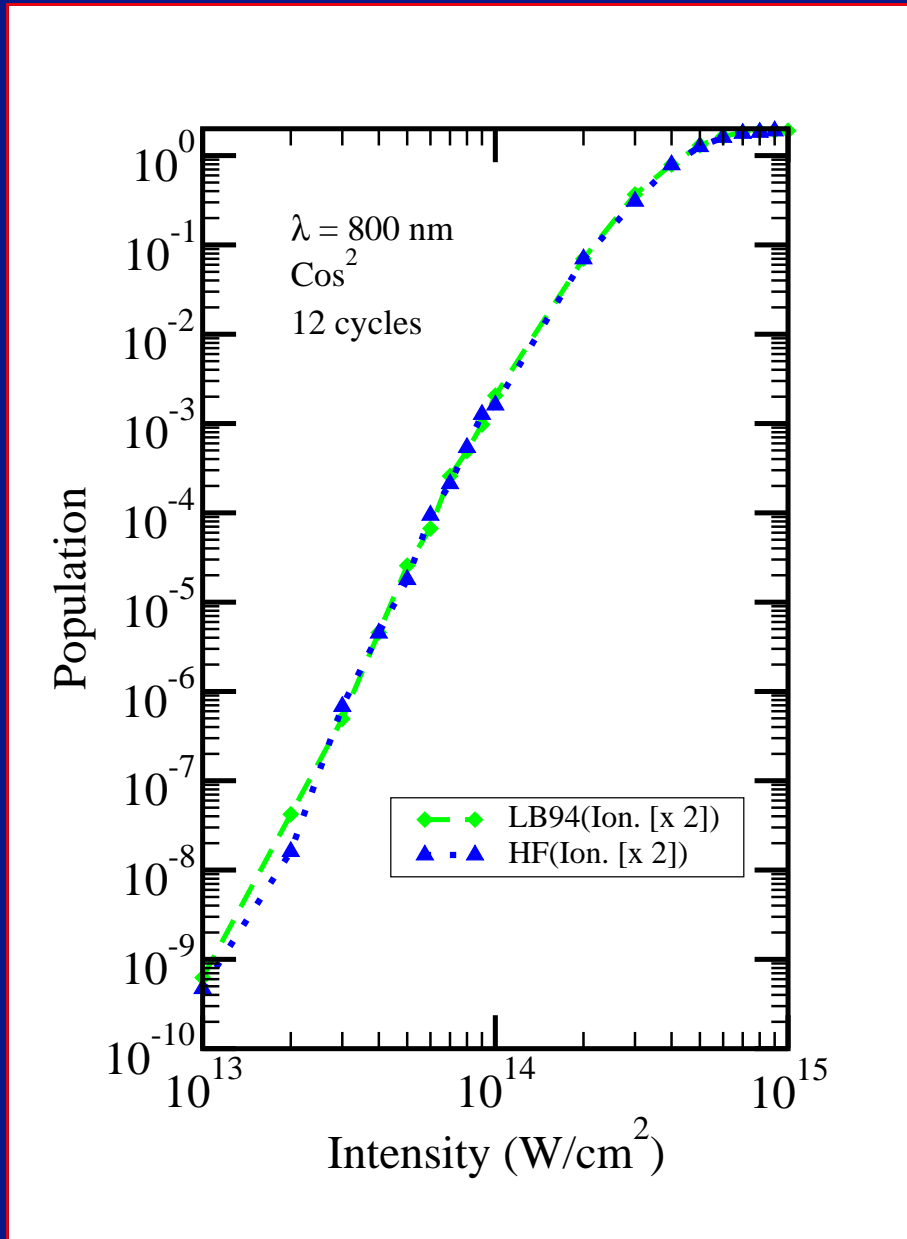
**Solid:** this method, **dashed:** explicitly-correlated basis functions + CSM [Saenz, *Phys. Rev. A* **67**, 033409 (2003)].

# Continuum transition moments for $\text{Na}_2^+$



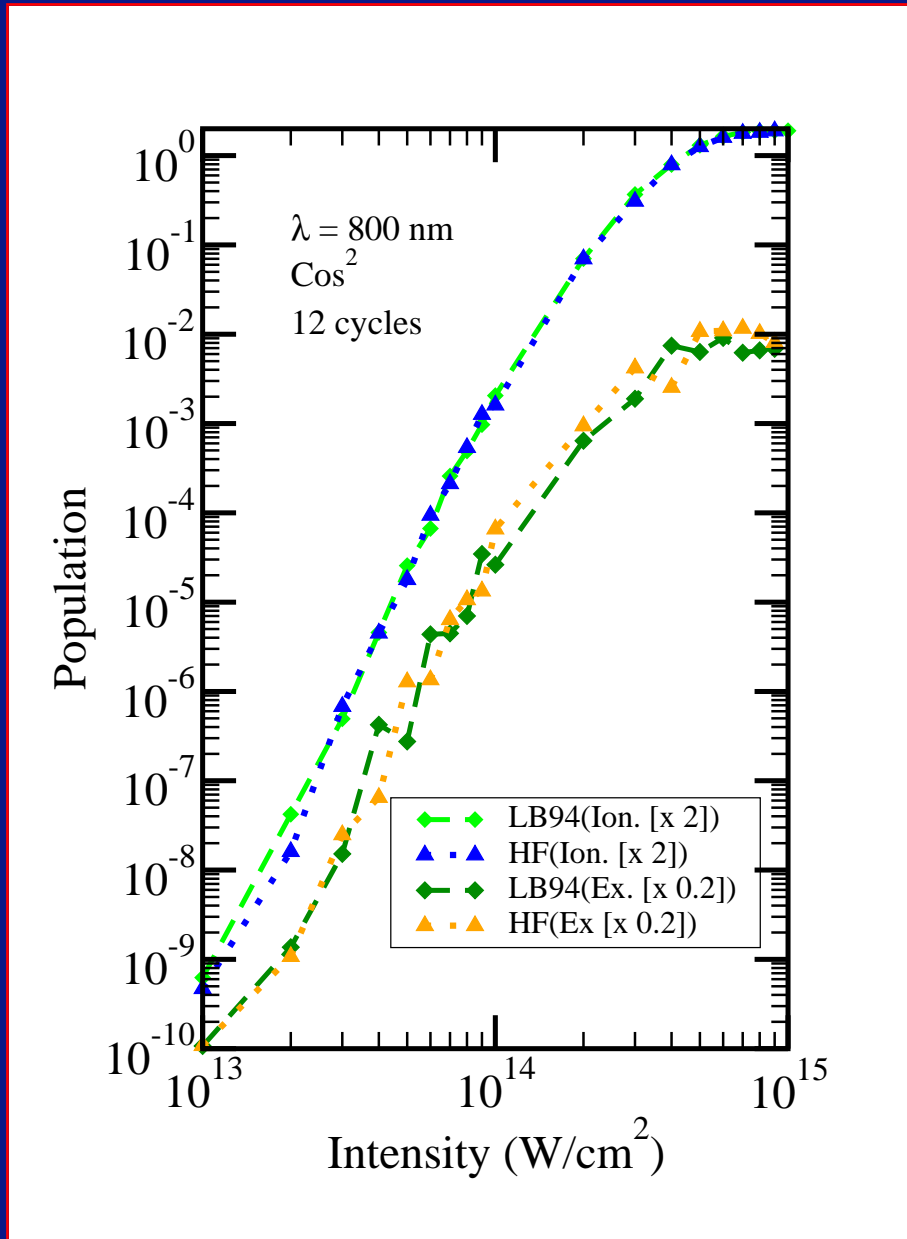
$$X^2\Sigma_g \rightarrow ^2\Sigma_u (R = 6.75 a_0)$$

## H<sub>2</sub>: Hartree-Fock vs. DFT core (ionization)



M. Awasthi et al.  
PRA **77**, 063403 (2008)

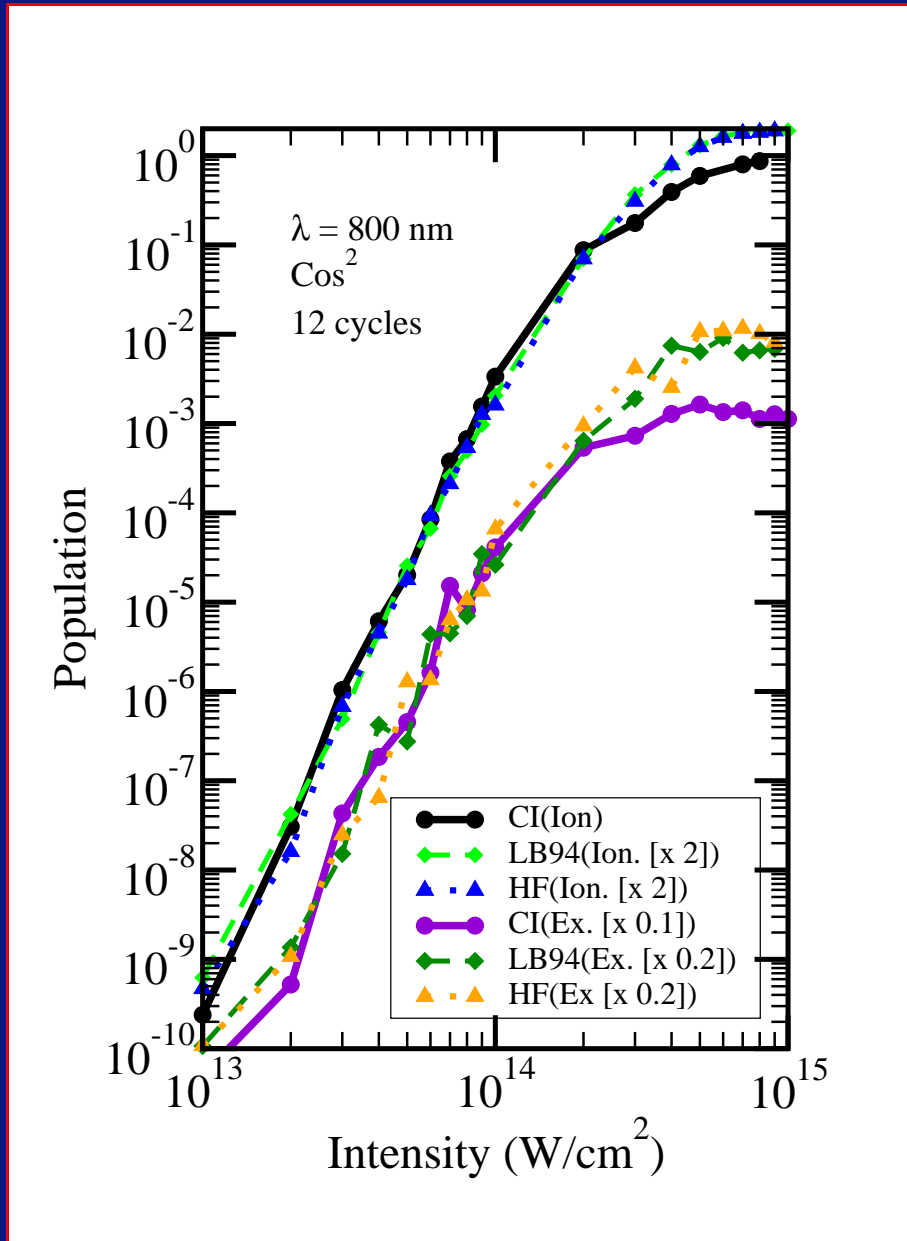
## H<sub>2</sub>: Hartree-Fock vs. DFT core (excitation)



M. Awasthi et al.  
PRA 77, 063403 (2008)

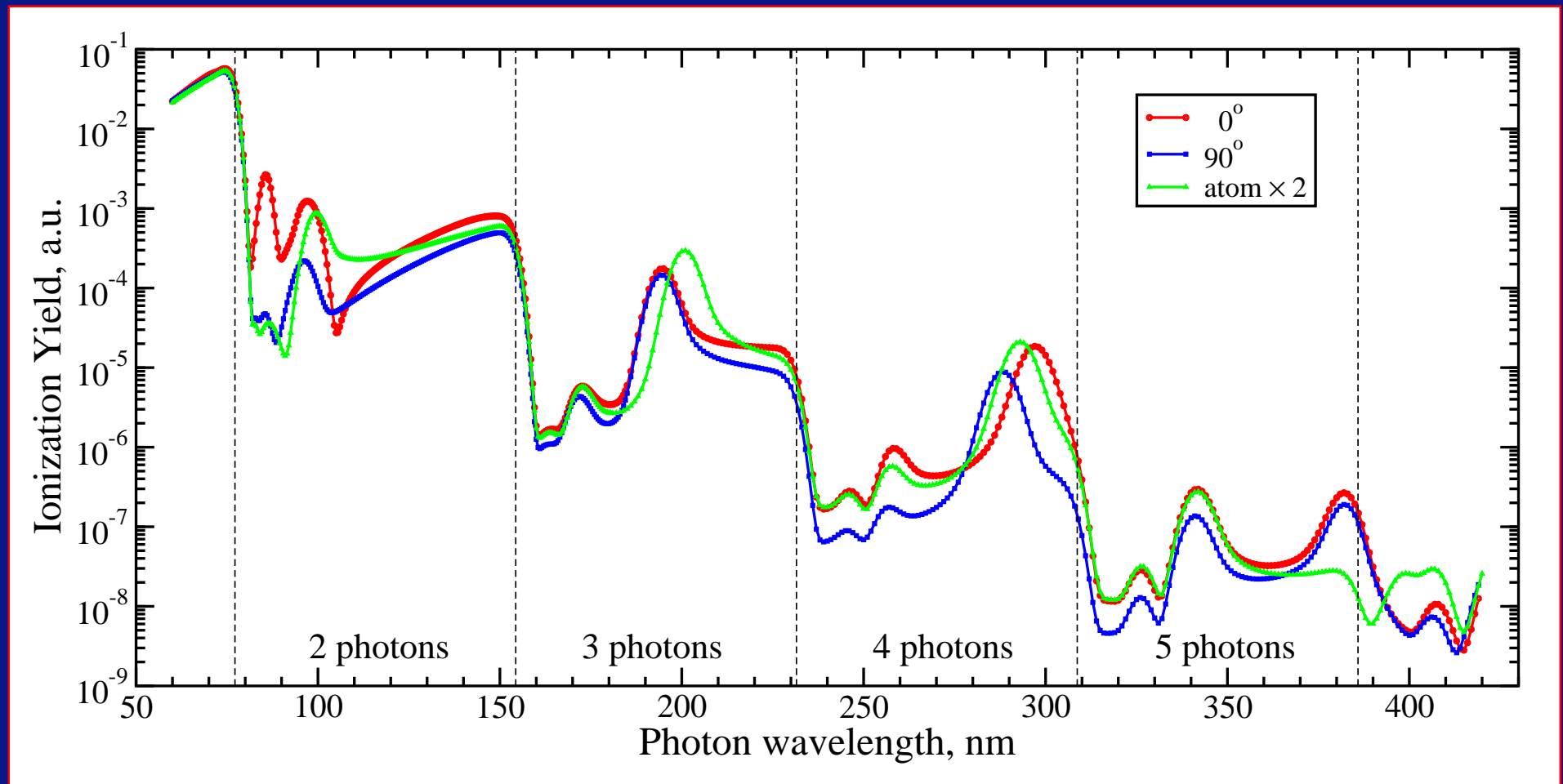


# Validity of the SAE approximation for H<sub>2</sub>



M. Awasthi et al.  
PRA 77, 063403 (2008)

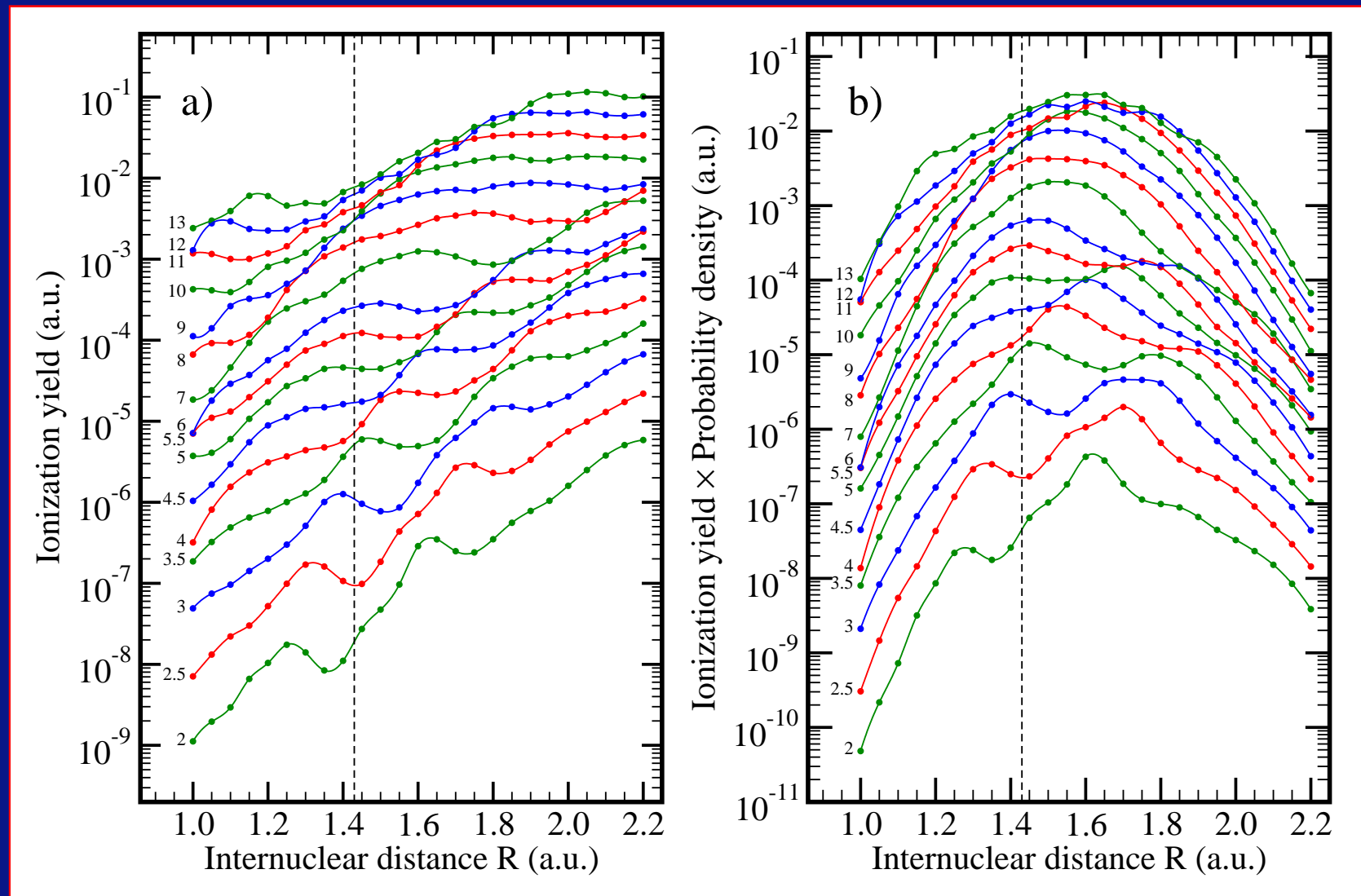
## 6D: Orientational dependent ion yield of H<sub>2</sub> ( $R = 1.4 a_0$ )



**Laser field:** 30-cycle ( $\cos^2$ ) pulses with peak intensity  $I = 5 \cdot 10^{12} \text{ W/cm}^2$ .

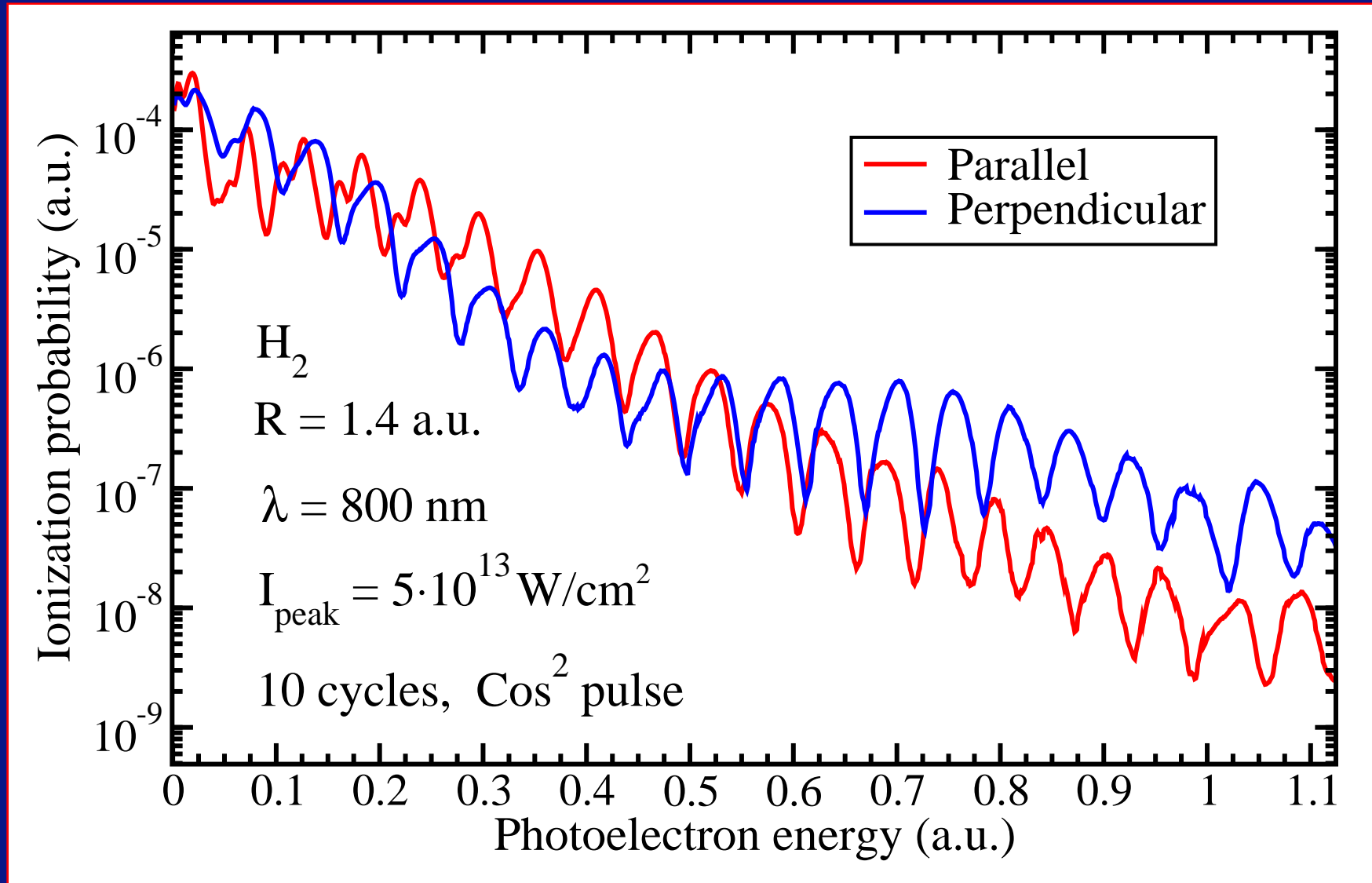
[Y. V. Vanne and A. Saenz, *J. Mod. Optics* **55**, 2665 (2008).]

# Internuclear-distance dependent ion yields of $H_2$ (800 nm, perp.)

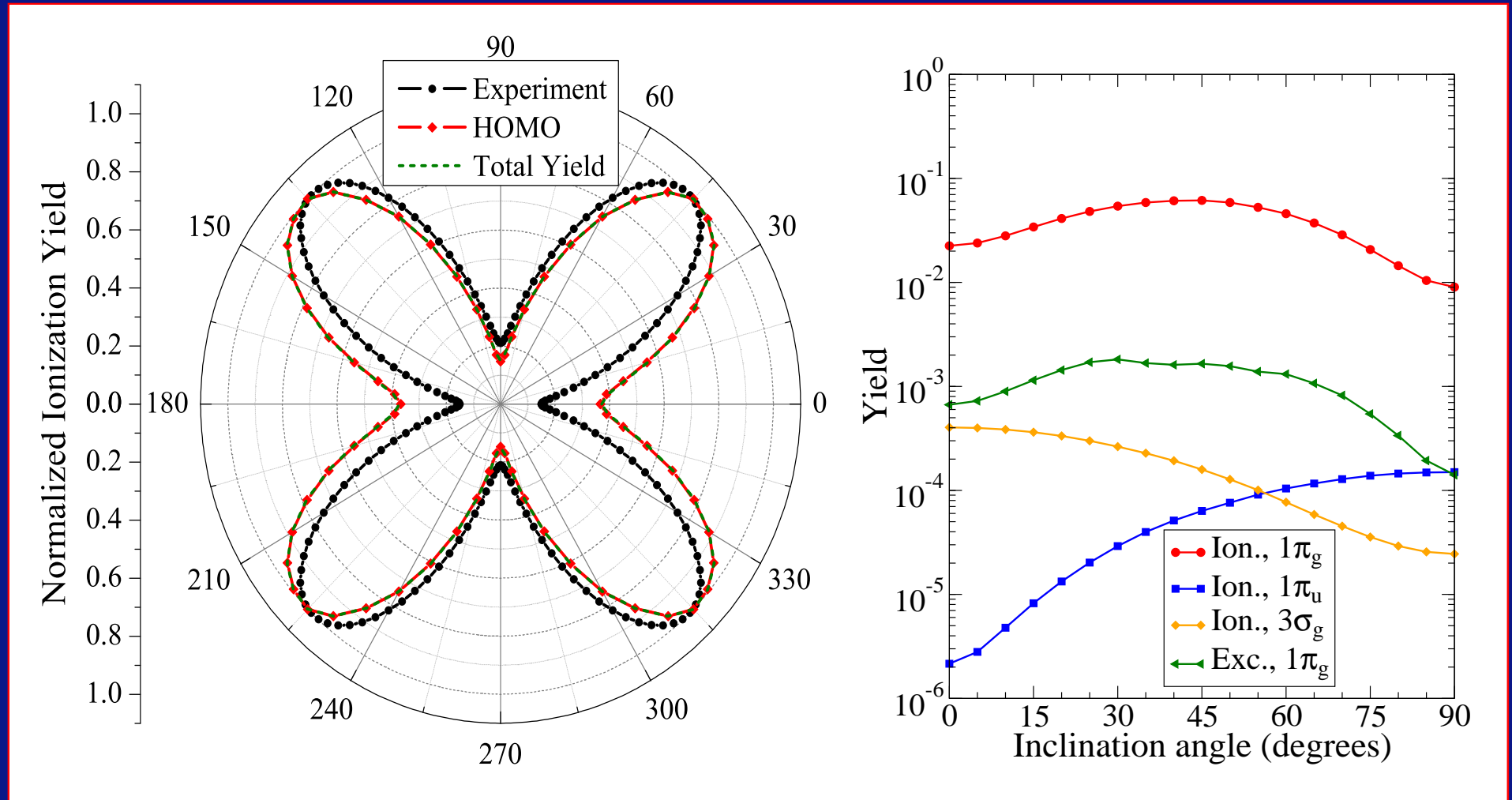


[for method see: Y.V. Vanne and A. Saenz, J. Modern Optics **55**, 2655 (2008); Phys. Rev. A **80**, 053422 (2009)]

# Energy-resolved electron spectra (ATI)

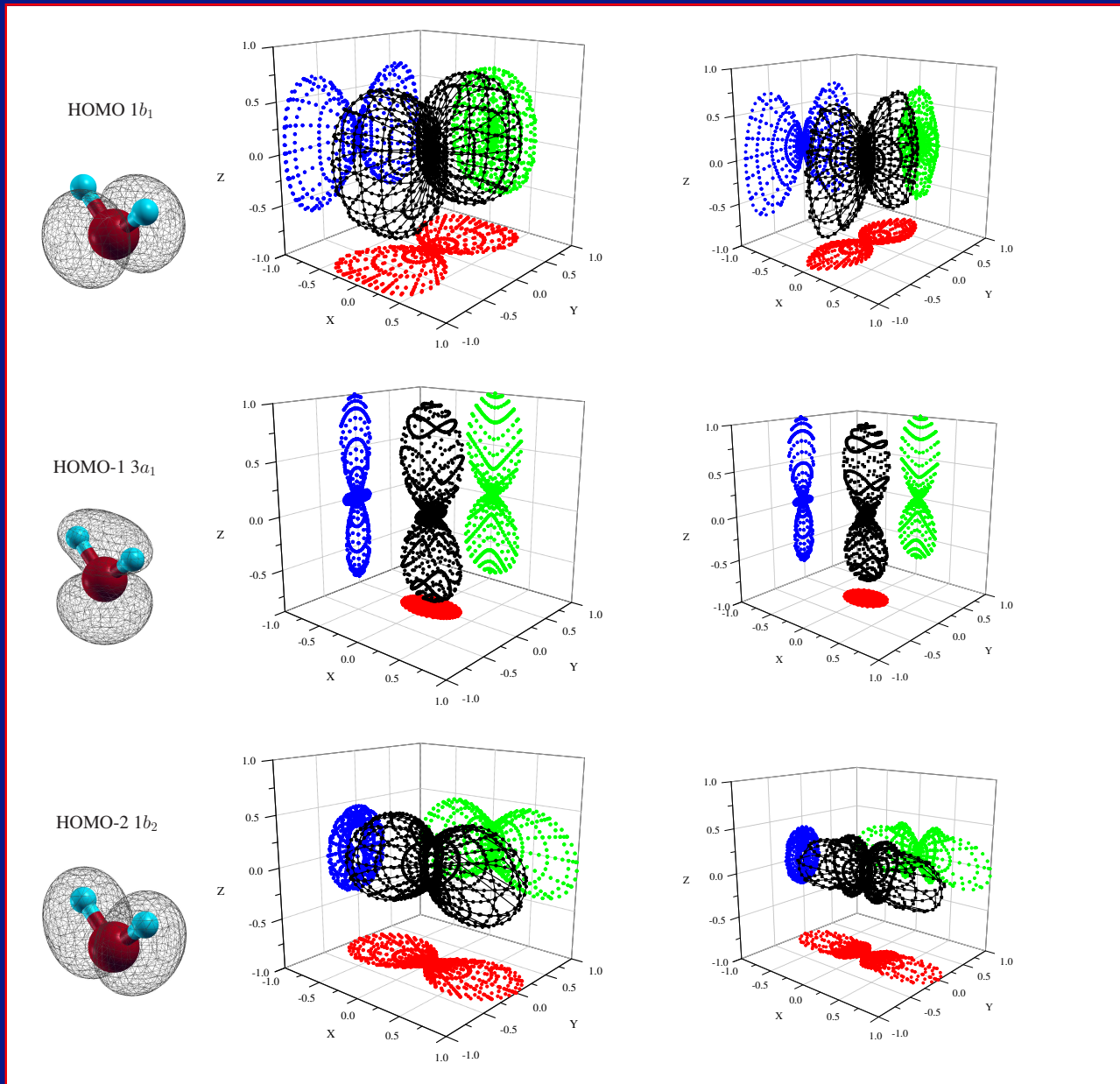


# Imaging (I): Orientational-dependent ionization of O<sub>2</sub> molecules



[Exp.: Pavicic et al., PRL **98**, 243001 (2007); Theory: Petretti et al., PRL **104**, 223001 (2010)]

# Imaging (II): Orientation-dependent ionization of H<sub>2</sub>O molecules



## Imaging:

enforced inversion symmetry.

## Short pulses:

carrier-envelope effects  
(interesting by itself),  
but limits time resolution!

[*Chem. Phys.* (2012)]