

# Dispersion anomalies in Cuprate Superconductors

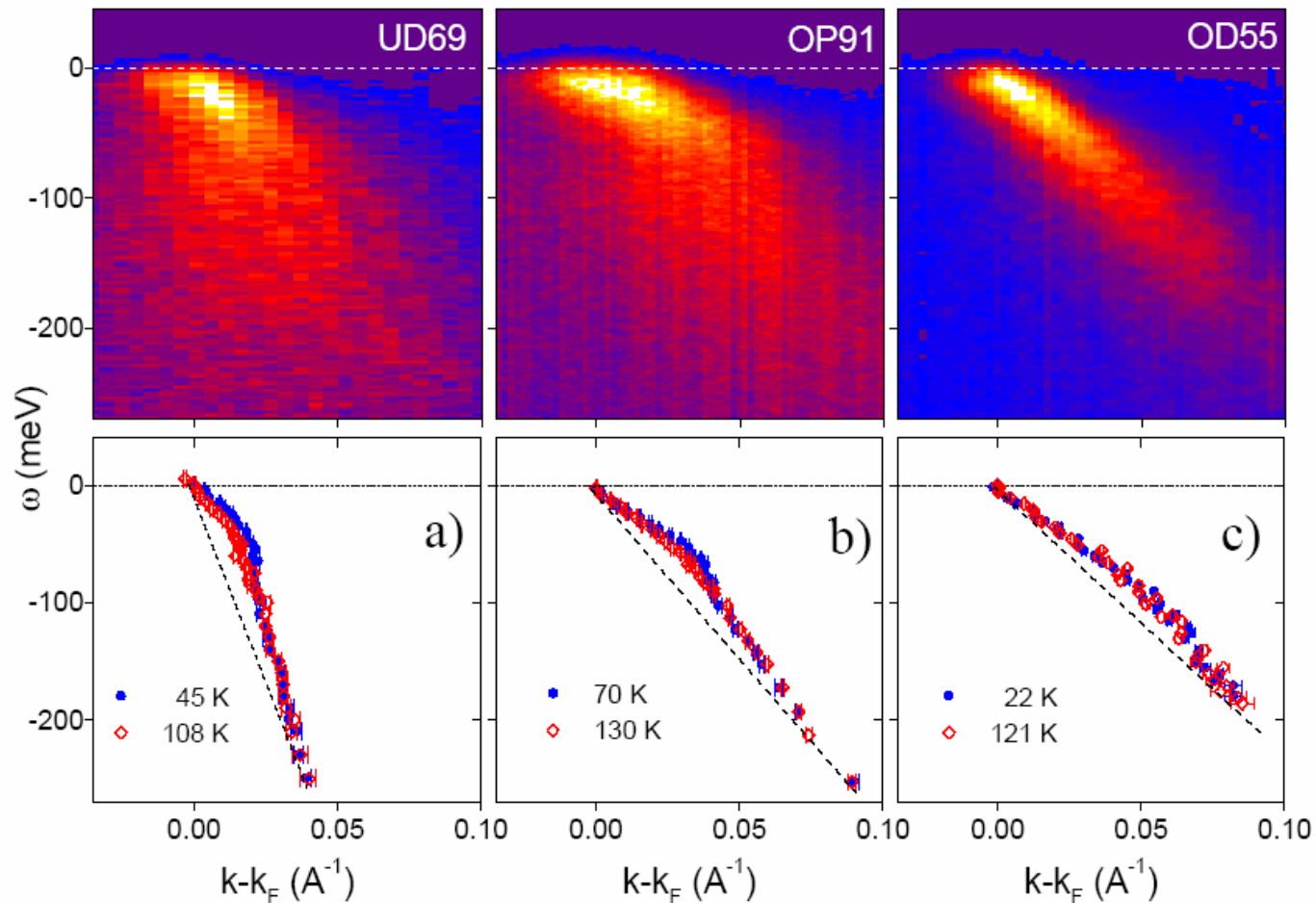
**Andrey Chubukov**

**University of Maryland**

**Mike Norman**

**Argonne National Lab**

# kink in the dispersion

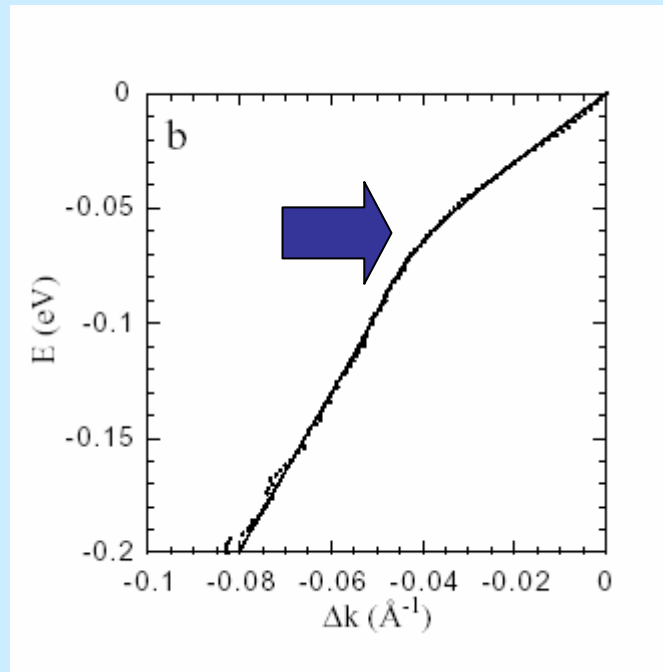


Nodal direction, Bi2212

P.D. Johnson, T. Valla, ...

# kink in the dispersion

Nodal direction



T. Valla et al,  
P. Bogdanov et al,  
A. Kaminski et al,  
A. Lanzara et al,  
P.D. Johnson et al,  
T. Sato et al,  
T.K. Kim et al,  
A.D. Gromko et al

Spin fluctuations or phonons?

# A simple theory – fermions coupled to a bosonic mode

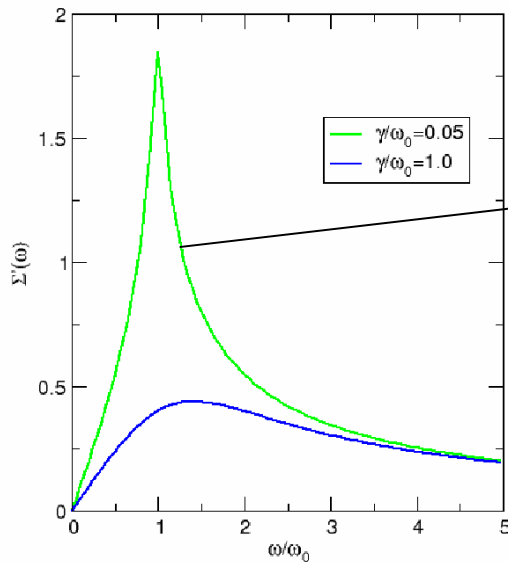
What should we expect?

$$\chi(\omega) = \frac{1}{(\omega + i\gamma)^2 - \omega_0^2}$$

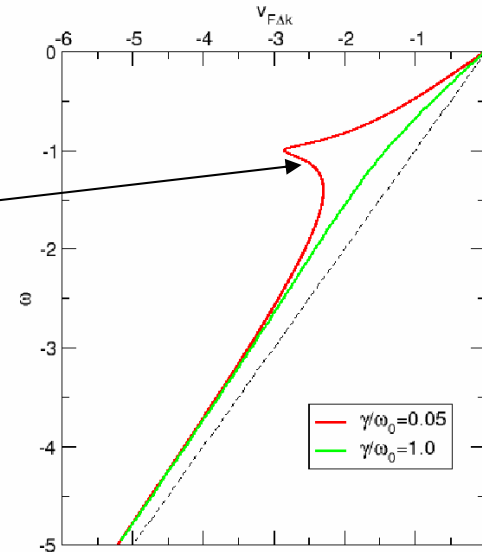
$$\Sigma(\omega) = \text{[Feynman diagram: a blue wavy boson line loop connected to a green fermion line with an arrow]} = \lambda T \sum_{\Omega} \int d\vec{q} \chi(\Omega) G(\vec{k} + \vec{q}, \omega + \Omega)$$

# Normal state

## The self-energy



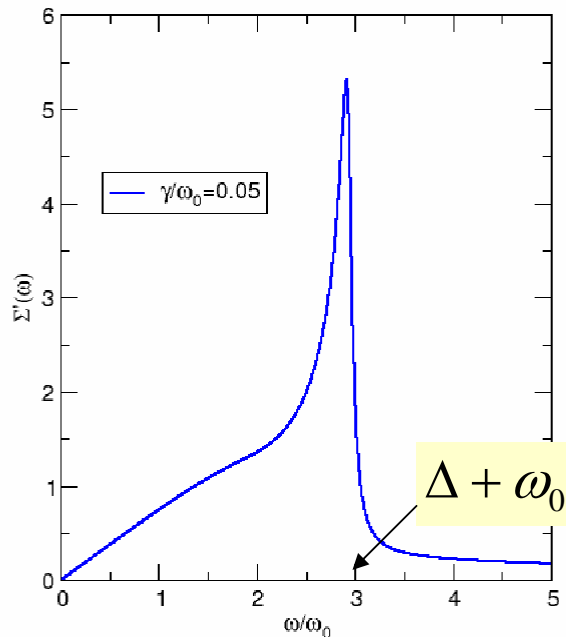
## The dispersion



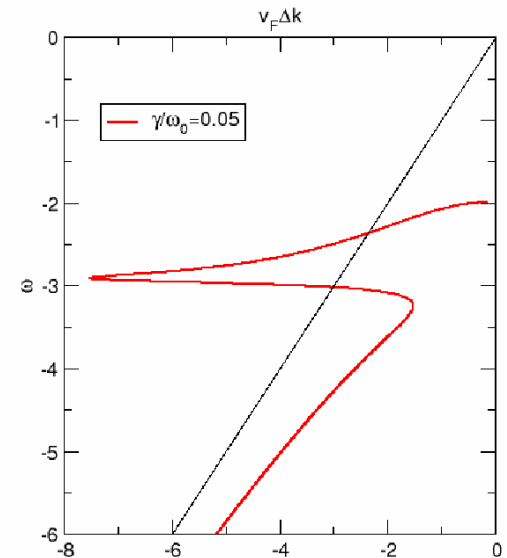
- the dispersion has an S-shape
- an S-shape can be eliminated by a large damping
- a large damping also reduces mass renormalization

# Superconducting state

## The self-energy

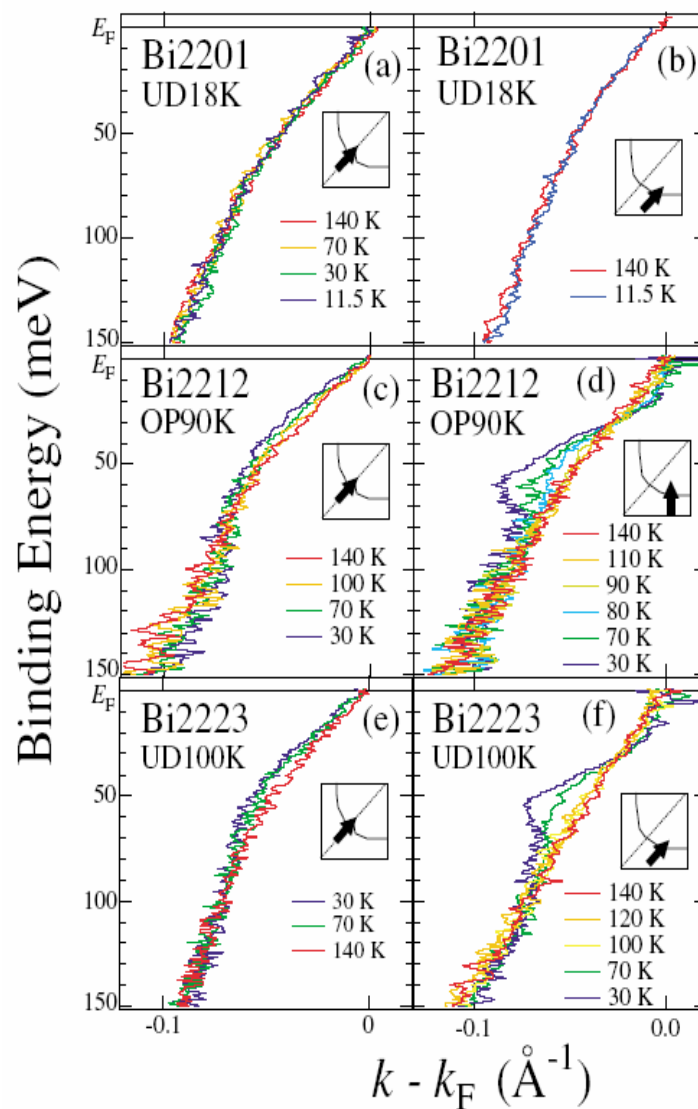
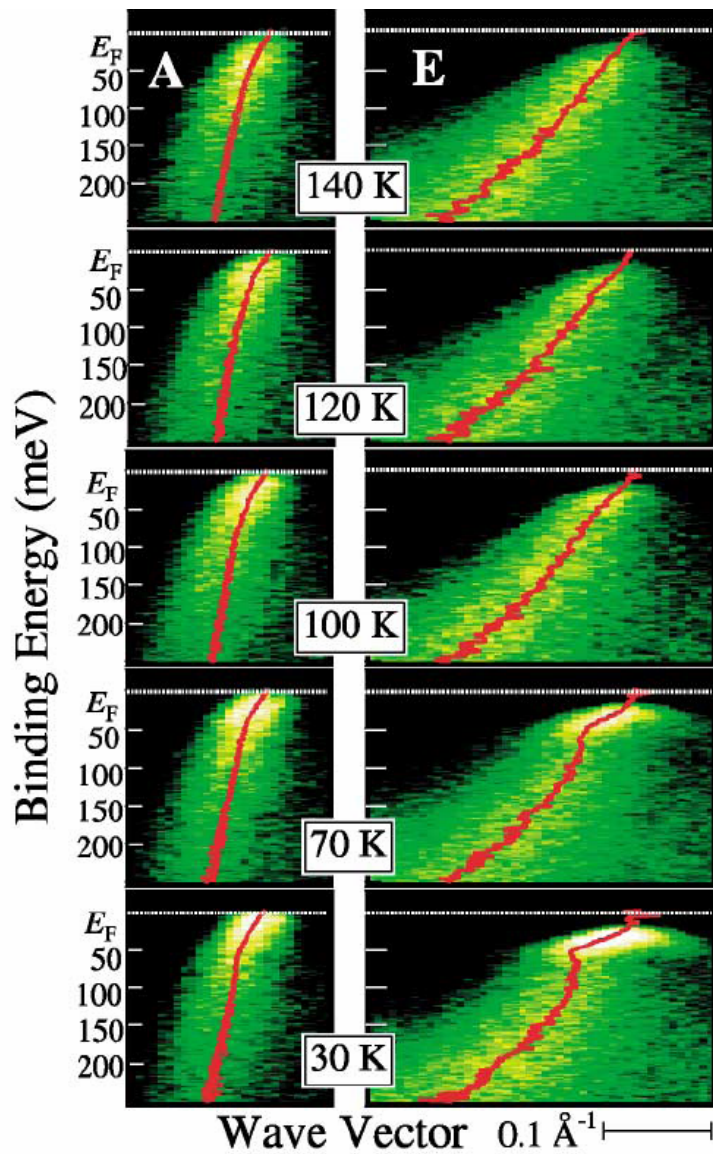


## The dispersion



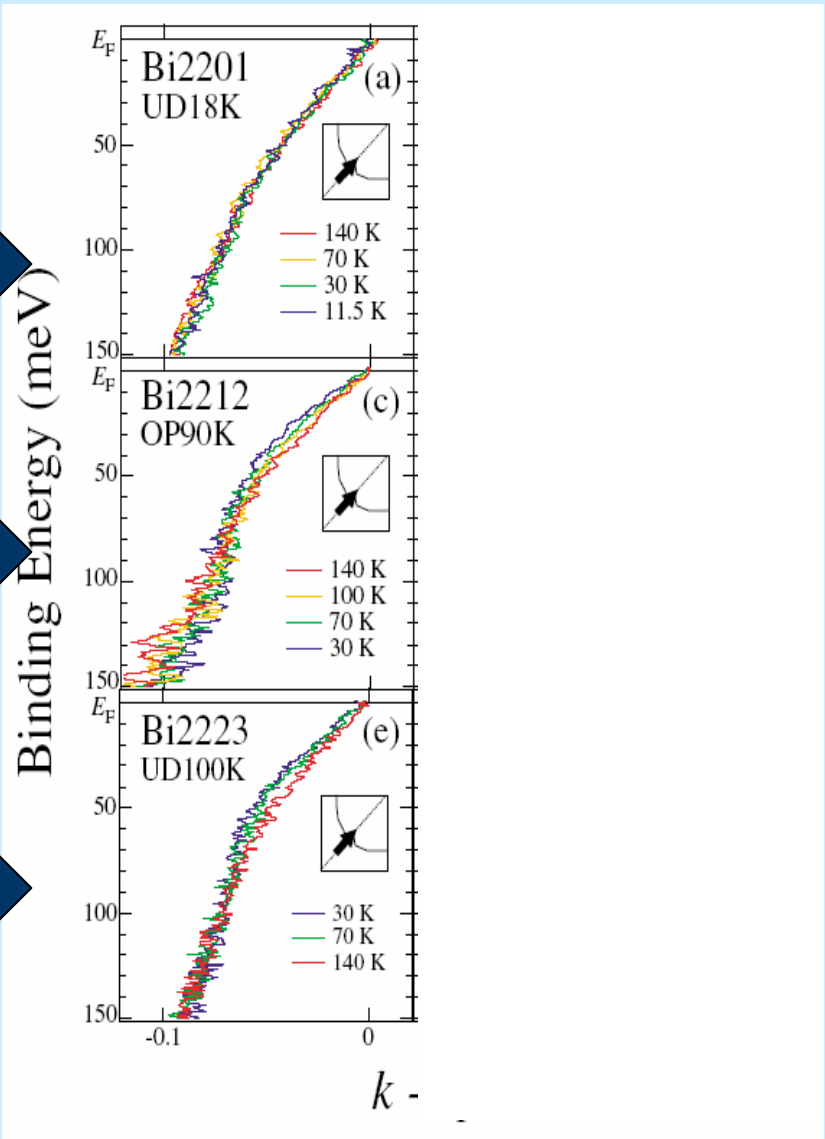
- the S-shape dispersion is even stronger
- high energy dispersion interpolates to  $k$  outside the Fermi surface

# Experiment



Sato et al, 2003

Nodal direction

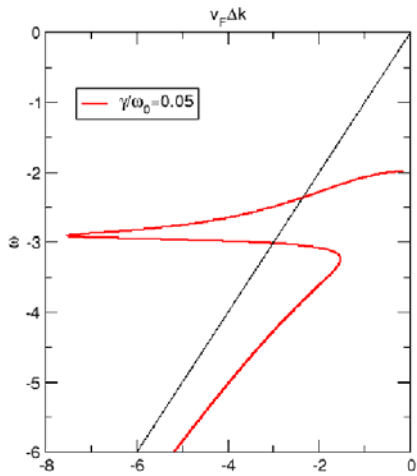


**a crossover  
(a smooth “kink”)  
in the normal state**

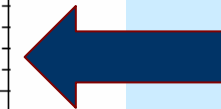
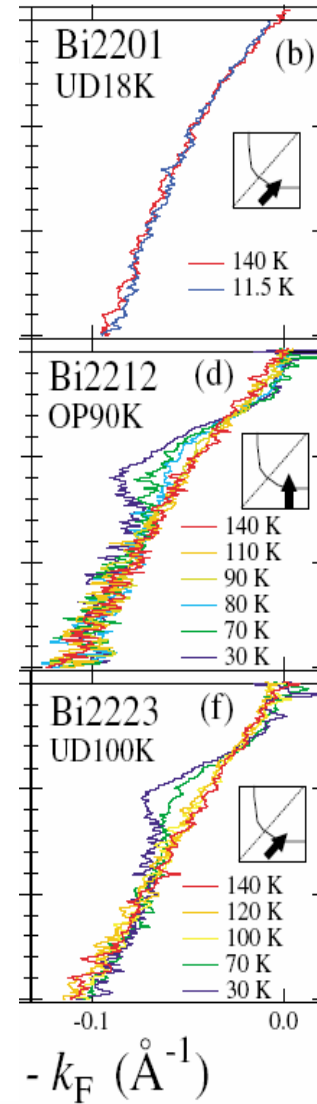
**some sharpening,  
but still rather smooth  
“kink” in the  
superconducting  
state**

**It doesn't look like a mode**





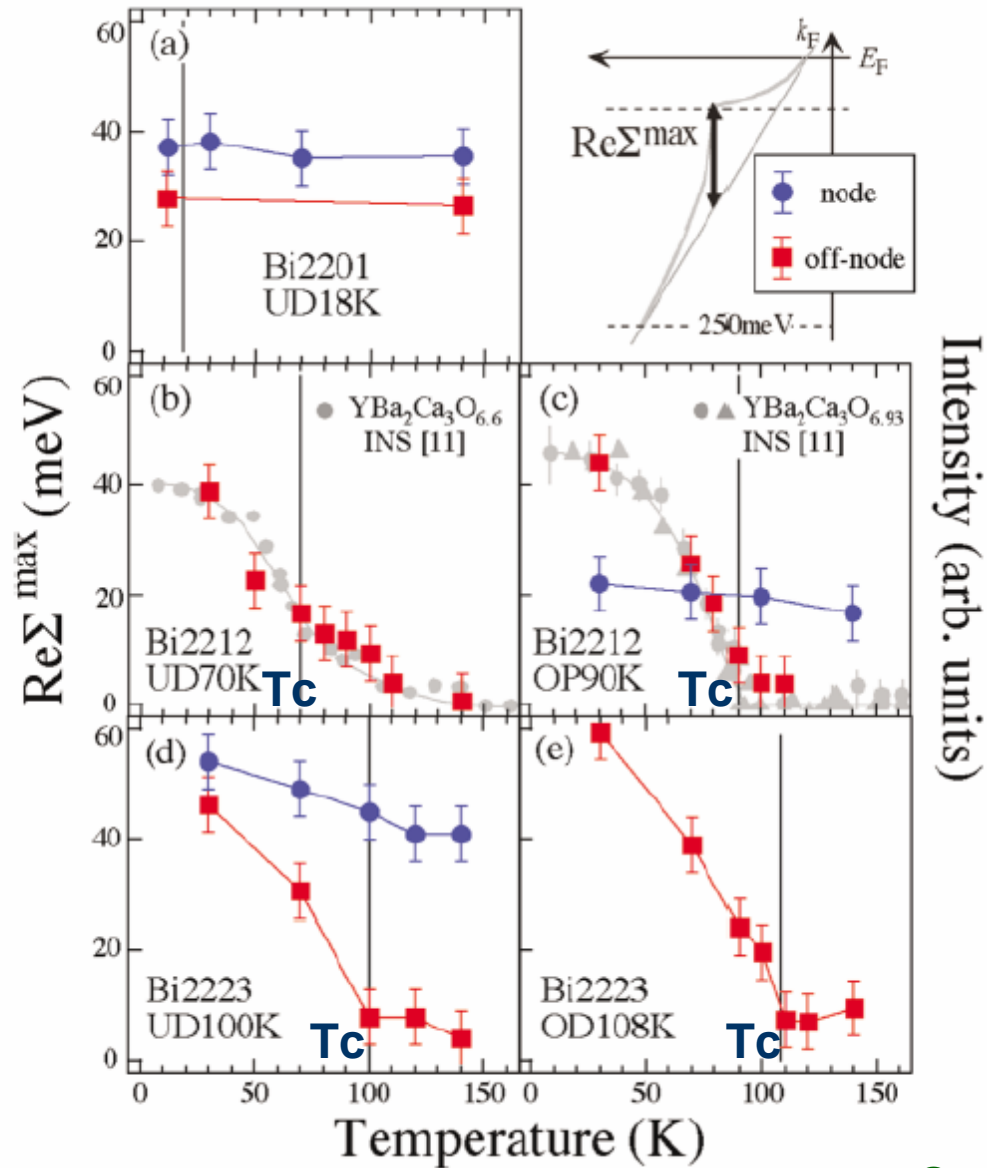
**High energy dispersion  
interpolates to  $k$  inside  
Fermi surface**



Antinodal  
direction

**This looks like the effect of a mode,  
but not completely**

**S-shape dispersion  
is correlated with  
 $T_c$   
(more accurately,  
with  $T^*$ )**



Sato et al, 2003

# Theory: what are the options

**Bosonic mode is independent on electrons (phonons)**

- one needs two different phonon modes (buckling and breathing)
- the coupling constant for the nodal direction must be small  $\lambda = 0.3$

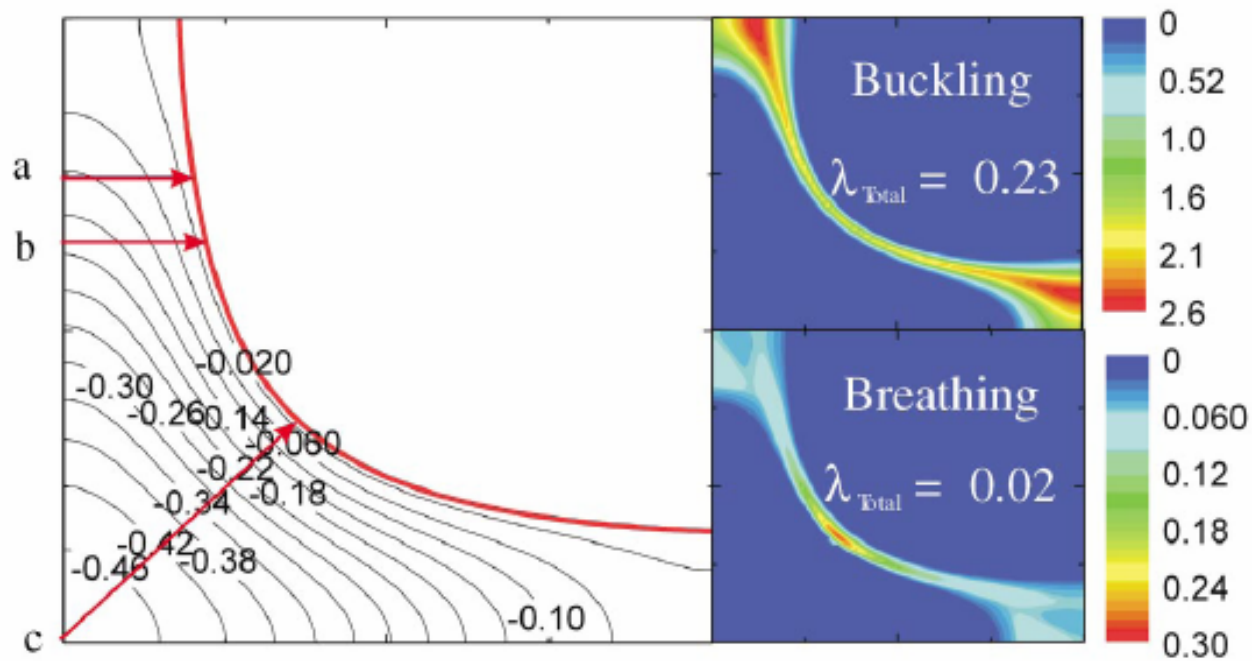
(Devereaux et al)

One needs some other mode to account for a large mass renormalization along the nodal direction

**Bosonic mode is a collective mode of electrons (spin fluctuations)**

“Mass media”: spin fluctuations == spin resonance  
(and as such, they are not that different from phonons)

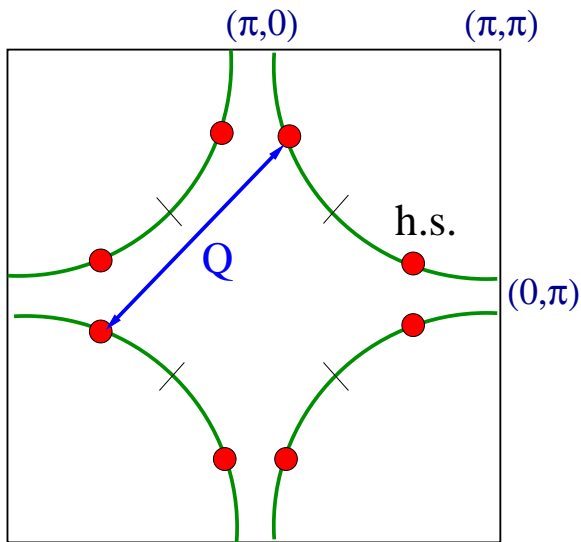
**Actually, spin fluctuations are more complex**



# Collective spin fluctuations



$$\chi(q, \omega) = \text{---} = \text{---} + \text{---} + \text{---} + \dots$$



In the normal state, spin fluctuations with momenta near  $(\pi, \pi)$  are Landau damped

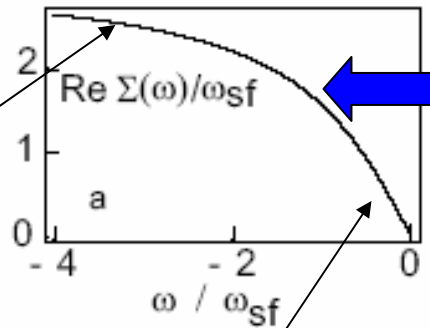
In the superconducting state spin damping is generally reduced due to gap opening, and spin fluctuations with momenta near  $(\pi, \pi)$  become sharp modes (spin waves)

# Normal state

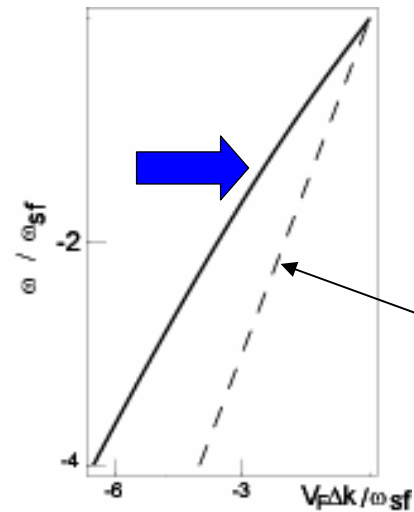
$$\chi^{-1}(q, \omega) \sim \left[ f(q) - i \frac{\omega}{\omega_{sf}} \right]$$

Overdamped mode

Self-energy



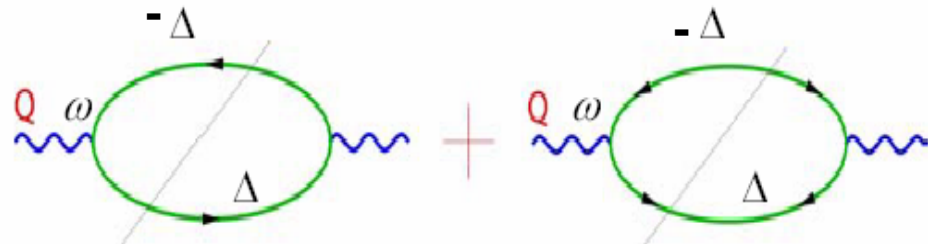
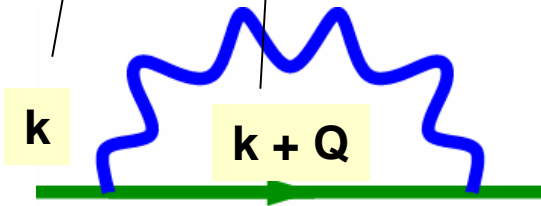
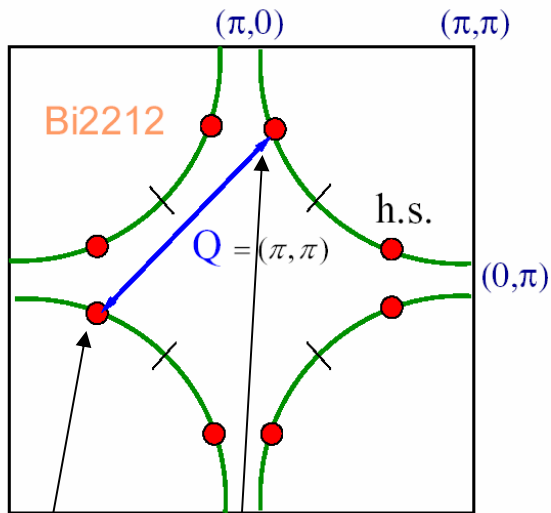
dispersion



a smooth crossover in the fermionic dispersion

# Superconducting state

## 1. antinodal region



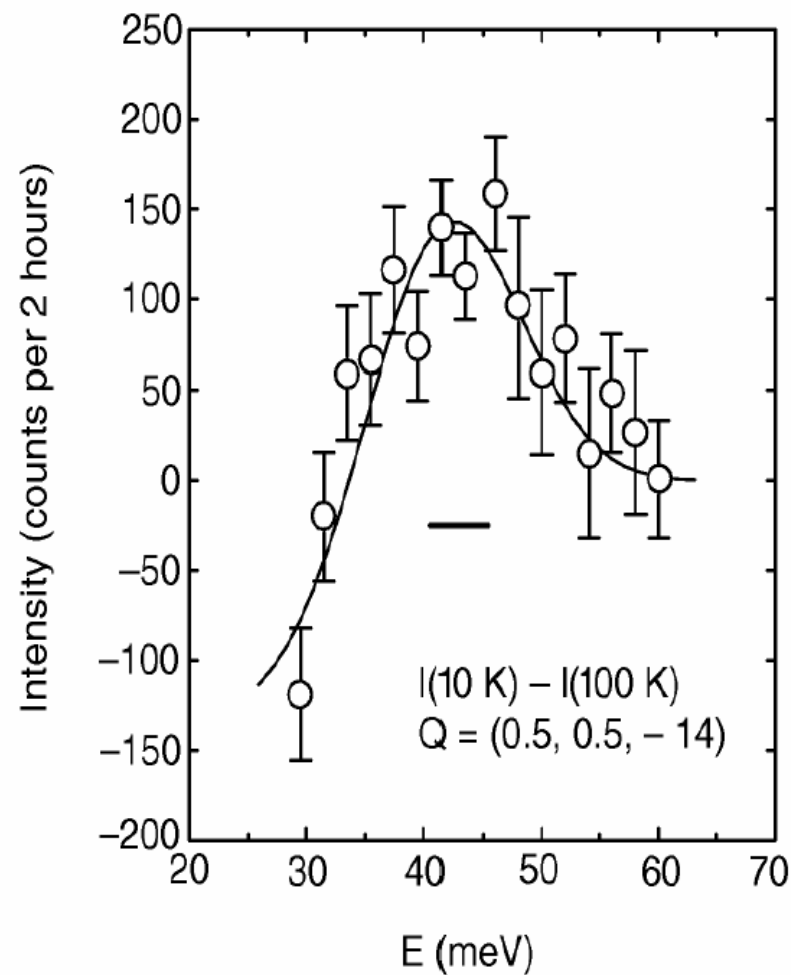
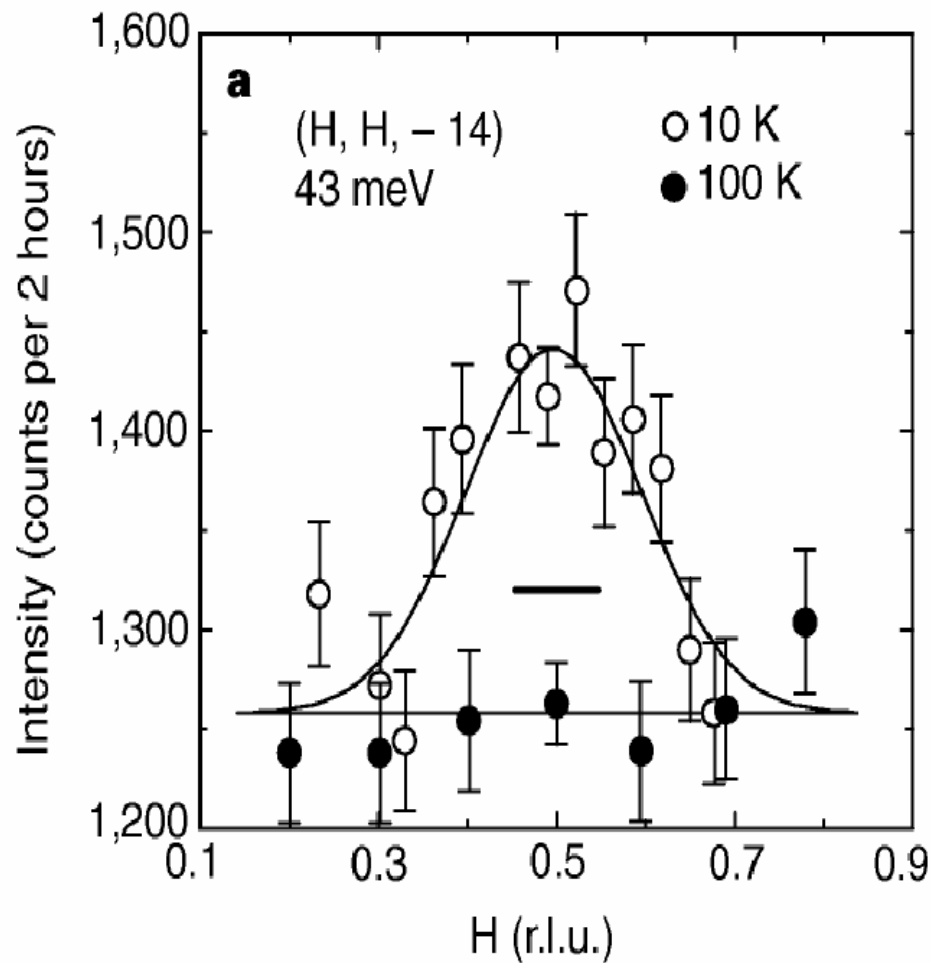
$$\chi(\omega) = \frac{Z}{\omega^2 - \omega_{\text{res}}^2}$$

$$Z \propto \Delta$$

A collective spin-fluctuation mode  
at an energy  $\omega_{\text{res}} < 2\Delta$

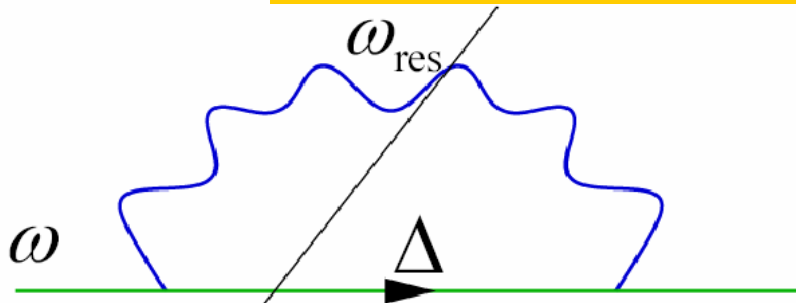
(a resonance peak)

# Resonance peak in a d-wave superconductor

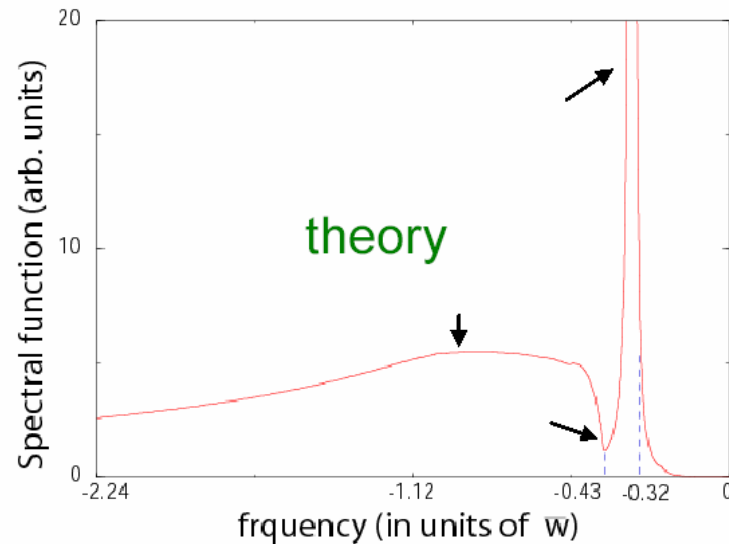
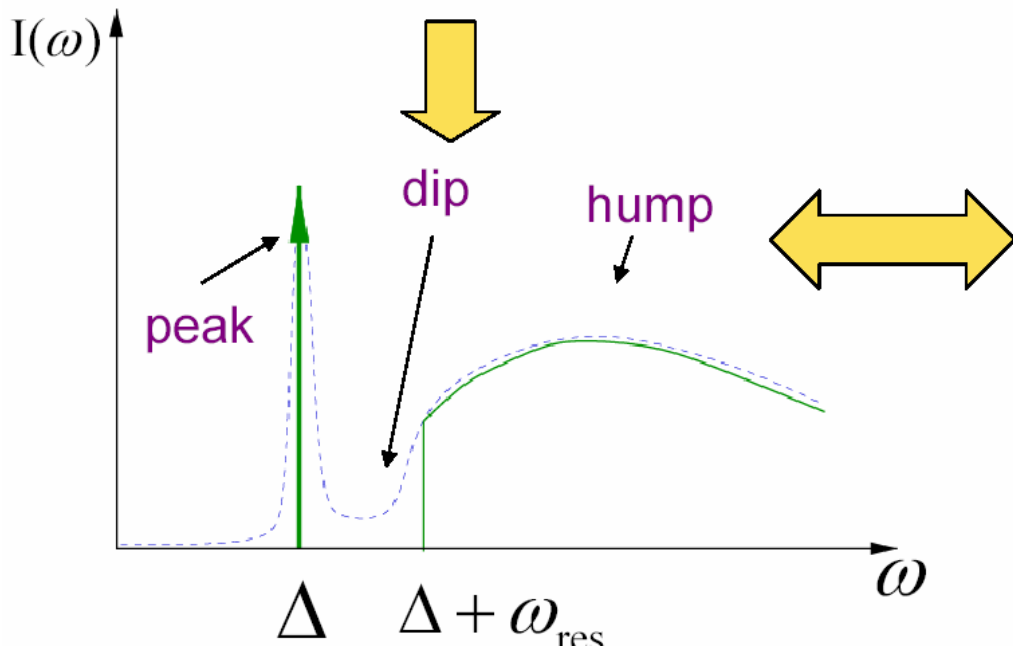
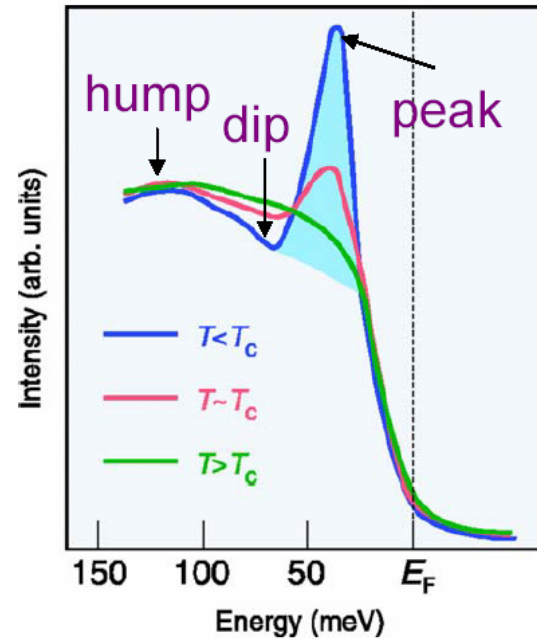




# Feedback on antinodal fermions

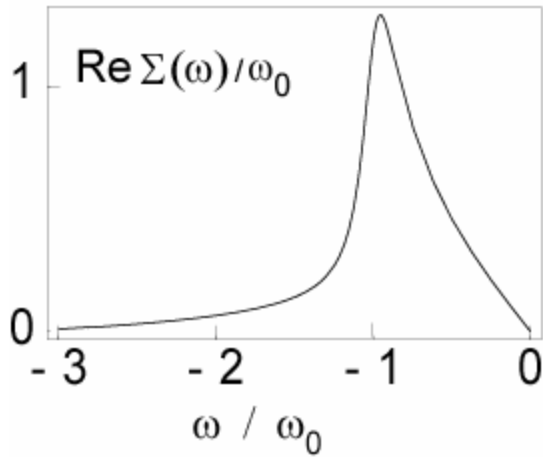


Threshold for fermionic damping at  $\Delta + \omega_{res}$



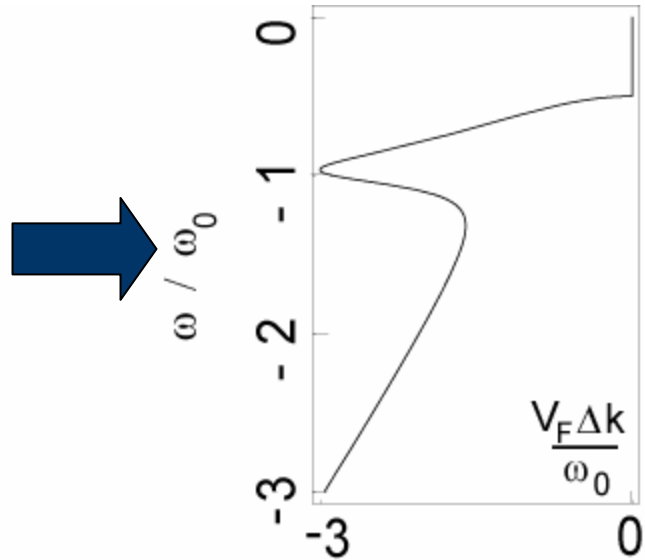
# Feedback effect on $\text{Re } \Sigma(\omega)$

## Antinodal self-energy



$$\omega_0 = \Delta + \omega_{\text{res}}$$

## Antinodal dispersion

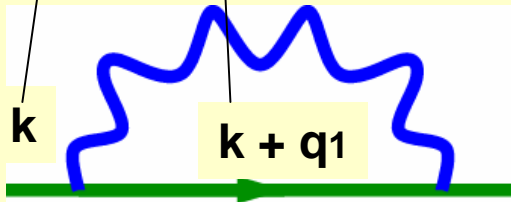
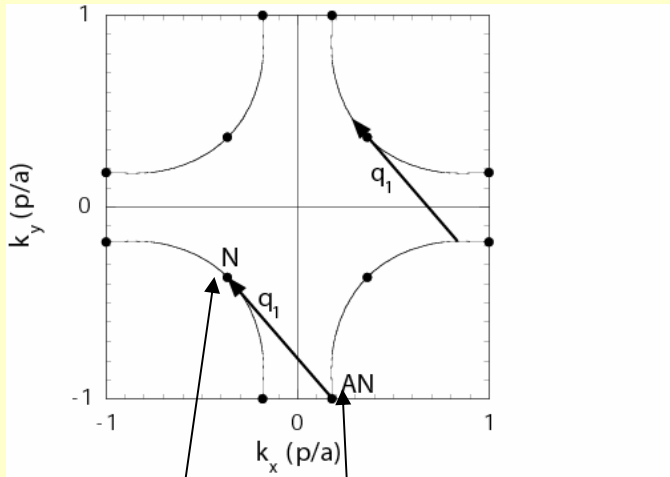


S – shape dispersion!

Interpolates to  $k$  inside Fermi surface

# Superconducting state

## 2. nodal direction

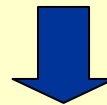


The main scattering is between nodal and antinodal regions

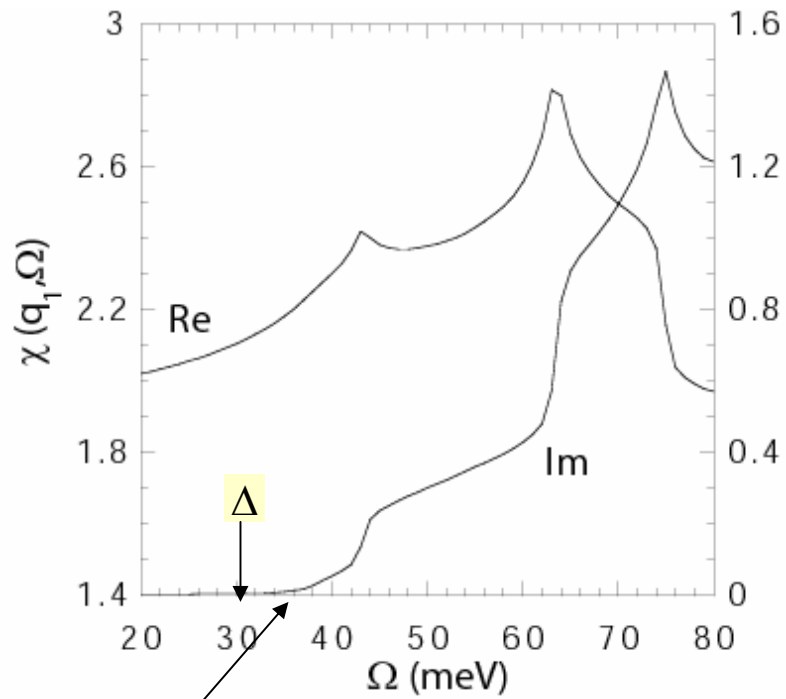
$$\chi_{q_1}(\Omega) = \frac{Z}{\Omega^2 - \omega_{\text{res}}^2}$$

Residue of the resonance peak at  $q_1$

$$Z \propto [\Delta_N \Delta_{AN}]^{1/2} = 0$$

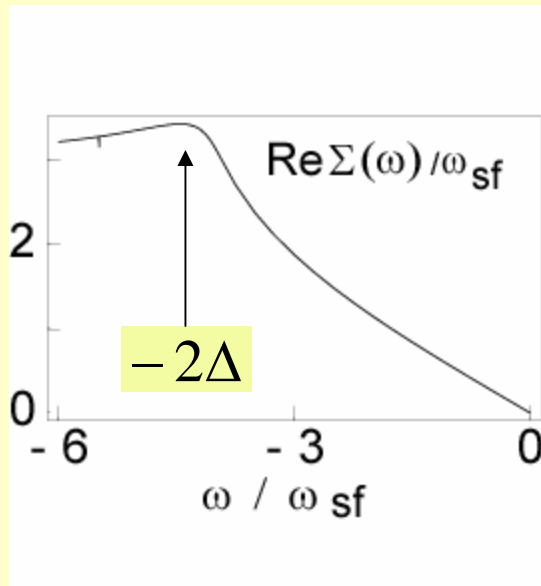


A nodal fermion DOES NOT couple to the resonance peak

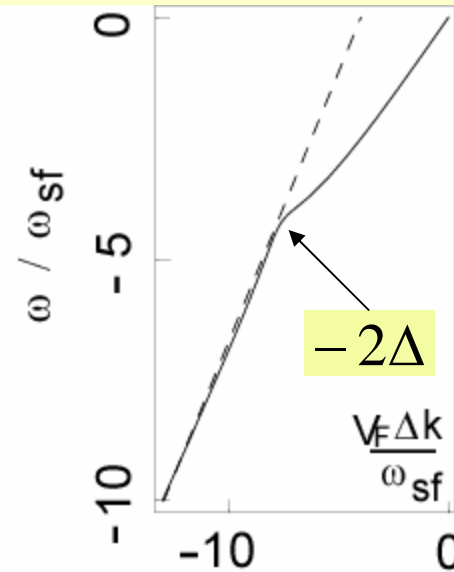


$$\chi^{-1}(\Omega) \propto 1 - i \sqrt{\Omega^2 - \Delta^2} / \omega_{sf}$$

## Nodal self-energy



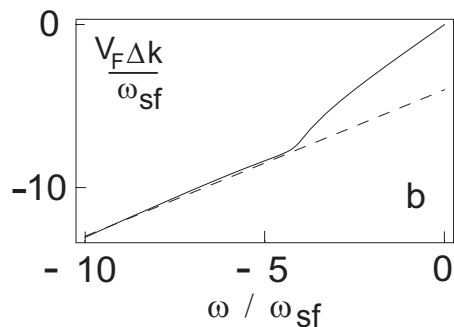
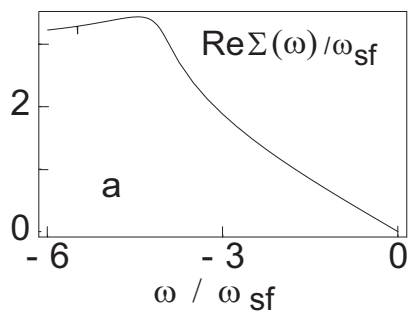
## Nodal dispersion



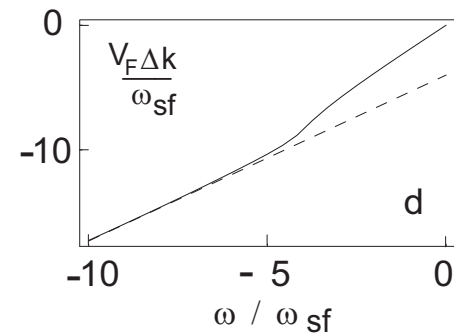
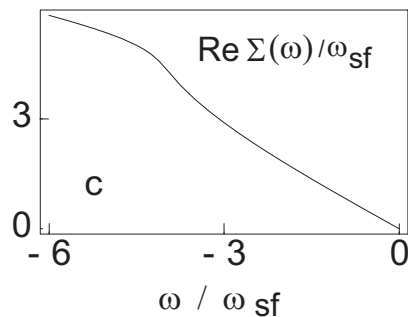
$$\Sigma'(\omega) - \Sigma'(-2\Delta) \propto (\omega + 2\Delta) \log \frac{\Delta}{|\omega + 2\Delta|}$$

**There is a true kink  
at  $\omega = 2\Delta$ , but no  
S-shape dispersion**

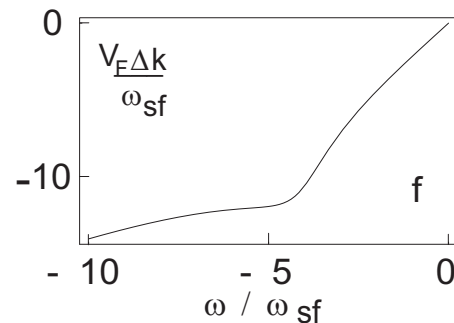
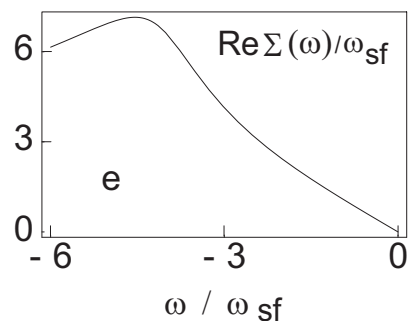
## Flat susceptibility



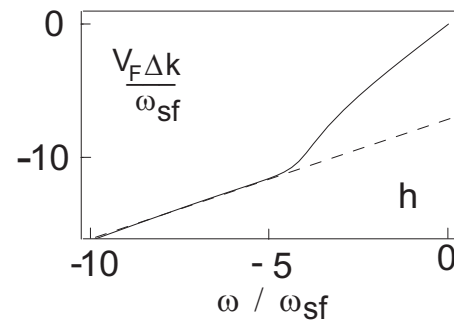
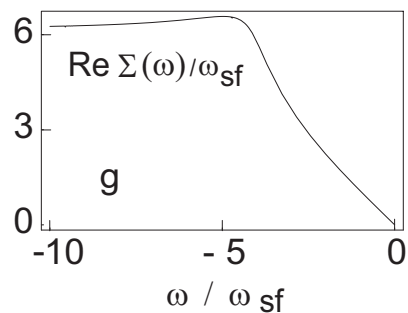
## Ornstein-Zernike, Eliashberg



## Ornstein-Zernike, anti- Eliashberg

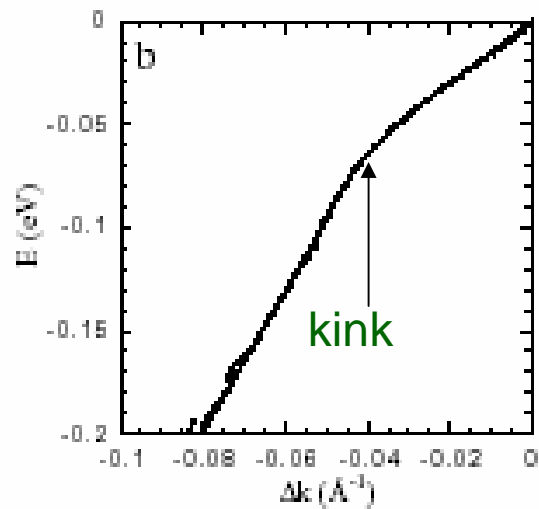
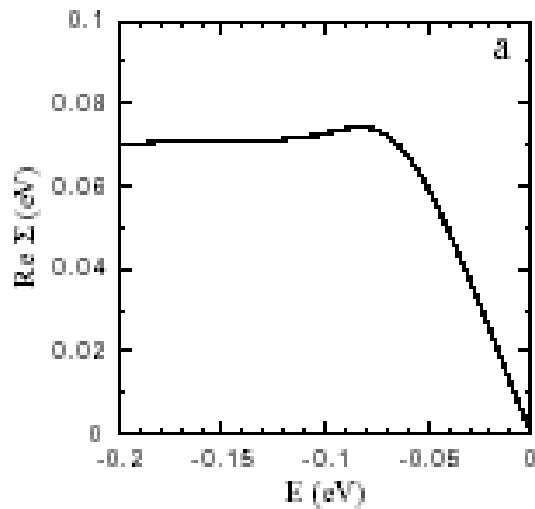


## With umklapp scattering

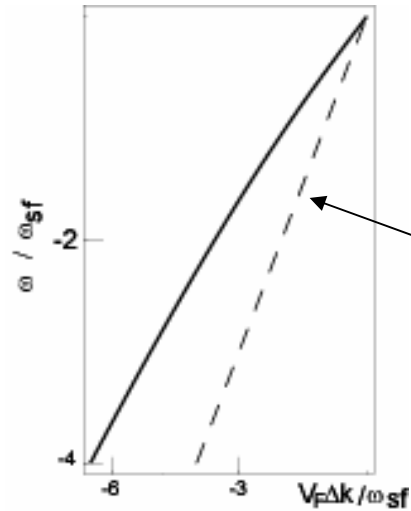
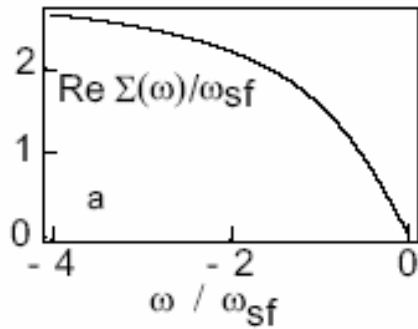


# Superconducting state

## Nodal dispersion

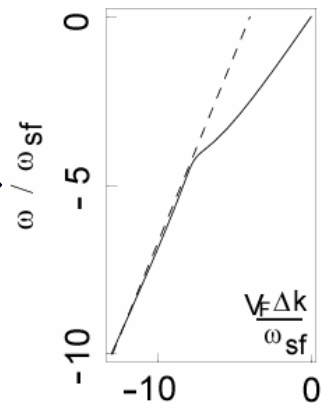
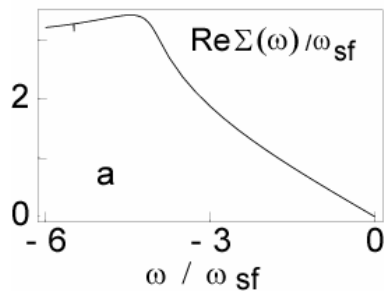


## Normal state



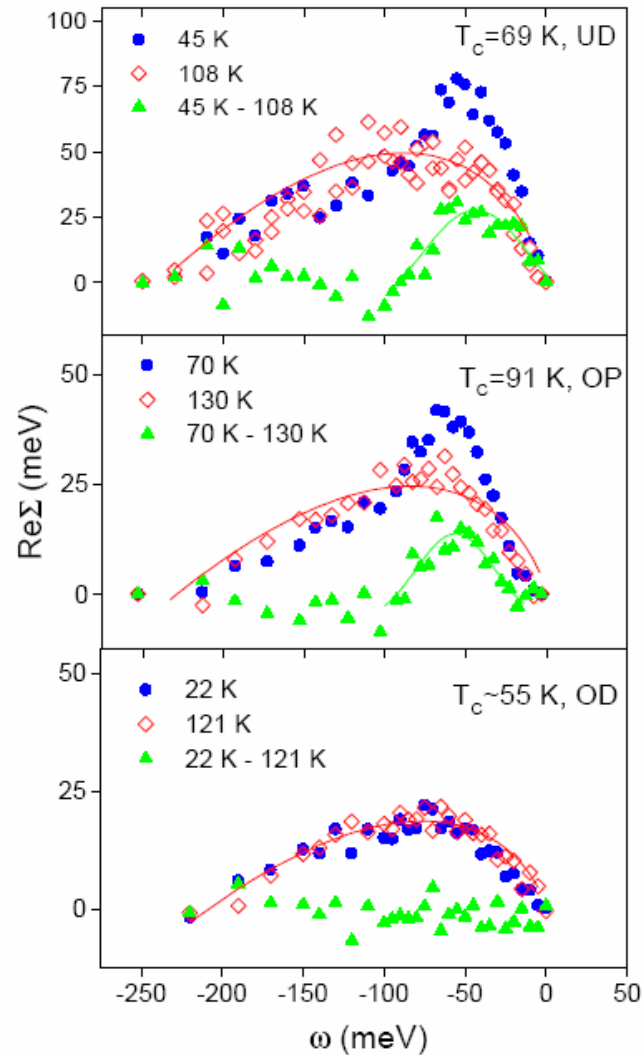
a bare dispersion

## Superconducting state, nodal direction

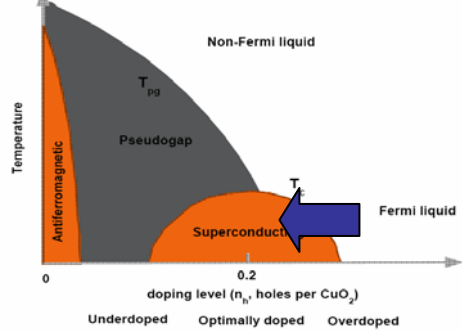




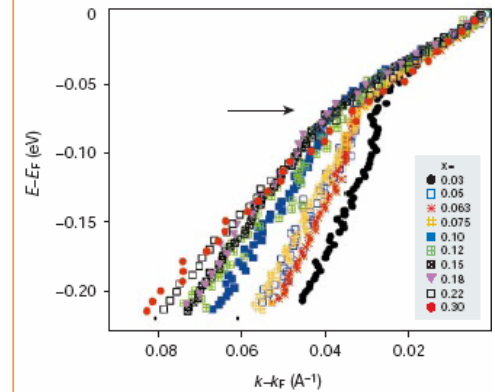
# Real part of the self-energy, extracted from ARPES data



P.D. Johnson et al



# The doping dependence



1. S-shape along antinode should increase with underdoping

The susceptibility at  $(\pi, \pi)$  increases

2. Nodal direction, low frequencies

$$V_F^* = V_F / (1 + \lambda(q_1)) \quad q_1 \neq (\pi, \pi), \quad \lambda(q_1) \sim 1$$

nodal velocity is weakly doping independent

(Dessau, Shen)

2. Nodal direction, high frequencies

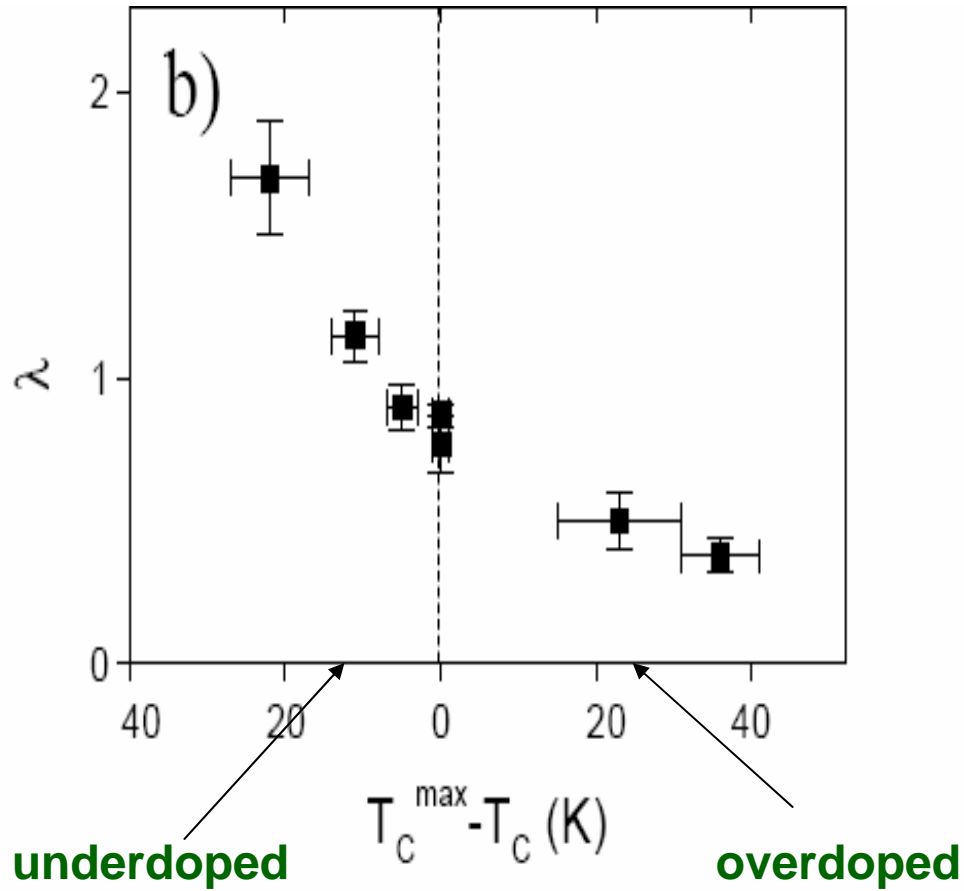
Mott (or SDW) physics

$$\Sigma(k, \omega) = \frac{\Delta^2}{\omega - \varepsilon_{k+\pi}}$$

$$\omega = \varepsilon_k - \frac{\Delta^2}{\varepsilon_{k+\pi} - \varepsilon_k}, \quad \varepsilon_k < 0, \quad \varepsilon_{k+\pi} > 0$$

Randeria et al

## Coupling vs doping (nodal direction)



# Conclusions

**Dispersion anomalies in the cuprates can be explained by the interaction with spin fluctuations**

## **Normal state:**

- overdamped spin fluctuations
- a smooth crossover in the dispersion

## **Superconducting state:**

- Antinodal fermions couple to spin resonance
- Nodal fermions do not couple to spin resonance

An S - shape dispersion  
near  $\Delta + \omega_{\text{res}}$

No S - shape dispersion  
A kink at  $2\Delta$