

# Kondo satellites in photoemission spectra of heavy fermion compounds

P. Wölfle, Universität Karlsruhe

J. Kroha, Universität Bonn



# Outline

- Introduction: Kondo resonances in the photoemission spectra of Ce systems
- Single ion Anderson model of 4f multiplets: slave boson mean field theory
- Spectral function from perturbative RG method
- Spectral function in non-crossing approximation
- Comparison with experiment and conclusion



## Collaborators:

### Theory

*S. Kirchner,*  
*G. Sellier,*

Universität Karlsruhe (till 2004)

### Experiment

*S. Hufner ,*  
*D. Ehm*  
*S. Schmidt*

Universität Saarbrücken

*F. Reinert*

Universität Würzburg

*O. Stockert,*  
*C. Geibel*

MPI-CPS, Dresden

*H. v. Löhneysen,*

Universität Karlsruhe

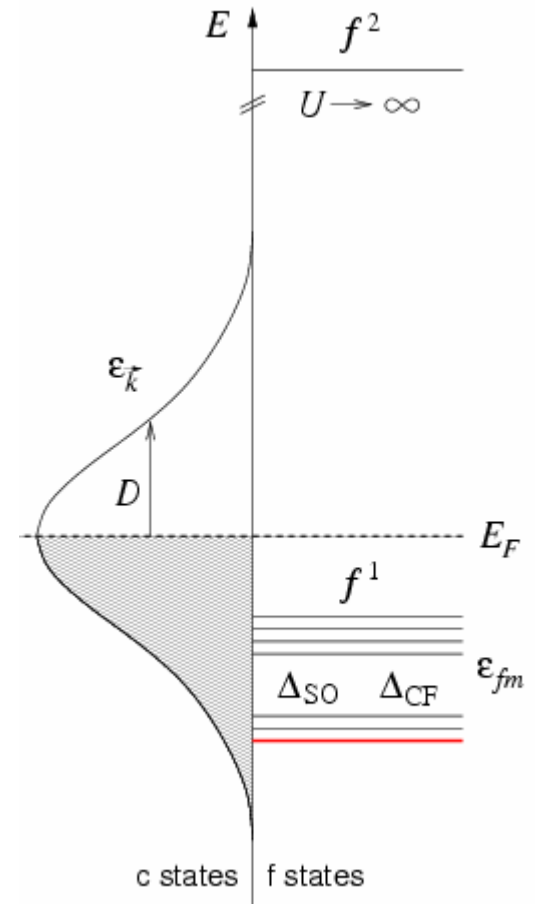


# Modelling Ce compounds: Periodic Anderson model

Localized 4f electrons hybridizing with conduction electron band

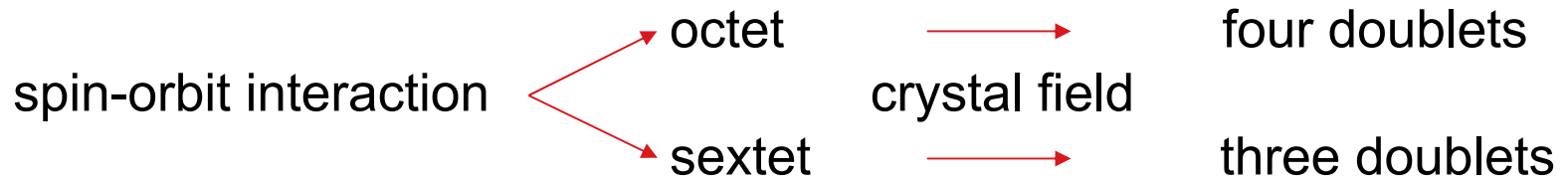
**Single ion physics** expected to dominate local properties observed in photoemission spectra

Lattice effects leading to coherent transport less important for PES



# Multiorbital single impurity Anderson model

4f-states  $|m, \sigma\rangle$  : 7 spin-degenerate levels split by



$$\begin{aligned}
 H = & H_0 + \sum_{m\sigma} \varepsilon_{fm} f_{m\sigma}^\dagger f_{m\sigma} + \sum_{\vec{k}m\sigma} (V_{\vec{k}m} c_{\vec{k}\sigma}^\dagger f_{m\sigma} + \text{h.c.}) \\
 & + \frac{U}{2} \sum_{(m\sigma) \neq (m'\sigma')} f_{m\sigma}^\dagger f_{m\sigma} f_{m'\sigma'}^\dagger f_{m'\sigma'} , \quad (1)
 \end{aligned}$$

$$H_0 = \sum_{\vec{k}\sigma} \varepsilon_{\vec{k}} c_{\vec{k}\sigma}^\dagger c_{\vec{k}\sigma} \quad \text{conduction band}$$

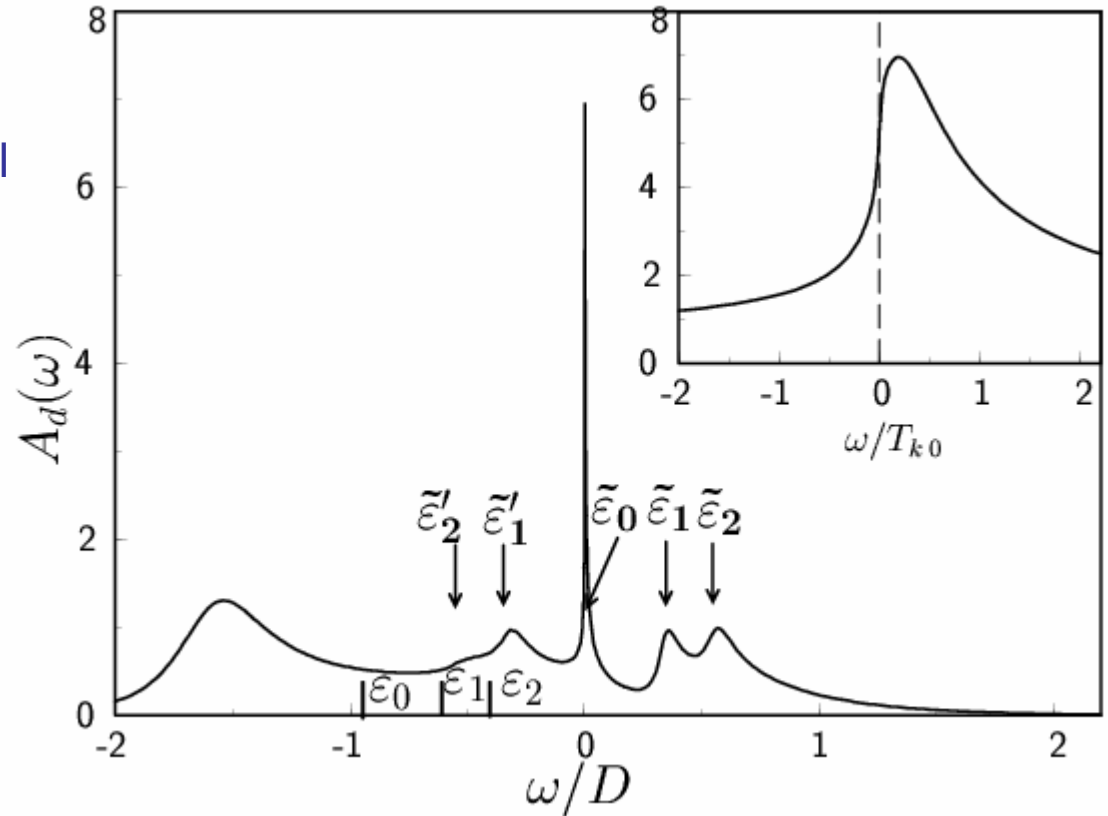


# Kondo resonances in the single particle spectra

Model system: triplet of localized states, occupation  $n=1$

## Kondo resonances

- main resonance at Fermi level
- above Fermi level : rigid shift of multiplet
- below Fermi level: mirror image of excited levels
- width of excited levels of order a few  $T_K$



# Methods: Pseudoparticle representation

Limit of infinite Coulomb interaction  $U$ : multiple occupancy of f-states excluded

Pseudoparticle operators:  $b^+$  (empty level),  $f_m^+$  (singly occupied level)

f-electron operator:  $f_{el,m}^+ = f_m^+ b$

Constraint:  $Q = b^+ b + \sum_m f_m^+ f_m = 1$

Implementation of constraint  $H \rightarrow H + \lambda Q$ ,  $\lambda \rightarrow \infty$



# Methods: Slave boson mean field theory

Replace Bose operators by mean field plus fluctuations:

$$\hat{b} = r + \hat{a}, \quad r = \langle b \rangle, \quad \hat{\lambda} = \lambda_0 + \hat{\tilde{\lambda}}$$

Mean field  
Hamiltonian

$$H = H_{\text{kin}} + \sum_{m\sigma} \tilde{\varepsilon}_{dm} f_{m\sigma}^\dagger f_{m\sigma} + \sum_{pm\sigma} [\tilde{V}_{mp} f_{m\sigma}^\dagger c_{p\sigma} + h.c.] \\ + (\tilde{\varepsilon}_{d0} - \varepsilon_{d0}) [r^2 + a^\dagger a - 1] + H_{\text{int}} \quad (3)$$

Mean field  
equations

$$r^2 + \langle a^\dagger a \rangle + \sum_{m\sigma} \langle f_{m\sigma}^\dagger f_{m\sigma} \rangle - 1 = 0 \\ (\tilde{\varepsilon}_{d0} - \varepsilon_{d0}) r + \sum_{pm\sigma} V_{mp} \langle f_{m\sigma}^\dagger c_{p\sigma} \rangle = 0 .$$

Level shifts:

$$\tilde{\varepsilon}_0 = \varepsilon_0 + \lambda_0 = \alpha T_K, \quad \alpha \ll 1$$

$$\tilde{\varepsilon}_m = \tilde{\varepsilon}_0 + (\varepsilon_m - \varepsilon_0)$$

Kondo temperature:

$$T_K = \left\{ \prod_{m=1}^{M-1} \left( \frac{D}{\varepsilon_m - \varepsilon_0} \right)^{\frac{\Gamma_{mm}}{\Gamma_{00}}} \right\} D e^{-\frac{\pi|\varepsilon_0|}{2\Gamma_{00}}}$$





# Electron spectral function: sb MFT + fluctuations

$$\begin{aligned}
 A_{d\sigma}(\omega) \simeq & \sum_m \frac{T_{K m}}{\Gamma_{m m}} \text{Im} G_{f m m \sigma}(\omega - i0) \\
 & - \left[ \Theta(\tilde{\varepsilon}_{d0} - \omega) - \frac{1}{2} \right] \text{Im} \left[ -\omega + \tilde{\varepsilon}_{d0} + \varepsilon_{d0} \right. \\
 & \left. - \sum_m \frac{\Gamma_{m m}}{\pi} \ln \frac{(\omega + \varepsilon_{d m} - \varepsilon_{d0})^2 + T_{K m}^2}{\tilde{\varepsilon}_{d m}^2 + T_{K m}^2} \right. \\
 & \left. - i \text{Im} \Sigma_a(\omega - i0) - i \text{sgn} \omega T_{K 0} \right]^{-1}.
 \end{aligned}$$

where

$$G_{f, \sigma, m, m'}(\omega) = [(\omega - \tilde{\varepsilon}_{d, m}) \delta_{m, m'} - i \tilde{\Gamma}_{m, m'}(\omega)]^{-1}$$

Resonance peaks at  $\tilde{\varepsilon}_{d, m} = \tilde{\varepsilon}_{d, 0} + (\varepsilon_{d, m} - \varepsilon_{d, 0})$  and at  $\tilde{\varepsilon}'_{d, m} = -(\varepsilon_{d, m} - \varepsilon_{d, 0})$

Resonance widths:  $\tilde{\Gamma}_{m, m} = r^2 \Gamma_{m, m} = O(T_K)$



weak dependence  
on  $\Delta\varepsilon$



bare level spacings



# Methods: Mapping on to Kondo model

Elimination of empty and multiply occupied states yields Kondo model :


$$H_K = H_0 + \sum_{\sigma, \sigma', m, m'} J_{m, m'} f_{m', \sigma'}^+ f_{m, \sigma} \sum_{k, k'} c_{k, \sigma}^+ c_{k', \sigma'} + \sum_m (\varepsilon_m - \varepsilon_0) f_{m, \sigma}^+ f_{m, \sigma}$$

where  $J_{m, m'} = 2V_m V_{m'}^* / (\varepsilon_m + \varepsilon_{m'})$       Constraint:  $\sum_{m, \sigma} f_{m, \sigma}^+ f_{m, \sigma} = 1$

Perturbative second order correction to exchange constants (  $\Delta\varepsilon_m = \varepsilon_m - \varepsilon_0$  ):

$$J_{mm'}^{(2)}(\omega) = -N(0) \sum_l J_{ml} J_{m'l} \left\{ \int_{-D}^D d\varepsilon \frac{f(\varepsilon)}{\varepsilon + \omega + \Delta\varepsilon_m - \Delta\varepsilon_l} + (\omega \rightarrow -\omega) \right\}$$

$$\approx -N(0) \sum_l J_{ml} J_{m'l} \ln \left\{ ([\Delta\varepsilon_m - \Delta\varepsilon_l]^2 - \omega^2) / D^2 \right\}$$

 Logarithmically divergent terms at  $\omega = \pm[\Delta\varepsilon_m - \Delta\varepsilon_l]$



# Methods: Renormalization group equations

- **Poor man's scaling** (Anderson, 1970)  
remove high energy states at  $\pm D$  and absorb change into coupling constant  $g(D)$ , depending on running cutoff  $D$ , take  $D \rightarrow 0$ .
- **Extend renormalization group meth. to energy dependent coupling functions**

$$\frac{dg_{\parallel m, m'}(\omega)}{d \ln D} = -2 \sum_l g_{\perp m, l} g_{\perp l, m'} \Theta(D - \Delta\omega) \quad \Delta\omega = |\omega - \Delta\varepsilon_l + \Delta\varepsilon_m|$$

$$\frac{dg_{\perp m, m'}(\omega)}{d \ln D} = -2 \sum_l g_{\perp m, l} g_{\parallel l, m'} \Theta(D - \Delta\omega)$$

$\Theta$ -(step)-functions account for absence of renormalization if energy is outside the bandwidth  $D$

Two levels, splitting  $\Delta\varepsilon$ :

$$g_{\parallel 01}(\omega) = \Theta(|\omega - \Delta\varepsilon| - \Delta\varepsilon) \frac{1}{2 \ln[|\omega - \Delta\varepsilon| / T_K]} + \Theta(\Delta\varepsilon - |\omega - \Delta\varepsilon|) \frac{1}{2 \ln(\Delta\varepsilon / T_K)} \left| \frac{\Delta\varepsilon}{\omega - \Delta\varepsilon} \right|^{1/\ln(\Delta\varepsilon / T_K)}$$



# Methods: Decoherence stops RG flow

Logarithmic divergencies involving excited states are cut off by the finite spin relaxation rate (even at  $T=0$ ):

$$\gamma \approx \frac{\Delta\varepsilon}{[\ln(\Delta\varepsilon/T_K)]^2}$$

To account for this effect, we replace the conduction electron energy in the

RG equations by  $\sqrt{\omega^2 + \gamma^2}$

 Width of Kondo satellite peaks  $\approx \gamma$ , increasing with  $\Delta\varepsilon$

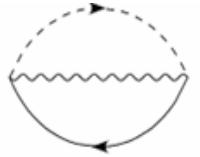
Systematic and controlled method, provided

$$\gamma \gg T_K, \quad \text{or} \quad \Delta\varepsilon/T_K \gg \frac{1}{\ln^2(\Delta\varepsilon/T_K)}$$



# Methods: Non-crossing approximation

Conserving approximation derived from generating functional  $\Phi =$



Pseudoparticle self energies:

$$\Sigma_{f,\sigma,m,m'}(\omega) = \text{Diagram: solid arc over wavy line} = \Gamma_{m,m'} \int d\varepsilon f(\varepsilon) A_{c,\sigma}(\varepsilon) G_b(-\varepsilon + \omega)$$

$$\Sigma_b = \text{Diagram: dashed arc over wavy line} = \sum_{m,m',\sigma} \Gamma_{m,m'} \int d\varepsilon f(\varepsilon) A_{c,\sigma}(\varepsilon) G_{f,\sigma,m,m'}(\varepsilon + \omega)$$

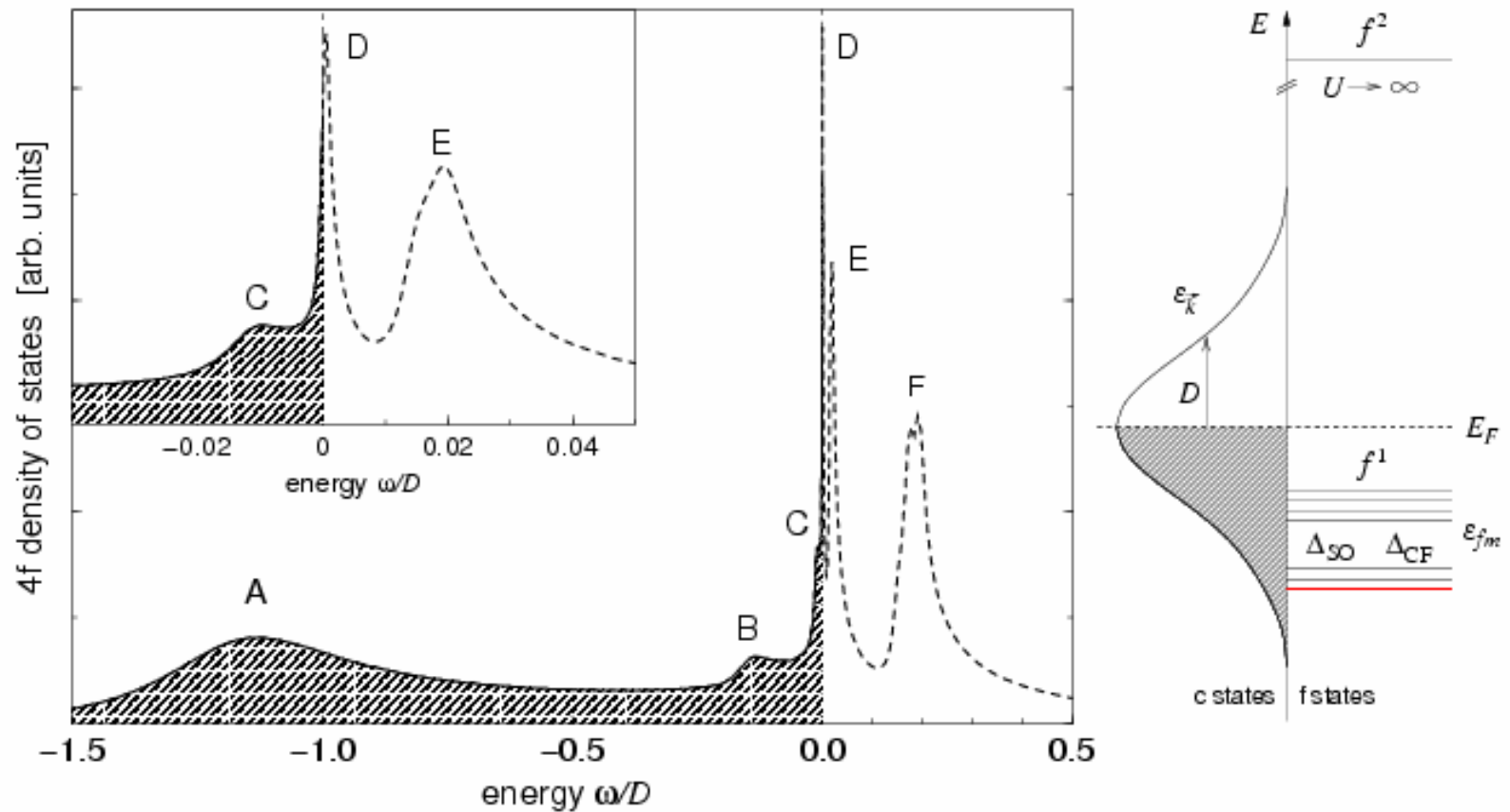
Conduction electron DOS  $A_{c,\sigma}(\varepsilon)$       Bare level broadening  $\Gamma_{m,m'}$

$$\text{Dashed arrow} \rightarrow G_{f,\sigma,m,m'}(\omega) = [(\omega - \lambda_0 - \varepsilon_{f,m})\delta_{m,m'} - \Sigma_{f,\sigma,m,m'}(\omega)]^{-1}$$

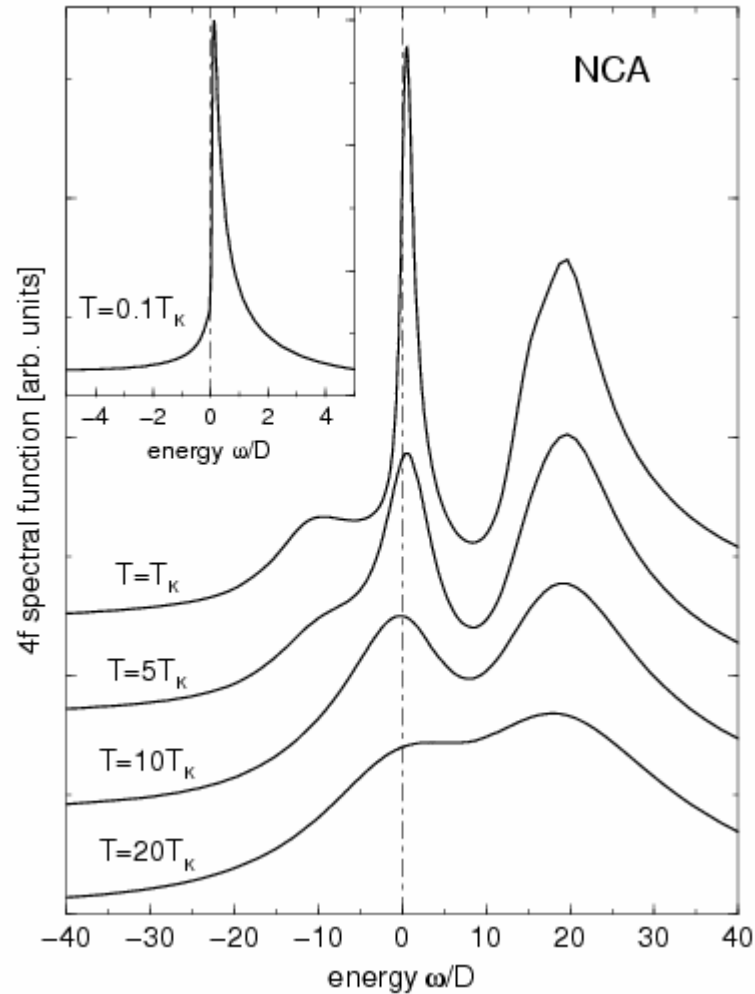
$$\text{Wavy line} \rightarrow G_b(\omega) = [\omega - \lambda_0 - \Sigma_b(\omega)]^{-1}$$

$$G_{f,\sigma,m,m'}^{el}(\omega) = \text{Diagram: dashed arc over wavy line}$$

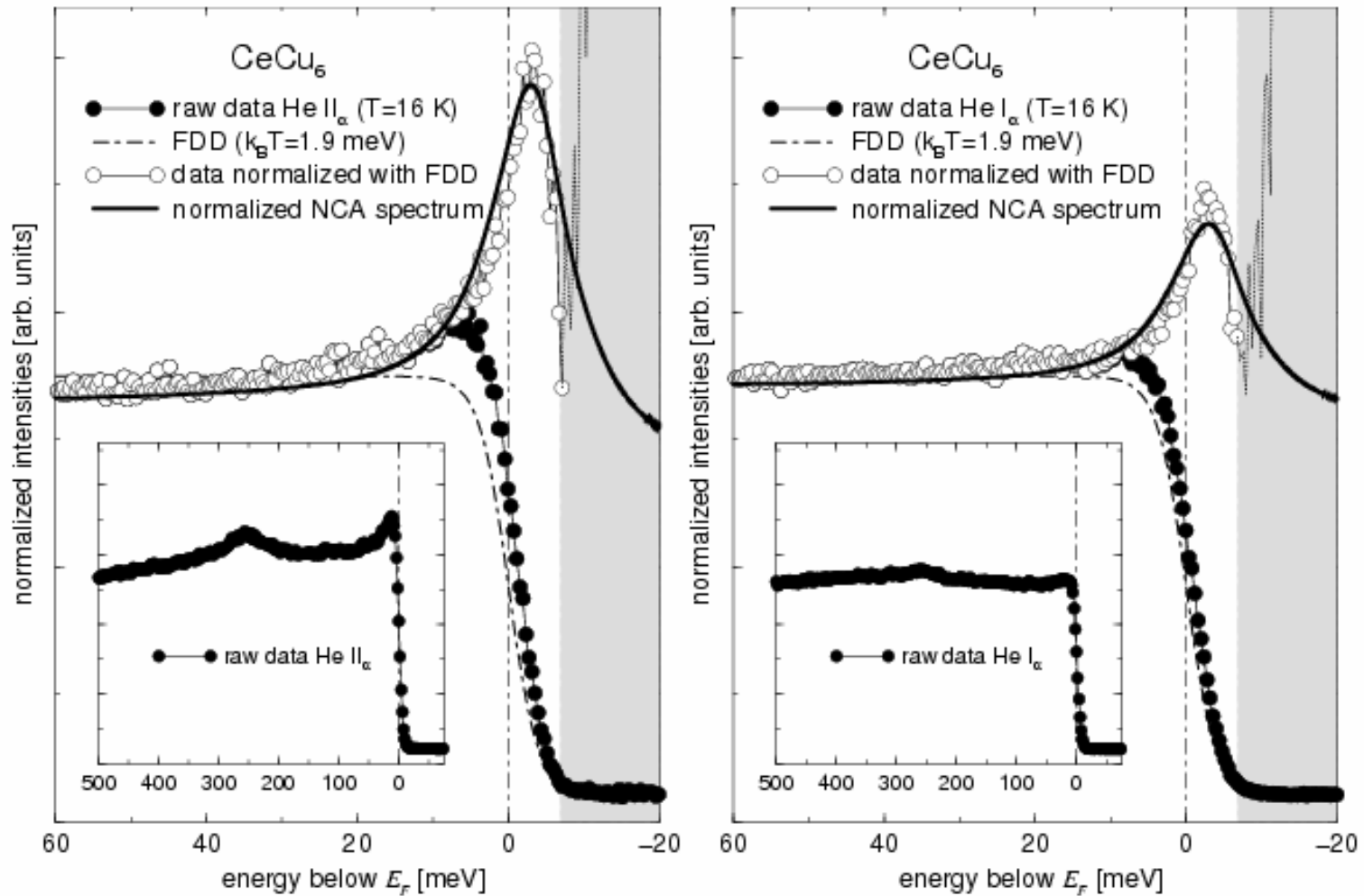
# Results: Theoretical 4f spectrum in NCA



# Results: Temperature dependence of 4f spectrum

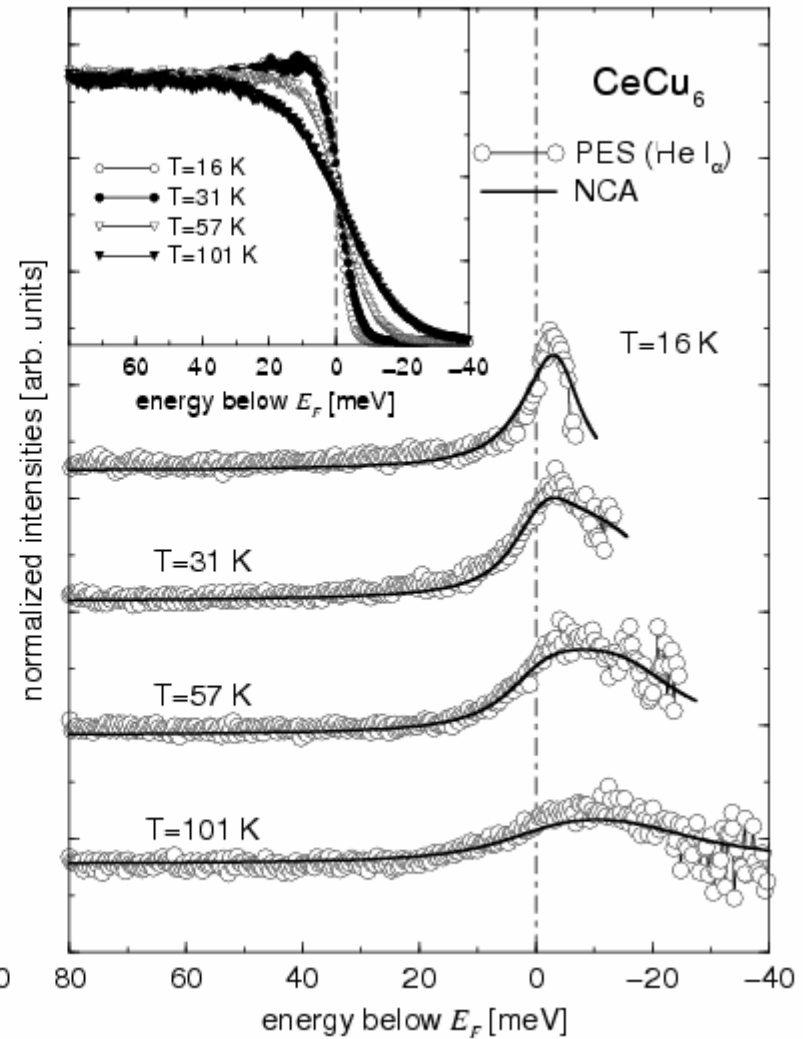
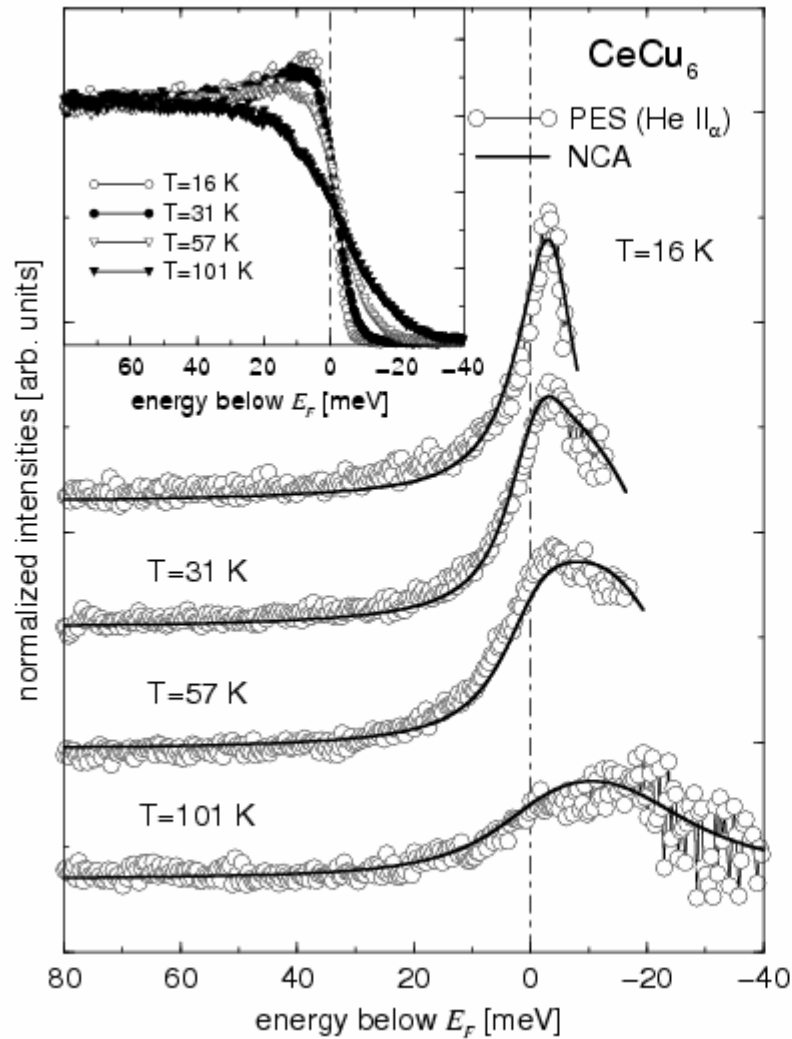


# Results: PES of CeCu<sub>6</sub>

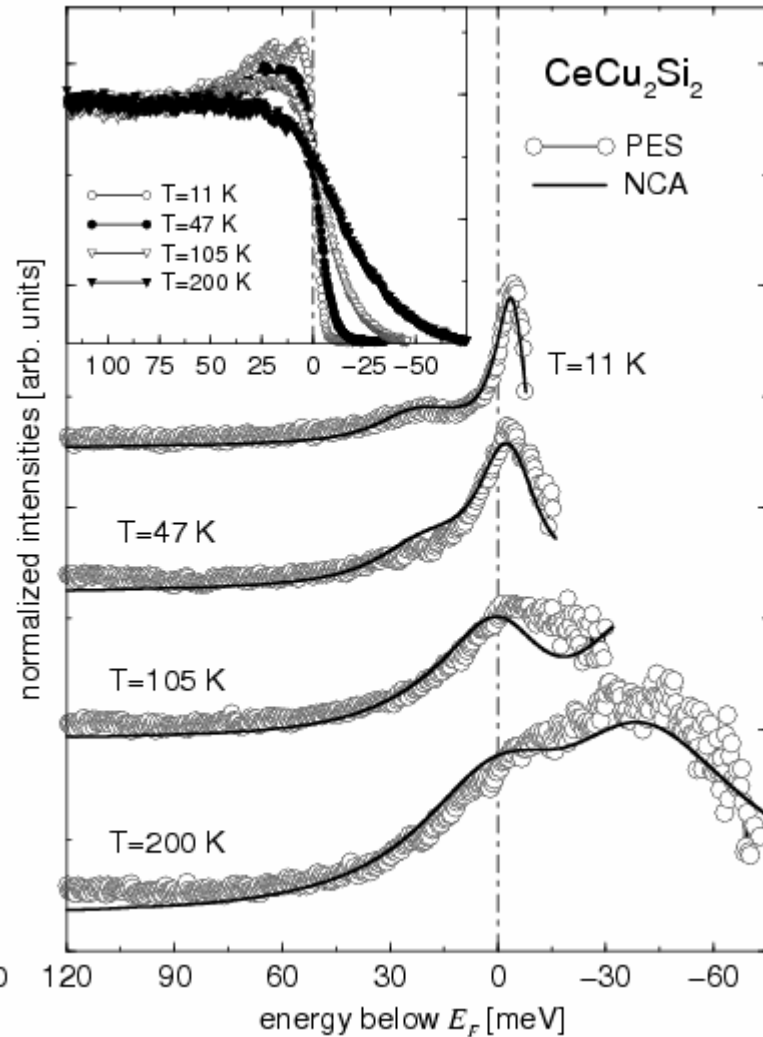
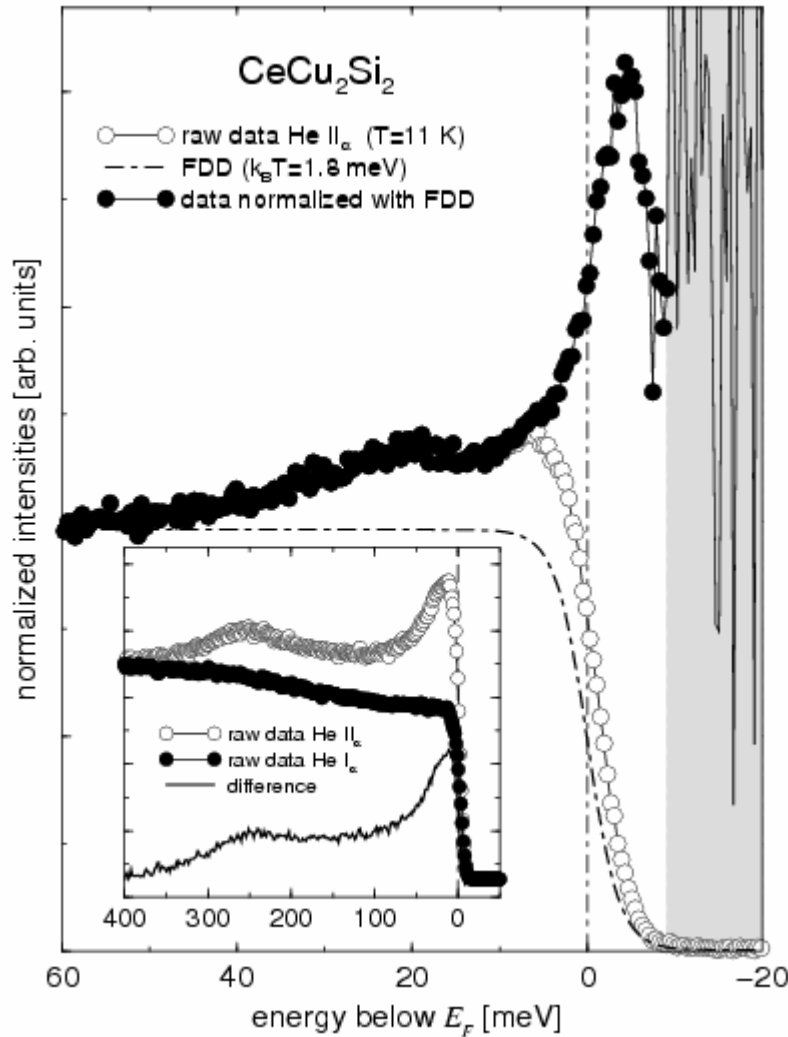




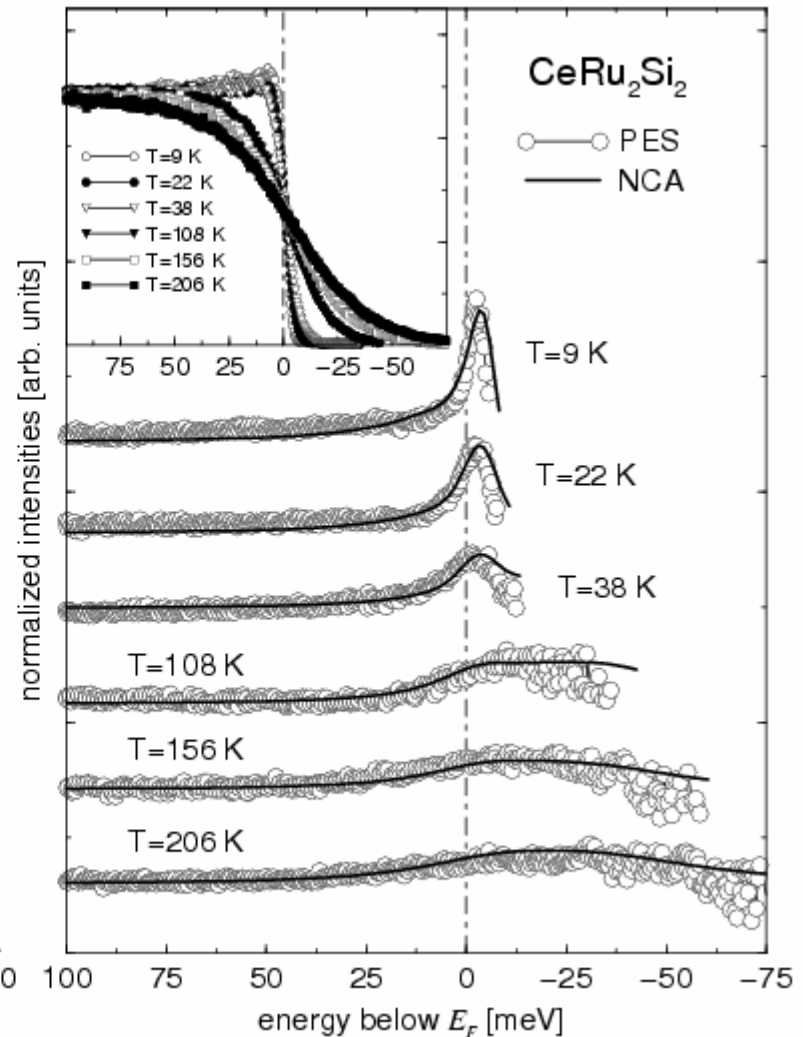
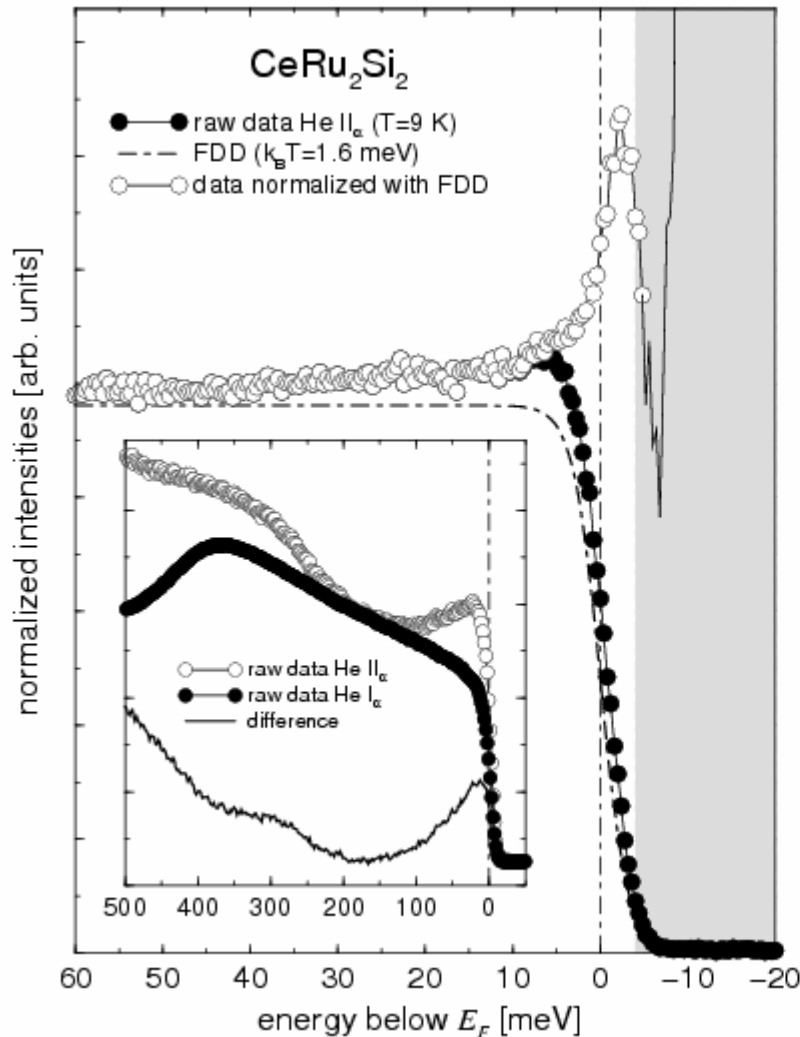
# Results: Temperature dependence of PES of CeCu<sub>6</sub>



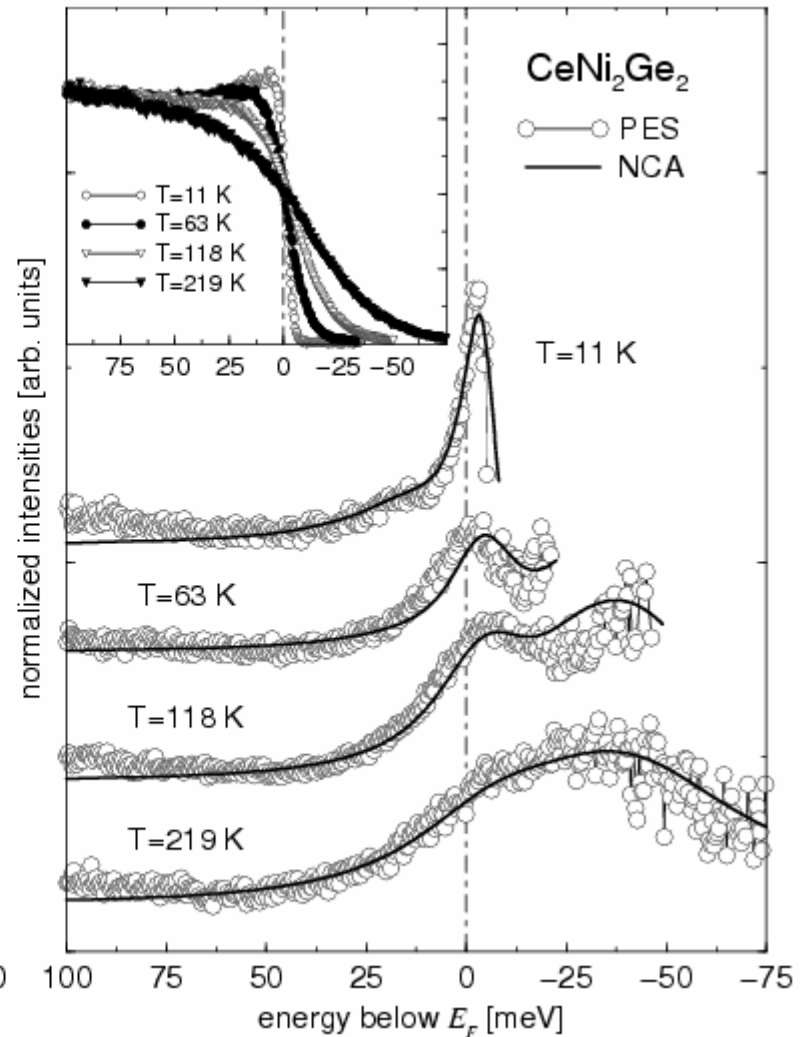
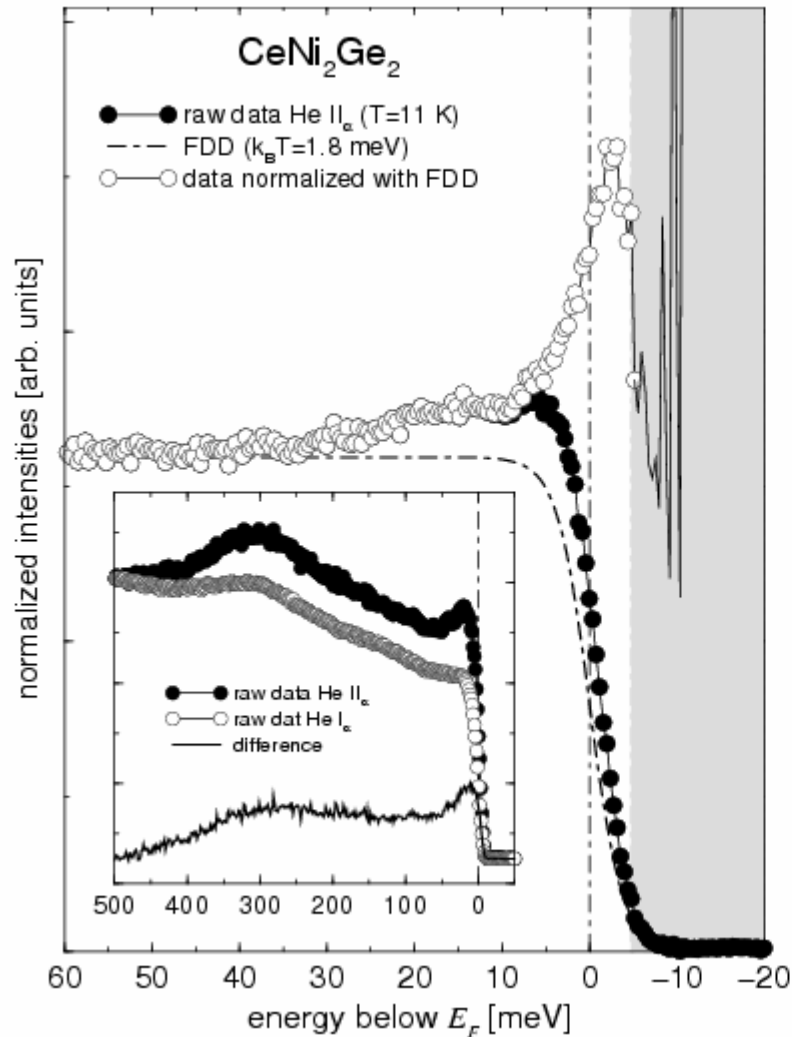
# Results: PES of $\text{CeCu}_2\text{Si}_2$



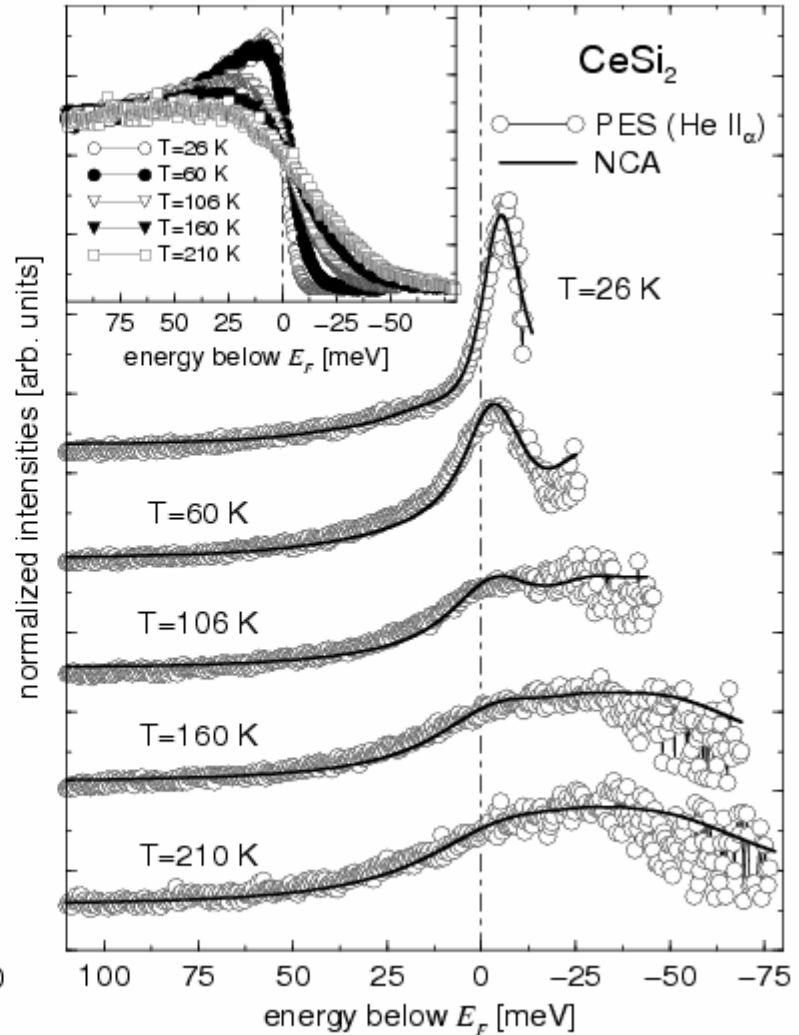
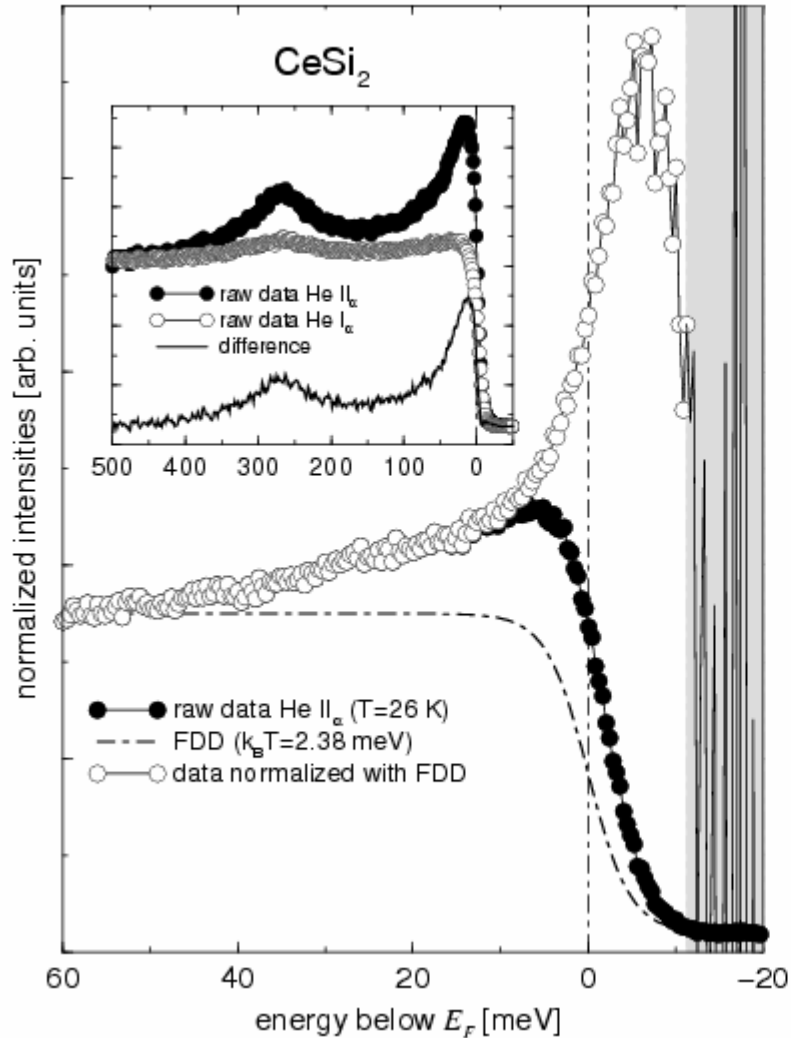
# Results: PES of $\text{CeRu}_2\text{Si}_2$



# Results: PES of $\text{CeNi}_2\text{Ge}_2$



# Results: PES of CeSi<sub>2</sub>



*D. Ehm et al. (2005)*



# Results: Kondo temperature and crystal field splittings

	from other experiments			from PES		
	$T_K$ [K]	CEF [meV]		$T_K$ [K]	CEF [meV]	
		$\Delta_{1-2}$	$\Delta_{1-3}$		$\Delta_{1-2}$	$\Delta_{1-3}$
CeCu <sub>6</sub>	$5.0 \pm 0.5^{(a)39}$	$7.0^{(a)40}$	$13.8^{(a)40}$	4.6	7.2	13.9
CeCu <sub>2</sub> Si <sub>2</sub>	$4.5^{(b)41} - 10^{(a)42}$	$30^{(a)40} - 36^{(c)43}$	—	6	32	37
CeRu <sub>2</sub> Si <sub>2</sub>	$16^{(a)44}$	$19^{(d)45}$	$34^{(d)45}$	16.5	18	33
CeNi <sub>2</sub> Ge <sub>2</sub>	$29^{(a)23}$	$(4)^{(a)46}$	$34^{(a)46}$	29.5	26	39
CeSi <sub>2</sub>	$22/41^{(a)47}$	$25^{(a)47}$	$48^{(a)47}$	35	25	48

TABLE I: Comparison of the Kondo temperatures  $T_K$  and the CF energies  $\Delta_{CF}$  determined from PES and from other experimental methods, namely from (a): INS studies, (b): specific heat measurements, (c): Raman scattering experiments, and (d): theoretical considerations based on specific heat measurements.

*D. Ehm et al. (2005)*

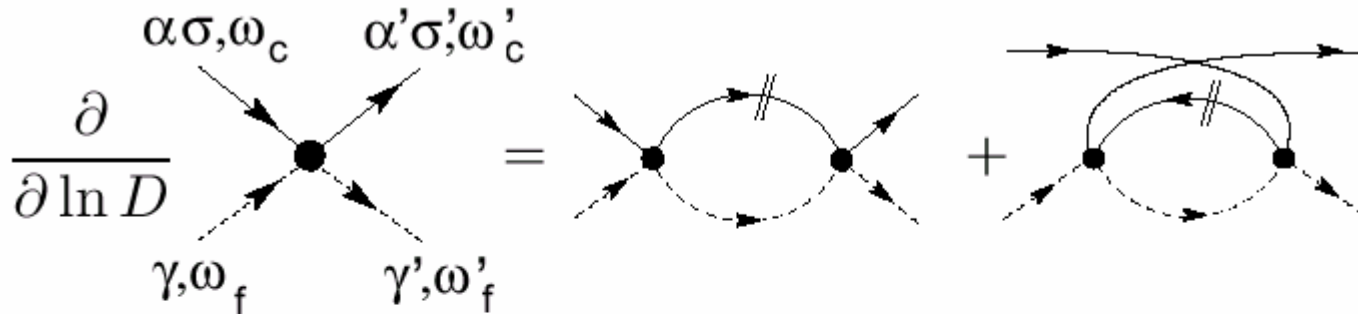


# Conclusion

- Photoemission spectra of Ce compounds may be modelled within single ion Anderson model of spin-orbit and crystal field split ionic states
- Excited crystal field split 4f-states lead to Kondo-type satellite resonance peaks in the single particle spectral function
- The peak positions are given by a rigid shift of the multiplet up to the Fermi level, as obtained by slave boson mean field theory
- The Kondo character of the peaks is apparent from logarithmically divergent terms in perturbation theory
- Summing the leading logarithms by renormalization group methods, observing the effect of phase decoherence for excited states yields resonance peaks of width increasing with level splitting
- Quantitative results were obtained within NCA, in excellent agreement with experiment



# Methods: Diagrams of one loop RG equation



Diagrammatic form of the RG equation. The strokes symbolize derivatives with respect to  $\ln D$  and  $\alpha, \alpha' = L/R$ ,  $\sigma, \sigma' = \uparrow/\downarrow$  and  $\gamma, \gamma' = \uparrow/\downarrow$  denote the quantum numbers of the incoming and outgoing conduction electrons and pseudo fermions, respectively. Their frequencies are given by  $\omega_c, \omega'_c, \omega_f, \omega'_f$  with  $\omega_c + \omega_f = \omega'_c + \omega'_f$ .

**Main contribution from Keldysh comp. of conduction electron  $G$  and real part of pseudofermion  $G$ :**

$$\frac{\partial}{\partial \ln D} \int_{-D}^D d\omega \frac{\text{sign } \omega}{\omega - \Delta\omega} \approx 2\Theta(D - |\Delta\omega|)$$

