

Dresden, 18 April 2007

**Signatures of the precursor
superconductivity above T_c**

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Outline

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Introduction

/ pairing in the many-body systems /

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Conventional superconductors

/ effects of small pair fluctuations /

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HTSC cuprates

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HTSC cuprates

⇒ basic properties

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⇒ description of pair fluctuations above T_c

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basic properties

description of pair fluctuations above T_c

⇒ results for ARPES, STM and collective phenomena

Outline



Introduction

/ pairing in the many-body systems /



Conventional superconductors

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HTSC cuprates

basic properties

description of pair fluctuations above T_c

results (ARPES, collective features, etc)



Summary

I. Introduction

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Very often formation of the fermion pairs goes hand in hand with **superconductivity/superfluidity** but it needs not be the rule.

Hamiltonian of the pairing interactions

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The momentum representation:

$$\hat{H} = \sum_{\mathbf{k}, \sigma} \xi_{\mathbf{k}} \hat{c}_{\mathbf{k}\sigma}^{\dagger} \hat{c}_{\mathbf{k}\sigma} + \sum_{\mathbf{k}, \mathbf{k}'} V_{\mathbf{k}, \mathbf{k}'} \hat{c}_{\mathbf{k}\uparrow}^{\dagger} \hat{c}_{-\mathbf{k}\downarrow}^{\dagger} \hat{c}_{-\mathbf{k}'\downarrow} \hat{c}_{\mathbf{k}'\uparrow}$$

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where $V_{\mathbf{k}, \mathbf{k}'} < 0$ (at least for some \mathbf{k}, \mathbf{k}' states)

The real space representation:

$$\hat{H} = \sum_{i,j} \sum_{\sigma} t_{i,j} \hat{c}_{i\sigma}^{\dagger} \hat{c}_{j\sigma} + \sum_{i,j} V_{i,j} \hat{c}_{i\uparrow}^{\dagger} \hat{c}_{i\uparrow} \hat{c}_{j\downarrow}^{\dagger} \hat{c}_{j\downarrow}$$

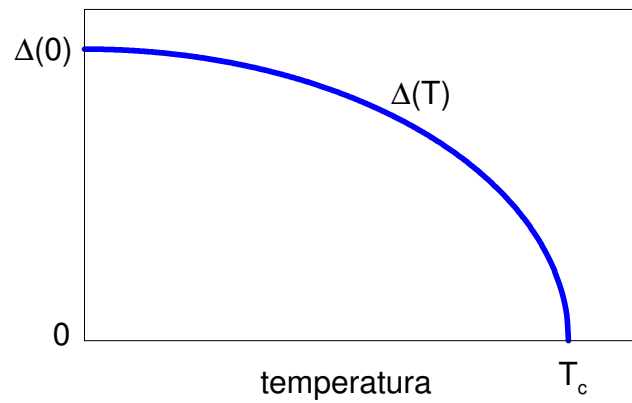
with attractive potential $V_{i,j} < 0$

II. Conventional superconductors

Major property

Major property

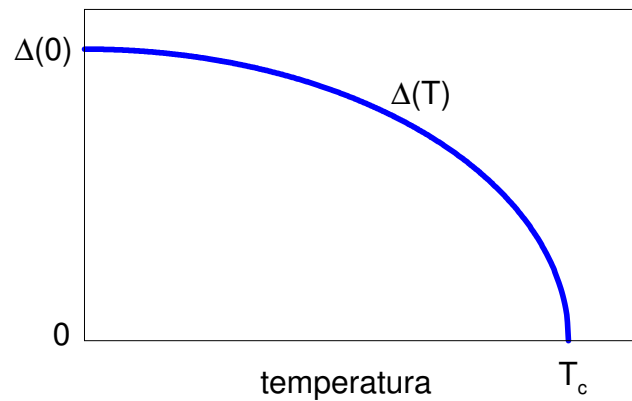
Pair formation and onset of their coherence coincide at T_c



Pairing is responsible for the gap in the single particle spectrum

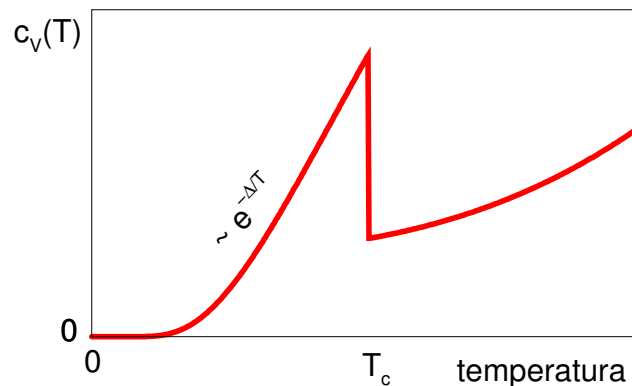
Major property

Pair formation and onset of their coherence coincide at T_c



Pairing is responsible for the gap in the single particle spectrum

The order parameter \longrightarrow 2-nd order phase transition



/ as classified by Landau /

Pair fluctuations

Pair fluctuations

Possible sources:

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1. quantum fluctuations

$$\hat{A}\hat{B} = \hat{A}\langle\hat{B}\rangle + \langle\hat{A}\rangle\hat{B} - \langle\hat{A}\rangle\langle\hat{B}\rangle + \delta\hat{A} \delta\hat{B}$$

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2. topological

In the low dimensional ($\text{dim} \leq 2$) systems ODLRO does not establish.

There can appear only the power-law behaviour

$$\langle\hat{\psi}_{\downarrow}(\mathbf{r}_1) \hat{\psi}_{\uparrow}(\mathbf{r}_2)\rangle \propto |\mathbf{r}_1 - \mathbf{r}_2|^{-\theta(T)}$$

with the phase stiffness $\theta \neq 0$ for $T \leq T_{KT}$.

J.M. Kosterlitz and P.J. Thouless, J. Phys. C 6, 1181 (1973).

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He predicted smearing of the specific heat jump near T_c but in an extremely narrow temperature region

$$\frac{\delta T}{T_c} \sim \left(\frac{a}{\xi} \right)^4 \sim 10^{-12} - 10^{-14}$$

a – interatomic distance,
 ξ – correlation length.

V.L. Ginzburg, Sov. Solid State 2, 61 (1968).

Historical remarks: 2

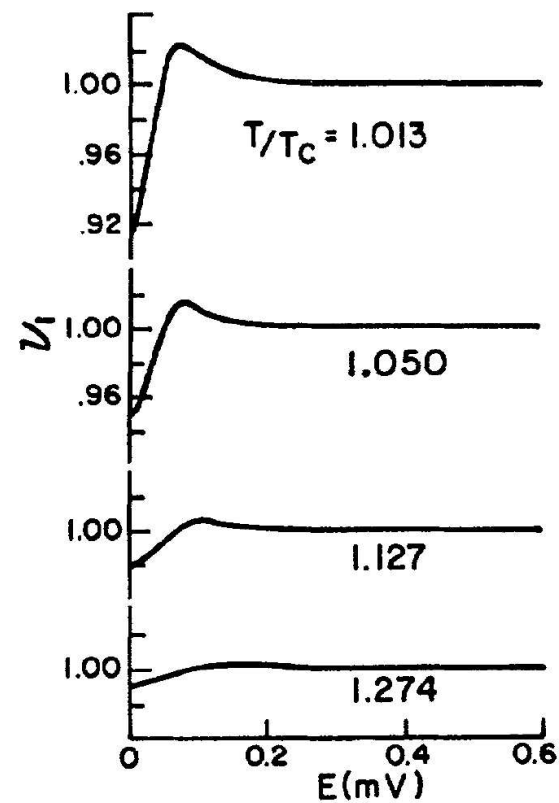
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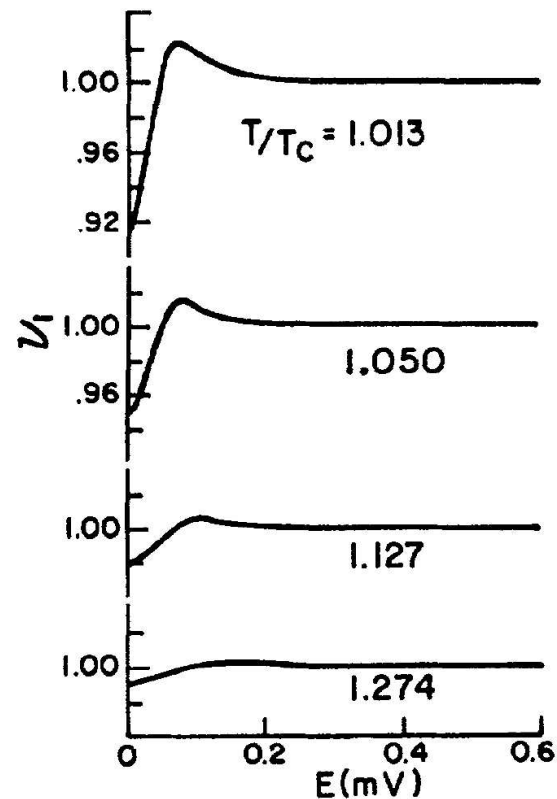
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R.W. Cohen and B. Abels, *Phys. Rev.* **168**, 444 (1968).

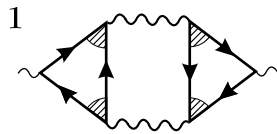
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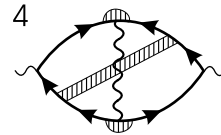
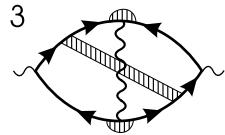
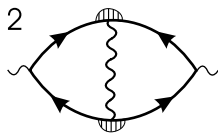
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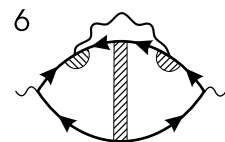
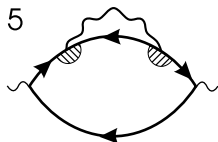
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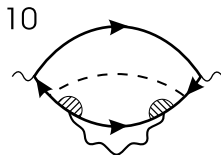
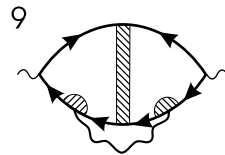
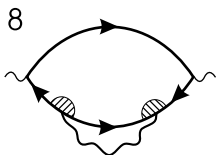
Aslamazov-Larkin



Maki - Thompson



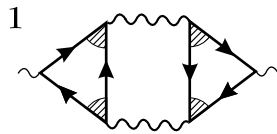
*diagrams giving
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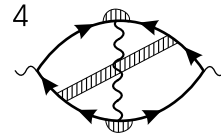
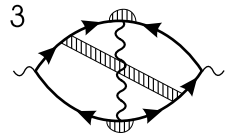
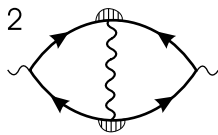
to the density of states

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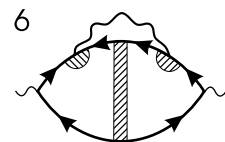
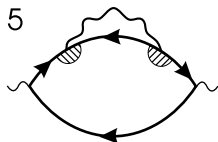
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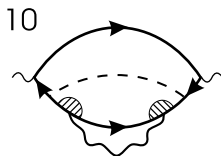
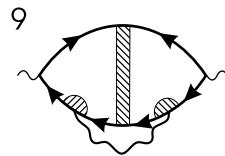
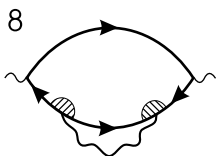
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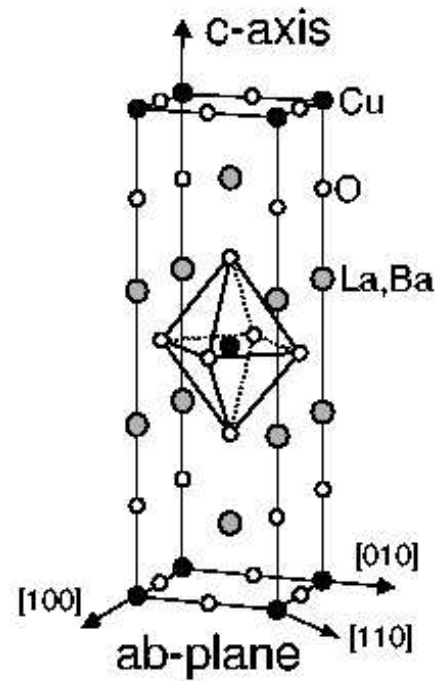


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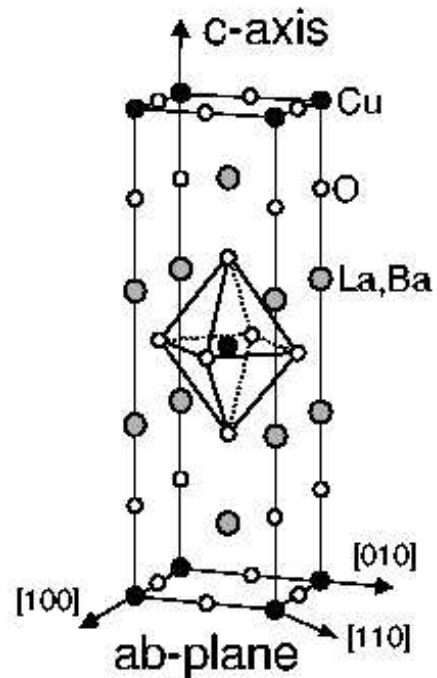
V.V. Dorin et al, *Phys. Rev. B* **48**, 12951 (1993).

III. HTSC cuprates

1. The parent compounds are quasi-2D Mott insulators



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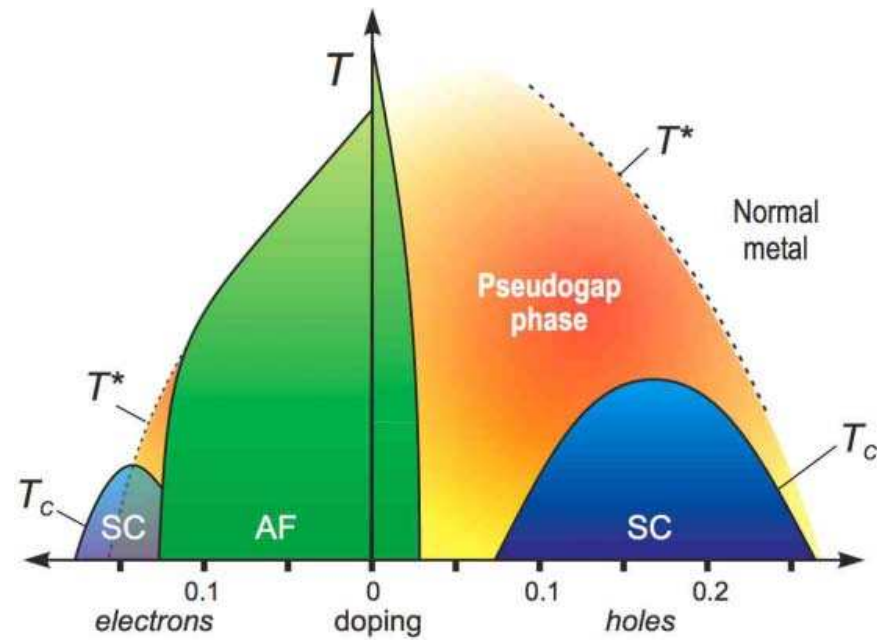
Important remark:

Spatial extent of the pairs is very short $\xi_{ab} \simeq 5 \text{ \AA}$

2. Superconductivity appears upon doping by

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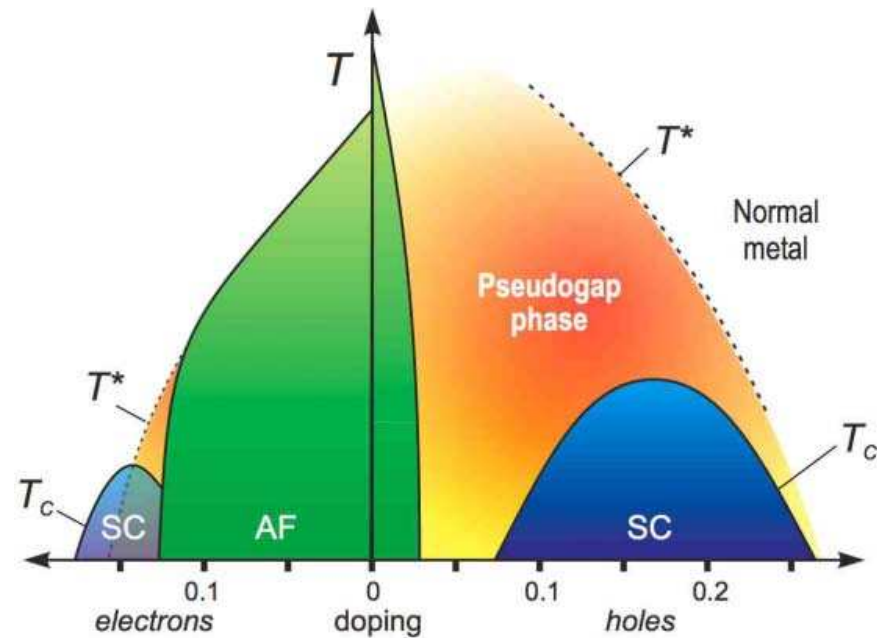
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O. Fisher et al, Rev. Mod. Phys. 79, 353 (2007).

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A puzzle:

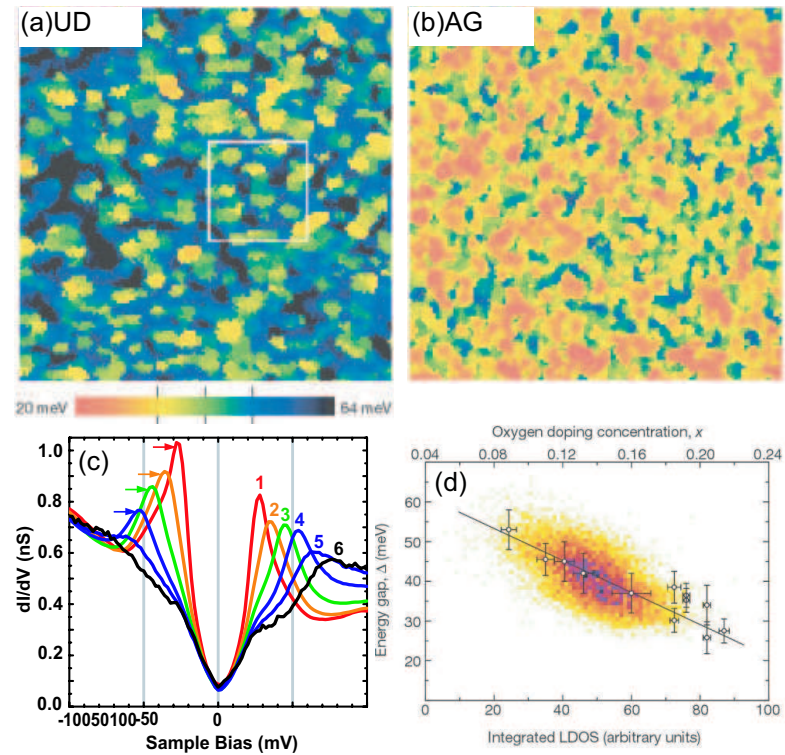
Why this diagram is asymmetric ?

3. Inhomogeneities

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The new STM spectroscopy finds that the energy gap is inhomogeneous.

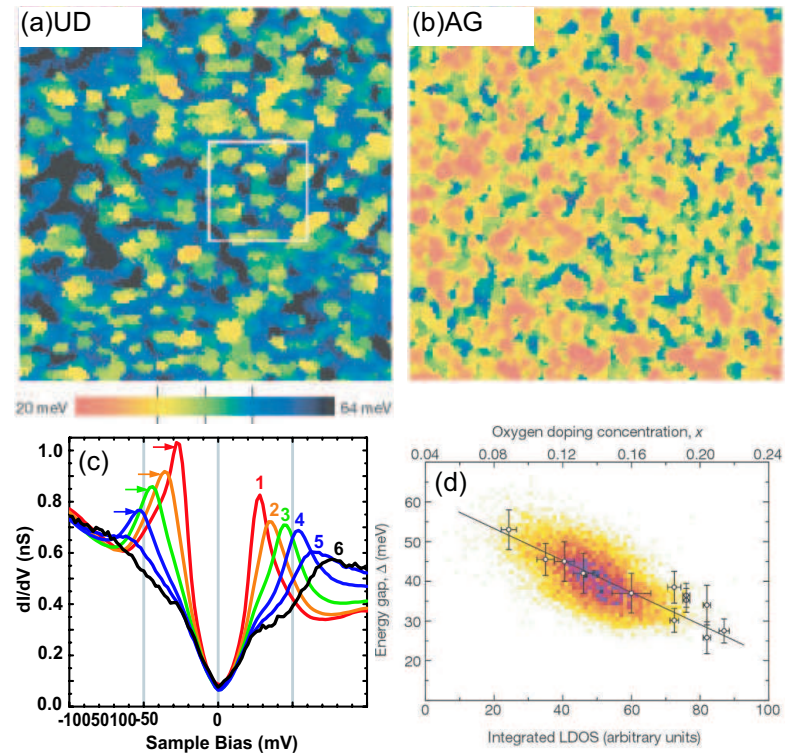
/ K. McElroy et al, Phys. Rev. Lett. 94, 197005 (2005) /



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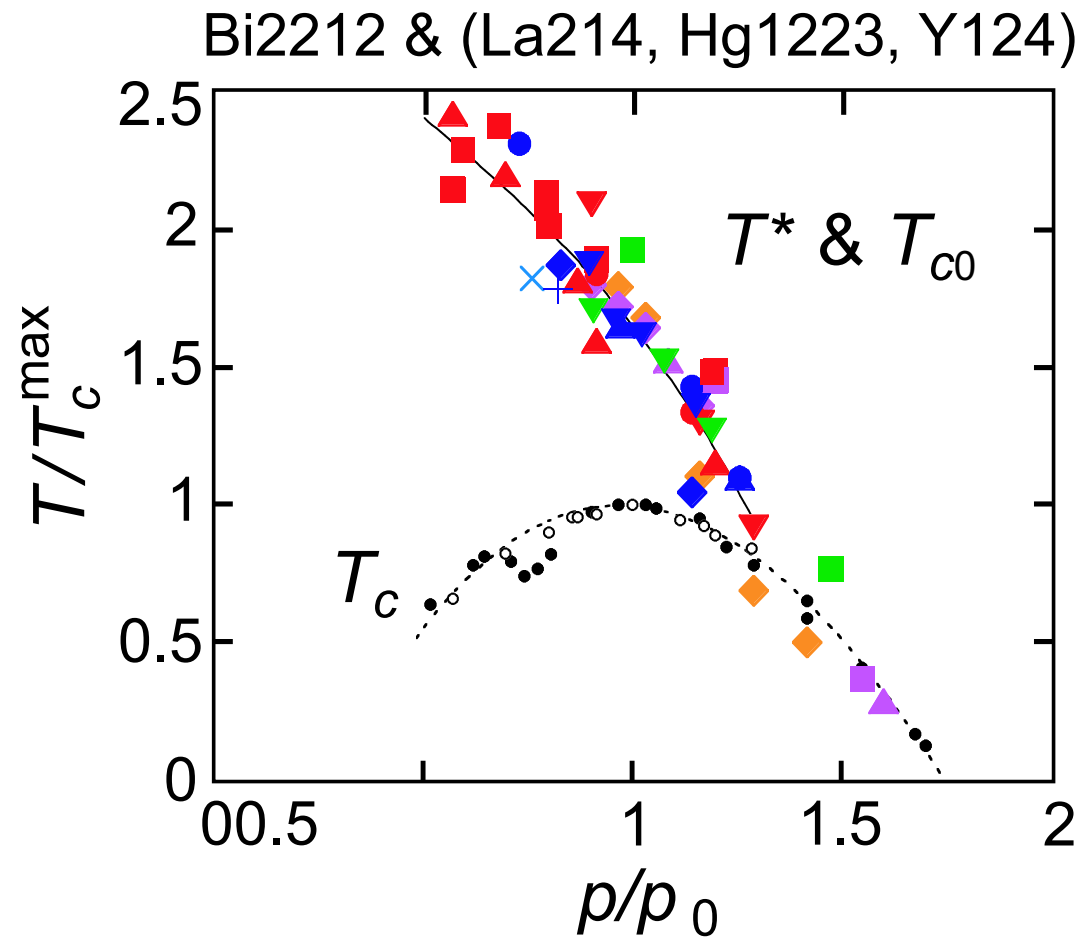


Notice:

Inhomogeneous structure promotes the fluctuations

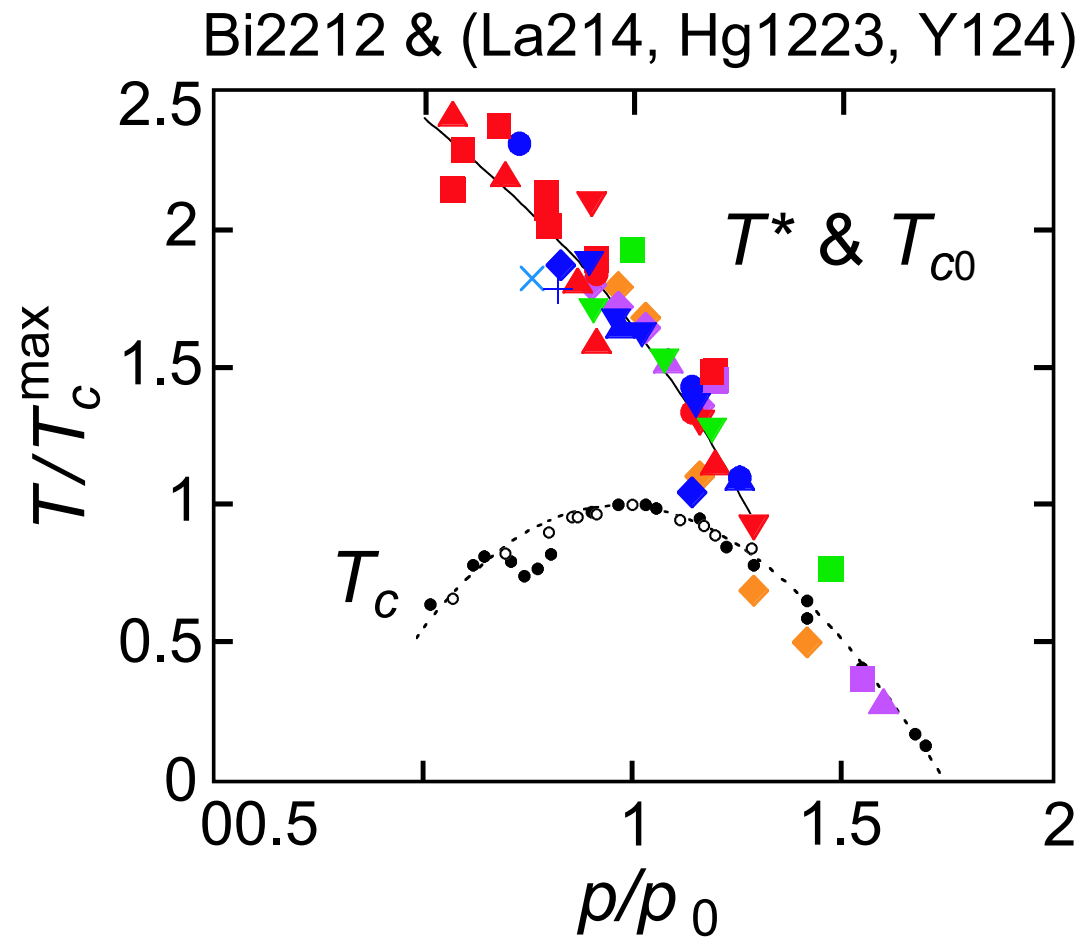
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T. Nakano et al, J. Phys. Soc. Jpn. 67, 2622 (2002).

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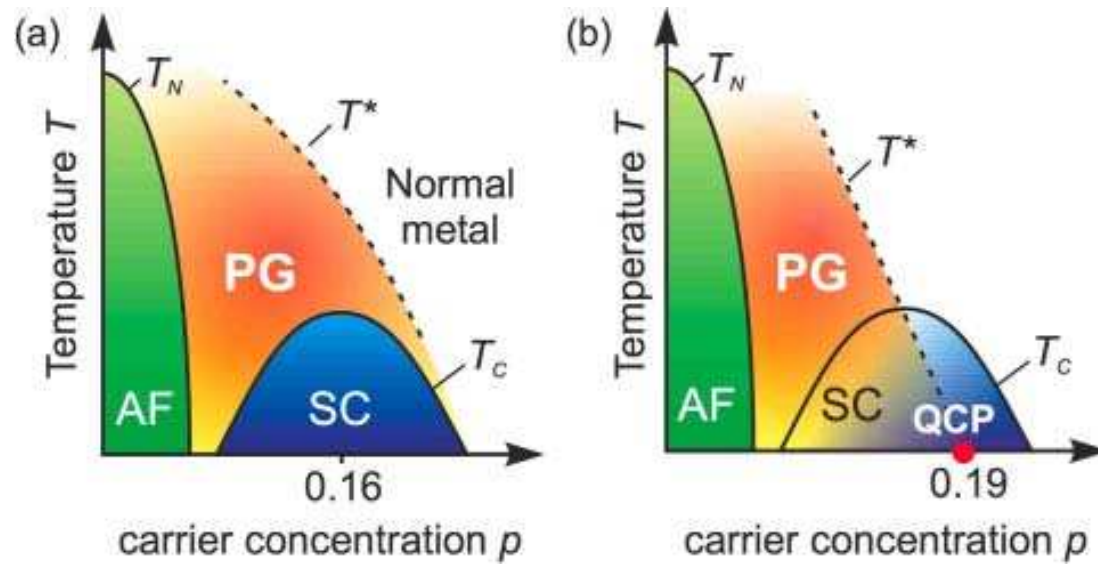


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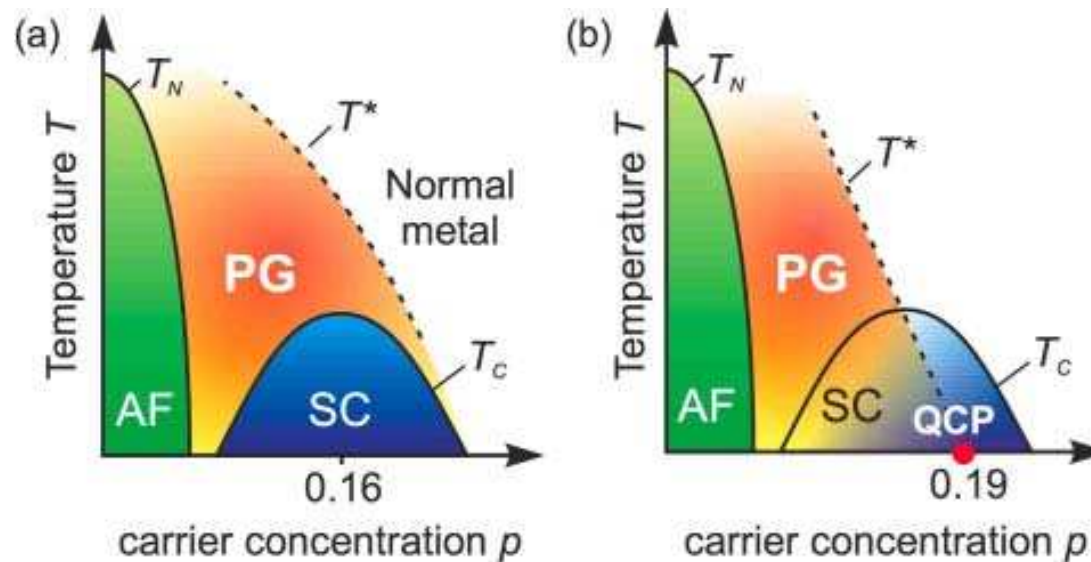
What kind of mechanism is responsible for the pseudogap ?

Theoretical concepts

Theoretical concepts



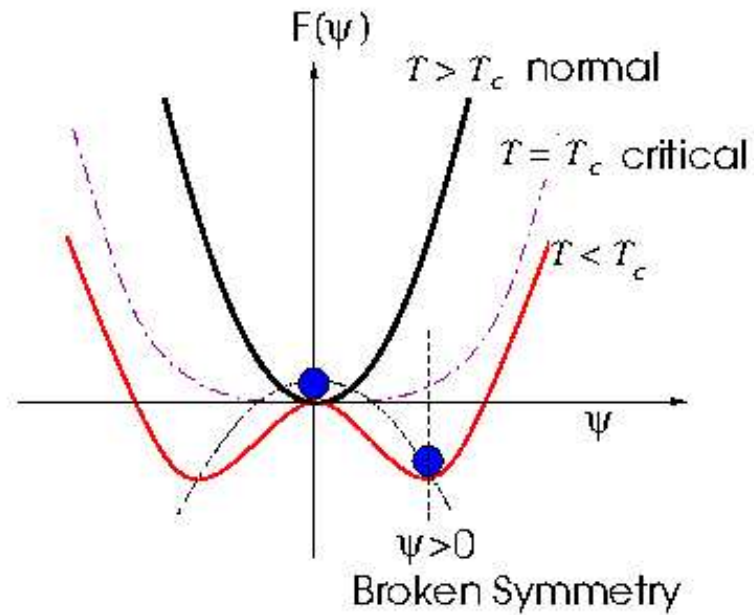
Theoretical concepts



- (a) Pseudogap is a precursor of the superconducting gap which is due to strong fluctuations (because of the reduced dimensionality, local pairing, etc).
- (b) Pseudogap is not related to sc gap. It represents some other type of ordering which is competing with the sc phase.

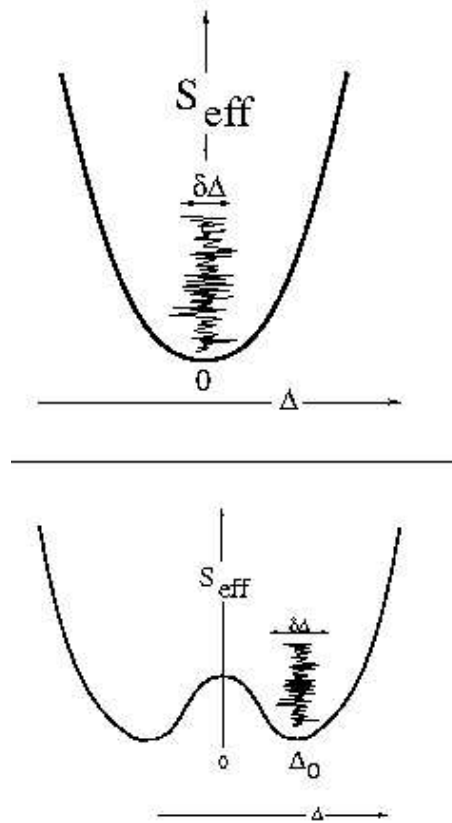
Pair fluctuations

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Usual mean field solution (i.e. the saddle point) neglects an influence of any fluctuations.

Pair fluctuations



One can describe the small fluctuations via Gaussian corrections.

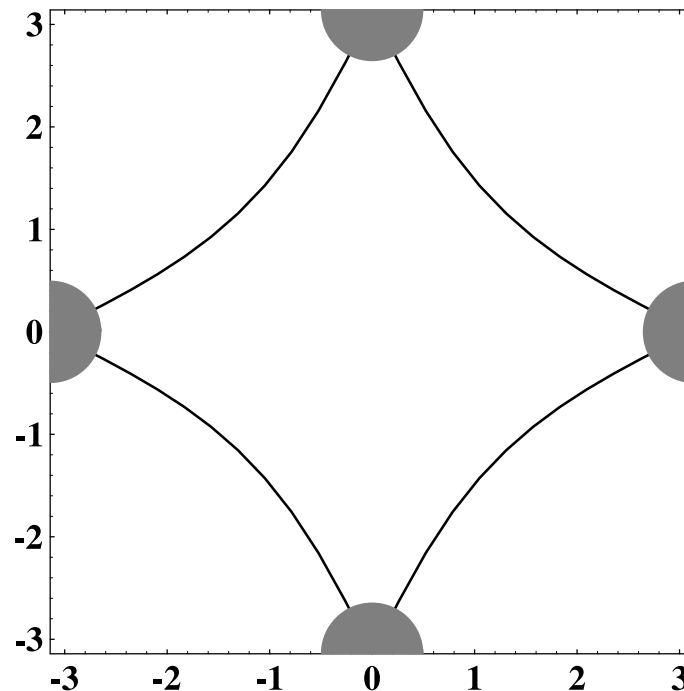
The effective physics

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Without a specific pairing mechanism being established we propose to describe the coherent (for $T < T_c$) or incoherent (for $T > T_c$) fermion pairs using the phenomenological boson-fermion model.

The effective physics

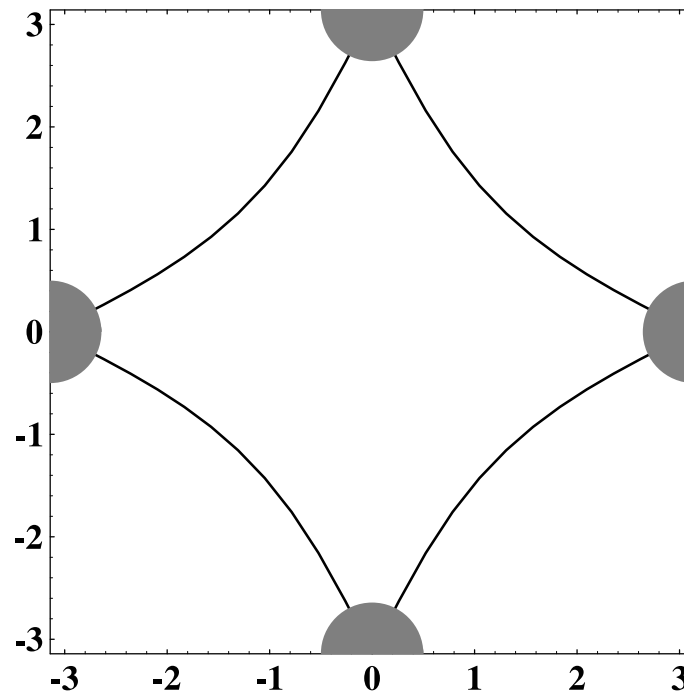
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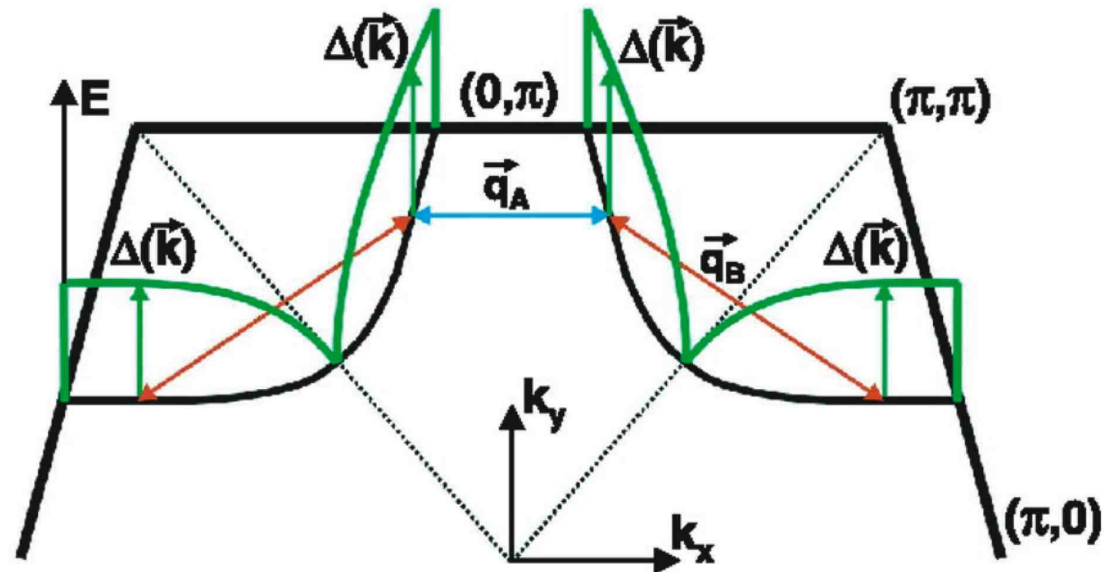
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V.B. Geshkenbein, L.B. Ioffe and A.I. Larkin, Phys. Rev. B 55, 3173 (1997).

Anisotropic energy gap



d-wave type symmetry $\Delta_{\mathbf{k}} = \Delta(\cos k_x - \cos k_y)$ of the gap

J.E. Hoffman et al, Science **297**, 1148 (2002).

The BF model (J. Ranninger, 1985):

$$H = \sum_{k\sigma} (\varepsilon_k - \mu) c_{k\sigma}^\dagger c_{k\sigma} + \sum_q (E_q - 2\mu) b_q^\dagger b_q + \frac{1}{\sqrt{N}} \sum_{k,q} v_{k,q} \left[b_q^\dagger c_{k,\downarrow} c_{q-k,\uparrow} + \text{h.c.} \right]$$

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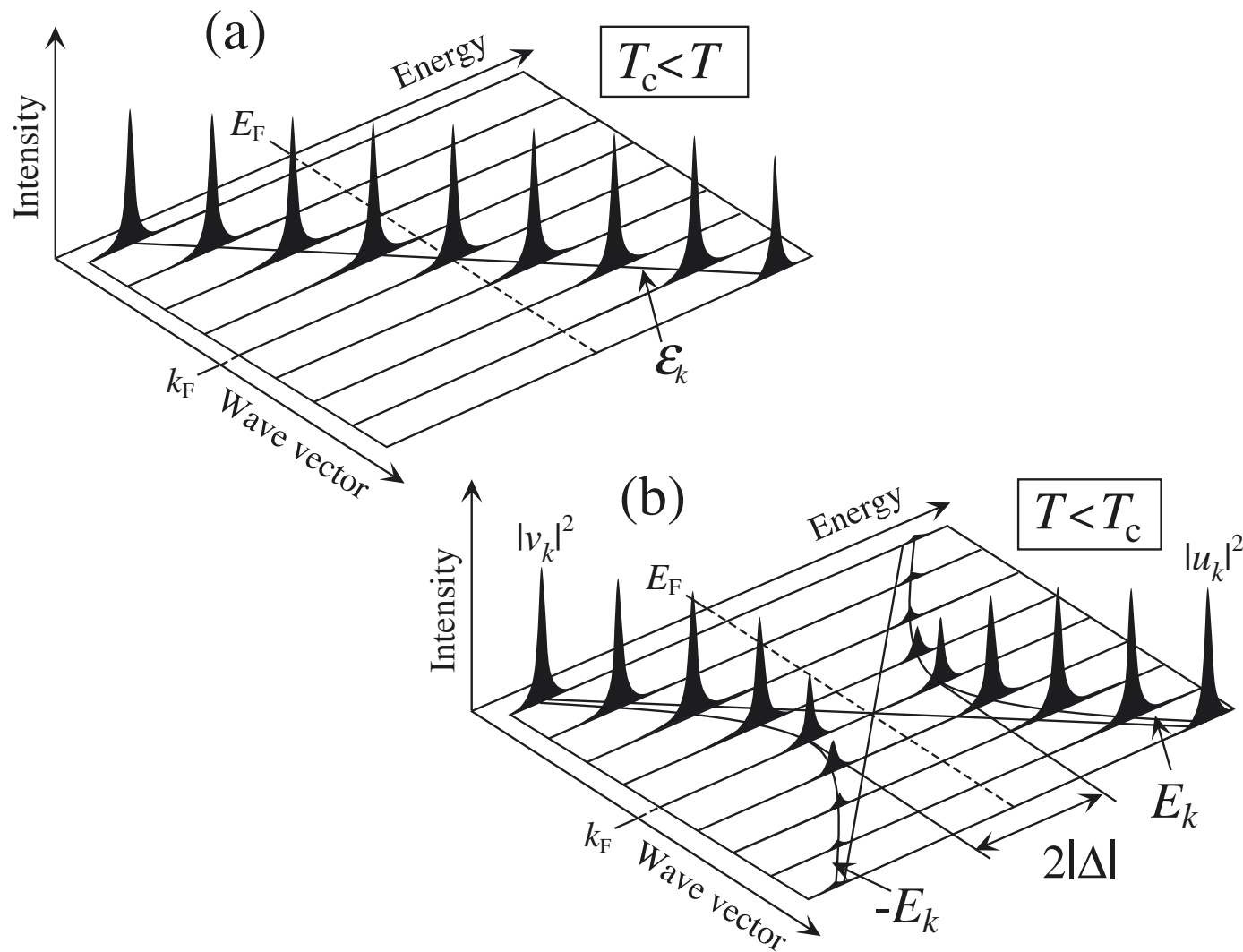
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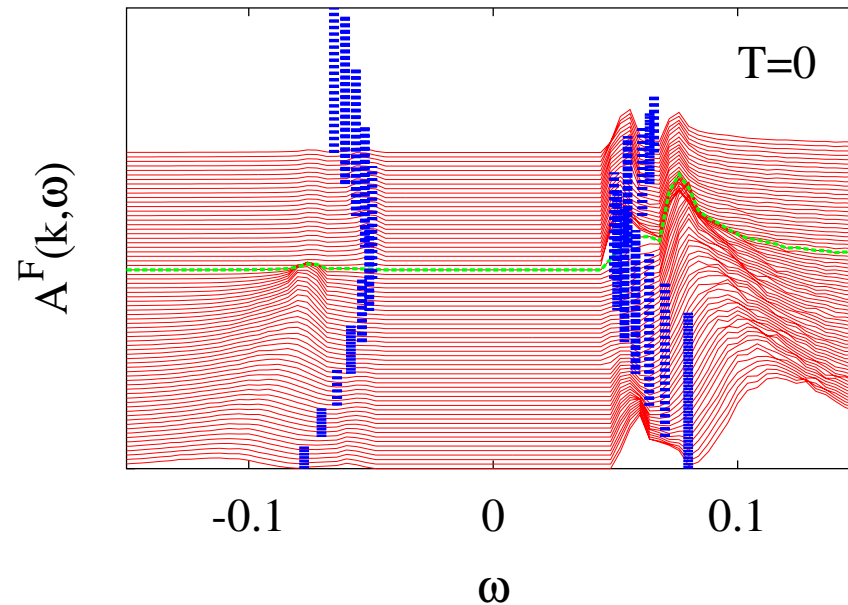
T. Domański and J. Ranninger, Phys. Rev. B 63, 134505 (2001).



Single particle spectrum of conventional superconductors consists of two Bogoliubov branches gapped around E_F (no fluctuation effects are here taken into account).

Bogoliubov-like spectrum

$$T = 0$$

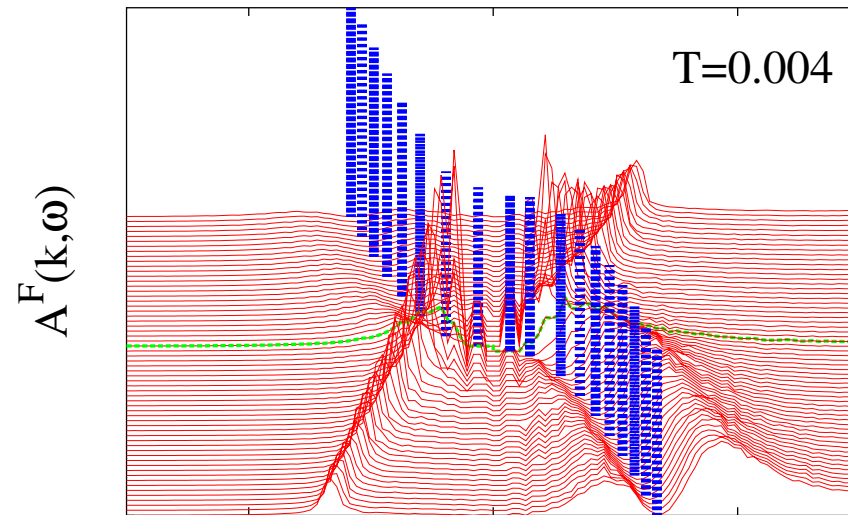


Below the critical temperature T_c there exist two branches of the single-particle excitation energies which (according to the BCS theory) occur at $\omega_{\mathbf{k}} = \pm \sqrt{(\varepsilon_{\mathbf{k}} - \mu)^2 + \Delta_{\mathbf{k}}^2}$.

*T. Domański and J. Ranninger, Phys. Rev. Lett. **91**, 255301 (2003).*

Bogoliubov-like spectrum

$$T_c < T < T^*$$

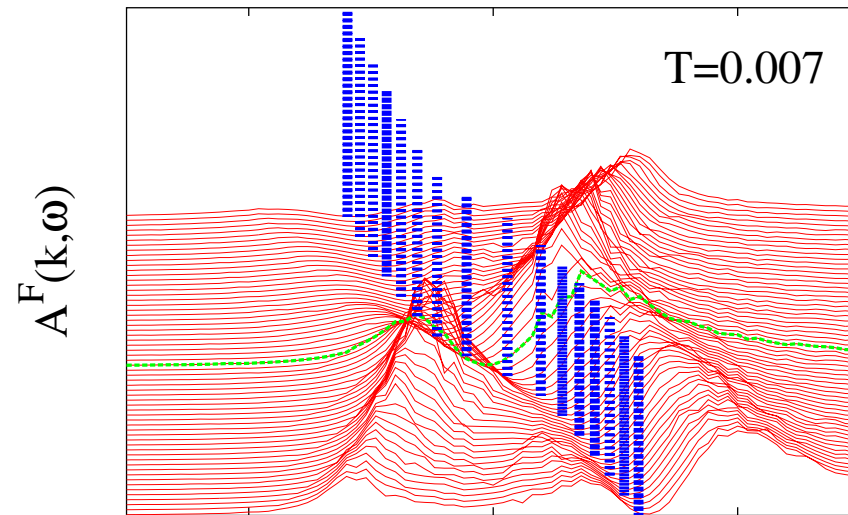


Above T_c the Bogoliubov-type spectrum still survives but one of the branches (the shadow) becomes overdamped. This means that fermion pairs have no longer an infinite life-time.

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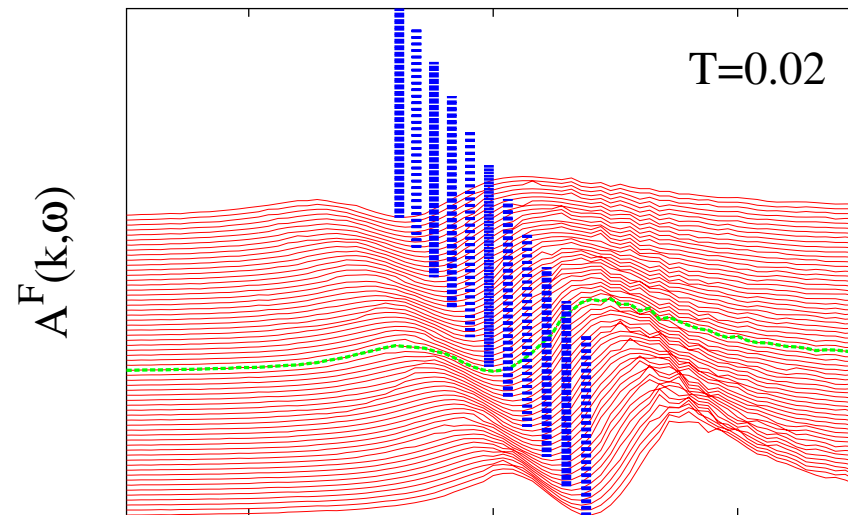


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Bogoliubov-like spectrum

$$T > T^*$$

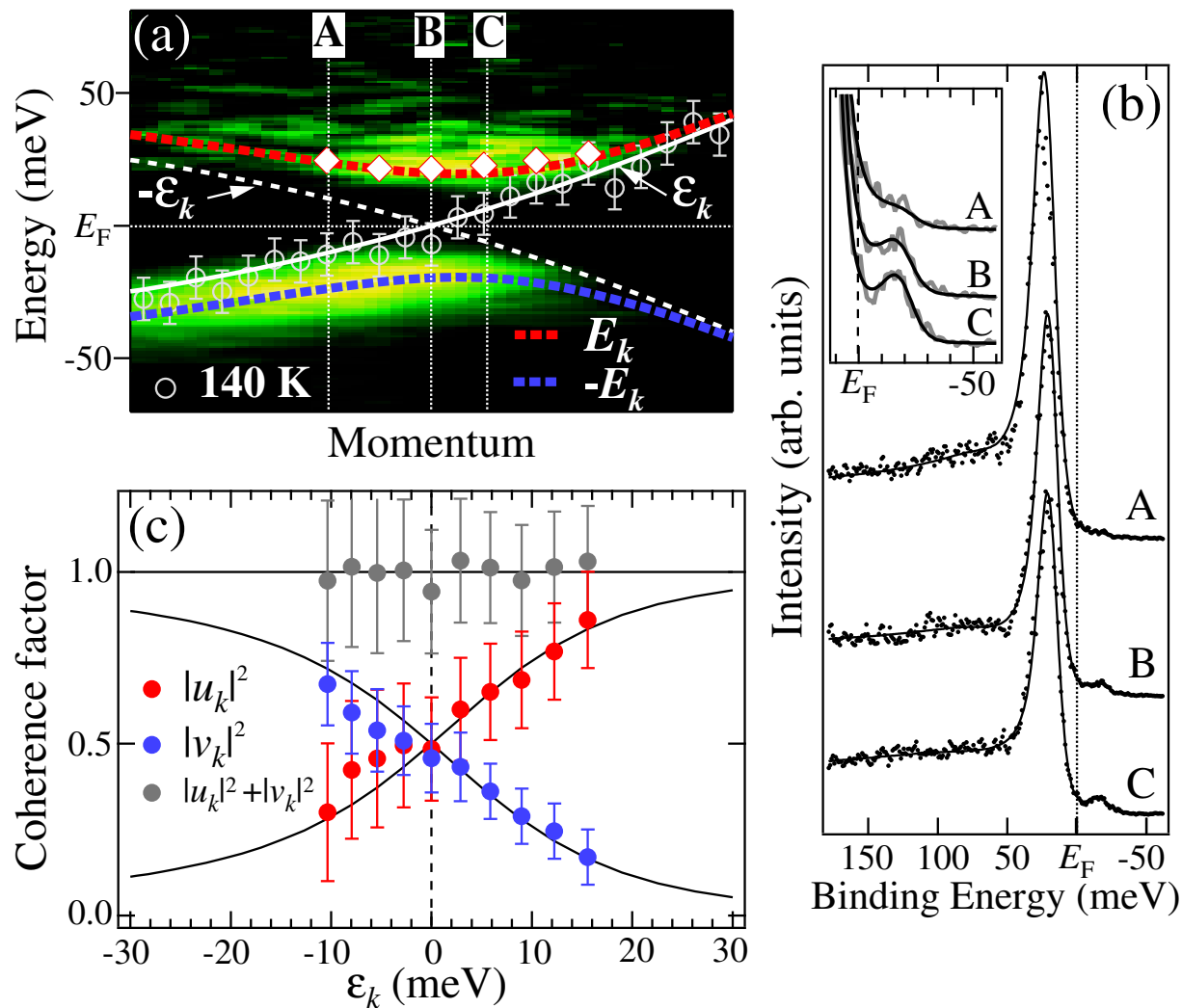


For temperatures far above T_c the Bogoliubov modes are gone and there remains only a single quasiparticle peak surrounded by an incoherent background.

*T. Domański and J. Ranninger, Phys. Rev. Lett. **91**, 255301 (2003).*

Experimental data (Japanese group)

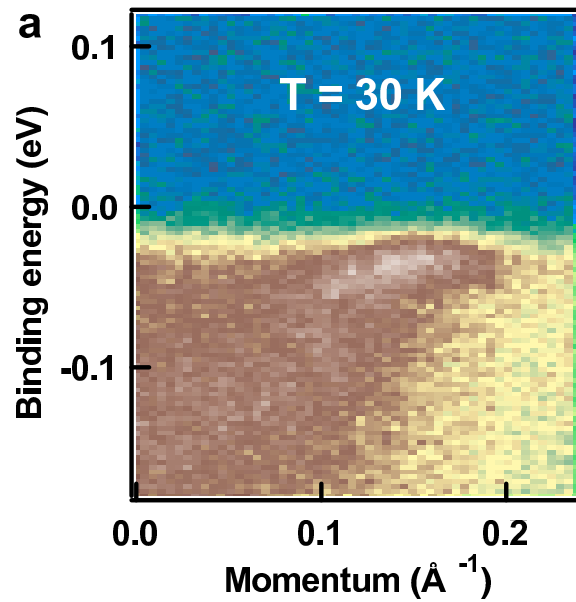
$$T < T_c$$



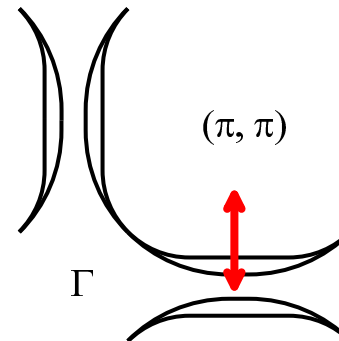
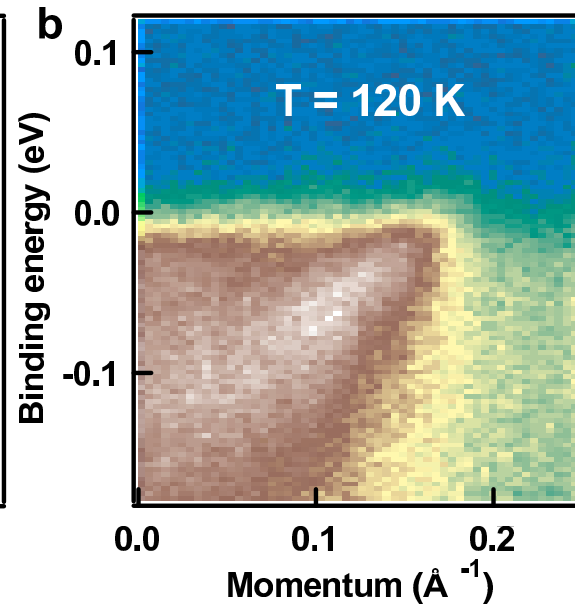
H. Matsui, T. Sato, and T. Takahashi, *Phys. Rev. Lett.* **90**, 217002 (2003).

Experimental data (Dresden group)

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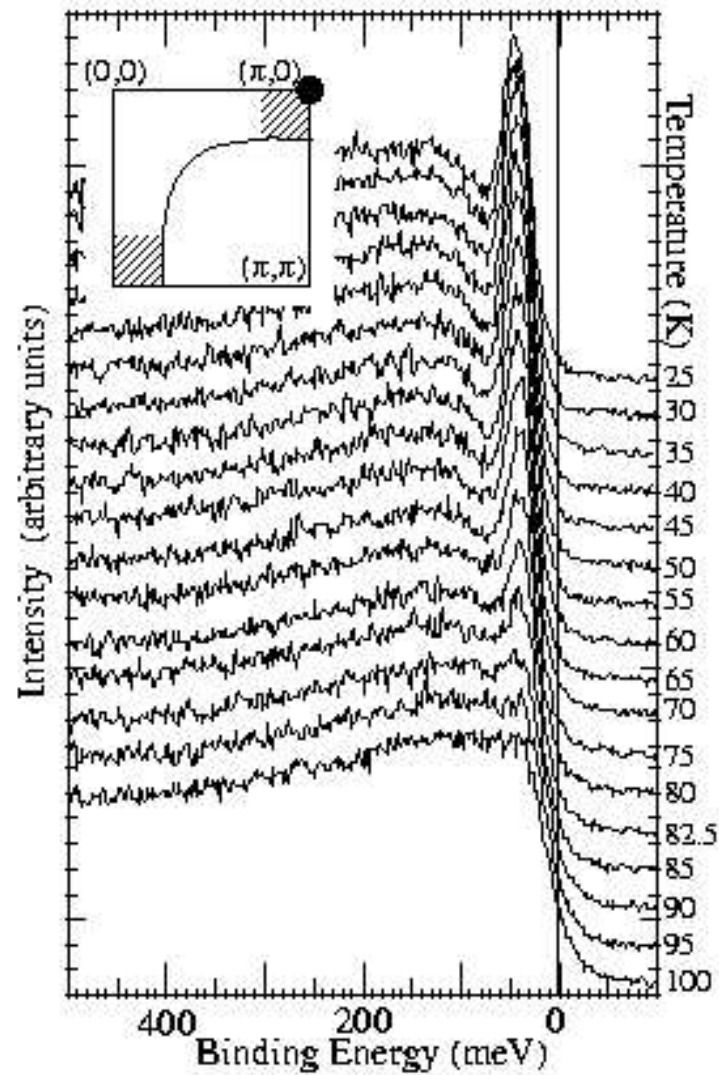


$$T_c < T$$



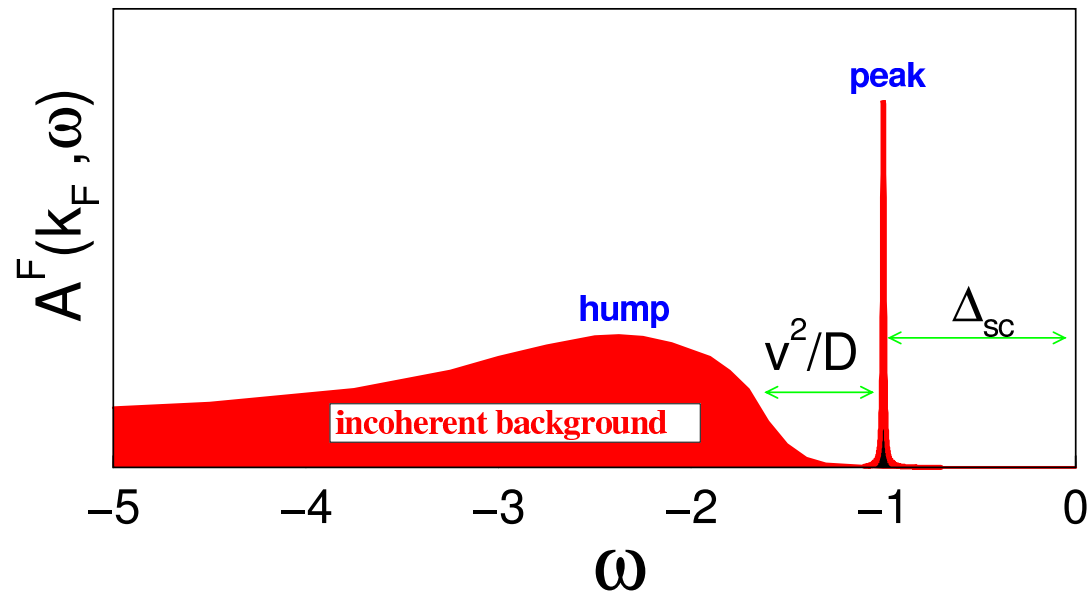
T. Eckl et al, Phys. Rev. B 70, 094522 (2004).

The peak-dip-hump structure



A.G. Loeser, Z.-X. Shen et al, *Phys. Rev. B* **56**, 14185 (1997).

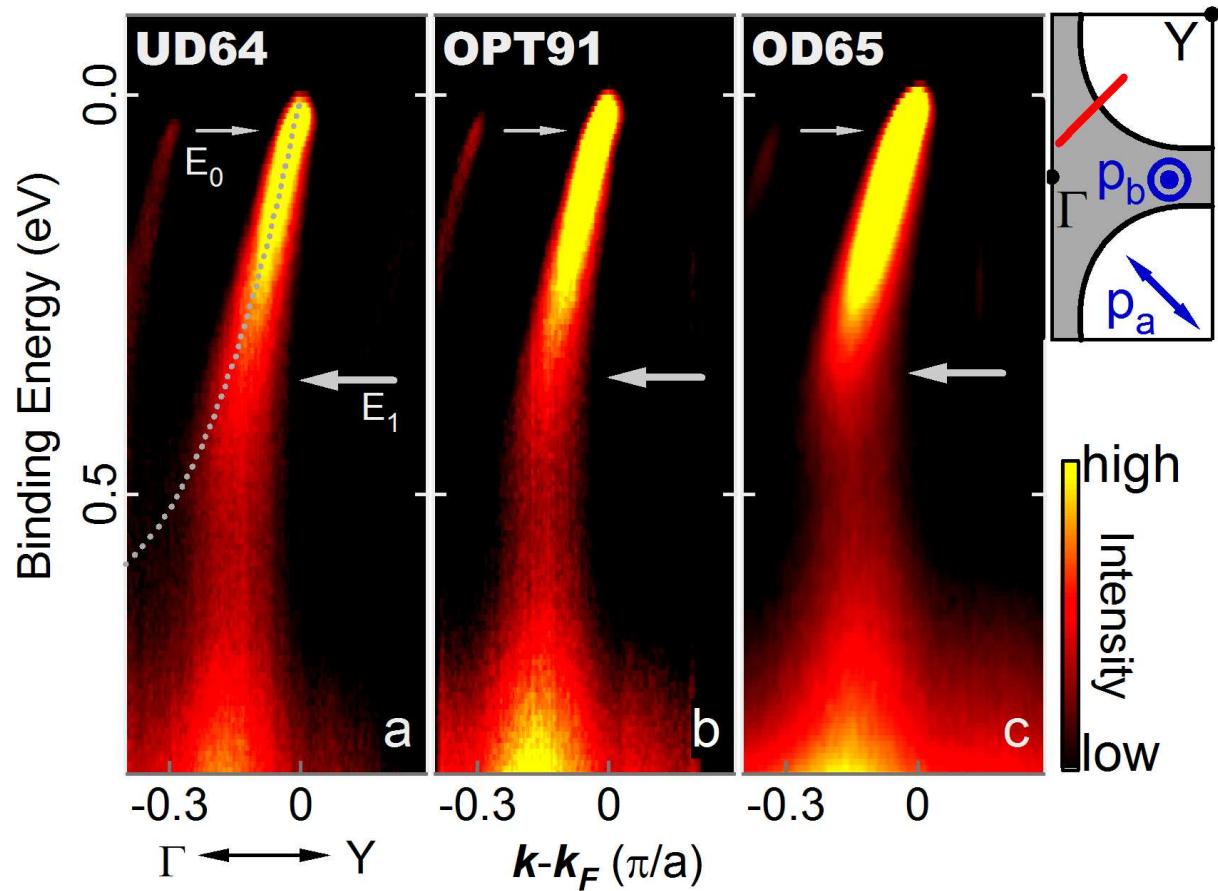
Spectral function for $T < T_c$



Schematic view of the spectral function in the antinodal direction for temperatures $T < T_c$ obtained using the boson-fermion model .

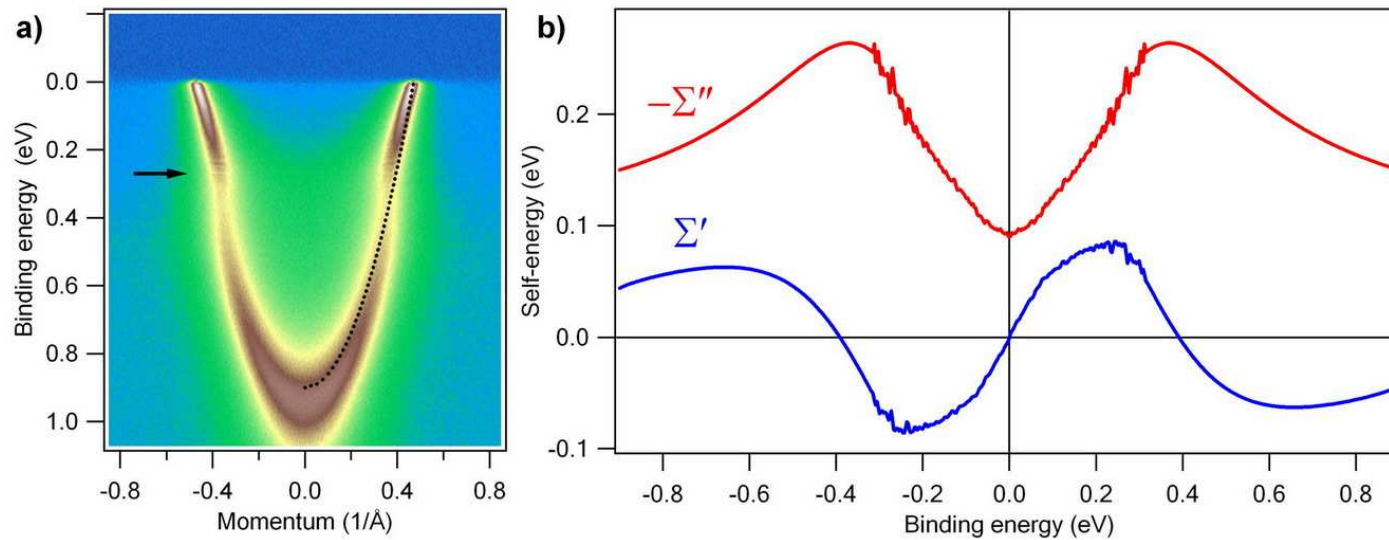
*T. Domański and J. Ranninger, Phys. Rev. B **70**, 184513 (2004).*

The phenomenon of "waterfalls"



J. Graf et al, Phys. Rev. Lett. 98, 067004 (2006).

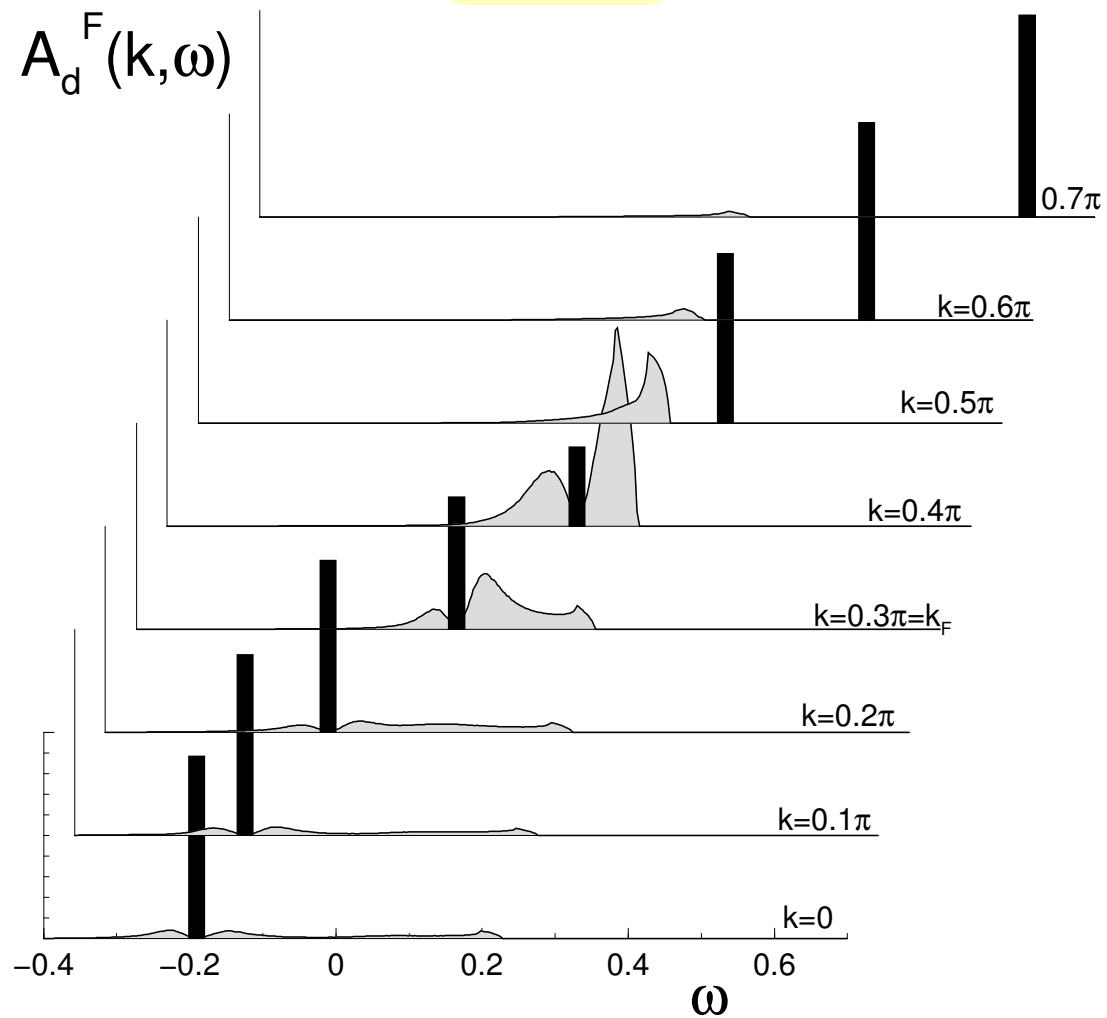
Physical implications of the "waterfalls"



D.S. Ionosov et al, cond-mat/0703223; A.A. Kordyuk et al, cond-mat/0702374.

"Waterfalls" can result from coupling to bosonic mode

$$T > T^*$$



T. Domański and J. Ranninger, Phys. Rev. B 70, 184503 (2004).

Collective phenomena

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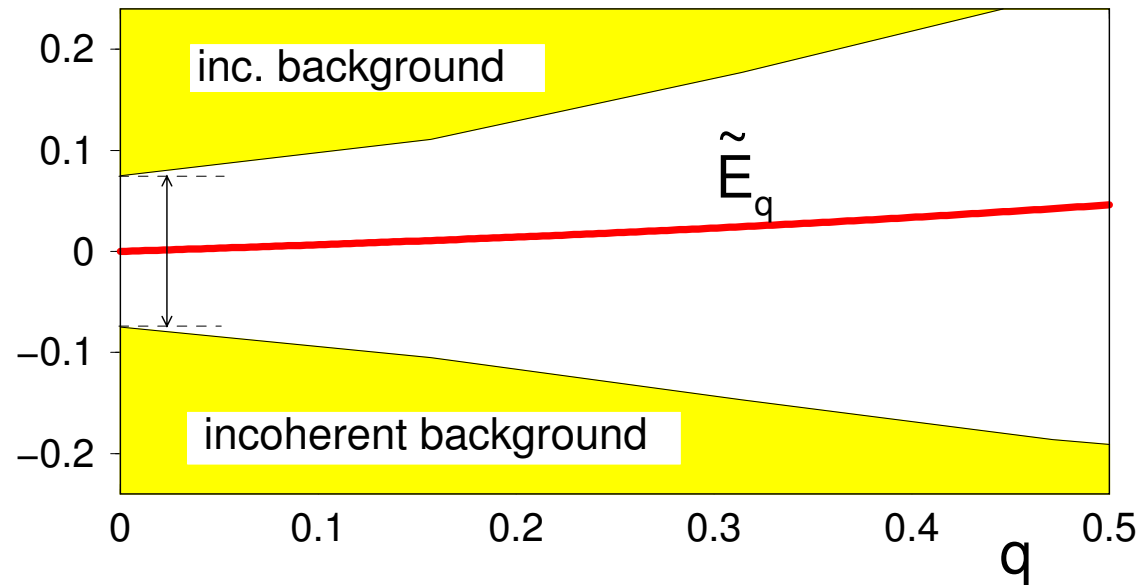
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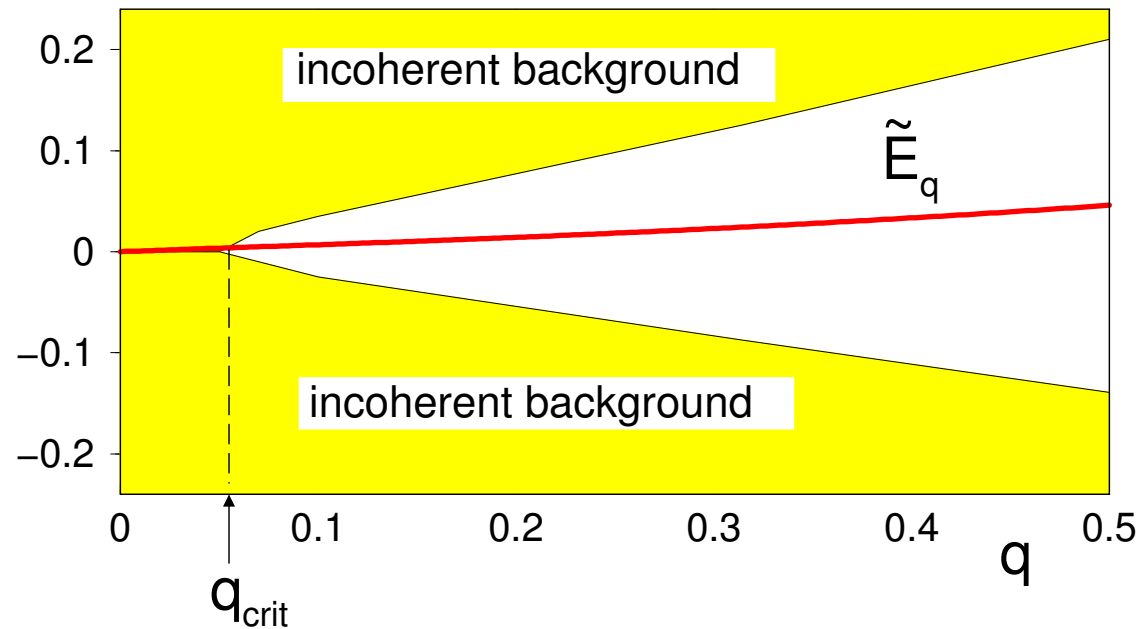
The pair spectrum for $T < T_c$



The quasiparticle peak is well separated from the incoherent background and, in the limit $q \rightarrow 0$, has a characteristic dispersion $\tilde{E}_q = c |q|$. This Goldstone mode is a hallmark of the symmetry broken state.

Such a unique situation could be observed in the case of ultracold fermion atoms, otherwise the Coulomb repulsions lift this mode to the high plasmon frequency.

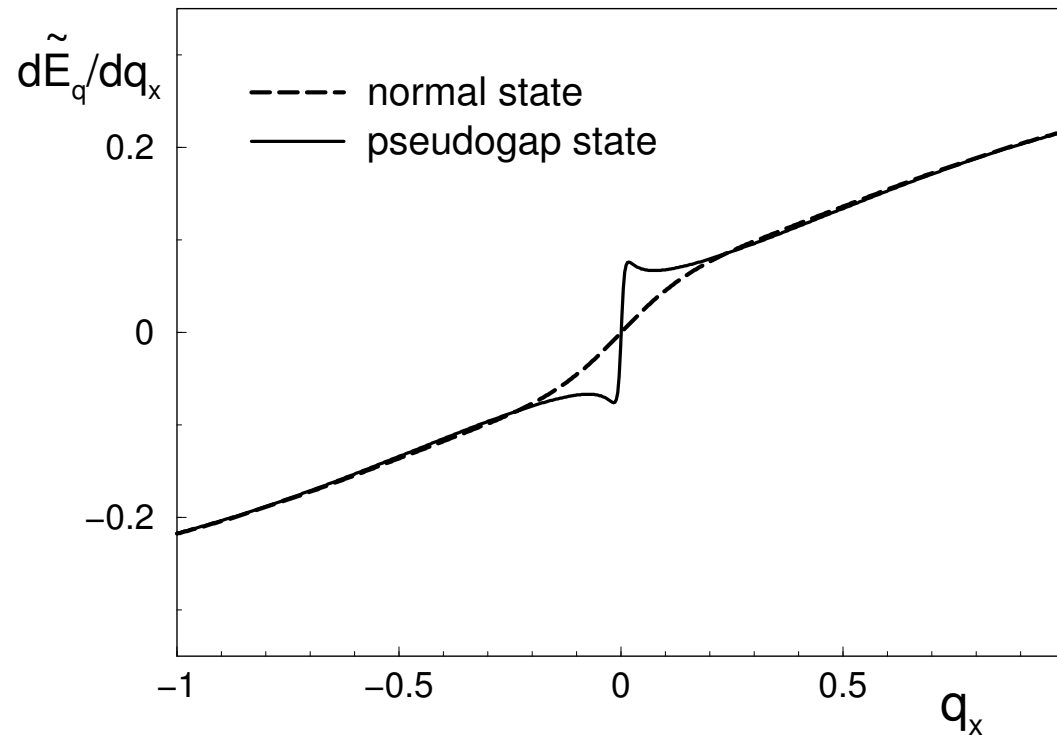
The pair spectrum for $T^* > T > T_c$



Above the transition temperature (for $T^ > T > T_c$):*

- ★ *the quasiparticle peak overlaps at small momenta with the incoherent background,*
- ★ *for $q \rightarrow 0$ the Goldstone mode disappears,*
- ★ *remnant of the Goldstone mode is seen above q_{crit} .*

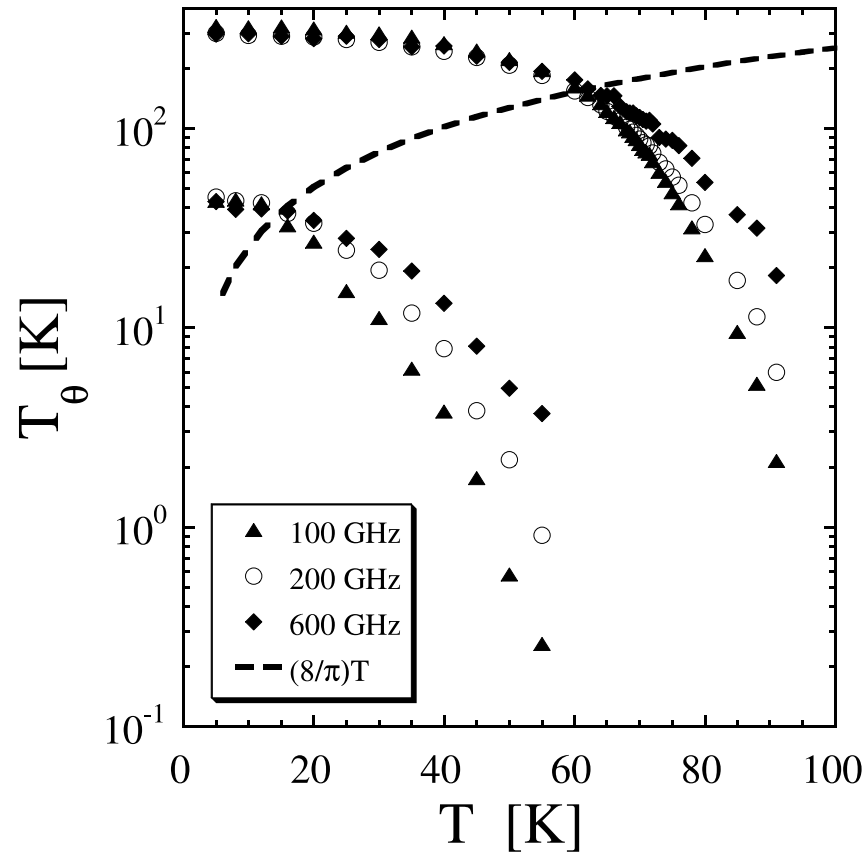
The pair spectrum for $T^* > T > T_c$



Remnant of the Goldstone branch in the dispersion of fermion pairs above T_c .

T. Domański and A. Donabidowicz, (2007) in print.

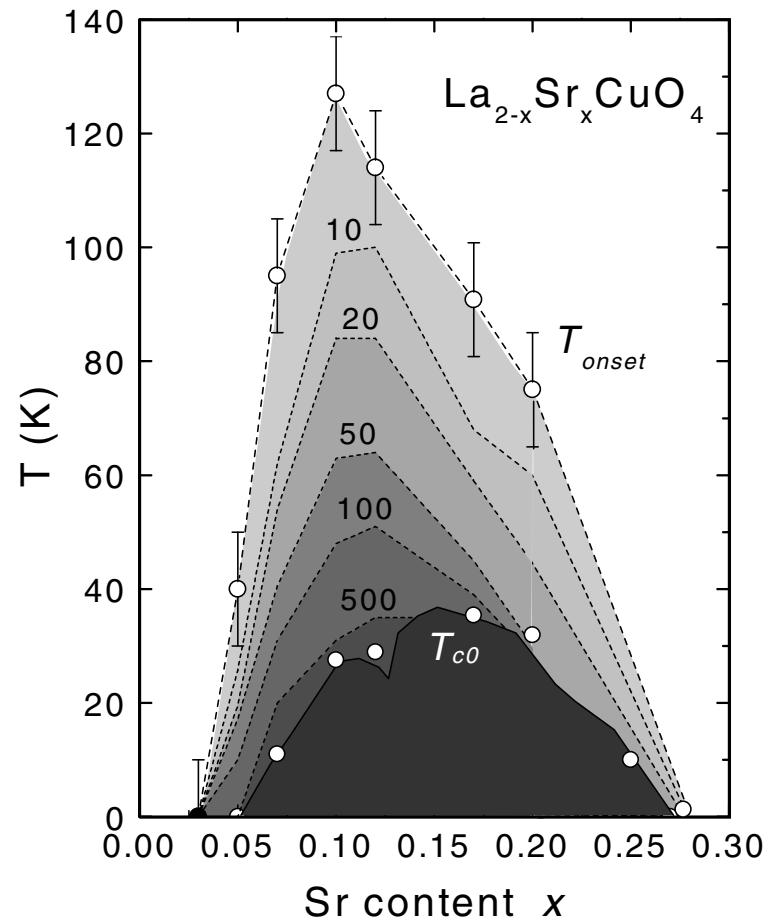
Experimental evidence: 1



Residual Meissner effect observed experimentally with use of the ultrafast magnetic fields.

J. Corson et al, Nature 398, 221 (1999).

Experimental evidence: 2



The large Nernst effect measured above T_c .

Y. Wang et al, *Science* **299**, 86 (2003).

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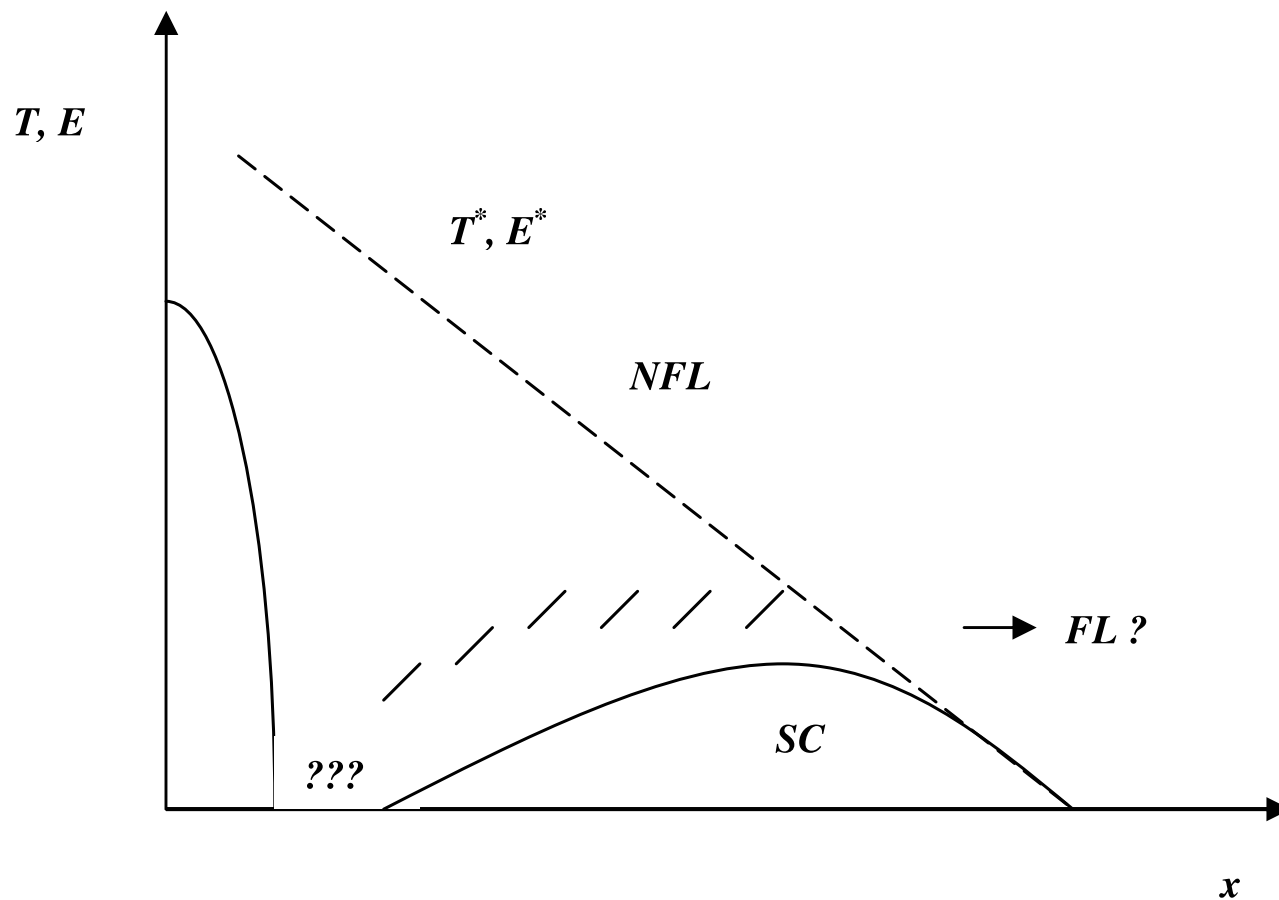
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Precursor effects are robust for all superconductors but their temperature extent varies from case to case.

Resonating Valence Bonds

Why not ?



P.W. Anderson, *Phys. Rev. Lett.* **96**, 017001 (2006).