Renormalization of the band structure and of the underlying Fermi surface in strongly correlated superconductors

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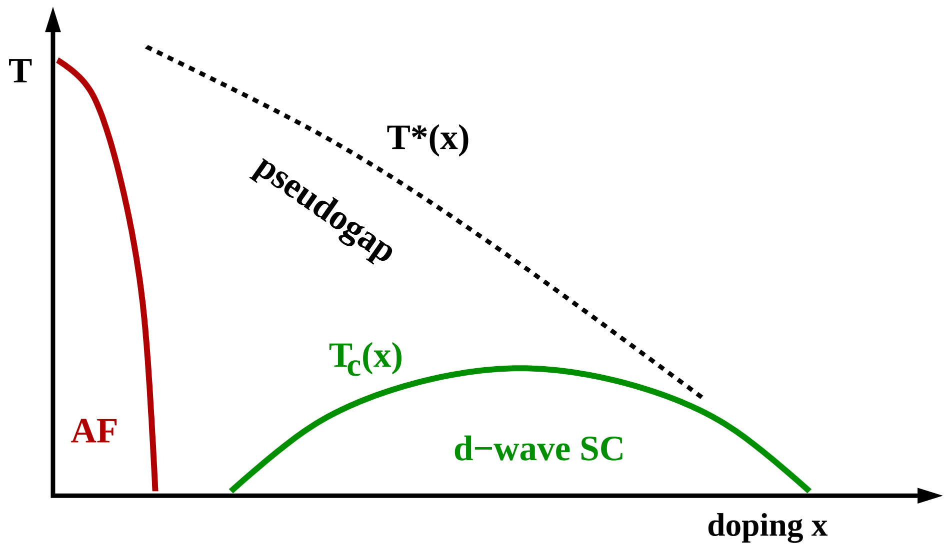
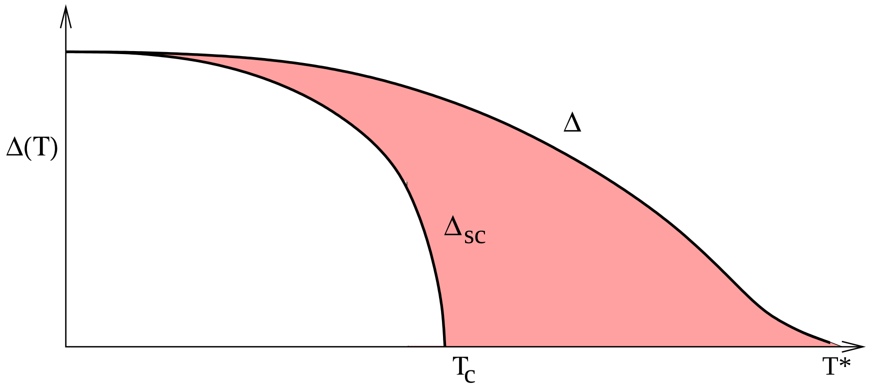
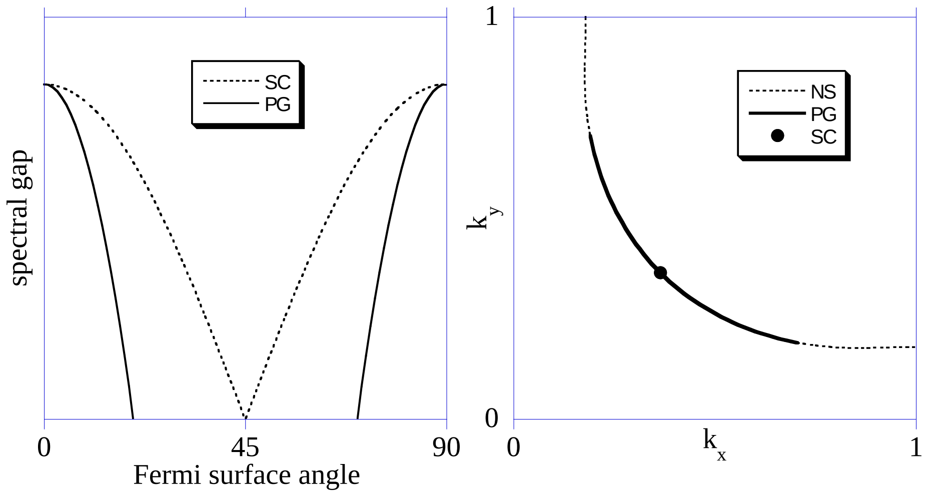
the pseudogap phase

What happens for $T > T_c$?







- one-particle excitations
  - ▷ transport, ARPES
  - ▷ pseudo $\Delta_k \approx \Delta(\cos k_x - \cos k_y)$

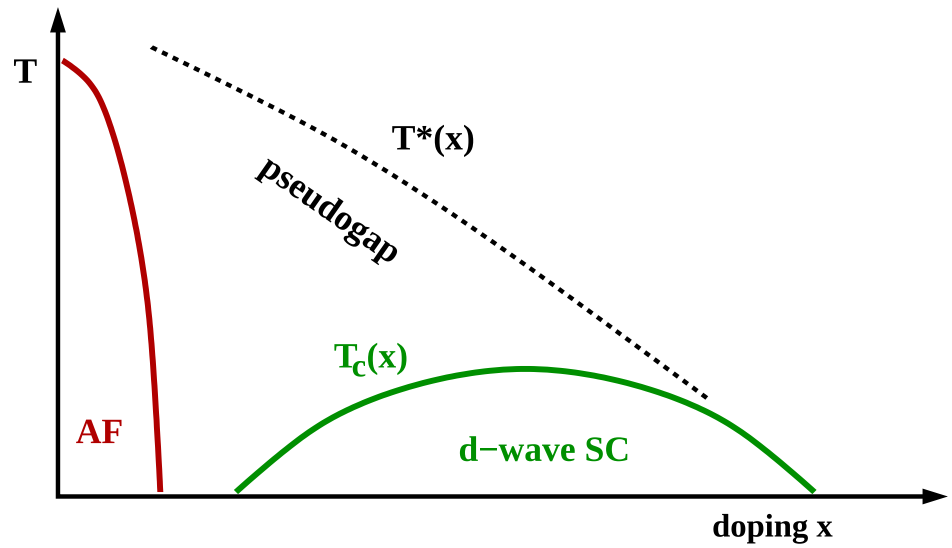
[Norman *et al* '98]

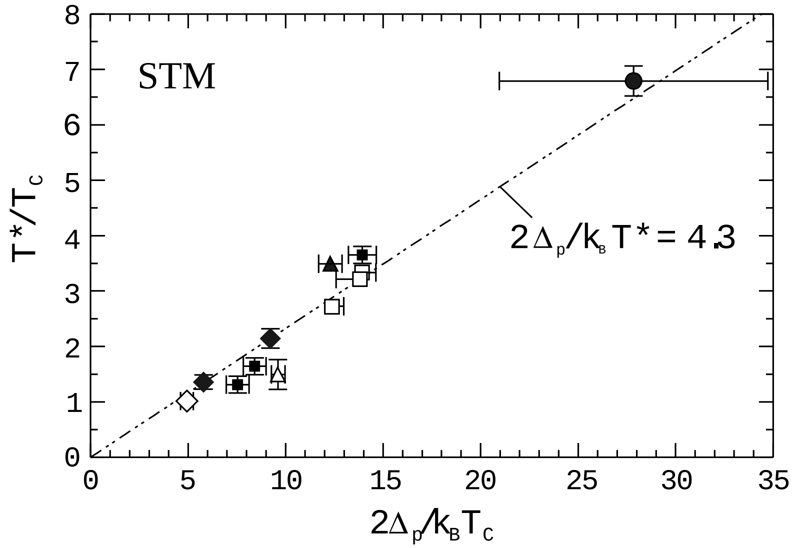
BCS ratio and preformed pairs

- universal for weak-coupling

$$\frac{2\Delta}{k_B T_c} = \begin{cases} 3.52 & \text{s-wave} \\ 4.3 & \text{d-wave} \end{cases}$$

[Kugler, Fischer, Renner, Ono, Ando '01]



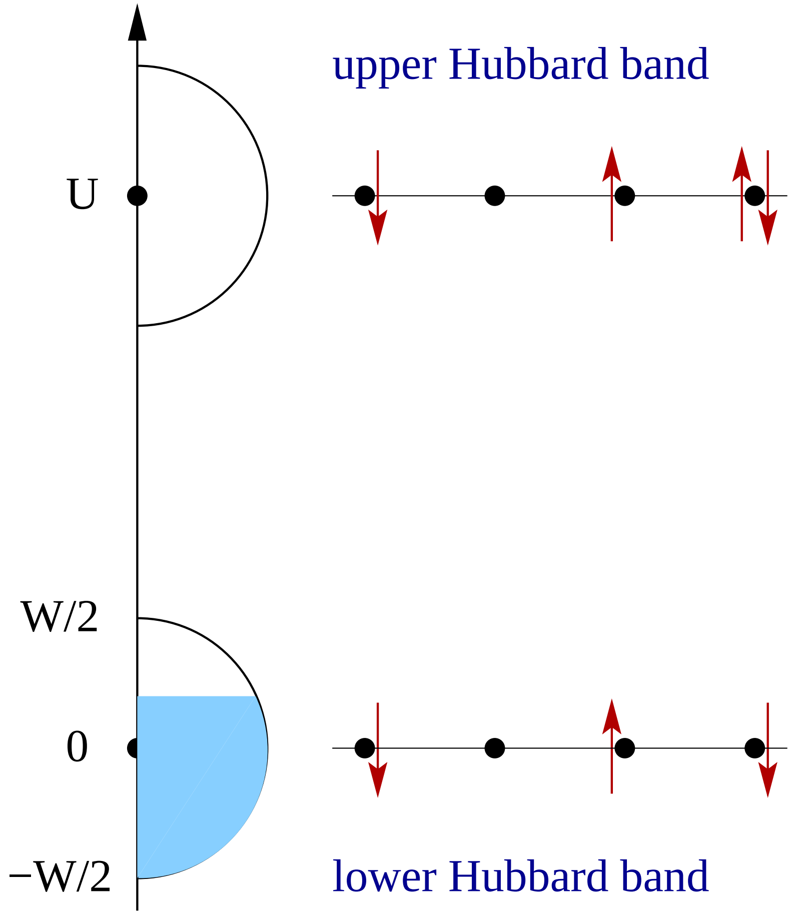


- high-temperature superconductors :

$$\boxed{\frac{2\Delta}{k_B T^*} = 4.3}$$

doped Mott-Hubbard insulators

$$H = -t \sum_{<i,j>,\sigma} \left(c^\dagger_{i\sigma} c_{j\sigma} + c^\dagger_{j\sigma} c_{i\sigma} \right) + U \sum_i n_{i,\uparrow} n_{i,\downarrow}$$



strong correlation

$U \gg t$: reduced double occupancy

$c^\dagger_{i\uparrow} c^\dagger_{i\downarrow} |0\rangle$ has energy U

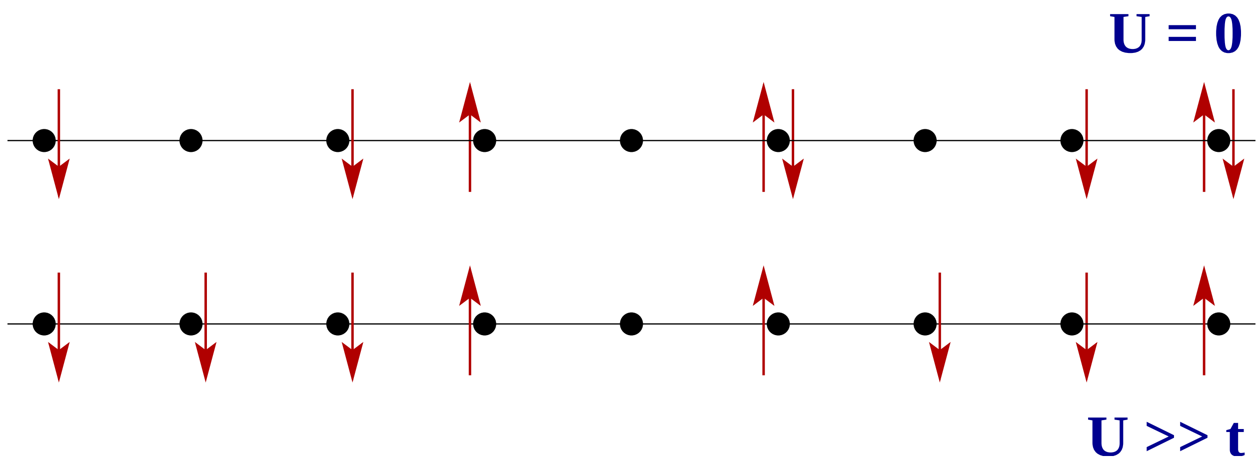
low-energy states

$c^\dagger_{i\sigma}|0\rangle$ singly-occupied

$|0\rangle$ empty sites

Mottness and enhanced phase fluctuations

reduction of particle number fluctuations by strong correlations

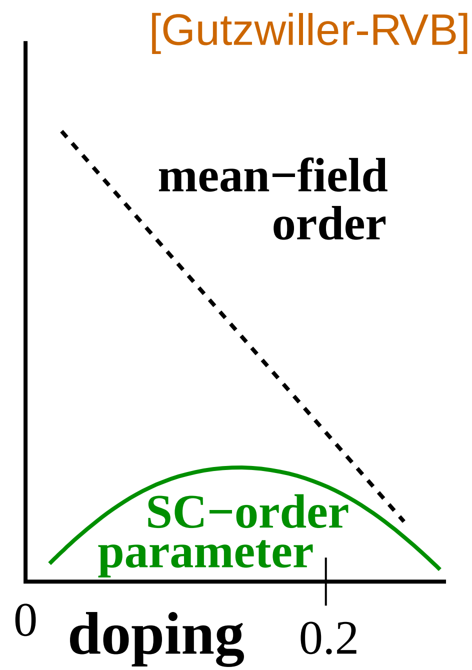


canonical conjugate variables $\langle \Delta N \rangle \langle \Delta \varphi \rangle \approx 1$

phase φ

particle number N

$$\left(\lim_{U/t \to \infty} \lim_{n \to 1} \langle \Delta N \rangle \to 0 \right) \iff \left(\lim_{U/t \to \infty} \lim_{n \to 1} \langle T_c \rangle \to 0 \right)$$



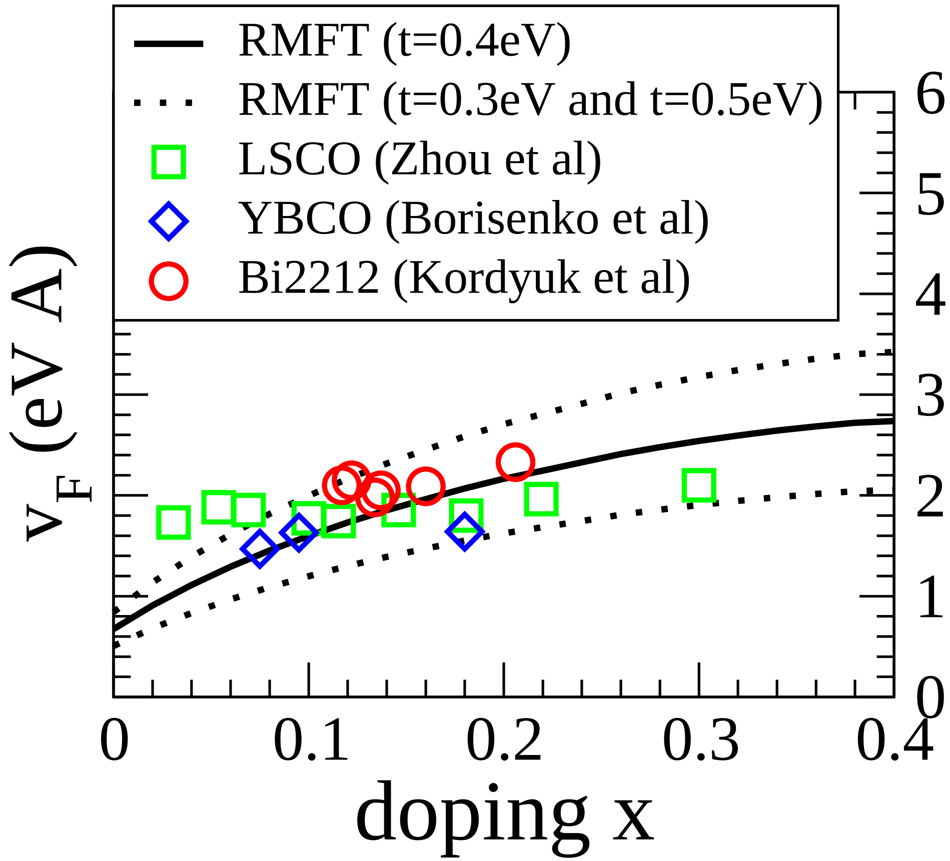
- Mottness results in diverging phase fluctuations

nodal Fermi velocity

- nodes along $(1,1)$ direction

$$v_F = \frac{d\varepsilon(k)}{dk} \approx \frac{m}{m^*}$$

[Edegger, Muthukumar, Gros, Anderson '06]



- RVB-Gutzwiller

$$\boxed{\lim_{U/t\to\infty}\lim_{n\to 1} v_F \propto J} = 4\frac{t^2}{U}$$

experiment & theory

$$\boxed{\lim_{U/t\to\infty}\lim_{n\to 1} v_F \longrightarrow \text{const}}$$

nodal quasiparticle weight renormalization

one particle Greens function

$$G(k,\omega) = \frac{1}{\omega - \xi_k - \Sigma(k,\omega)} \approx \frac{Z}{\omega - v_F(k-k_F) + i\delta_k} + G_{inc}(k,\omega)$$

- quasiparticle weight

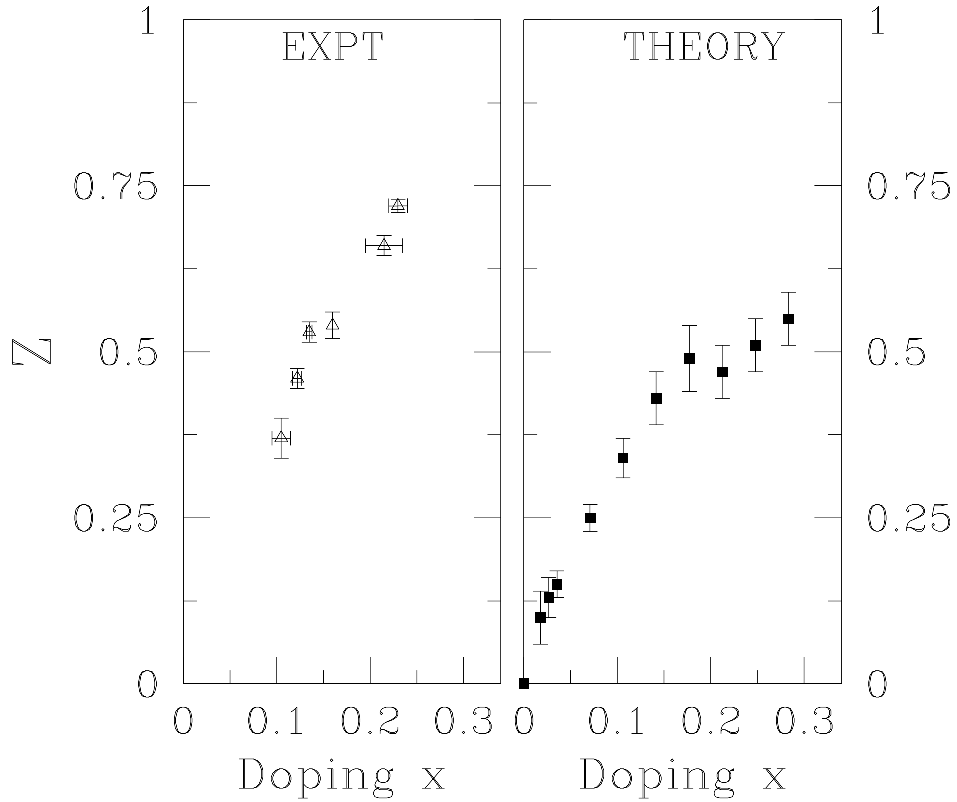
$$Z = \frac{1}{1 - \partial Re\Sigma/\partial\omega}$$

- Fermi velocity

$$v_F = Z\left(v_F^0 + \frac{\partial Re\Sigma}{\partial k}\right)$$

[Johnson *et al.* '01]

[Randeria, Paramekanti, Trivedi '04]



theory & experiment(?)

$$\lim_{U/t\to\infty} \lim_{n\to 1} Z \to 0$$

diverging momentum dependence of self energy

theory & experiment

$$\lim_{U/t\to\infty}\lim_{n\to 1} v_F \longrightarrow \text{const}$$

$$\lim_{U/t\to\infty}\lim_{n\to 1} Z \longrightarrow 0$$

- Fermi velocity

$$v_F = Z\left(v_F^0 + \frac{\partial Re\Sigma}{\partial k}\right)$$

singular momentum dependence (nodal)

$$\lim_{U/t\to\infty}\lim_{n\to 1} \frac{\partial Re\Sigma}{\partial k} \propto \frac{1}{x} \longrightarrow \infty$$

- doping $x = 1 - n$

d-wave superconductivity in the Hubbard model

.. as predicted by strong coupling theories in the late eighties

[Gros; Kotliar & Liu; Ogata & Shiba; Zhang, Gros, Rice & Shiba, ..]

induced by antiferromagnetic exchange $\sim J \sum_{<i,j>} \mathbf{S}_i \cdot \mathbf{S}_j$

.. not explicitly present in the original Hubbard model $J = 4t^2/U$

approaches

- numerical simulations

[Maier, Jarrell & Scalapino; Kotliar , ..]

- small-U RG

[Honerkamp, Salmhofer, Furukawa & Rice; Halboth & Metzner , ..]

- large-U canonical transformation

$$H_{t-J} = e^{iS} H e^{-iS} \approx -t \sum_{<i,j>,\sigma} \left(c_{i\sigma}^{\dagger} c_{j\sigma} + c_{j\sigma}^{\dagger} c_{i\sigma} \right) + J \sum_{<i,j>} \mathbf{S}_i \cdot \mathbf{S}_j + \ldots$$

Gutzwiller approximation

projected wavefunctions

$$|\Psi\rangle = P\,|\Psi_0\rangle = \prod_i (1 - n_{i\uparrow} n_{i\downarrow})\,|\Psi_0\rangle$$

projected Hilbert space : $|\Psi\rangle$

pre-projected Hilbert space : $|\Psi_0\rangle$

renormalization scheme

$$\frac{\langle\Psi|\hat{O}|\Psi\rangle}{\langle\Psi|\Psi\rangle} \approx g\,\frac{\langle\Psi_0|\hat{O}|\Psi_0\rangle}{\langle\Psi_0|\Psi_0\rangle} \qquad g_t = \frac{1-n}{1-n/2}$$

renormalization factors

Hilbert space counting arguments

renormalization scheme

un-projected Hilbert space — Hubbard Hamiltonian

$\Downarrow$ canonical transformation $\mathrm{e}^{iS} H \mathrm{e}^{-iS}$ $\Downarrow$

projected Hilbert space — t-J Hamiltonian

$\Downarrow$ Gutzwiller renormalization g $\Downarrow$

pre-projected Hilbert space — renormalized Hamiltonian

$$\frac{\langle\Psi|H_{t-J}|\Psi\rangle}{\langle\Psi|\Psi\rangle} \approx \frac{\langle\Psi_0|H_{t-J}^{(renor)}|\Psi_0\rangle}{\langle\Psi_0|\Psi_0\rangle}$$

$$H_{t-J}^{(renor)} = g_t T_e + g_s J \sum_{<i,j>} \mathbf{S}_i \cdot \mathbf{S}_j$$

$$g_t = \frac{1-n}{1-n/2}, \qquad g_s = \frac{1}{(1-n/2)^2}$$

renormalized molecular-field theory

pre-projected Hilbert-space

$$|0\rangle,\ |\uparrow\rangle,\ |\downarrow\rangle,\ |\uparrow\downarrow\rangle$$

decoupling

$$S_i^+ S_j^- \;=\; c_{i\uparrow}^\dagger c_{i\downarrow} c_{j\downarrow}^\dagger c_{j\uparrow} \;\approx\; < c_{i\uparrow}^\dagger c_{j\uparrow} > c_{i\downarrow} c_{j\downarrow}^\dagger - < c_{i\uparrow}^\dagger c_{j\downarrow}^\dagger > c_{i\downarrow} c_{j\uparrow} + \ldots$$

molecular fields

hopping-amplitude: $\xi_{i-j} = < c_{i\uparrow}^\dagger c_{j\uparrow} >$

pair-amplitude: $\Delta_{i-j} = < c_{i\uparrow}^\dagger c_{j\downarrow}^\dagger >$

ground-state wavefunction

BCS-wavefunction

$$|\Psi_0\rangle \;=\; \prod_k \left(u_k + v_k c_{k\uparrow}^\dagger c_{k\downarrow}^\dagger \right) |0\rangle$$

strong-coupling approach via RMFT

Hubbard Hamiltonian

canonical transformation $e^{iS}He^{-iS}$ $\Updownarrow$

t-J Hamiltonian

Gutzwiller renormalization g $\Downarrow$

renormalized Hamiltonian

mean-field decoupling ξ, Δ $\Downarrow$

renormalized mean-field Hamiltonian

ground state $|\Psi_0\rangle$ $\Downarrow$

trial projected wavefunction $|\Psi\rangle = P|\Psi_0\rangle$

Variational Monte Carlo

numerical evaluation

$$\frac{\langle\Psi|H_{t-J}|\Psi\rangle}{\langle\Psi|\Psi\rangle} = \frac{\langle\Psi_0|PH_{t-J}P|\Psi_0\rangle}{\langle\Psi_0|PP|\Psi_0\rangle}$$

[Zhang, Gros, Rice & Shiba ‘88]

order-parameter renormalization

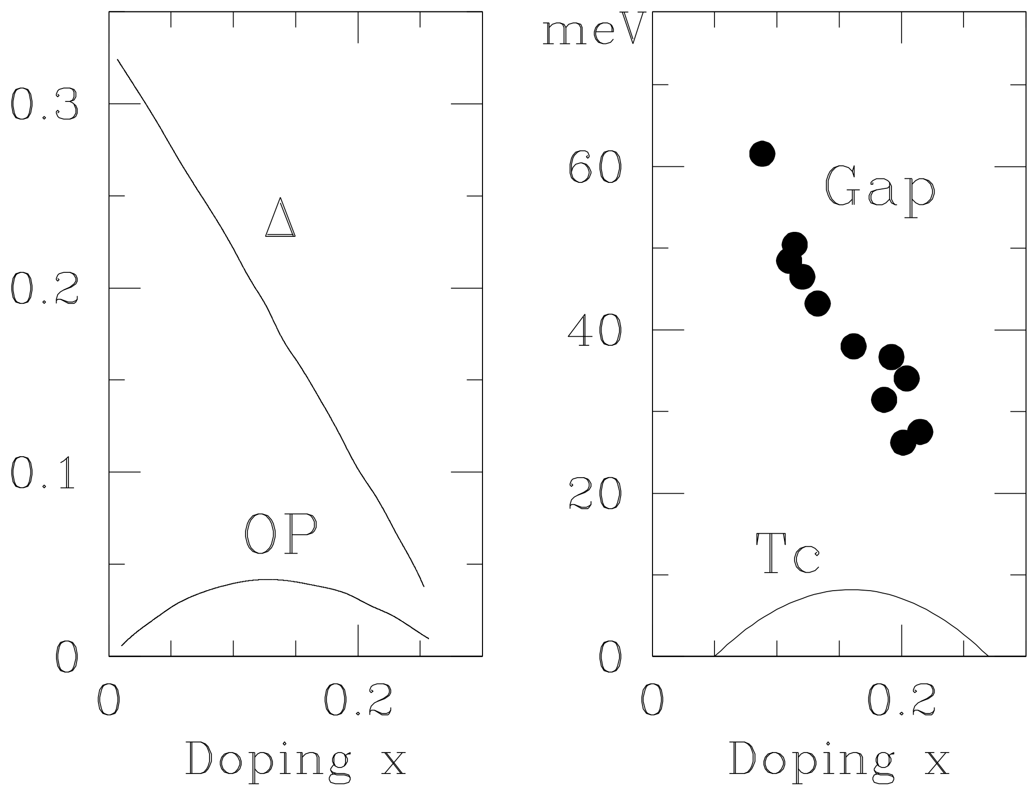
$$< c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} >_\Psi = g_t < c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} >_{\Psi_0}$$

$$< \Delta >_\Psi = g_t < \Delta >_{\Psi_0}$$

- Hubbard-U suppresses particle-number fluctuations

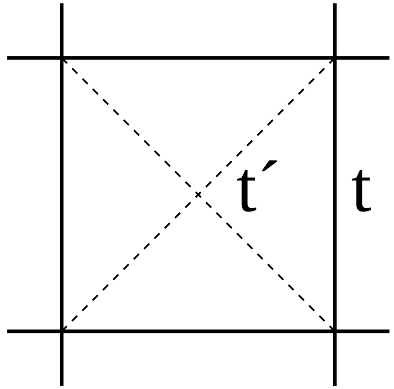
$$g_t = \frac{1-n}{1-n/2}$$

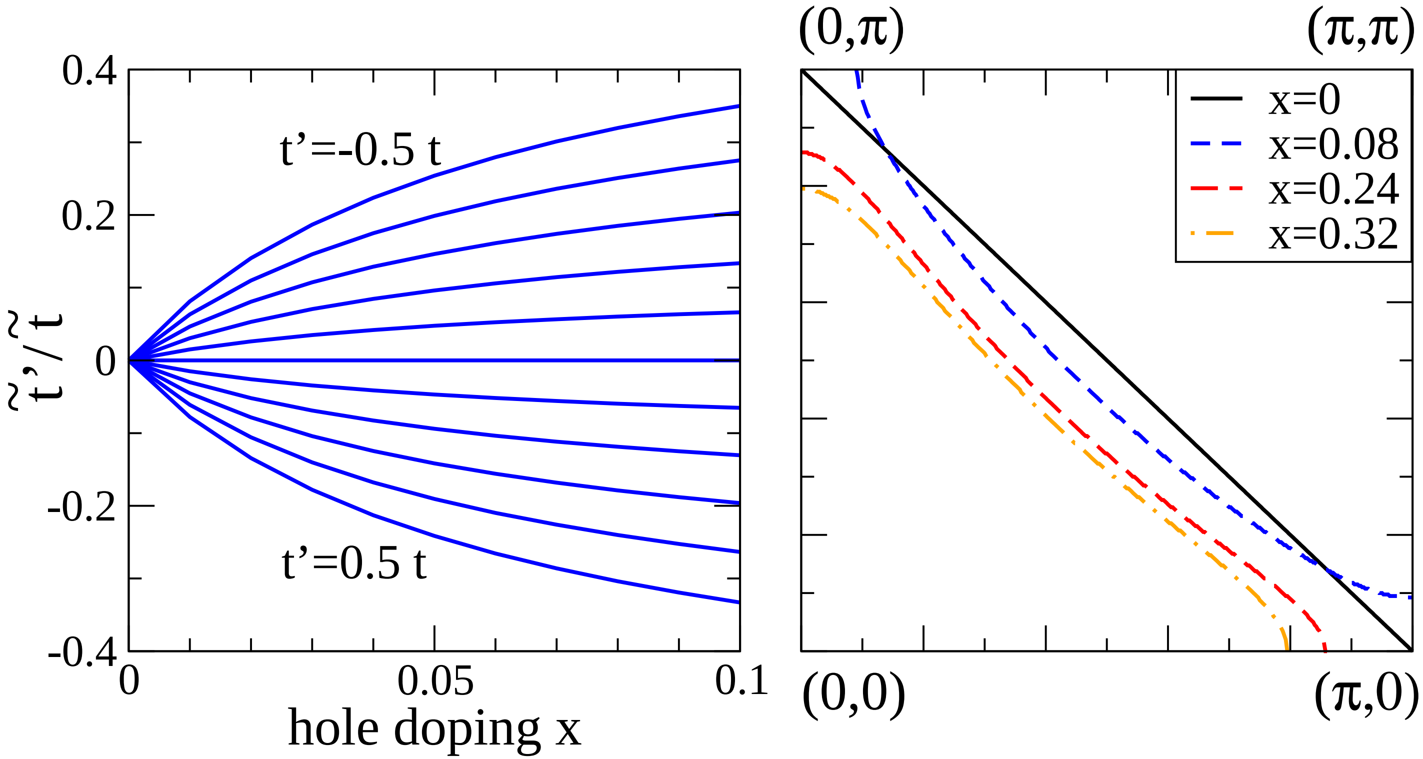
[Anderson *et al.* '04]



renormalization towards perfect nesting

RMFT $\lim_{n\to 1} \begin{cases} \tilde{t} = (t)_{\text{renorm}} & \to \quad J \\ \tilde{t}' = (t')_{\text{renorm}} & \to \quad 0 \end{cases}$





$t' = -t/4, \; U = 12t$

[Gros, Edegger, Muthukumar & Anderson, PNAS ‘06]

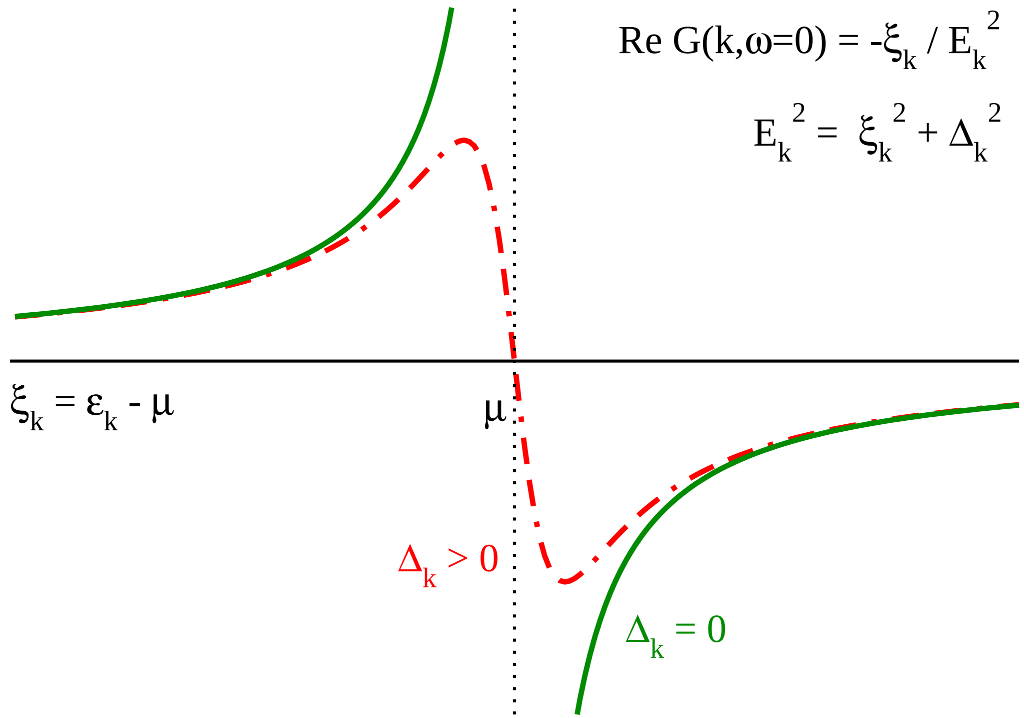
Luttinger vs. Fermi surface

gap / pseudogap

- Fermi surface (FS) not defined
- underlying Fermi surface = ?

Luttinger surface

- $Re G(k, \omega = 0)$ changes sign
- underlying Fermi surface
  ≙ Luttinger surface



BCS superconductor

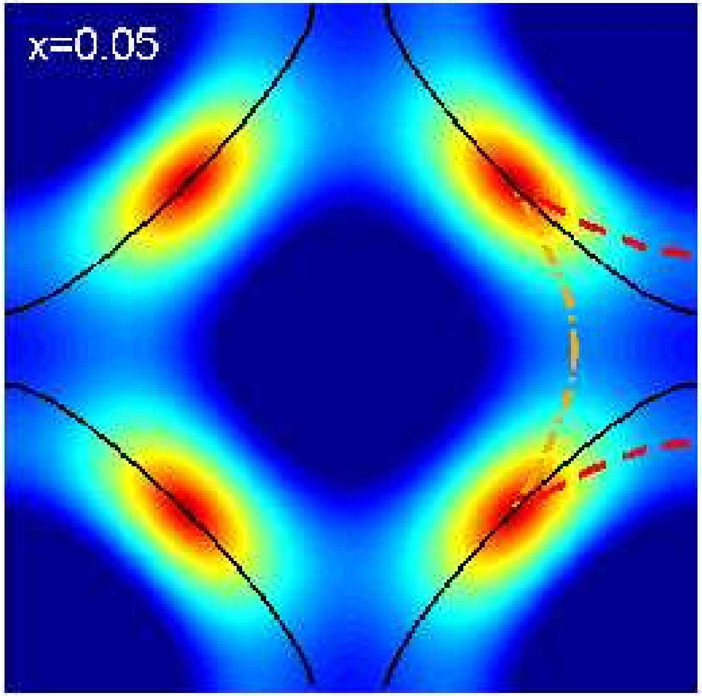
$$\mathrm{Re}\, G(k, \omega = 0) = -\frac{\xi_k}{E_k^2}$$

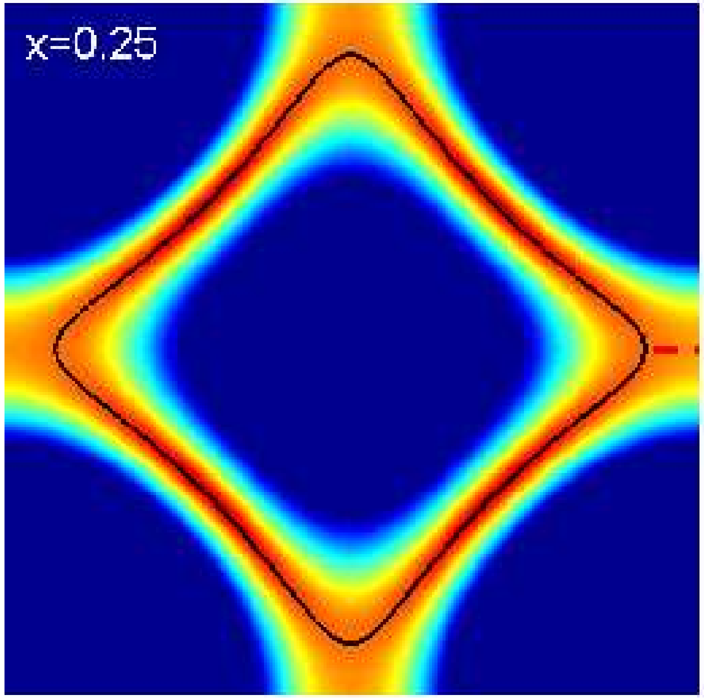
Underlying Fermi surface determination

Luttinger surface vs. maximal spectral intensity

maximal inensity surface $\neq$ Luttinger surface

- large, momentum dependent gap Δ_k





intensity plots

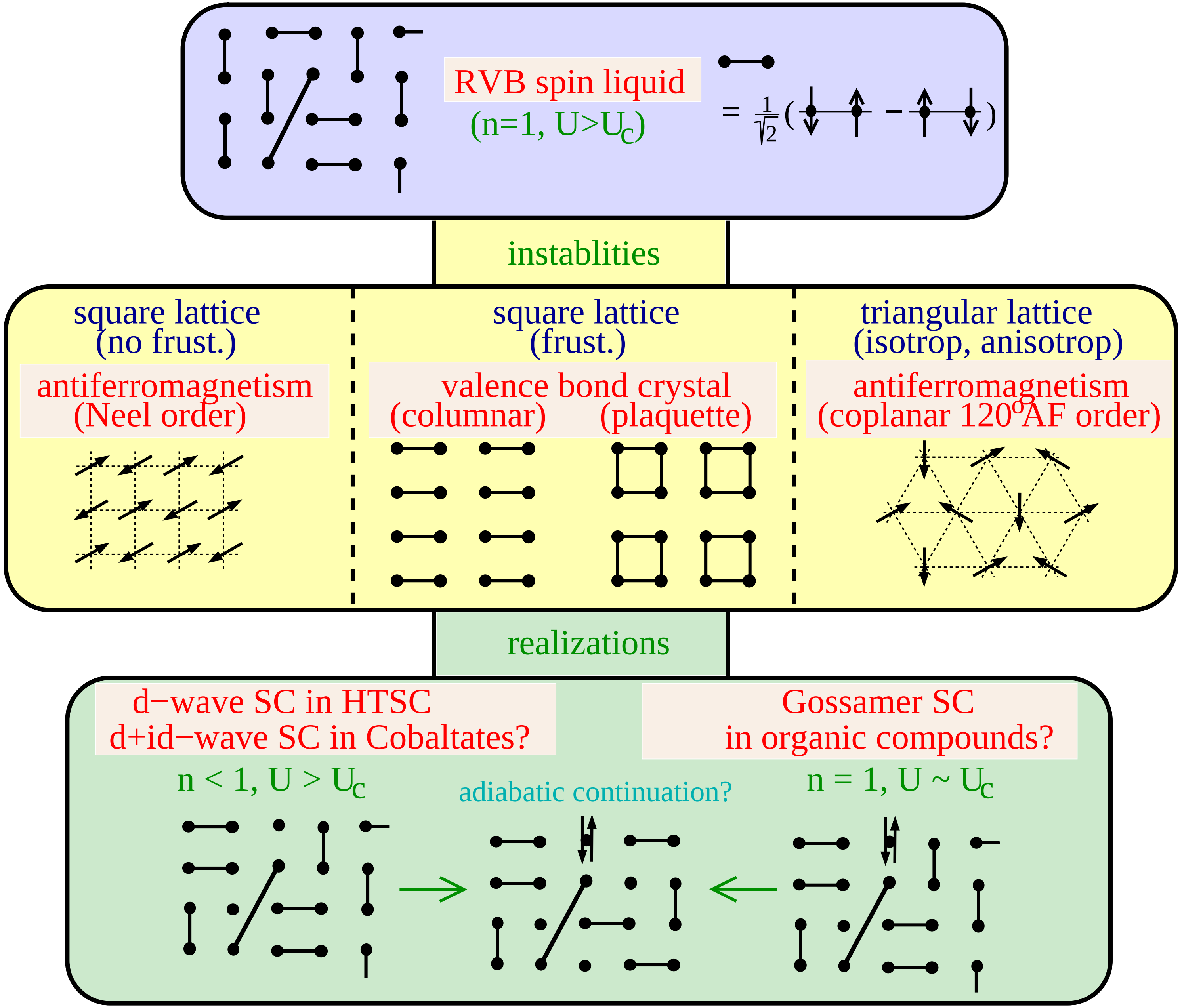
$$\sim \frac{\Gamma}{E_k^2+\Gamma^2}$$

$$E_k = \sqrt{\xi_k^2+\Delta_k^2}$$

line $\equiv$ Luttinger surf.

[Gros, Edegger, Muthukumar & Anderson, PNAS '06]

RVB as an unstable fixpoint

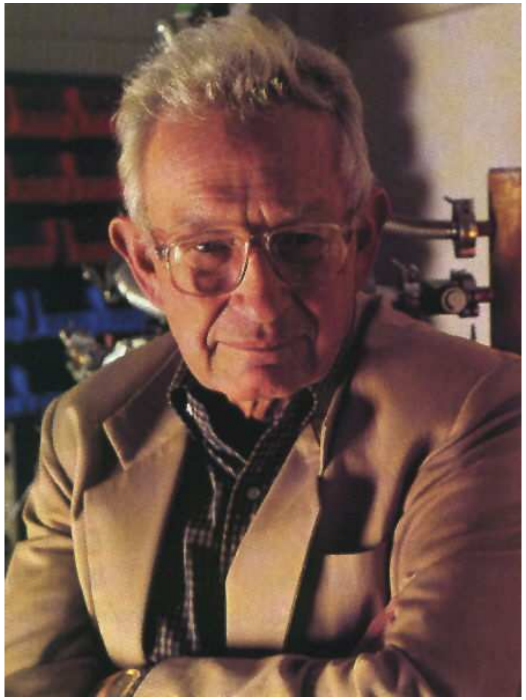


outlook

It's all fun together with ...

Bernhard Edegger

V.N. Muthukumar



P.W. Anderson

[PRL '06, PNAS '06, review on RVB-Gutzwiller '07]