



# Hole dynamics in frustrated antiferromagnets: Coexistence of many-body and free-like excitations

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**CORPES'07 - April 17.2007**

# Outline

➡ **Introduction: Hole dynamics in antiferromagnets**

➡ **Hole motion in different magnetic backgrounds**

**Frustration effects: weakening of AF correlations, competing correlations, and a new mechanism for hole motion**

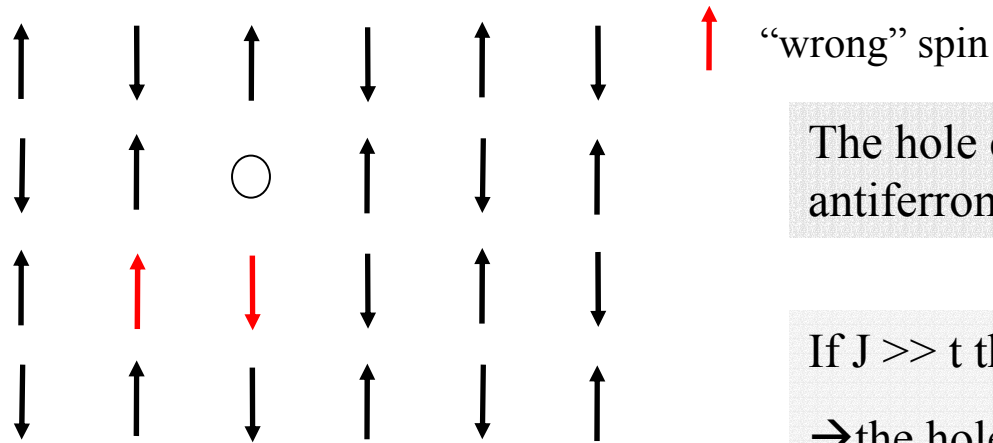
➡ **t-J models solved with the self-consistent Born approximation (SCBA)**

➡ **Hole spectral functions: spin polaron quasiparticle excitation at low energy and broad resonances at higher energies.**

➡ **Conclusions**

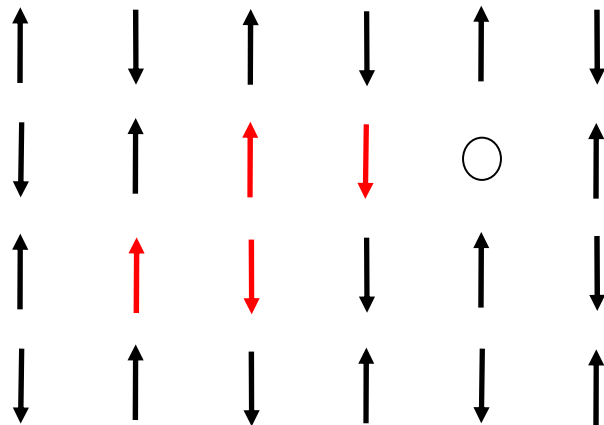
# A single hole dynamics in an antiferromagnet

***t*-*J* model** 
$$H = - \sum_{ij\sigma} t_{ij} \hat{c}_{i\sigma}^\dagger \hat{c}_{j\sigma} + \sum_{\langle ij \rangle} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$



The hole can move only by disturbing the antiferromagnetic background

If  $J \gg t$  then  $\tau_{\text{exch}} \sim 1/J \ll \tau_{\text{hopp}} \sim 1/t$   
 $\rightarrow$  the hole can propagate "easily"

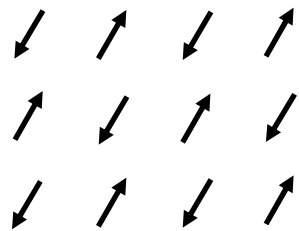


**Hole + surrounding cloud of spin flips = quasiparticle or spin polaron**

If  $J \ll t$  then  $\tau_{\text{exch}} \gg \tau_{\text{hopp}}$   
 $\rightarrow$  the hole will leave behind a string of "wrong" spins, increasing its effective mass

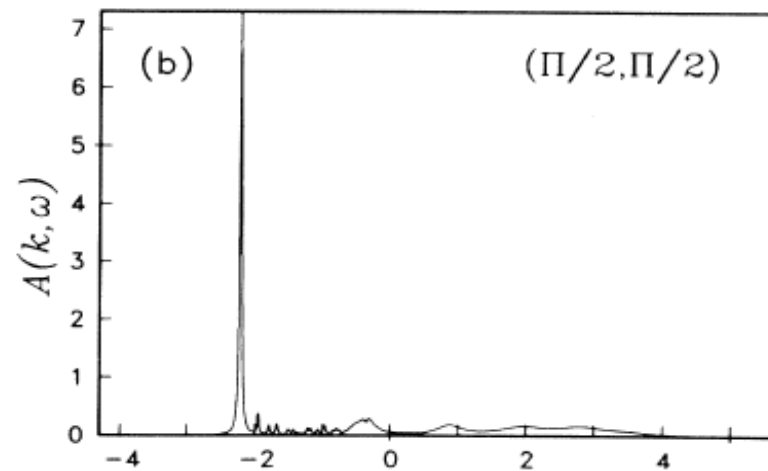
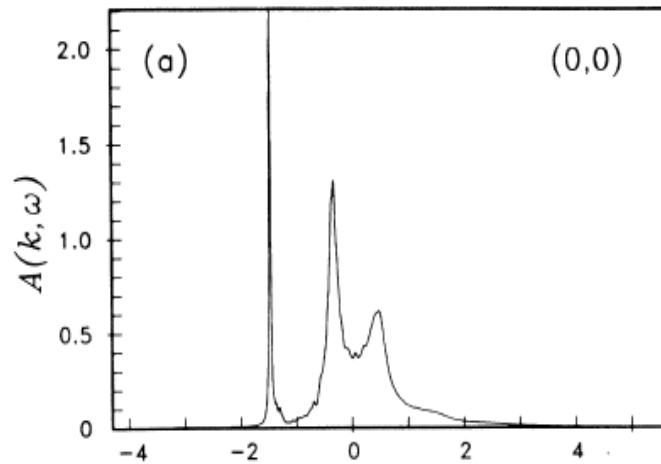
# Hole motion and magnetic order: non-frustrated lattices

The hole motion will strongly depend on the magnetic correlations of the underlying magnetic order



In the square lattice antiferromagnet the spin polaron is always well defined, for all momenta and  $J > 0$

Martinez and Horsch PRB 44, 317 (1991) ; Dagotto RMP 66, 763 (1994); Brunner *et al* PRB 62, 15480 (2000)



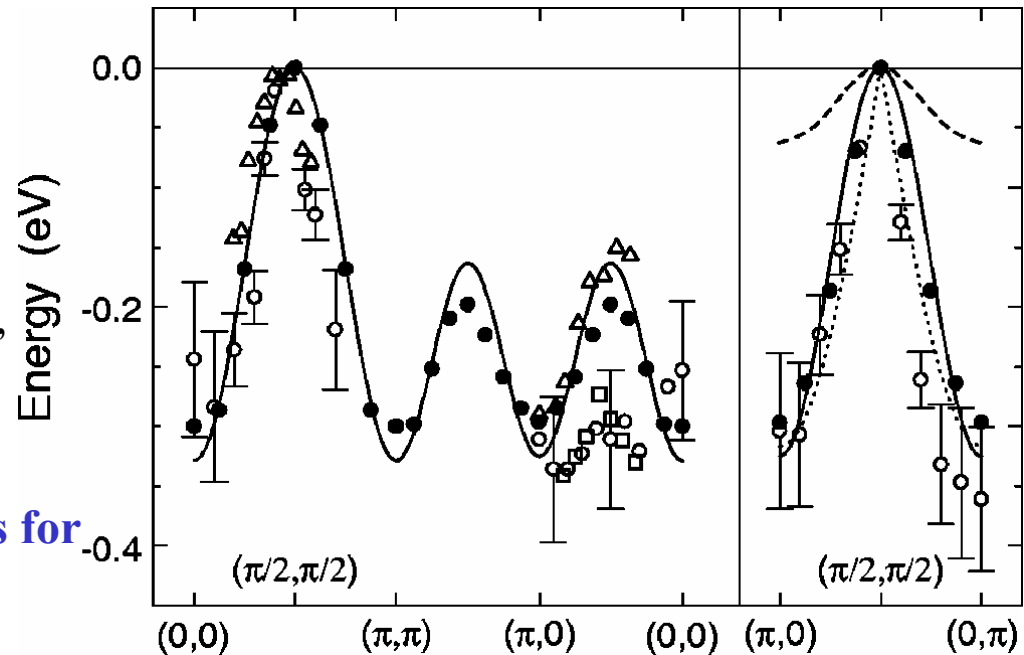
# Experimental

Electronic dispersions for  $\text{Sr}_2\text{CuO}_2\text{Cl}_2$  and  $\text{Ca}_2\text{CuO}_2\text{Cl}_2$  measured by ARPES seem to confirm this picture

Wells et al, PRL 74, 964, (1995); Ronning et al, Science 282, 2067 (1998)

ARPES data and SCBA results for the  $t$ - $t'$ - $t''$ - $J$  model

( $t=0.35$  eV,  $t'=-0.12$  eV,  $t''=0.08$  eV, and  $J=0.14$  eV)

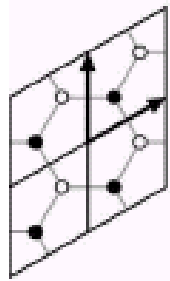


$$\text{min : } E(0,0) \sim E(\pi,0)$$

$$\text{max : } E\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$E\left(\frac{\pi}{2}, \frac{\pi}{2}\right) - E(0,0) \sim 2[E\left(\frac{\pi}{2}, 0\right) - E(0,0)]$$

But the width of the peaks is too large to correspond to physical lifetimes of QP! Polaronic effects? (Ronning, Rosch, Gunnarsson, etc)



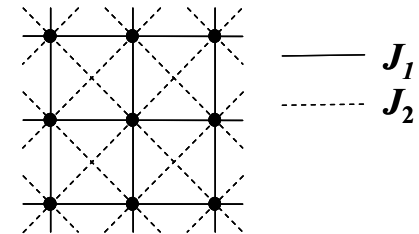
## Another non-frustrated lattice: honeycomb lattice

A. Luscher et al, PRB 73, 155118 (2006)

SCBA, series expansions, and exact diagonalization results show well defined quasiparticle peaks at the bottom of the spectrum throughout the whole Brillouin zone

## Frustrated lattices: weakly frustrated $J_1$ - $J_2$ model

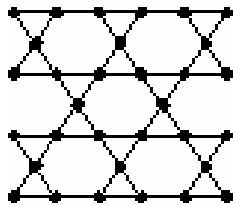
Y. Shibata, T. Tohyama, and S. Maekawa, PRB 59, 1840 (1999)



$J_2$  weakens the AF spin background. The frustration suppresses the QP weight and makes the spectrum broad for small momentum

## A highly frustrated lattice: kagomé lattice

A. Lauchli and D. Poilblanc, PRL 92, 236404 (2004)



Lanczos exact diagonalization results show no QP peaks for  $J/t=0.4$  and all momenta, for both signs of  $t$

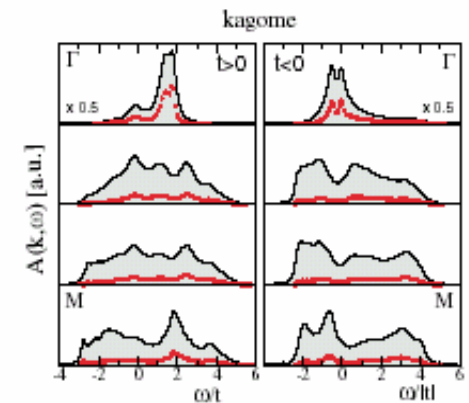
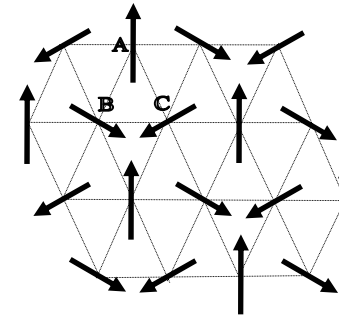


FIG. 5 (color online). Single hole spectral functions (black lines) along the line  $\Gamma \rightarrow M$  computed on a 27 site kagome cluster for  $t = +1$  (left panel) and for  $t = -1$  (right panel). In both cases  $J/|t| = 0.4$ . The red circles denote pole locations and their residues. Note that no quasiparticle peaks are visible for all momenta.

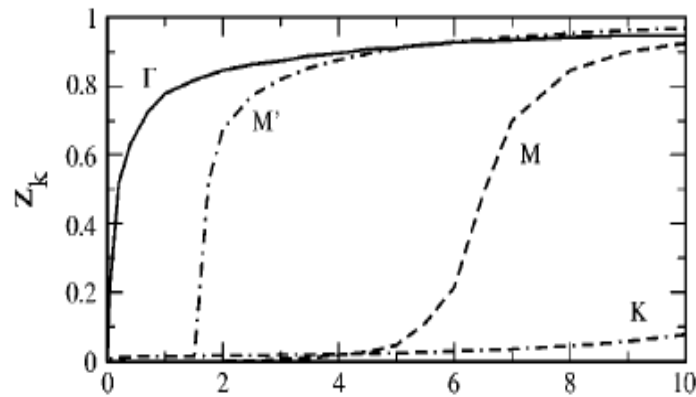
# Hole dynamics in the triangular lattice

A. Trumper, C. Gazza, and L.O.M., PRB 69, 184407 (2004)

The ground state is a “simple” semiclassical  $120^\circ$  Néel order

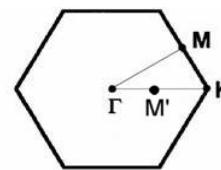
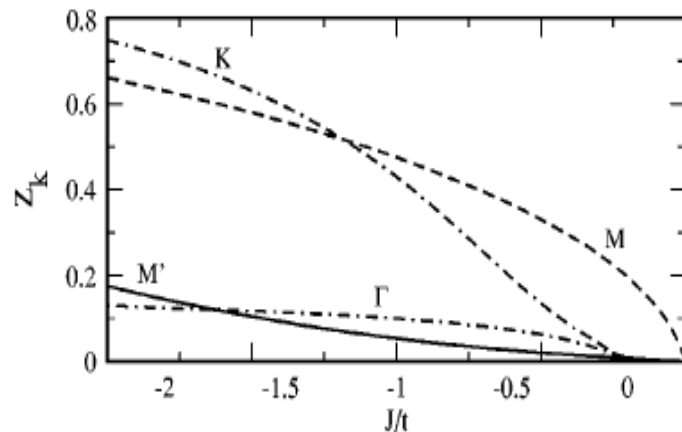


$t > 0$



SCBA results show no QP only for  $t > 0$ , and for momenta away from the magnetic Goldstone modes

$t < 0$



# Model and method

We use the  $t$ - $J$  model in local spin quantization axis, assuming a semiclassical magnetic order

Representations: hole  $\rightarrow$  spinless fermion

spin fluctuations  $\rightarrow$  Holstein-Primakov bosons

$$\hat{c}_{i\uparrow} = h_i^\dagger \quad \hat{c}_{i\downarrow} = h_i S_i^-$$

$$S_i^x \sim \frac{1}{2}(a_i^\dagger + a_i) \quad S_i^y \sim \frac{i}{2}(a_i^\dagger - a_i) \quad S_i^z = \frac{1}{2} - a_i^\dagger a_i$$

## Effective Hamiltonian

$$H = \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} h_{\mathbf{k}}^\dagger h_{\mathbf{k}} + \sum_{\mathbf{q}} \omega_{\mathbf{q}} \alpha_{\mathbf{q}}^\dagger \alpha_{\mathbf{q}} - t \sqrt{\frac{3}{N_s}} \sum_{\mathbf{k}, \mathbf{q}} \left[ M_{\mathbf{k}\mathbf{q}} h_{\mathbf{k}}^\dagger h_{\mathbf{k}-\mathbf{q}} \alpha_{\mathbf{q}} + h.c. \right]$$

Free hopping (due to the ferromagnetic component)

Free magnon energy

hole-magnon interaction



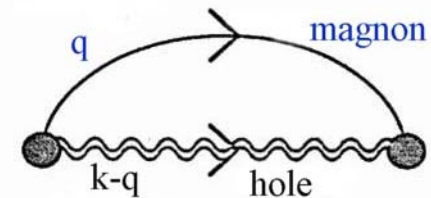
# Self-consistent Born approximation (SCBA)

We calculate the hole **spectral function**

$$A_{\mathbf{k}}(\omega) = -\frac{1}{\pi} \text{Im} G_{\mathbf{k}}^h(\omega)$$

$$G_{\mathbf{k}}^h(\omega) = \langle AF | h_{\mathbf{k}} \frac{1}{(\omega + i\eta^+ - H)} h_{\mathbf{k}}^\dagger | AF \rangle$$

solving the self-consistent equation for the self-energy



$$\Sigma_{\mathbf{k}}(\omega) = \frac{3t^2}{N_s} \sum_{\mathbf{q}} \frac{|M_{\mathbf{k}\mathbf{q}}|^2}{\omega - \omega_{\mathbf{q}} - \epsilon_{\mathbf{k}-\mathbf{q}} - \Sigma_{\mathbf{k}-\mathbf{q}}(\omega - \omega_{\mathbf{q}})}$$

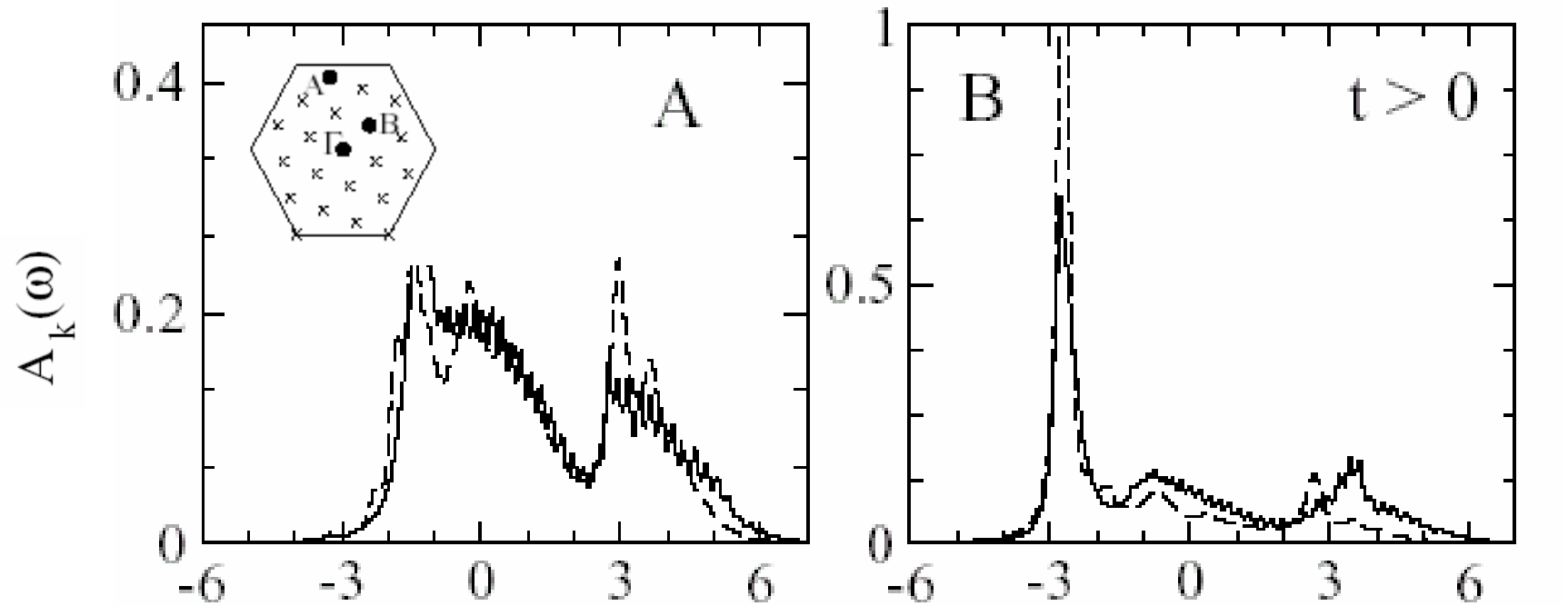
**Quasiparticle weight**

(How much of the hole survives)

$$z_{\mathbf{k}} = \left( 1 - \frac{\partial \Sigma_{\mathbf{k}}(\omega)}{\partial \omega} \right)^{-1} \Big|_{E_{\mathbf{k}} = \Sigma_{\mathbf{k}}(E_{\mathbf{k}})}$$

# Comparison SCBA vs exact results

**N = 21 sites**



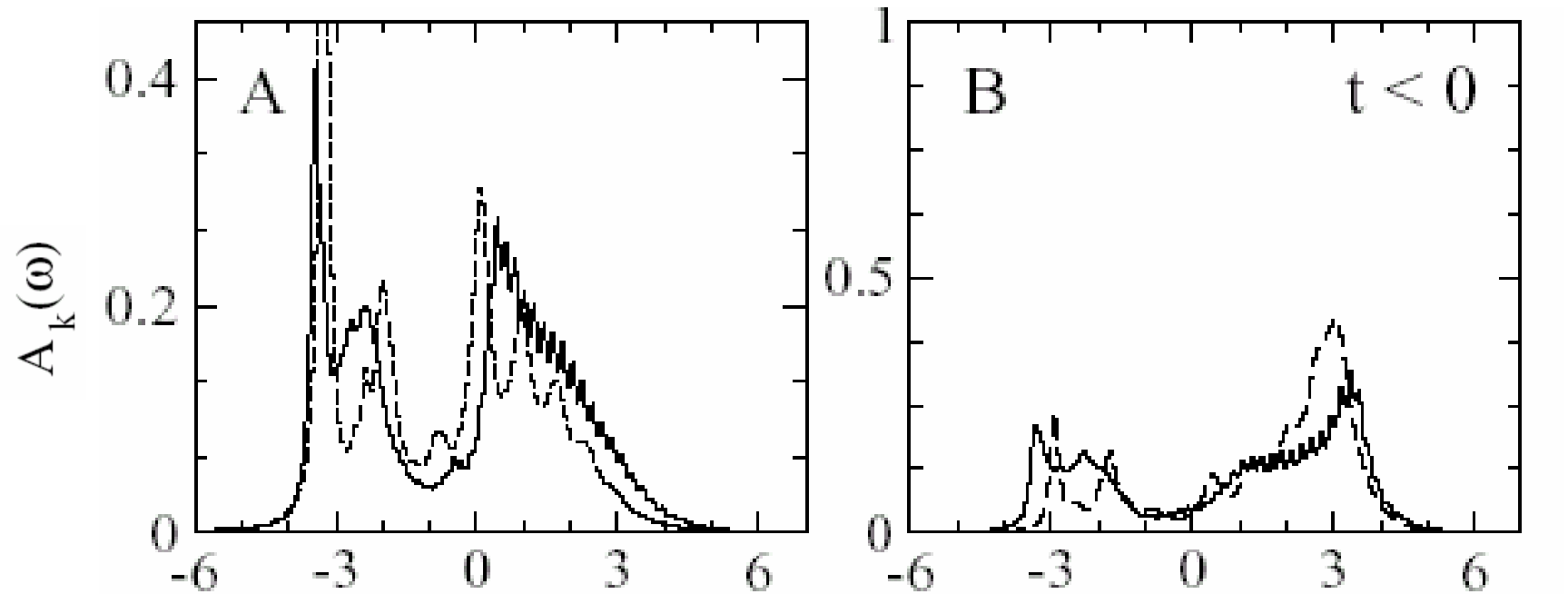
► Positive  $t$

►  $J/t=0.4 \rightarrow$  strong coupling regime

————— Lanczos  
- - - - - SCBA

# SCBA vs exact results

**N = 21 sites**



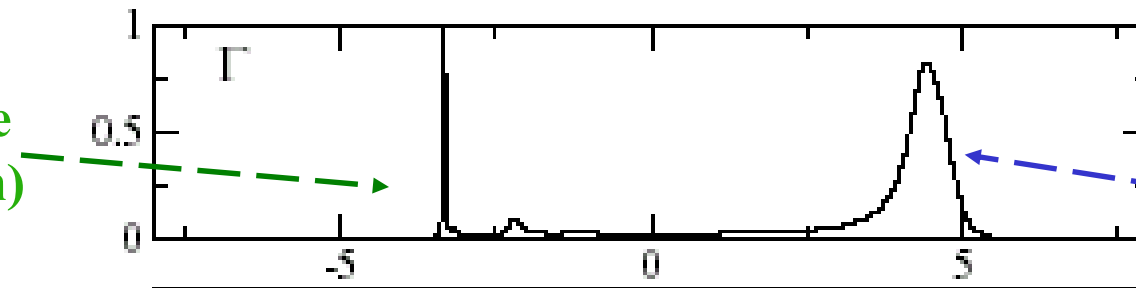
▶ Negative t

▶  $J/|t|=0.4$

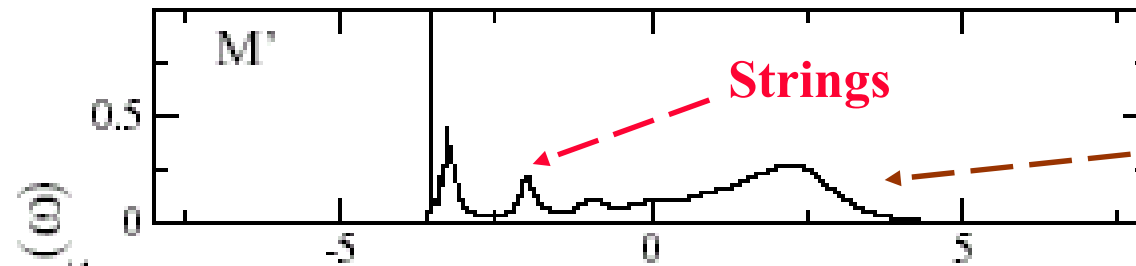
———— Lanczos  
----- SCBA

# Hole spectral functions: negative $t$

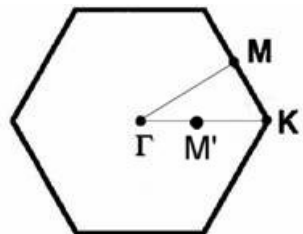
Quasiparticle  
(spin polaron)



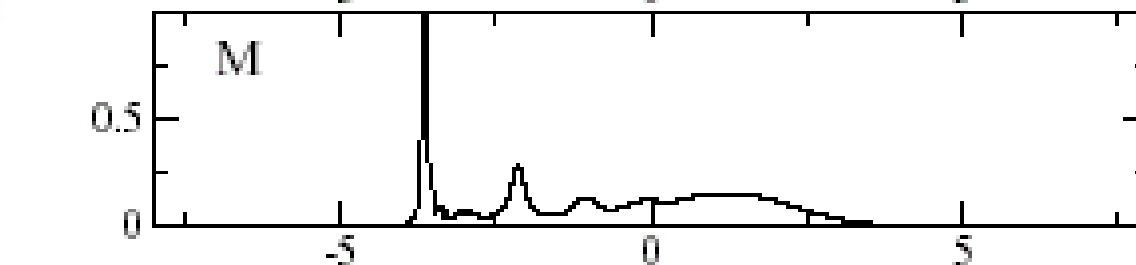
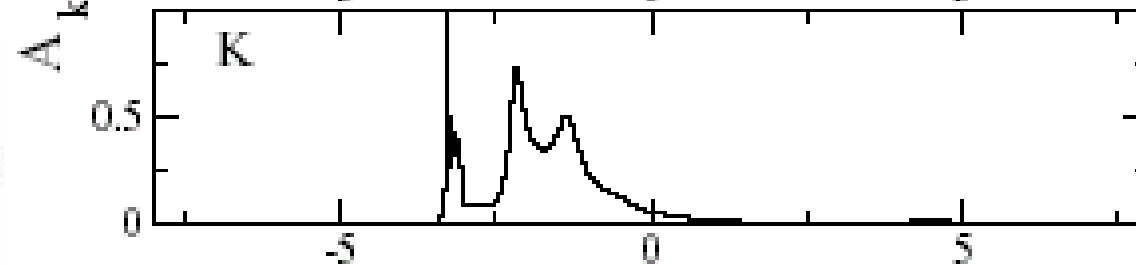
$t$ -resonance:  
free hopping



Incoherent  
background



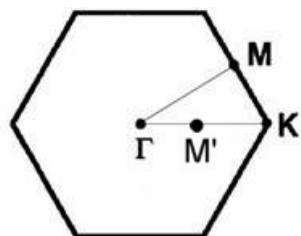
$J/|t|=0.4$



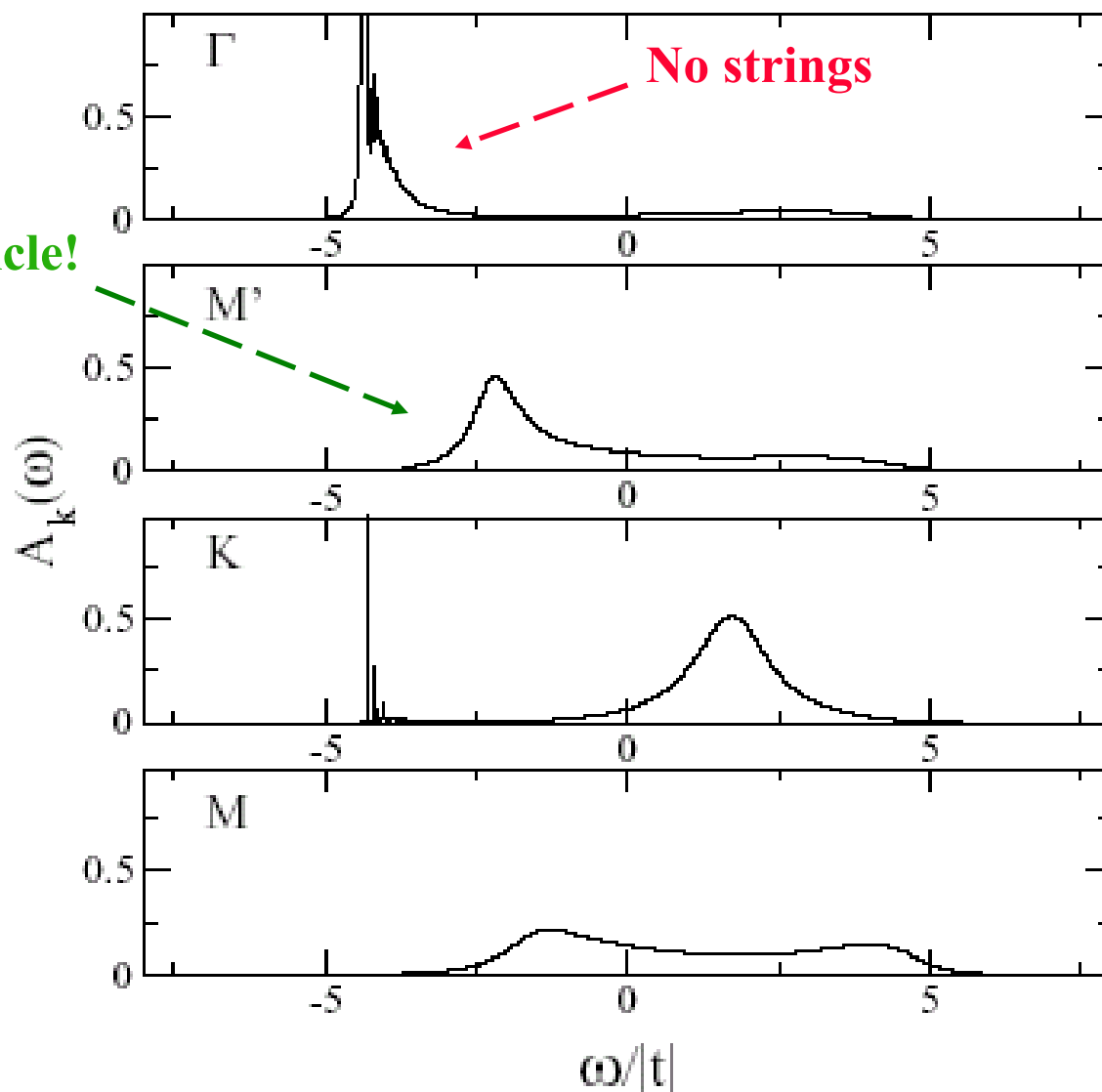
$\omega/|t|$

# Hole spectral functions: positive $t$

No quasiparticle!



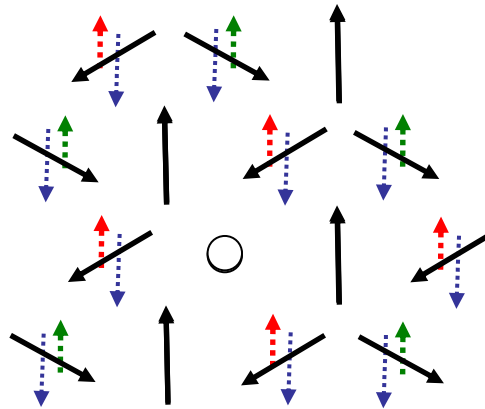
$J/t=0.4$



Sign reversal of  $t$  is not trivial!

# Triangular lattice

Triangular lattice:  
semiclassical  $120^\circ$   
order



Decomposing the spins in an up-down basis

$$\begin{aligned} \swarrow &= \downarrow + \uparrow \\ \nwarrow &= \uparrow + \downarrow \end{aligned}$$

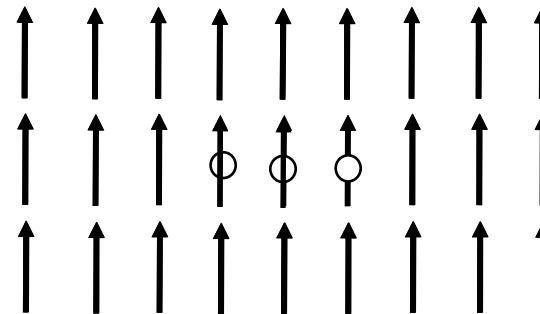
## Two mechanisms for hole motion

Magnon-assisted hopping  
(hole-magnon interaction)



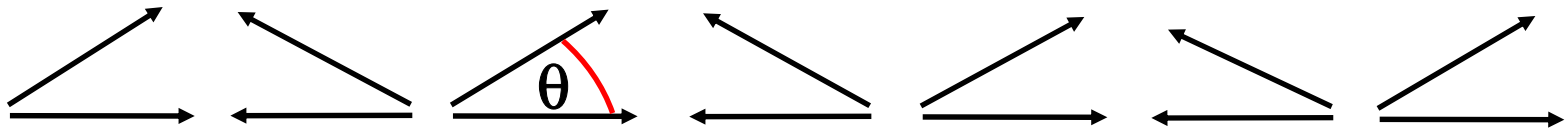
spin-polaron origin in  
non-frustrated  
antiferromagnets

Free hopping: no absorption or emission of  
magnons (due to the ferromagnetic component of  
the magnetic order)



# These two mechanisms for hole motion will interference

To study this interference we can go from the pure AF state (only magnon-assisted propagation) to the pure ferromagnetic state (only free hopping propagation) by canting the AF order



We solve the  $t$ - $J$  model with a Zeeman term that couples only with spin, to stabilize the canted phase, using the SCBA

$$H = H_t + H_J = -t \sum_{\langle i,j \rangle} (\hat{c}_{i,\sigma}^\dagger \hat{c}_{j,\sigma} + h.c.) + \\ + J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + B \sum_i S_i^z$$

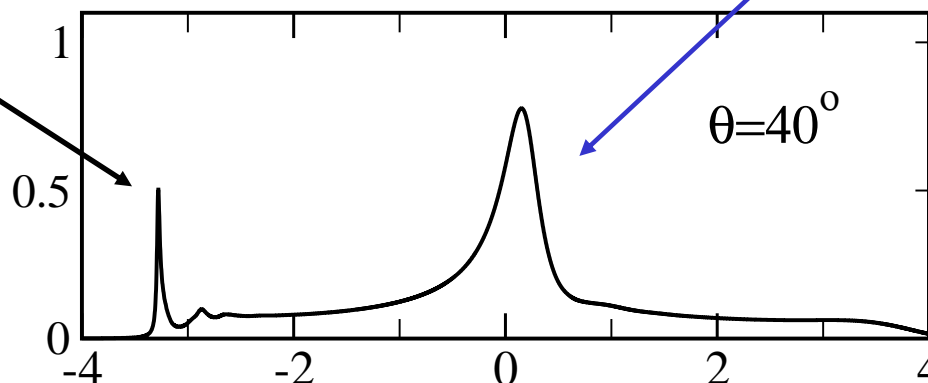
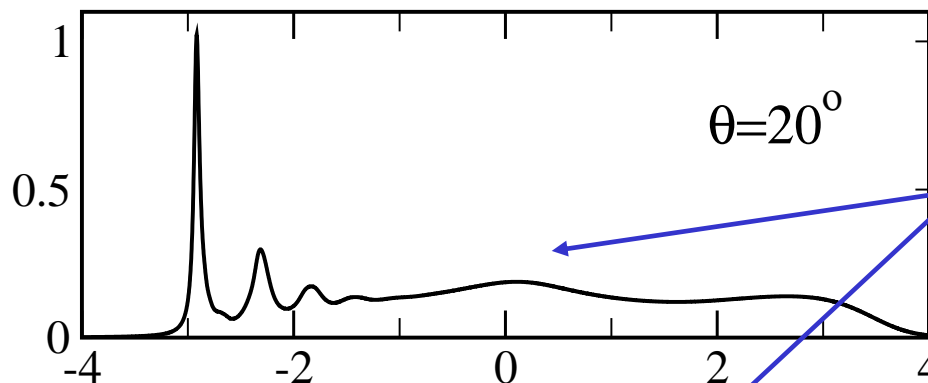
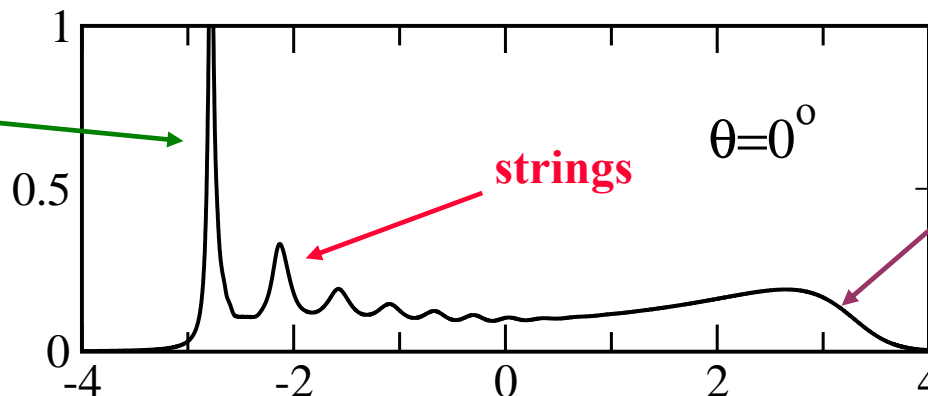
# Hole spectral functions: $\mathbf{k}=(\pi/2,\pi/2)$

Quasiparticle (spin polaron): always magnon assisted

Propagation along ferromagnetic clusters induced by spin fluctuations

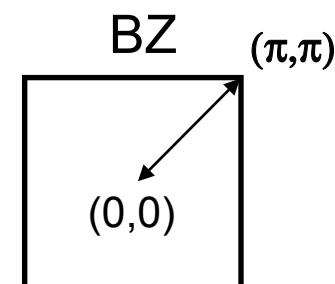
$J/t=0.1$

$A_{\mathbf{k}}(\omega)$



As the angle increases the QP weight decreases

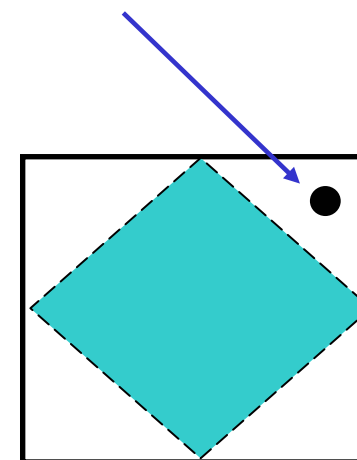
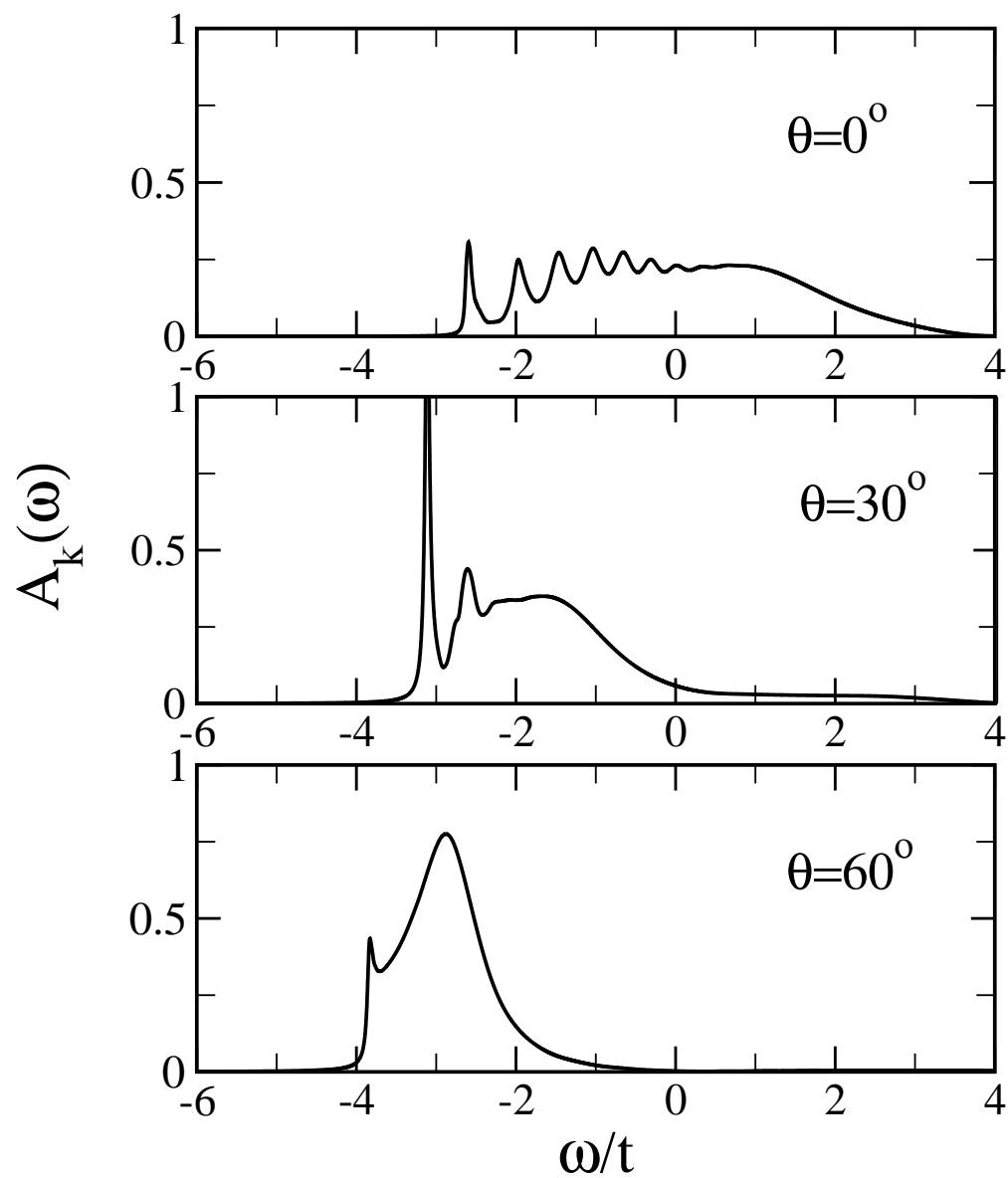
Free hopping (classical ferro. component): t-resonance



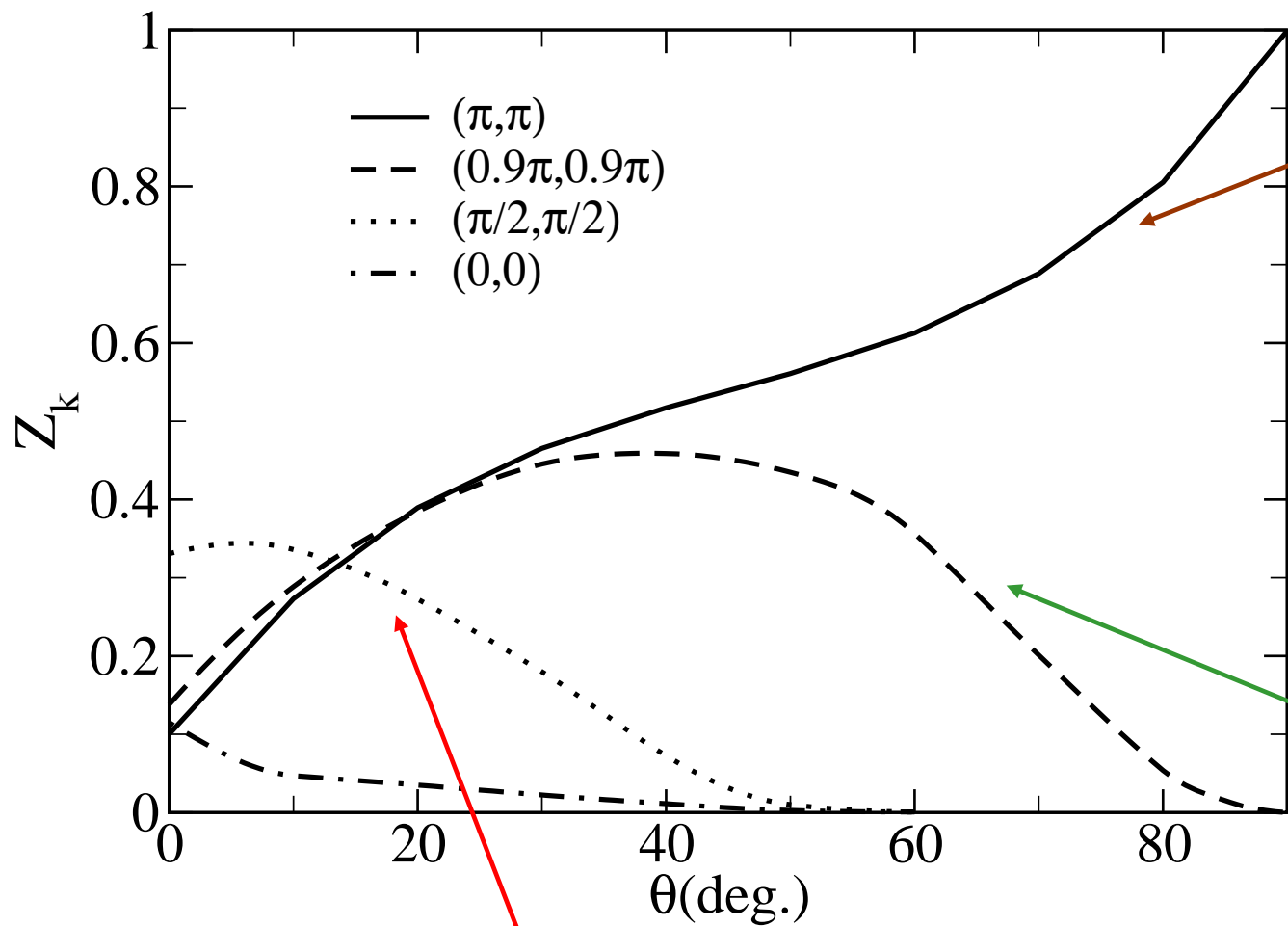
$\omega/t$



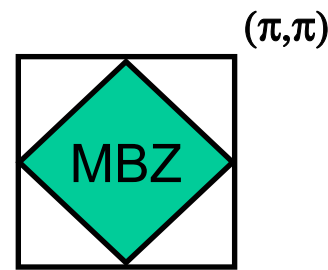
# Hole spectral functions: $\mathbf{k}=(0.8\pi,0.8\pi)$



$\mathbf{J}/t=0.1$



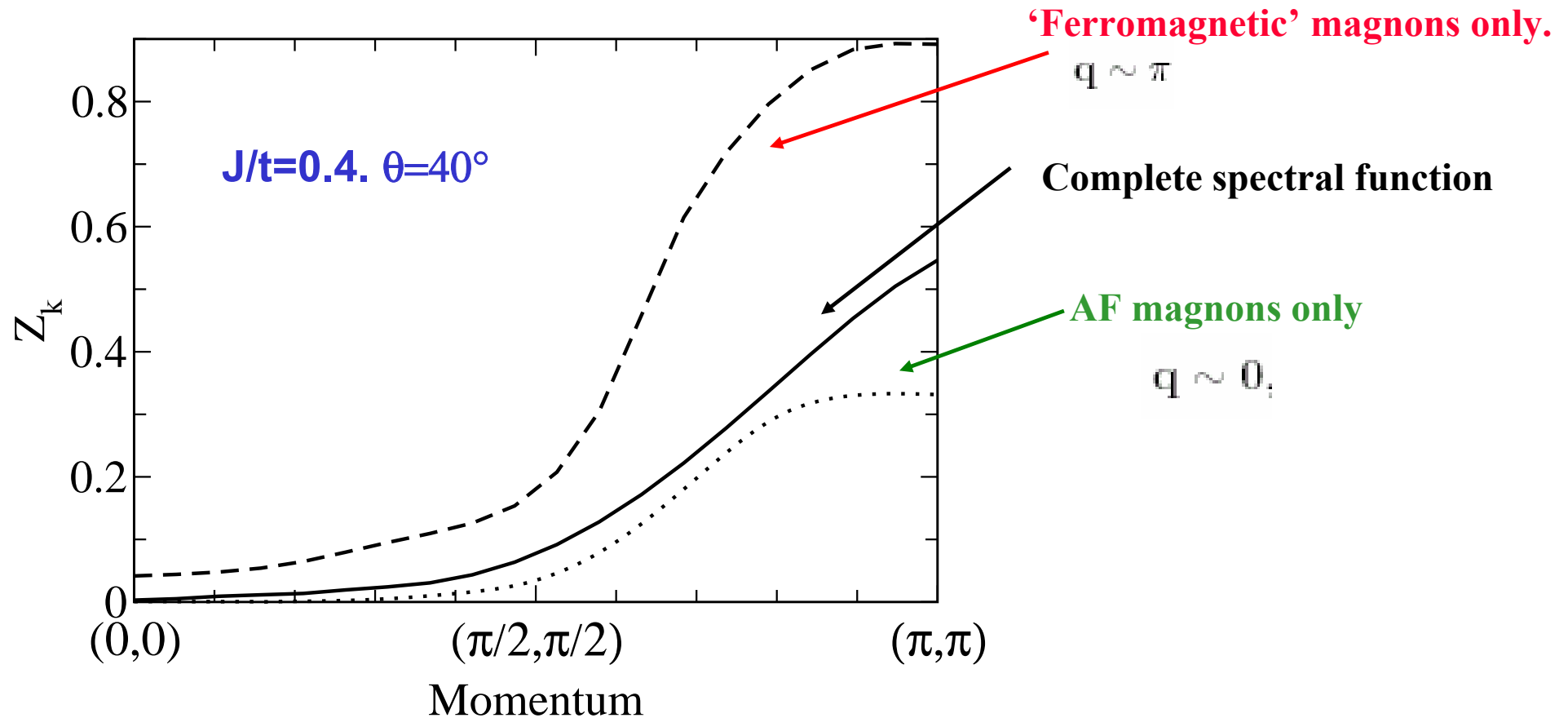
$(\pi, \pi)$  is a unique case: constructive interference



Outside the MBZ the QP weight goes to zero only for  $\theta=90^\circ$

Inside the magnetic BZ the QP weight goes to zero at  $60^\circ$

## Contributions of the magnetic bands to the hole spectral function



AF: For all  $\mathbf{k}$  and  $q \sim 0$ ,  $M_{\mathbf{k},q} \sim \sqrt{q}$

F: For all  $\mathbf{k}$  and  $q \sim \pi$ ,  $M_{\mathbf{k},q} \sim \text{const} + |q - \pi|$

**The coupling with ferromagnetic magnons is more coherent: more spectral weight.**

# J/t dependence of QP excitations.

As J/t increases, there is a crossover from

**Strong coupling:  $J/t < 1$**  → **QP: many-body state: hole coupled with magnons**

**Weak coupling:  $J/t > 1$**  → **One hole + one magnon**

$$|\Psi_{\mathbf{k}}\rangle = a_{\mathbf{k}}^{(0)} h_{\mathbf{k}}^{\dagger} |AF\rangle + \frac{1}{\sqrt{N}} \sum_{\mathbf{q}_1} a_{\mathbf{k},\mathbf{q}_1}^{(1)} h_{\mathbf{k}-\mathbf{q}_1}^{\dagger} \alpha_{\mathbf{q}_1}^{\dagger} |AF\rangle + \frac{1}{N} \sum_{\mathbf{q}_1, \mathbf{q}_2} a_{\mathbf{k},\mathbf{q}_1, \mathbf{q}_2}^{(2)} h_{\mathbf{k}-\mathbf{q}_1-\mathbf{q}_2}^{\dagger} \alpha_{\mathbf{q}_2}^{\dagger} \alpha_{\mathbf{q}_1}^{\dagger} |AF\rangle + \dots$$

↑  
**bare hole**

↑  
**one magnon**

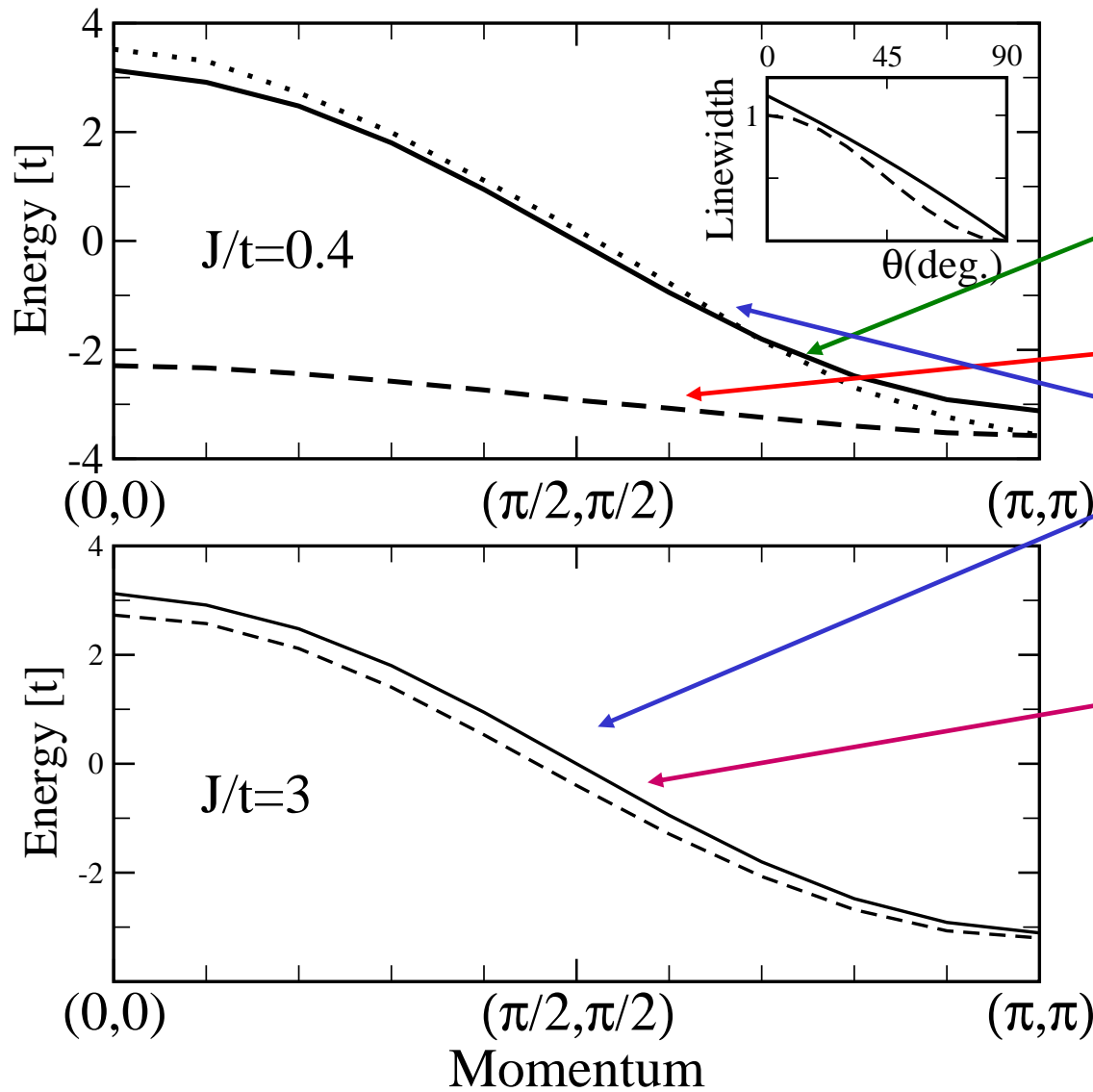
↑  
**multi-magnon**

$$z_{\mathbf{k}} \rightarrow 1 \text{ and } E_{\mathbf{k}} \rightarrow \epsilon_{\mathbf{k}}$$

**In weak coupling (Rayleigh-Schrodinger)**

$$a_{\mathbf{k}\mathbf{q}}^{(1\sigma)} = M_{\mathbf{k},\mathbf{q}}^{\sigma} / (\epsilon_{\mathbf{k}} - \epsilon_{\mathbf{k}-\mathbf{q}} - \omega_{\mathbf{q}}^{\sigma})$$

↑  
**Free hole, weakly renormalized by one magnon excitation**



**Bare hole and t-resonance**

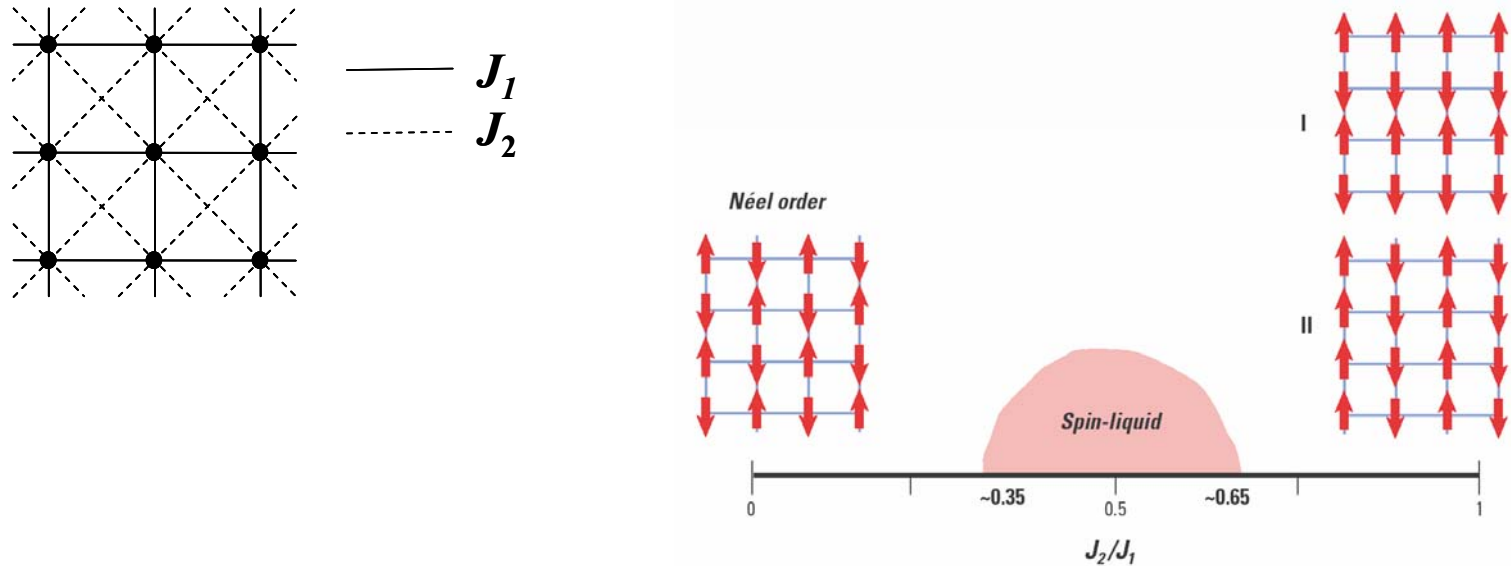
**QP energy**

**The t-resonance is always the bare hole weakly perturbed by a**

**magnon**  
**QP, t-resonance and bare hole are the same**

# J1-J2 Heisenberg model: Collinear phases

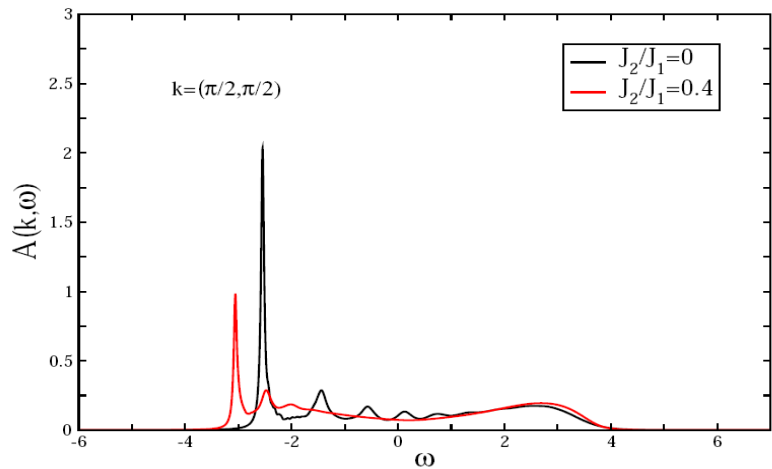
I. Hamad, A. Trumper, L.O.M., Physica B (2007)



**Experimental realization:  $\text{Li}_2\text{VOSiO}_4$**   
(see Trumper's poster next week)

**What happens when antiferromagnetic and ferromagnetic chains coexist?**

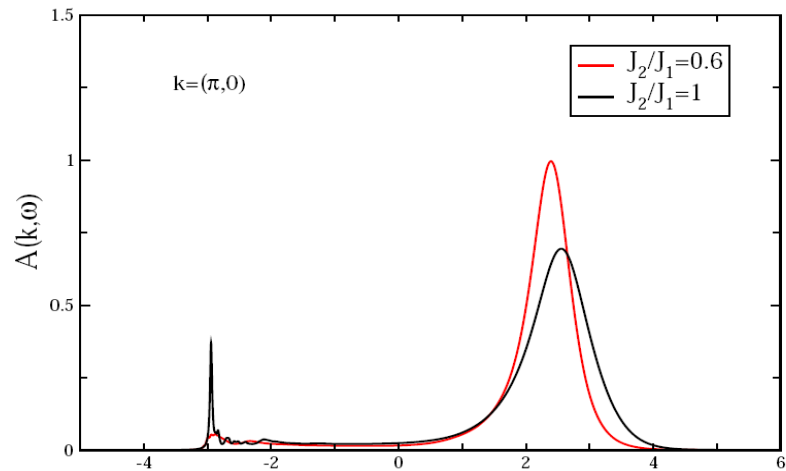
Néel phase ( $J_2 < 0.5J_1$ )  
 Frustration  $\rightarrow$  weakened QP  
 spectral weight



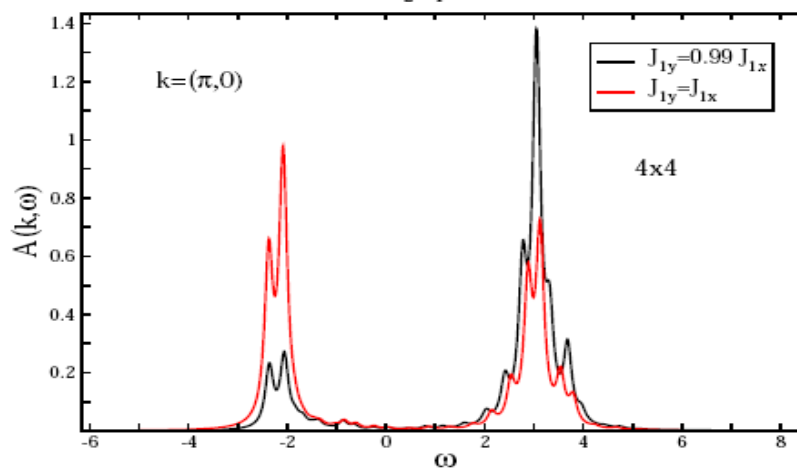
$J_1 = 0.4t$

Lanczos results confirm the SCBA  
 picture

Collinear phase ( $J_2 > 0.5J_1$ )  
 Frustration  $\rightarrow$  weakened QP  
 spectral weight and prominent  
*t-resonance*



$J_2/J_1 = 0.7$



# Conclusions

- **Competing frustrated interactions can induce ferromagnetic correlations, resulting in two mechanisms for hole motion:**  
*A magnon assisted propagation, due to AF fluctuations of the background.*  
*A free-like hopping mechanism due to the ferromagnetic component of the magnetic order.*
- **As a consequence of the competition between both mechanisms, the QP spectral weight vanishes in some cases (triangular lattice for  $t > 0$ , canted phase for  $\theta \geq 60^\circ$ , etc.)**
- **In the strong coupling regime,  $t > J$ , the hole propagates preferably at two well separated energies**  
**At low energies as a coherent spin polaron.**  
**At higher energies as a free hole weakly renormalized by magnons.**
- **For  $t < J$  there is a crossover of the QP excitation from a many body state to a quasi-free hole.**