

# Theory of electronic spectrum in cuprate superconductors

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- Motivation:

Is it possible to explain ARPES results  
(*'arc' Fermi surface and pseudogap*)

and high- $T_c$  superconductivity within a microscopic theory  
for an effective Hubbard model for the  $\text{CuO}_2$  plane?

- Conclusion:

self-consistent solution of the Dyson equation for a single  
particle Green function in the limit of strong electron  
correlations for the Hubbard model provides such a  
possibility

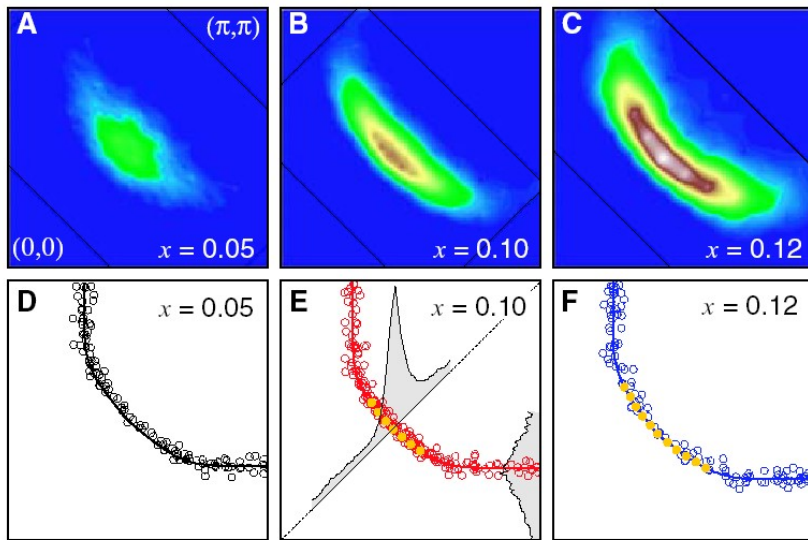
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## Outline

- ARPES and theory of SCES
  - Effective p-d Hubbard model for the  $\text{CuO}_2$  plane
  - Projection technique for Green functions:
    - Dyson equation
    - Self-energy in NCA
  - Dispersion and spectral functions
  - Fermi surface and **arcs**
  - Self – energy: coupling constants and **kinks**
  - Conclusion
-

# ARPES

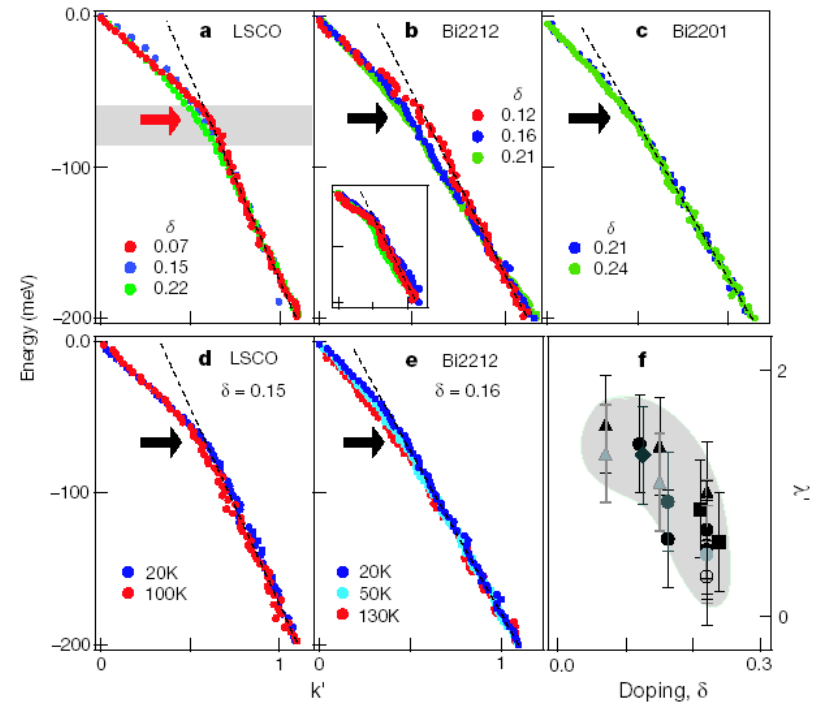
“Destruction” of FS – “arc” FS



cupric oxychloride  $\text{Ca}_{2-x}\text{Na}_x\text{CuO}_2\text{Cl}_2$

*K. M. Shen, Science 307 901 (2005).*

“Kink” phenomenon

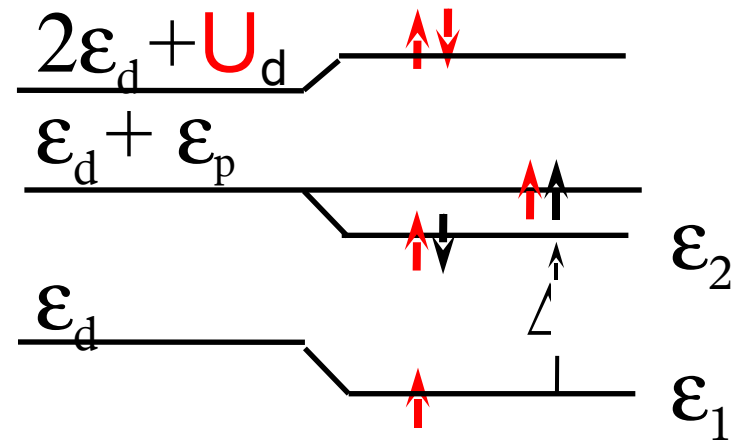
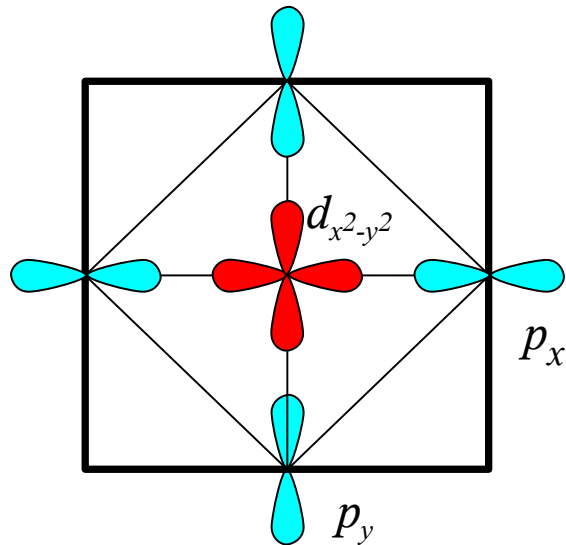


*A. Lanzara, et al., Nature 412 (2001)*

## Theory of SCES

- DMFT –  $\mathbf{q}$ -independent self-energy,  $d \gg 1$ , (*kinks* – Kollar, et al.),
  - Momentum decomposition for GF (*K. Matho et al.*)
  - Quantum cluster theories – (*review by Maier et al. RMP 2005*)
    - Quantum MC, ED (*Scalapino, Dagotto, Maekawa, Tohyama, Prelovsek*)
    - DCA – dynamical cluster approximation (*Hettler, Jarrel, et al.*)
    - CDMFT – Cellular DMFT (*Kotliar, Civelli, et al.*)
    - VCA – variational cluster approximation (*Potthoff et al.*)
    - Two-Particle Self-Consistent approach (TPSC) (*Tremblay et al.*)
  - Perturbative technique
    - Phenomenological approaches (spin-fermion models)  
· (*Pines, Norman, Chubukov, Eschrig, Sadovskii, et al.*)
      - FLEX (weak correlations,  $U < W$ ) (*Bickers et al., Manske, Eremin*)
      - Strong correlations: Hubbard operator technique:
        - Diagram approach (involved) (*Zaitsev, Izyumov, et al.*)
        - Equation of motion method for HOs (Mori-type projection technique)  
· (*Plakida, Mancini, Avella, Kakehashi – Fulde, et al.*)
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## Effective Hubbard p-d model for CuO<sub>2</sub> plane



Model for CuO<sub>2</sub> plane:

**Cu-3d** ( $\epsilon_d$ ) and **O-2p** ( $\epsilon_p$ ) hole states,

with  $U_d > \Delta = \epsilon_p - \epsilon_d \approx 2 t_{pd} \approx 3$  eV



In the strong correlation limit:  $U_{\text{eff}} = \Delta > W$  it is convenient to start from the atomic basis within a two-subband Hubbard model in terms of the projected, Hubbard operators:

$$c_{i\sigma} = c_{i\sigma} (1 - n_{i-\sigma}) + c_{i\sigma} (n_{i-\sigma}) = X_i^{0\sigma} + X_i^{-\sigma 2}, \quad n_{i\sigma} = c_{i\sigma}^\dagger c_{i\sigma}$$

Two subbands:

LHB – one-hole **d** - like state  $| \sigma \rangle$ :  $\epsilon_1 = \epsilon_d - \mu$

UHB – two-hole (**p** - **d**) ZR singlet state:  $| \uparrow \downarrow \rangle$ :  $\epsilon_2 = 2 \epsilon_1 + \Delta$

For these 4 states we introduce the Hubbard operators:

$$X_i^{\alpha\beta} = | i\alpha \rangle \langle i\beta |$$

where  $| \alpha \rangle = | 0 \rangle$ ,  $| \sigma \rangle = | \uparrow \rangle$ ,  $| \downarrow \rangle$ , and  $| 2 \rangle = | \uparrow \downarrow \rangle$

Hubbard operators rigorously obey the constraint:

$$X_i^{00} + X_i^{\uparrow\uparrow} + X_i^{\downarrow\downarrow} + X_i^{22} = 1$$

- only one quantum state can be occupied at any site  $| i \rangle$



Commutation relations for the Hubbard operators:

anticommutator for the Fermi-like operators

$$\{X_i^{0\sigma}, X_j^{\sigma'0}\} = \delta_{ij} (\delta_{\sigma'\sigma} X_i^{00} + X_i^{\sigma'\sigma}),$$

commutator for the Bose-like operators

$$[X_i^{\sigma\sigma'}, X_j^{\sigma''\sigma}] = \delta_{ij} (\delta_{\sigma'\sigma''} X_i^{\sigma\sigma} - X_i^{\sigma''\sigma'})$$

These commutation relations result in the **kinematic interaction**.

Spin operators in terms of HOs:

$$S_i^z = (1/2) (X_i^{++} - X_i^{--}), \quad S_i^+ = X_i^{+-}, \quad S_i^- = X_i^{-+},$$

Number operator

$$N_i = (X_i^{++} + X_i^{--}) + 2 X_i^{22}$$



The two-subband effective Hubbard model for holes

$$\begin{aligned}
 H = & \varepsilon_1 \sum_{i,\sigma} X_i^{\sigma\sigma} + \varepsilon_2 \sum_i X_i^{22} + \sum_{i \neq j, \sigma} \{ t_{ij}^{11} X_i^{\sigma 0} X_j^{0\sigma} \\
 & + t_{ij}^{22} X_i^{2\sigma} X_j^{\sigma 2} + 2\sigma t_{ij}^{12} (X_i^{2\bar{\sigma}} X_j^{0\sigma} + \text{H.c.}) \},
 \end{aligned}$$

One-hole and two-hole single-site energies

$$\varepsilon_1 = \varepsilon_d - \mu, \quad \varepsilon_2 = 2\varepsilon_1 + U_{\text{eff}}, \quad U_{\text{eff}} = \Delta = \varepsilon_p - \varepsilon_d$$

Hopping parameters for n.n.  $t$  and n.n.n sites  $t'$ ,  $t''$ :

$$t(\mathbf{k}) = 4t \gamma(\mathbf{k}) + 4t' \gamma'(\mathbf{k}) + 4t'' \gamma''(\mathbf{k})$$

Average number of **holes** is defined by the chemical potential  $\mu$ :

$$n = \langle N_i \rangle = \left\langle \sum_{\sigma} X_i^{\sigma\sigma} + 2X_i^{22} \right\rangle = 1 + \delta \leq 2$$



## Single-particle two-subband thermodynamic (retarded) Green functions

$$\begin{aligned} \hat{G}_{ij\sigma}(t-t') &= \langle\langle \hat{X}_{i\sigma}(t) | \hat{X}_{j\sigma}^\dagger(t') \rangle\rangle = \langle\langle \begin{pmatrix} X_i^{\sigma 2} \\ X_i^{0\bar{\sigma}} \end{pmatrix} | (X_i^{2\sigma} \ X_i^{\bar{\sigma}0}) \rangle\rangle \\ &= -i\theta(t-t') \langle\{ \hat{X}_{i\sigma}(t), \hat{X}_{j\sigma}^\dagger(t') \}\rangle \end{aligned}$$

Mori-type projection technique for equations of motion:

$$i \text{d} X_{i\sigma} / \text{d}t = \hat{Z}_{i\sigma} = [\hat{X}_{i\sigma}, H] = \sum_j \hat{\varepsilon}_{ij\sigma} \hat{X}_{j\sigma} + \hat{Z}_{i\sigma}^{(ir)},$$

orthogonality  
condition:

$$\langle\{ \hat{Z}_{i\sigma}^{(ir)}, \hat{X}_{j\sigma}^\dagger \}\rangle = \langle\hat{Z}_{i\sigma}^{(ir)} \hat{X}_{j\sigma}^\dagger + \hat{X}_{j\sigma}^\dagger \hat{Z}_{i\sigma}^{(ir)}\rangle = 0$$

Frequency matrix – QP spectra in MFA:  $\hat{\varepsilon}_{ij} = \langle\{ [\hat{X}_{i\sigma}, H], \hat{X}_{j\sigma}^\dagger \}\rangle \hat{Q}^{-1}$

$$\hat{Q} = \langle\{ \hat{X}_{i\sigma}, \hat{X}_{i\sigma}^\dagger \}\rangle = \begin{pmatrix} Q_2 & 0 \\ 0 & Q_1 \end{pmatrix} \quad \begin{array}{l} \text{where spectral weights} \\ \text{for Hubbard subbands:} \end{array} \quad \begin{array}{l} Q_2 = n/2 \\ Q_1 = 1 - n/2 \end{array}$$



Equation for GF:  $\hat{G}_\sigma(\mathbf{k}, \omega) = \hat{G}_\sigma^0(\mathbf{k}, \omega) + \hat{G}_\sigma^0(\mathbf{k}, \omega) \hat{Q}^{-1} \langle \langle \hat{Z}_{\mathbf{k}\sigma}^{\text{irr}} | \hat{X}_{\mathbf{k}\sigma}^\dagger \rangle \rangle_\omega$

where GF in MFA:  $\hat{G}_\sigma^0(\mathbf{k}, \omega) = (\omega \hat{\tau}_0 - \hat{\epsilon}(\mathbf{k}))^{-1} \hat{Q}$ ,

Differentiation of the many-particle GF  $\langle \langle \hat{Z}_{i\sigma}^{\text{irr}}(t) | \hat{X}_{j\sigma}^\dagger(t') \rangle \rangle$  over  $t'$  and carrying-out the projection results in the Dyson equation:

$$G_\sigma(\mathbf{k}, \omega) = \left\{ \omega \hat{\tau}_0 - \hat{\epsilon}(\mathbf{k}) - \hat{\Sigma}_\sigma(\mathbf{k}, \omega) \right\}^{-1} \hat{Q}$$

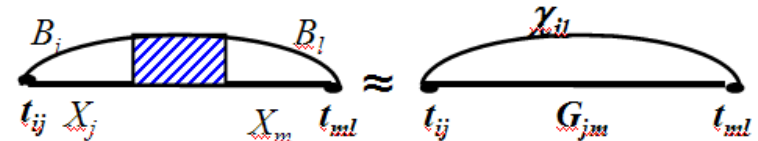
where the self-energy (SE)  $\hat{\Sigma}_\sigma(\mathbf{k}, \omega) = \langle \langle \hat{Z}_\sigma^{(\text{ir})} | \hat{Z}_\sigma^{(\text{ir})\dagger} \rangle \rangle_{\mathbf{k}, \omega}^{(\text{prop})} \hat{Q}^{-1}$  is the many-particle GF

**Kinematic interaction:**  $(\text{id}/dt) X_i^{\sigma 2} = [X_i^{\sigma 2}, H] = (\varepsilon_1 + \Delta) X_i^{\sigma 2}$   
 $+ \sum_{l \neq i, \sigma'} \left( t_{il}^{22} B_{i\sigma\sigma'}^{22} X_l^{\sigma' 2} - 2\sigma t_{il}^{21} B_{i\sigma\sigma'}^{21} X_l^{0\bar{\sigma}'} \right) - \sum_{l \neq i} X_i^{02} \left( t_{il}^{11} X_l^{\sigma 0} + 2\sigma t_{il}^{21} X_l^{2\bar{\sigma}} \right)$

$$B_{i\sigma\sigma'}^{22} = (N_i/2 + S_i^z) \delta_{\sigma'\sigma} + S_i^\sigma \delta_{\sigma'\bar{\sigma}} \quad B_{i\sigma\sigma'}^{21} = (N_i/2 + S_i^z) \delta_{\sigma'\sigma} - S_i^\sigma \delta_{\sigma'\bar{\sigma}}$$

## Self-consistent system of equations for GF and SE

Non-crossing approximation (NCA)  
for SE is given by the decoupling



for Fermi and Bose-like operators in the two-time correlation functions:

$$\langle B_{1'}(t) X_1(t) | B_{2'} X_2 \rangle \simeq \langle X_1(t) X_2 \rangle \langle B_{1'}(t) B_{2'} \rangle_{(1' \neq 1)}$$

SE in NCA for two Hubbard subbands reads:

$$\begin{aligned} \tilde{\Sigma}(\mathbf{k}, \omega) = & \frac{1}{N} \sum_{\mathbf{q}} |t(\mathbf{q})|^2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{d\nu dz}{\omega - z - \nu} \frac{1}{2} \left( \tanh \frac{z}{2T} + \coth \frac{\nu}{2T} \right) \\ & \times (1/\pi) \text{Im} \chi_{\text{sc}}(\mathbf{k} - \mathbf{q}, \nu) (1/\pi) \text{Im} \{ \tilde{G}_1(\mathbf{q}, z) + \tilde{G}_2(\mathbf{q}, z) \} \end{aligned}$$

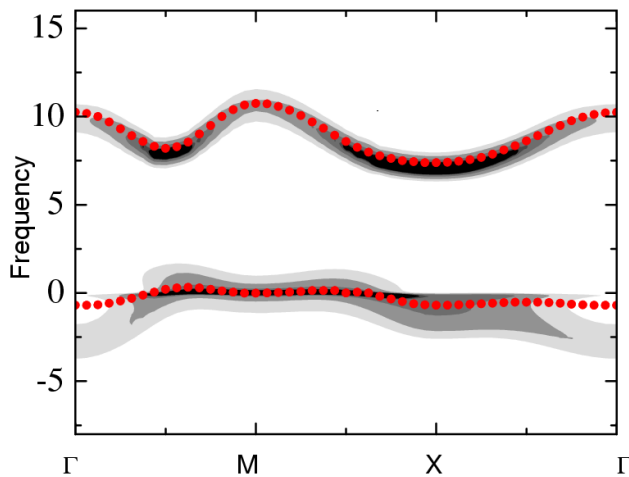
The interaction is specified by the hopping parameter  $t(\mathbf{q})$  and the

spin-charge susceptibility  $\chi_{\text{sc}}(\mathbf{q}, \nu) = (1/4) \langle \langle N_{\mathbf{q}} | N_{-\mathbf{q}} \rangle \rangle_{\nu} + \langle \langle S_{\mathbf{q}} | S_{-\mathbf{q}} \rangle \rangle_{\nu}$

where the GFs for two subbands  $\tilde{G}_{1(2)}(\mathbf{q}, \omega) = \frac{1}{\omega - \tilde{\epsilon}_{1(2)}(\mathbf{q}) - \tilde{\Sigma}(\mathbf{q}, \omega)}$

## Spectrum in MFA

$$\hat{G}_\sigma^0(\mathbf{k}, \omega) = (\omega \hat{\tau}_0 - \hat{\varepsilon}(\mathbf{k}))^{-1} \hat{Q},$$



Dispersion curves ( $\delta = 0.1$ ) along the symmetry directions

$\Gamma (0,0) \rightarrow M(\pi,\pi) \rightarrow X(\pi,0) \rightarrow \Gamma (0,0)$  in MFA (●●●) and with SE corrections (contour plot) for  $U=8t$

$$\tilde{\varepsilon}_{1,2}(\mathbf{k}) = (1/2)[\omega_2(\mathbf{k}) + \omega_1(\mathbf{k})] \mp (1/2)\Lambda(\mathbf{k}),$$

$$\Lambda(\mathbf{k}) = \{[\omega_2(\mathbf{k}) - \omega_1(\mathbf{k})]^2 + 4|W(\mathbf{k})|^2\}^{1/2},$$

$$\omega_1(\mathbf{k}) = 4t \alpha_1 \gamma(\mathbf{k}) + 4t' \beta_1 \gamma'(\mathbf{k}) - \mu,$$

$$\omega_2(\mathbf{k}) = \Delta + 4t \alpha_2 \gamma(\mathbf{k}) + 4t' \beta_2 \gamma'(\mathbf{k}) - \mu,$$

$$|W(\mathbf{k})| = 4t \alpha_{12} \gamma(\mathbf{k}) + 4t' \beta_{12} \gamma'(\mathbf{k})$$

where

$$\gamma(\mathbf{k}) = (1/2)(\cos k_x + \cos k_y),$$

$$\gamma'(\mathbf{k}) = \cos k_x \cos k_y,$$

Renormalization parameters

$$\alpha_{1(2)} = Q_{1(2)} [1 + C_1 / Q_{1(2)}^2],$$

$$\alpha_{12} = \sqrt{Q_1 Q_2} [1 - C_1 / Q_1 Q_2],$$

$$\beta_{1(2)} = Q_{1(2)} [1 + C_2 / Q_{1(2)}^2],$$

$$\beta_{12} = \sqrt{Q_1 Q_2} [1 - C_2 / Q_1 Q_2]$$

$$C_1 = \langle \mathbf{S}_i \mathbf{S}_{i \pm a_x / a_y} \rangle, \quad C_2 = \langle \mathbf{S}_i \mathbf{S}_{i \pm a_x \pm a_y} \rangle$$



1. Strong spectrum renormalization by the short-range **static** antiferromagnetic correlations (**missed in DMFT**)

$$\omega_2(\mathbf{k}) = \Delta + 4t \alpha_2 \gamma(\mathbf{k}) + 4t' \beta_2 \gamma'(\mathbf{k}) - \mu,$$

where  $\gamma(\mathbf{k}) = (1/2)(\cos k_x + \cos k_y)$ ,  $\gamma'(\mathbf{k}) = \cos k_x \cos k_y$ ,

$$\alpha_{1(2)} = Q_{1(2)}[1 + C_1/Q_{1(2)}^2], \quad \beta_{1(2)} = Q_{1(2)}[1 + C_2/Q_{1(2)}^2],$$

AF spin correlation functions:  $C_1 = \langle \mathbf{S}_i \mathbf{S}_{i \pm a_x/a_y} \rangle$ ,  $C_2 = \langle \mathbf{S}_i \mathbf{S}_{i \pm a_x \pm a_y} \rangle$

Close to half-filling,  $n = 1.05$ ,  $Q_2 = n/2$ ,  $C_1 \approx -0.26$ ,  $C_2 \approx 0.16$

hopping for the nearest neighbor sites is suppressed:

$$\alpha_2 \approx 0.1, \quad t_{ren} = 0.1 t \ll t$$

So, the dispersion is given by the next nearest neighbor hopping

$$\omega_1(\mathbf{k}) = 4 t' \beta_1 \cos k_x \cos k_y, \quad \beta_1 \approx 1.6, \quad t'_{ren} = 1.6 t' > t_{ren}$$



2. Self-energy in a static approximation (*Pines et al., Sadovskii et al.*)  
In the classical limit  $kT \gg \omega_s$  we get for the self-energy

$$\Sigma(\mathbf{k}, i\omega_n) \simeq |g(\mathbf{k} - \mathbf{Q})|^2 \frac{T}{N} \sum_{\mathbf{p}} \frac{1}{\kappa^2 + p^2} \\ \times [G_1(\mathbf{k} - \mathbf{Q} - \mathbf{p}, i\omega_n) + G_2(\mathbf{k} - \mathbf{Q} - \mathbf{p}, i\omega_n)]$$

For  $\kappa = 1/\xi \rightarrow 0$  for the GF we get equation (in one subband)

$$[G(\mathbf{k}, \omega)]^{-1} \approx \{ \omega - \varepsilon(\mathbf{k}) - |g(\mathbf{k} - \mathbf{Q})|^2 / [ \omega - \varepsilon(\mathbf{k} - \mathbf{Q}) - \Sigma(\mathbf{k} - \mathbf{Q}, \omega) ] \}$$

This results in the AF gap in the spectrum (neglecting  $\Sigma(\mathbf{k} - \mathbf{Q}, \omega)$ )

$E_{1,2} = (1/2) [\varepsilon(\mathbf{k}) + \varepsilon(\mathbf{k} - \mathbf{Q})] \pm (1/2) \{ [\varepsilon(\mathbf{k}) - \varepsilon(\mathbf{k} - \mathbf{Q})]^2 + 4 |g(\mathbf{k} - \mathbf{Q})|^2 \}^{1/2}$   
or a pseudogap for finite  $\xi$  and finite  $\Sigma(\mathbf{k} - \mathbf{Q}, \omega)$  close to X ( $\pi, 0$ ) region.

Thus, the pseudogap appears due to **AF short-range correlations**  
**in our theory -- dynamical short-range spin fluctuations**



## Numerical Results

The system of equations for GFs and SE was solved self-consistently by using imaginary frequency representation .

Model for the dynamical spin-susceptibility function in SE

$$\begin{aligned}\text{Im } \chi_s(\mathbf{q}, \nu) &= \frac{\chi_0}{1 + \xi^2(1 + \gamma(\mathbf{q}))} \tanh \frac{\nu}{2T} \frac{1}{1 + (\nu/\omega_s)^2} \\ \chi_0 &= \frac{3(1 - |\delta|)}{2\omega_s} \left\{ \frac{1}{N} \sum_{\mathbf{q}} \frac{1}{1 + \xi^2[1 + \gamma(\mathbf{q})]} \right\}^{-1}\end{aligned}$$

Spin-susceptibility shows a maximum at AF wave-vector  $\mathbf{Q}_{\text{AF}} = (\pi, \pi)$ .

where  $\gamma(\mathbf{q}) = (1/2)(\cos q_x + \cos q_y) = -1$

Two fitting parameters: AF correlation length  $\xi$  and energy  $\omega_s \sim J = 0.4 t$ , while constant  $\chi_0$  is defined by the equation:  $\langle \mathbf{S}_i \mathbf{S}_i \rangle = (3/4)(1 - |\delta|)$



Static AF correlation functions  $C_1, C_2$  and correlation length  $\xi$

$\delta =$	0.03	0.05	0.10	0.15	0.20	0.30
$C_1$	-0.36	-0.26	-0.21	-0.18	-0.14	-0.10
$C_2$	0.27	0.16	0.11	0.09	0.06	0.04
$C(\xi)$	22.0	5.91	3.58	2.67	1.93	1.40
$\xi$	8.0	3.40	2.50	2.10	1.70	1.40

where

$$C_1 = \langle \mathbf{S}_i \mathbf{S}_{i \pm a_x / a_y} \rangle, \quad C_2 = \langle \mathbf{S}_i \mathbf{S}_{i \pm a_x \pm a_y} \rangle$$

$$C_1 = \frac{1}{N} \sum_{\mathbf{q}} C_{\mathbf{q}} \gamma(\mathbf{q}), \quad C_2 = \frac{1}{N} \sum_{\mathbf{q}} C_{\mathbf{q}} \gamma'(\mathbf{q}).$$

$$C_{\mathbf{q}} = \langle \mathbf{S}_{\mathbf{q}} \mathbf{S}_{-\mathbf{q}} \rangle = \frac{C(\xi)}{1 + \xi^2 [1 + \gamma(\mathbf{q})]}, \quad C(\xi) = \chi_0 (\omega_s / 2).$$

Spectral function for electrons  $A_{\text{el}}(\mathbf{k}, \omega) = A_{\text{h}}(\mathbf{k}, -\omega)$  where

$$A_{(\text{h})}(\mathbf{k}, \omega) = \{1/2 - P(\mathbf{k})\} \tilde{A}_1(\mathbf{k}, \omega) + \{1/2 + P(\mathbf{k})\} \tilde{A}_2(\mathbf{k}, \omega),$$

$$\tilde{A}_{1(2)}(\mathbf{k}, \omega) = -(1/\pi) \text{Im} \tilde{G}_{1(2)}(\mathbf{k}, \omega + i0^+)$$

where  $P(\mathbf{k})$  is the hybridization contribution.

Electron occupation numbers  $n_{\sigma}^{(\text{el})}(\mathbf{k}) = 1 - n_{\sigma}^{(\text{hole})}(\mathbf{k})$

where hole  
numbers

$$n_{\sigma}^{\text{hole}}(\mathbf{k}) = \int_{-\infty}^{\infty} \frac{d\omega}{e^{\omega/T} + 1} A_{(\text{h})}(\mathbf{k}, \omega)$$

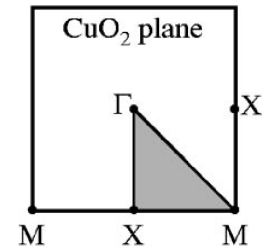
$$= \{1/2 - P(\mathbf{k})\} \tilde{N}_1(\mathbf{k}) + \{1/2 + P(\mathbf{k})\} \tilde{N}_2(\mathbf{k})$$

Parameters:  $t \approx 0.4 \text{ eV}$ ,  $t' = -0.3 t$ ,  $U_{\text{eff}} = 8 t$

$t \approx 0.6 \text{ eV}$ ,  $t' = -0.13 t$ ,  $t'' = 0.16 t$ ,  $U_{\text{eff}} = 4 t$



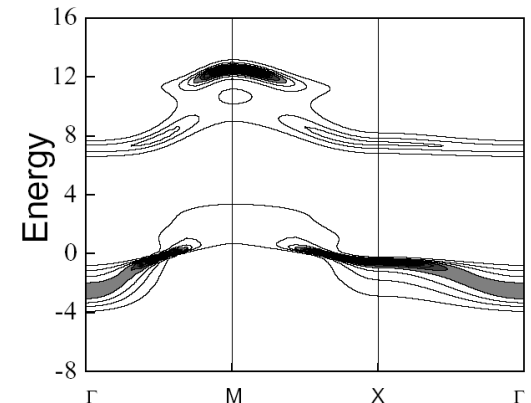
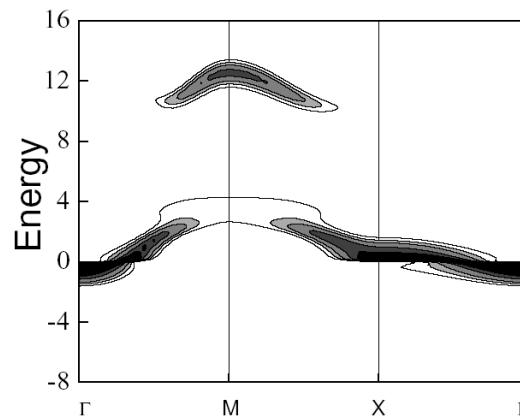
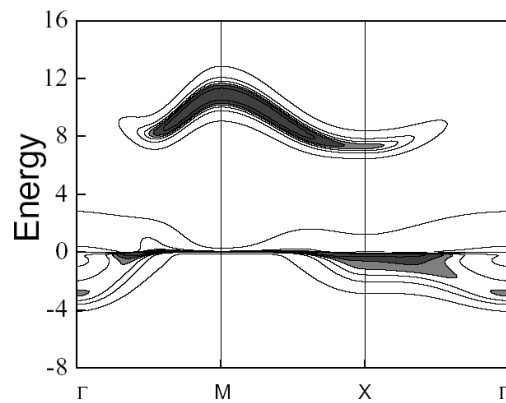
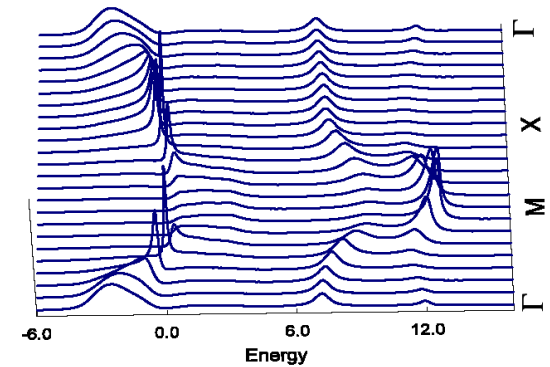
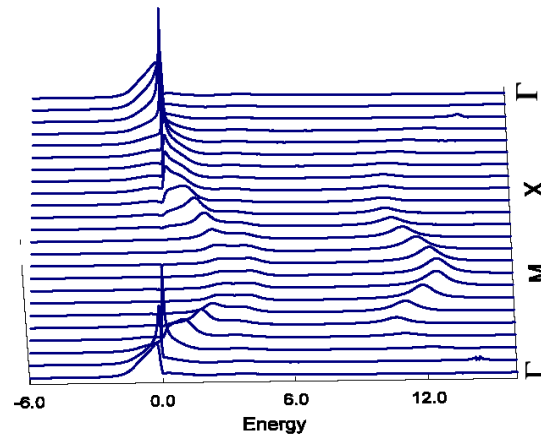
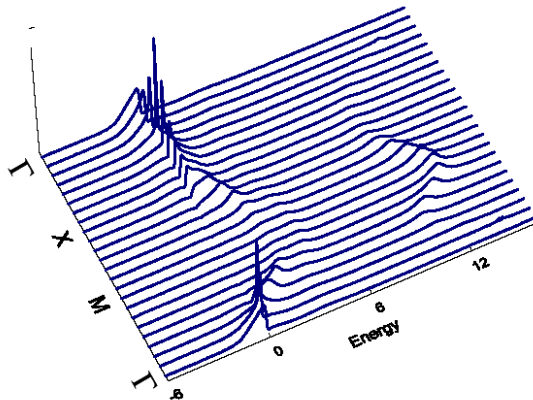
Spectral functions  $A(k, \omega)$  and dispersion curves along symmetry directions  $\Gamma (0,0) \rightarrow M(\pi,\pi) \rightarrow X(\pi,0) \rightarrow \Gamma (0,0)$



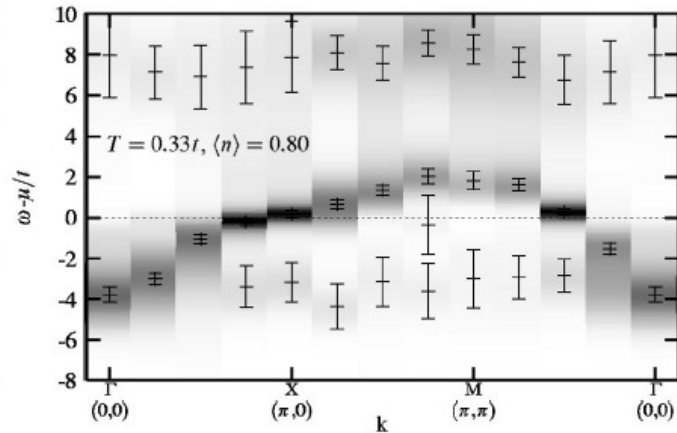
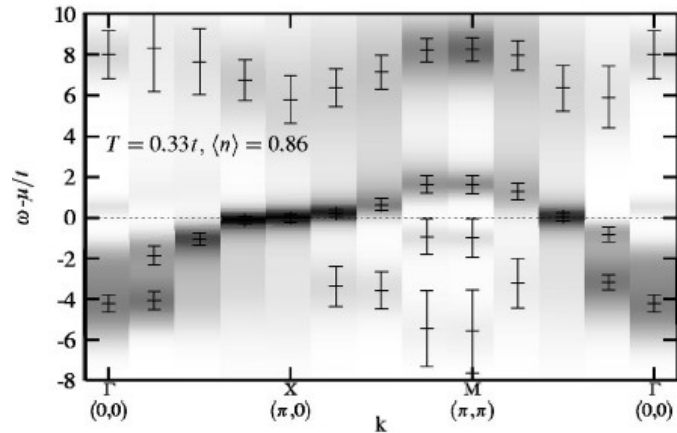
$\delta = 0.1$  ( $T \approx 0.03 t$ )

$\delta = 0.3$  ( $T \approx 0.03 t$ )

$\delta = 0.1$  ( $T \approx 0.3 t$ )

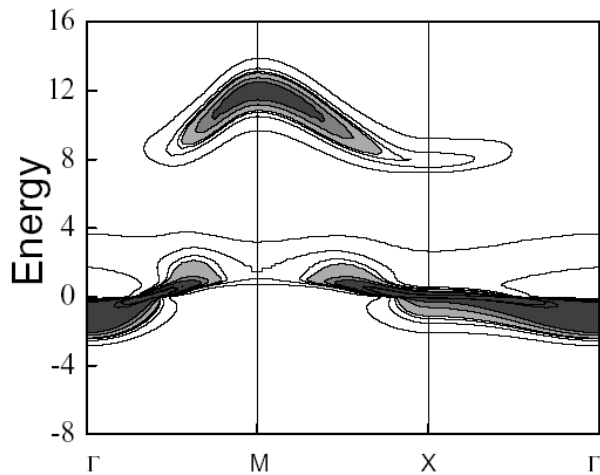


MC study of  $A(k, \omega)$  for 8x8 cluster at  $T=0.33 t$ ,  $U = 8t$  [Grober et al. (2000)]

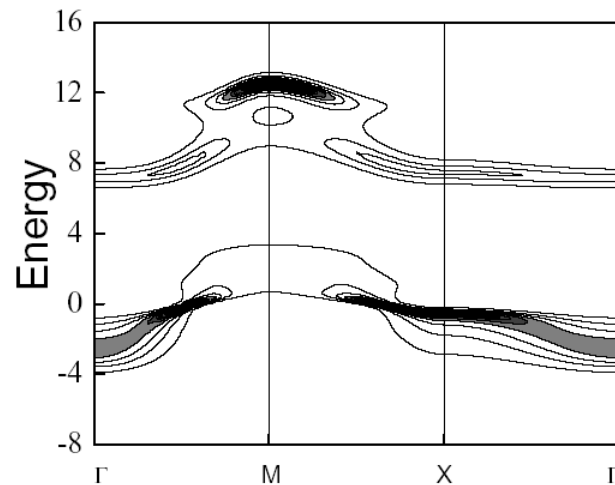


$\delta = 0.14$

$\delta = 0.20$



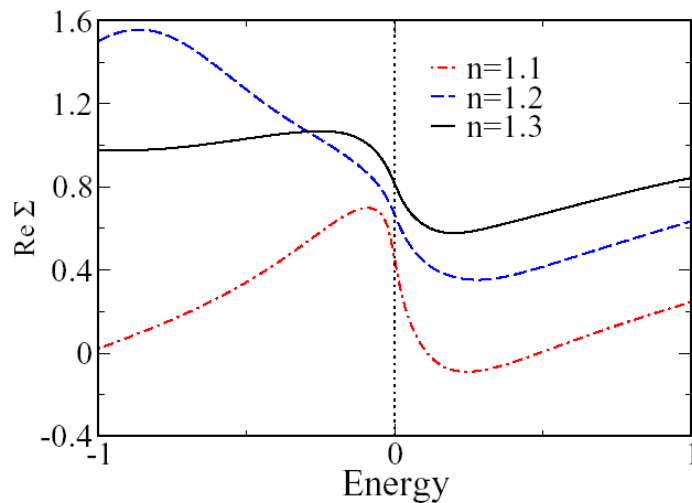
$\delta = 0.2 (T \approx 0.03 t)$



$\delta = 0.1 (T \approx 0.3 t)$

## Coupling constant

$$\lambda(\mathbf{k}) = -(\partial \text{Re}\Sigma(\mathbf{k}, \omega) / \partial \omega)_{\omega=0}$$

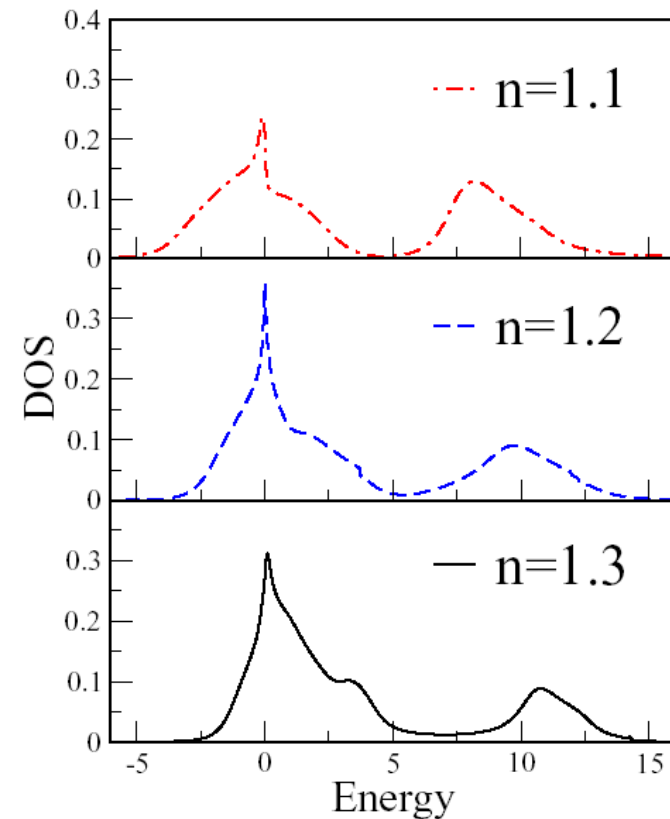


$$\lambda(k_F) = 7.86 \text{ at } \delta = 0.1$$

$$\lambda(k_F) = 3.3 \text{ at } \delta = 0.3$$

## Density of states $A(\omega)$ :

Spectral weight transfer with doping

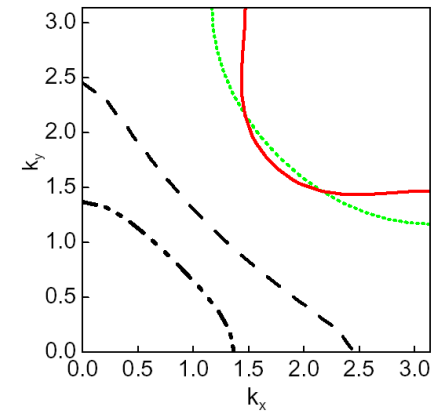


## Fermi surface: contour plot of equation

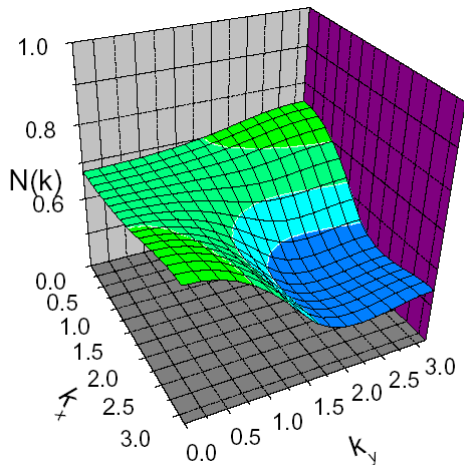
$$\tilde{\epsilon}_2(\mathbf{k}_F) + \tilde{\Sigma}(\mathbf{k}_F, \omega = 0) = 0$$

( $T \approx 0.03 t$ ):  $\delta = 0.1$  —  $\delta = 0.2$  - -  $\delta = 0.3$  - - - -

( $T \approx 0.3 t$ ):  $\delta = 0.1$  —

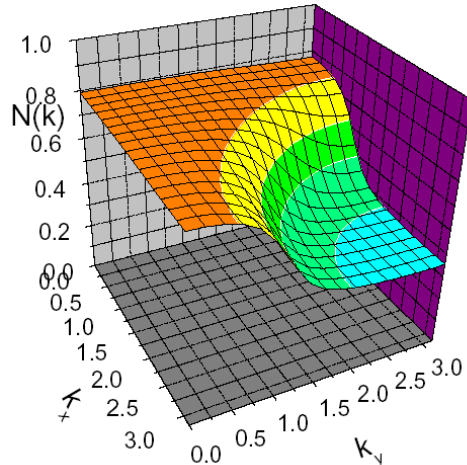


## Electron occupation numbers $n_{el}(\mathbf{k}) = 1 - n_h(\mathbf{k})$



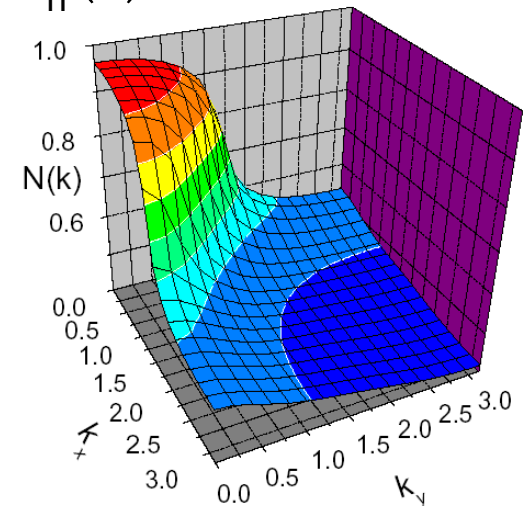
$\delta = 0.1$  ( $T \approx 0.03 t$ )

$\Delta n \approx 0.15$



$\delta = 0.1$  ( $T \approx 0.3 t$ )

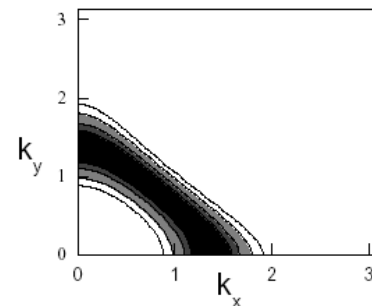
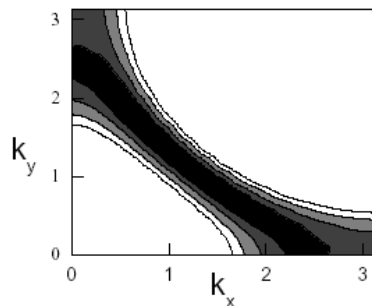
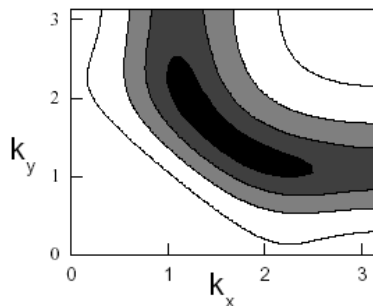
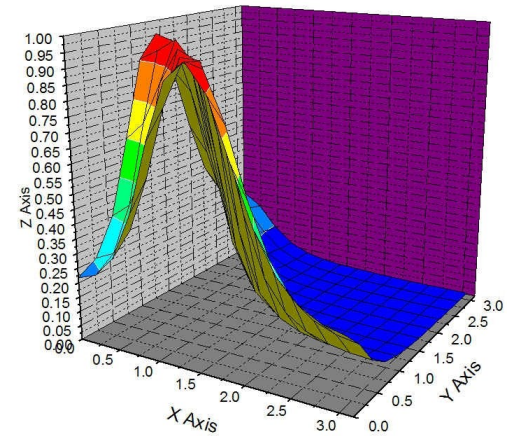
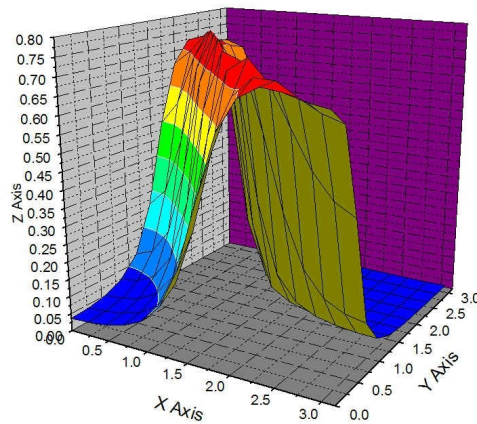
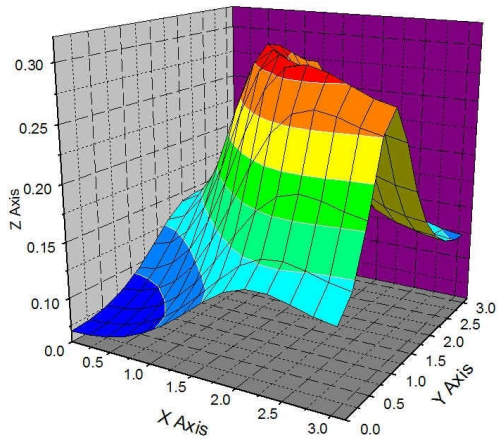
$\Delta n \approx 0.45$



$\delta = 0.3$  ( $T \approx 0.03 t$ )

$\Delta n \approx 0.55$

Fermi surface: maximum values of  $A_{(el)}(k_F, \omega = 0)$

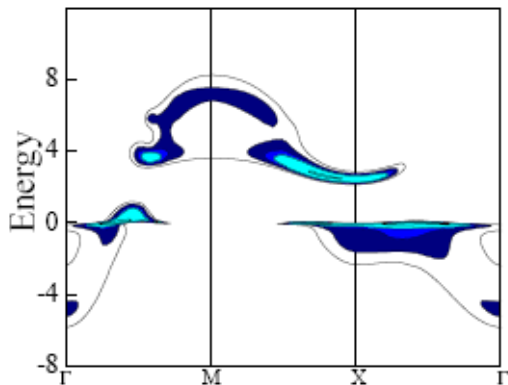


( $T \approx 0.03 t$ ):  $\delta = 0.1$

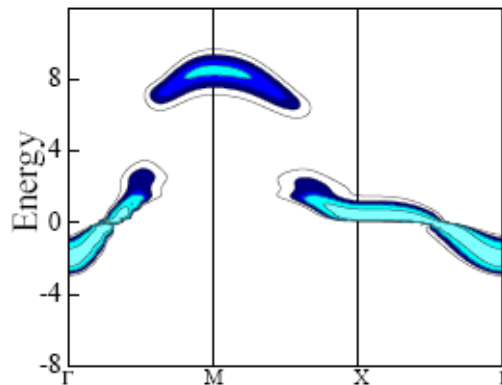
$\delta = 0.2$

$\delta = 0.3$

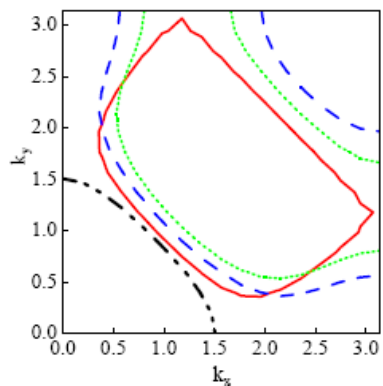
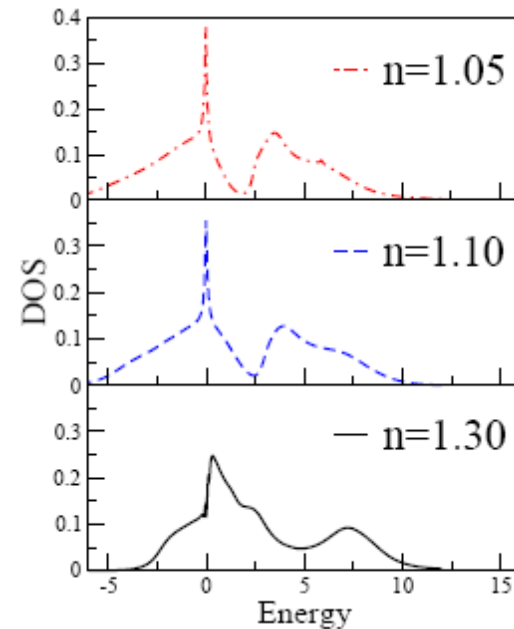
Numerical solution for  $U_{\text{eff}} = 4t$



$\delta = 0.05$  ( $T \approx 0.03 t$ )

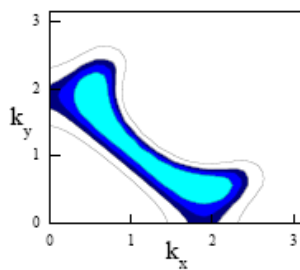


$\delta = 0.3$  ( $T \approx 0.03 t$ )

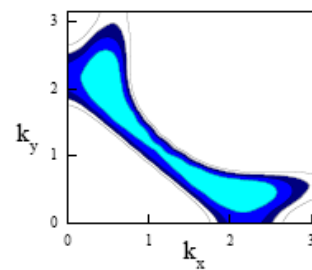


( $T \approx 0.03 t$ ):  
 $\delta = 0.1$  —  
 $\delta = 0.2$  - - -  
 $\delta = 0.3$  - - -  
 ( $T \approx 0.3 t$ ):  
 $\delta = 0.1$  - - -

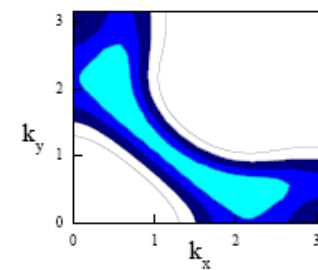
Fermi surface:  $A(k, 0)=0$



$\delta = 0.05$   
 ( $T \approx 0.03 t$ )



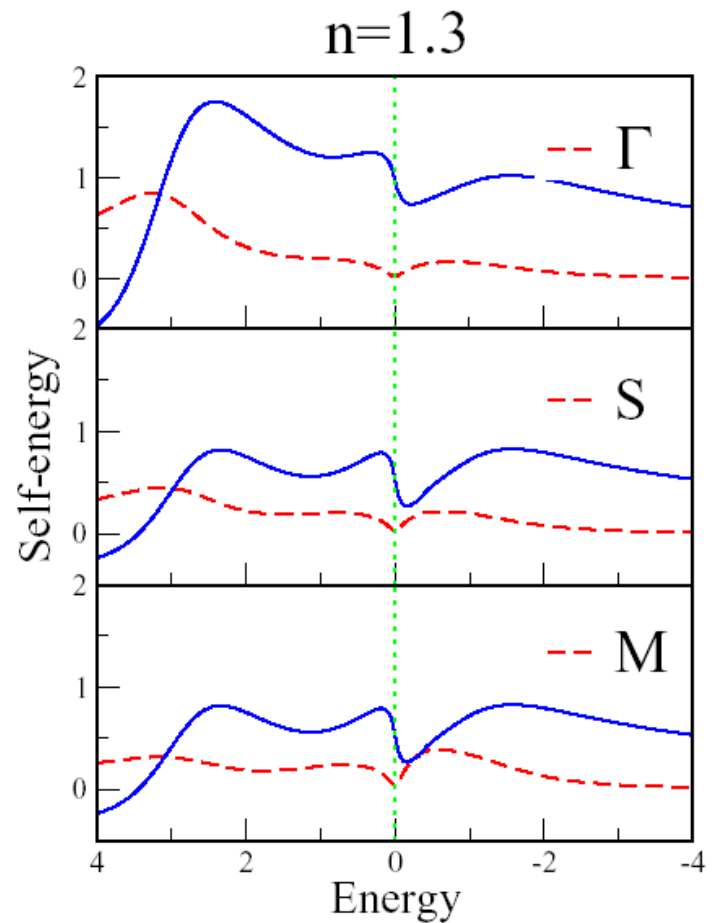
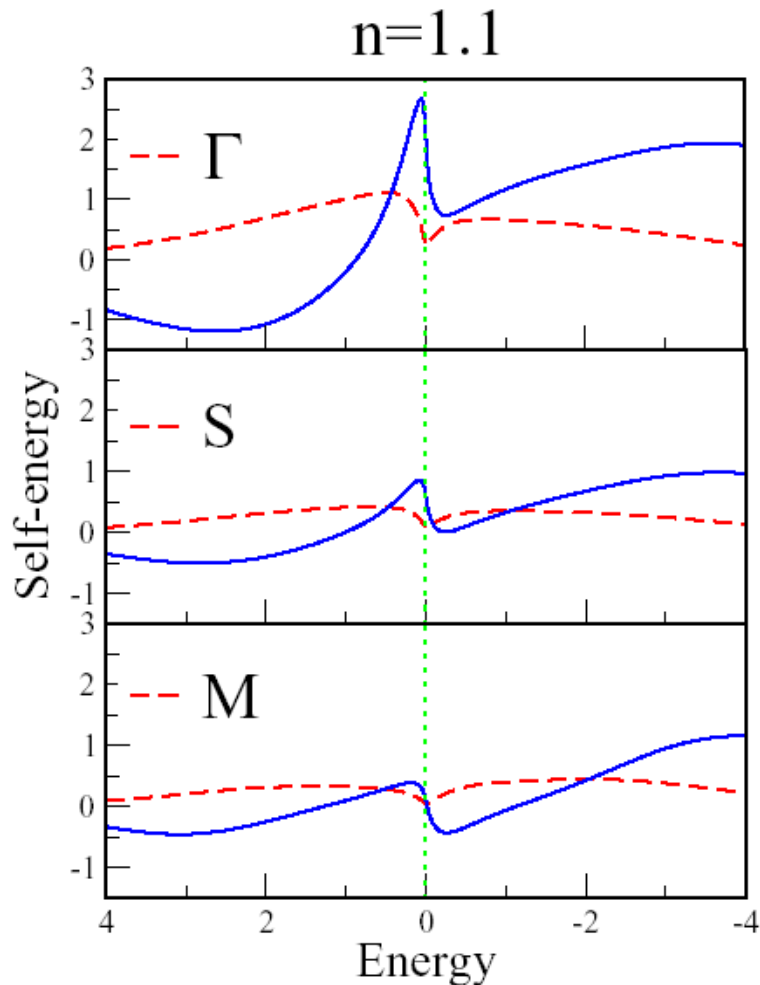
$\delta = 0.1$   
 ( $T \approx 0.03 t$ )



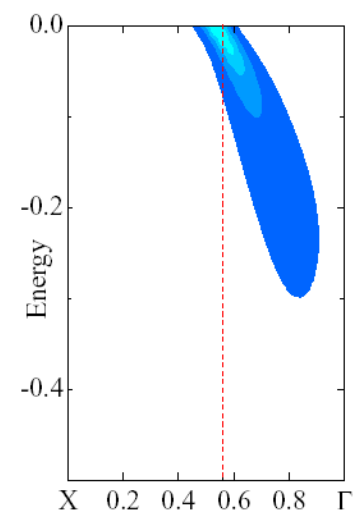
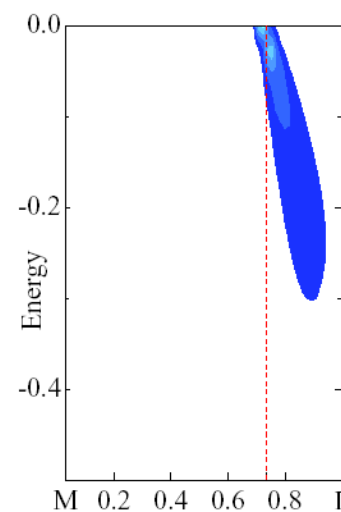
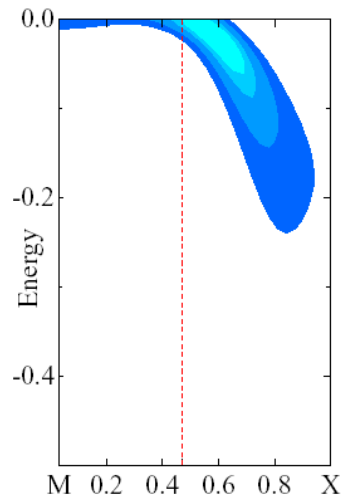
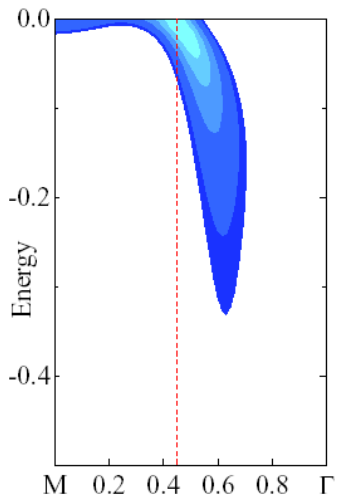
$\delta = 0.1$   
 ( $T \approx 0.3 t$ )



**Self-energy:** real (---) and imaginary (---) parts at  $\Gamma(0,0)$ ,  $S(\pi/2,\pi/2)$ , and  $M(\pi,\pi)$  points of BZ



## “Kink” in the dispersion curves



$M(\pi, \pi) \rightarrow \Gamma(0,0)$       $M(\pi, \pi) \rightarrow X(0, \pi)$   
 Dispersion along symmetry directions  
 at doping  $\delta = 0.1$

$M(\pi, \pi) \rightarrow \Gamma(0,0)$       $X(0, \pi) \rightarrow \Gamma(0,0)$   
 Dispersion along symmetry directions  
 at doping  $\delta = 0.3$

No well defined kink energy due a continuum spectrum of spin  
 fluctuations up to  $\omega_s \sim J = 0.4 t \sim 160 \text{ meV}$

Numerical solution (direct diagonalization) of the SC gap equation

$$\phi(\mathbf{k}, i\omega_n) = -T \sum_{\mathbf{q}} \sum_m K(\mathbf{k} - \mathbf{q}, \mathbf{q} | i\omega_n, i\omega_m) F(\mathbf{q}, i\omega_m)$$

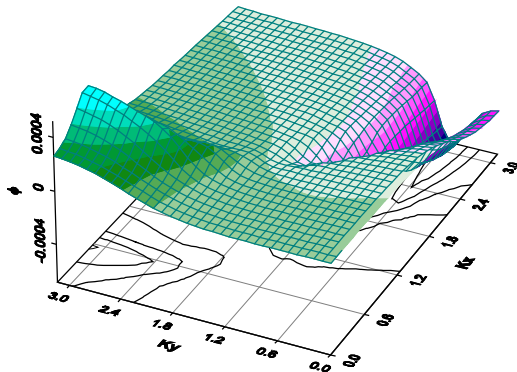
$$K(\mathbf{k} - \mathbf{q}, \mathbf{q} | i\omega_n, i\omega_m) = [J(\mathbf{k} - \mathbf{q}) + \lambda(\mathbf{q}, \mathbf{k} - \mathbf{q} | i\omega_n - i\omega_m)] F(\mathbf{q}, i\omega_m)$$

with interaction  $\lambda(\mathbf{q}, \mathbf{k} - \mathbf{q} | i\omega_n) = -|t(\mathbf{q})|^2 \chi_S(\mathbf{k} - \mathbf{q} | i\omega_n)$

for the linearized anomalous GF

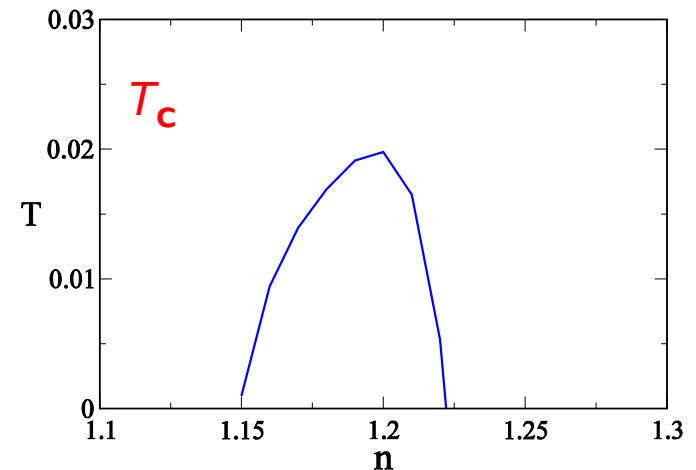
$$F(\mathbf{q}, i\omega_m) = -G(\mathbf{q}, -i\omega_m) \phi(\mathbf{q}, i\omega_m) G(\mathbf{q}, i\omega_m)$$

results in *d*-wave pairing:



$$T_c \sim 0.02t$$

$$\sim 100 \text{ K}$$



## Conclusion

- The proposed microscopic theory provides an explanation for doping and temperature dependence of electronic spectrum in cuprates as controlled by the AF spin correlations.
- Self-consistent solution of the Dyson equation for GF and SE in NCA reproduces the gross features of the electronic spectra:
  - pseudogap formation and arc-type FS in the underdoped region,
  - doping dependence of the dispersion and QP weight at the FS,
  - weight transfer of the subband spectral density with doping
- To perform quantitative comparison with ARPES data contributions from charge fluctuations and electron-phonon interaction should be taken into account

Publications: *N.M. Plakida, et al. JETP* **97**, 331 (2003). *Exchange and spin-fluctuation mechanisms of superconductivity in cuprates.*

*N. M. Plakida, V. S. Oudovenko, JETP* **104**, 230 (2007): *Electronic spectrum in high-temperature cuprate superconductors.*

M.V.Sadovskii (1974): “Toy” 1D pseudogap model (2D -- hot spots )

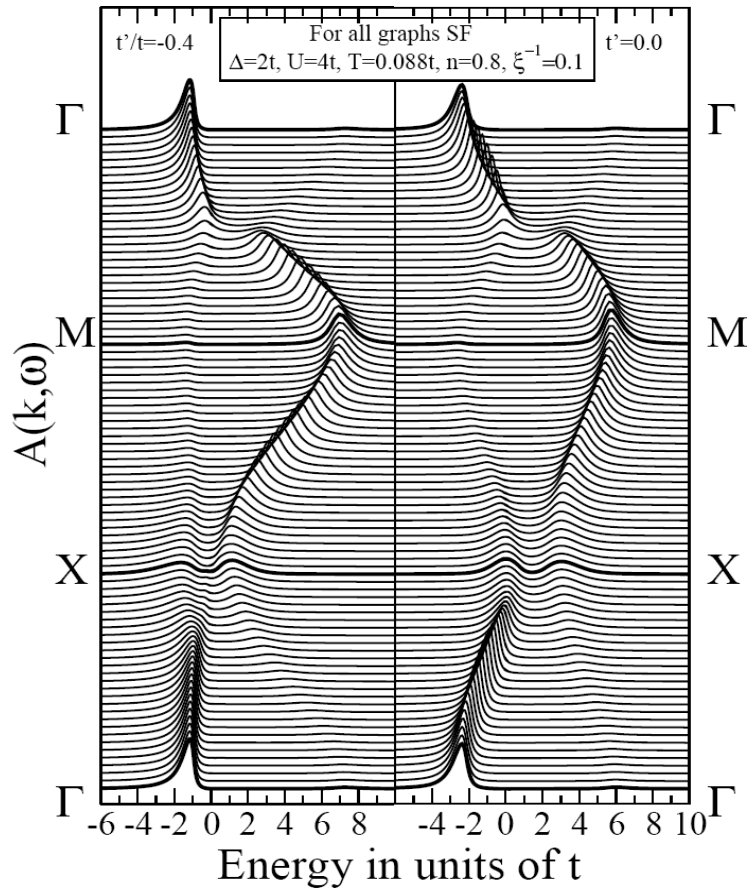
$$\begin{aligned} \Sigma(\varepsilon_n p) &= \int \frac{dQ}{2\pi} V_{eff}(Q) \frac{1}{i\varepsilon_n - \xi_{p-Q}} \approx 2W^2 \int_{-\infty}^{\infty} \frac{dx}{2\pi} \frac{\kappa}{x^2 + \kappa^2} \frac{1}{i\varepsilon_n + \xi_p - v_F x} = \\ &= 2W^2 \int_{-\infty}^{\infty} \frac{dx}{2\pi} \frac{\kappa}{(x - i\kappa)(x + i\kappa)} \frac{1}{i\varepsilon_n + \xi_p - v_F x} = \\ & \text{where} \end{aligned}$$

$$V_{eff}(Q) = 2W^2 \left\{ \frac{\kappa}{(Q - 2p_F)^2 + \kappa^2} + \frac{\kappa}{(Q + 2p_F)^2 + \kappa^2} \right\} = \frac{W^2}{i\varepsilon_n + \xi_p + i v_F \kappa}$$

M.V.Sadovskii et al. (2005): DMFT +  $\Sigma_{\mathbf{k}}$  approach

$$G_{\mathbf{k}}(i\omega) = \frac{1}{i\omega + \mu - \varepsilon(\mathbf{k}) - \Sigma(i\omega) - \Sigma_{\mathbf{k}}(i\omega)}$$

$$\Sigma_n(i\omega, \mathbf{k}) = \Delta^2 \frac{s(n)}{i\omega + \mu - \Sigma(i\omega) - \varepsilon_n(\mathbf{k}) + i n v_n \kappa - \Sigma_{n+1}(i\omega, \mathbf{k})}$$



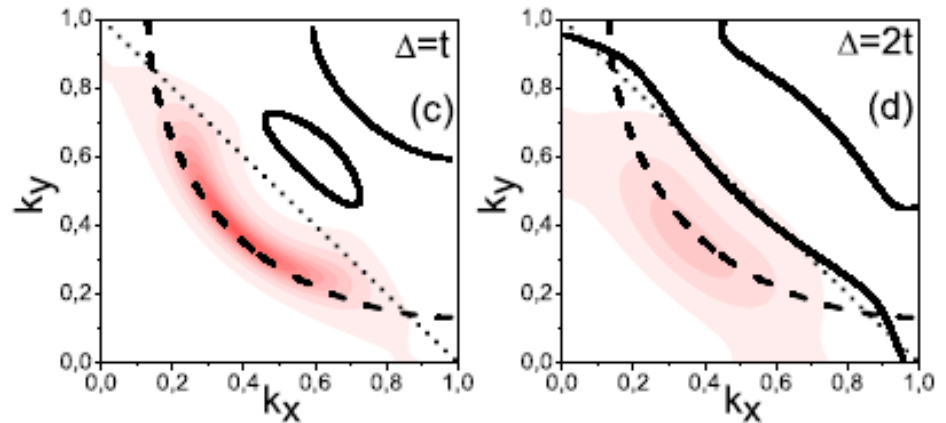
*Sadovskii et al.*

*Phys. Rev. B 72, 155105 (2005)*

Pseudogaps in strongly correlated metals: A generalized dynamical mean-field theory approach

*E. Z. Kuchinskii et al.,*

*JETP Letters, 82, 198 (2005)*

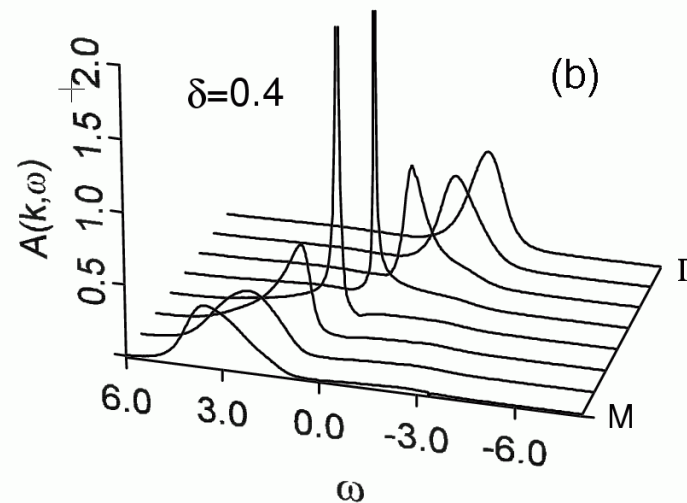
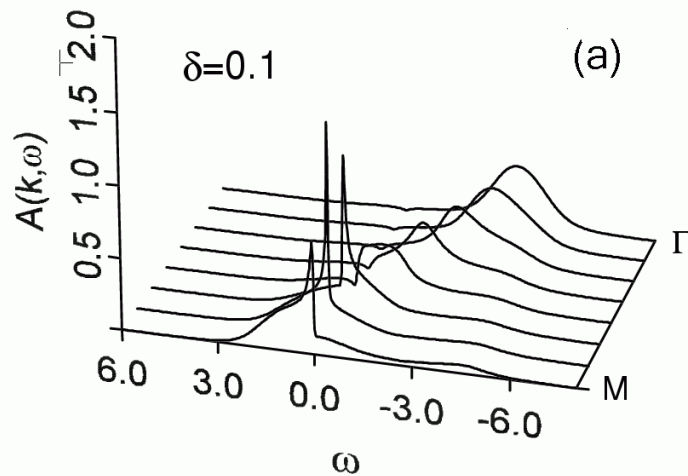


## Comparison with t-J model

*N.M. Plakida, V.S. Oudovenko, Phys. Rev. B 59, 11949 (1999)*

Electron spectrum and superconductivity in the t-J model at moderate doping.

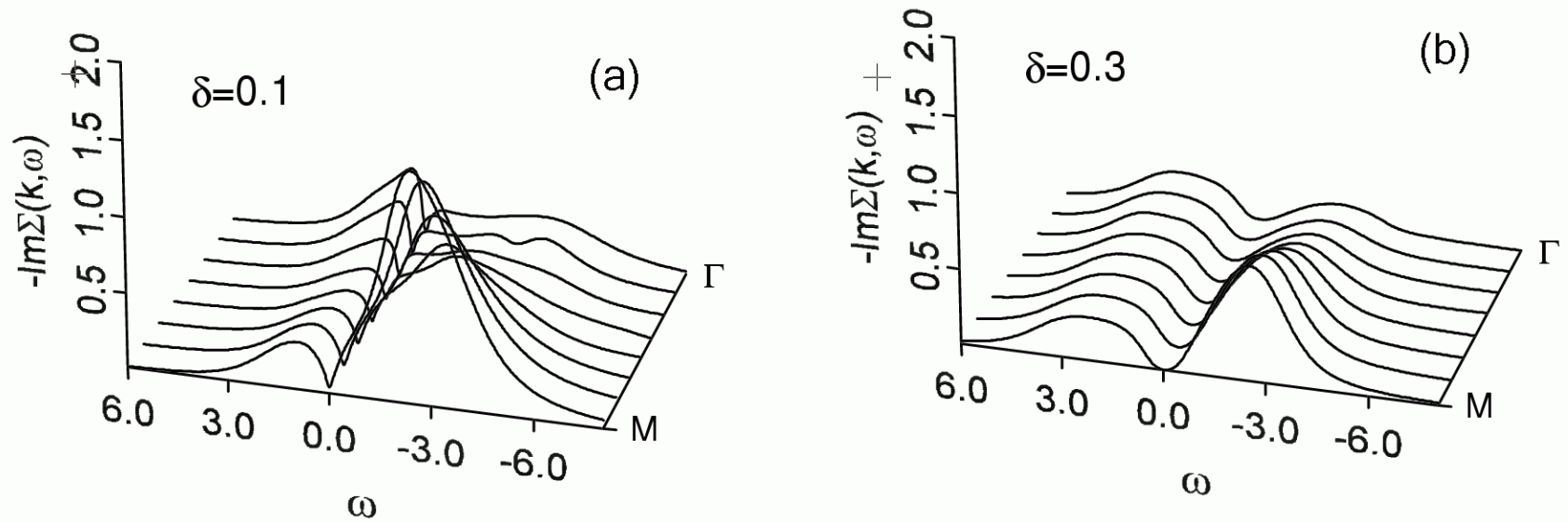
### 1. Spectral functions $A(k, \omega)$



Spectral function for the t-J model in the symmetry direction

$\Gamma(0,0) \rightarrow M(\pi,\pi)$  at doping  $\delta = 0.1$  (a) and  $\delta = 0.4$  (b).

## 2. Self-energy, $\text{Im } \Sigma(k, \omega)$

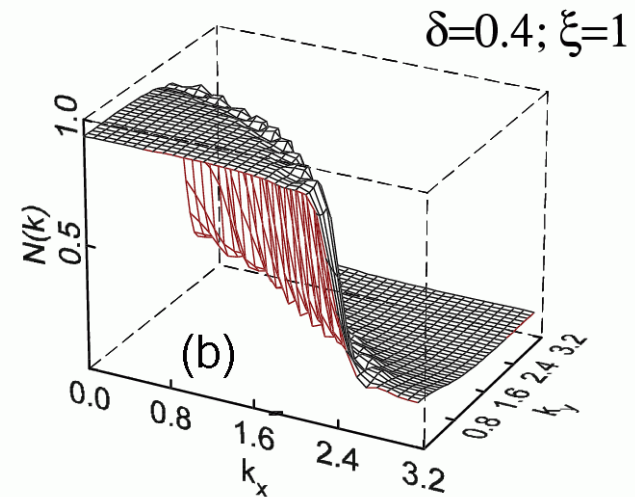
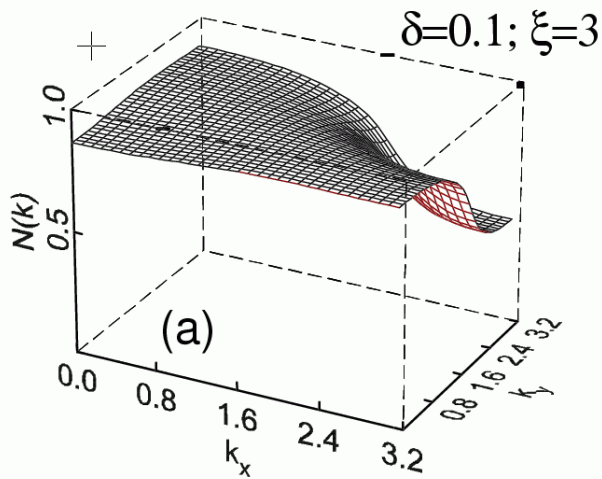


Self-energy for the t-J model in the symmetry direction

$\Gamma(0,0) \rightarrow M(\pi,\pi)$  at doping  $\delta = 0.1$  (a) and  $\delta = 0.4$  (b) .



### 3. Electron occupation numbers $N(k) = n(k)/2$



Electron occupation numbers for the t-J model in the quarter of BZ, ( $0 < k_x, k_y < \pi$ ) at doping  $\delta = 0.1$  (a) and  $\delta = 0.4$  (b).