Entanglement

and

Rydberg interaction

Antoine Browaeys, Institut d'Optique, CNRS

Outline

1. Basics of entanglement

2. Entanglement and 2-atom gate using Rybderg blockade

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1. Basics of entanglement

2. Entanglement and 2-atom gate using Rybderg blockade

Goal: control Rydberg interaction between few (10 – 100) atoms
(No ensemble average)

Quantum state engineering

Outline

1. Basics of entanglement

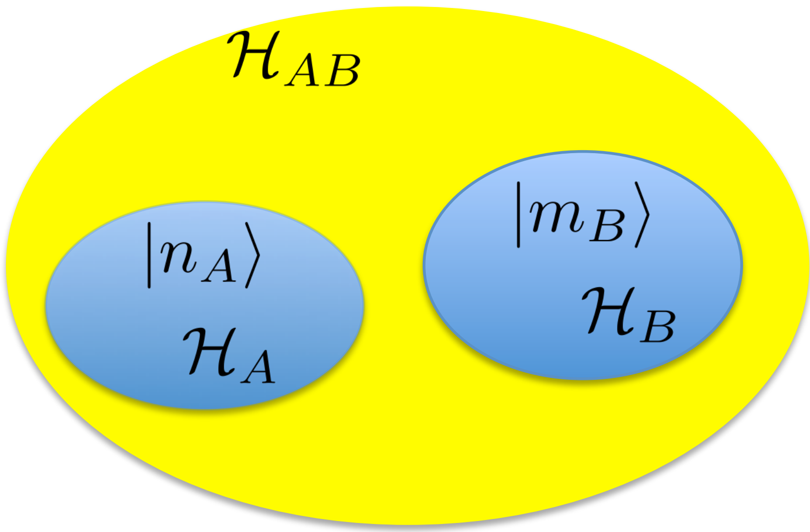
2. Entanglement and 2-atom gate using Rybderg blockade

What is entanglement?

2 systems A, B

2 d° of freedom

2 modes



Superposition principle ⇒ $$|\phi_{AB}\rangle = \sum_{nm} c_{nm} |n_A\rangle \otimes |m_B\rangle$$

Separable: $$|\phi_{AB}\rangle = |\phi_A\rangle \otimes |\phi_B\rangle$$

Non-separable = **entangled:** $$|\phi_{AB}\rangle \neq |\phi_A\rangle \otimes |\phi_B\rangle$$

MOST states are entangled!!

Two-level systems

$|\uparrow\rangle$

$|\downarrow\rangle$

2-level systems: atom, quantum circuits, spin, polarization of photon

Rotation (e.g. polarizer, Raman, microwaves…)

$$R(\theta,\varphi) = \begin{pmatrix} \cos\frac{\theta}{2} & ie^{i\varphi}\sin\frac{\theta}{2} \\ ie^{-i\varphi}\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix}_{|\uparrow\rangle,|\downarrow\rangle}$$

Ω Rabi frequency

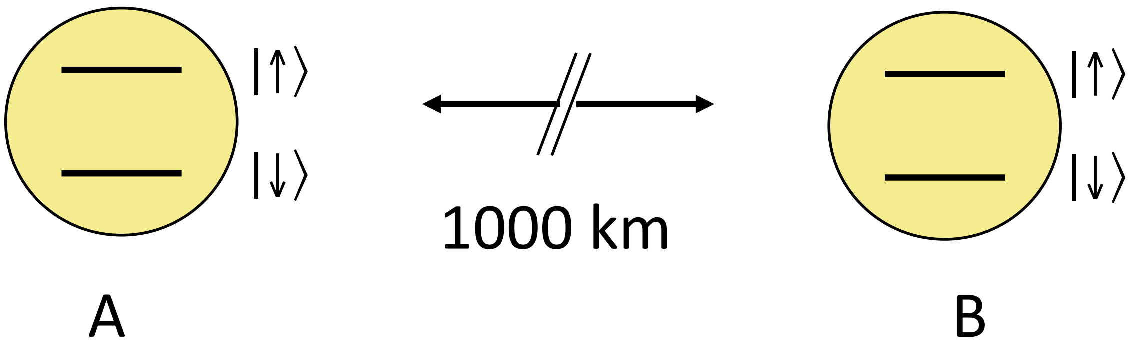
$$\theta = \Omega t$$

Bell states

$$|\psi_\pm\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow,\downarrow\rangle \pm |\downarrow,\uparrow\rangle\right)$$

$$|\phi_\pm\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow,\uparrow\rangle \pm |\downarrow,\downarrow\rangle\right)$$

What is so special about entanglement?



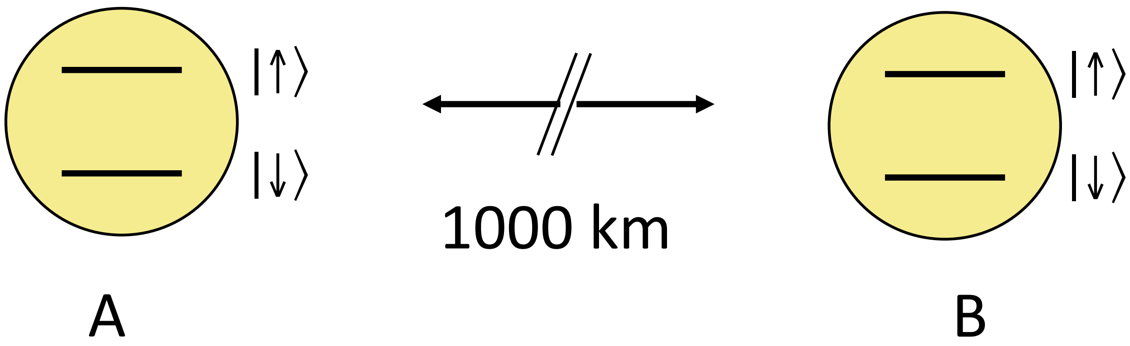
$$\frac{1}{\sqrt{2}}(|\downarrow_{\mathrm{A}},\uparrow_{\mathrm{B}}\rangle + |\uparrow_{\mathrm{A}},\downarrow_{\mathrm{B}}\rangle)$$

1. Perfect **correlations** (even at very large distance…!)

$$P(\uparrow_A \mid \downarrow_B) = \frac{P_{\uparrow_A,\downarrow_B}}{P_{\downarrow_B}} = \frac{|\langle\uparrow,\downarrow|\psi_+\rangle|^2}{P_{\downarrow_B}} = \frac{1/2}{1/2} = 1$$

Cond. proba.

What is so special about entanglement?



$$\frac{1}{\sqrt{2}}(|\downarrow_{\rm A},\uparrow_{\rm B}\rangle+|\uparrow_{\rm A},\downarrow_{\rm B}\rangle)$$

1. Perfect **correlations** (even at very large distance…!)

$$P(\downarrow_A|\uparrow_B)=1 \qquad P(\downarrow_A|\downarrow_B)=0$$

$$P(\uparrow_A|\downarrow_B)=1 \qquad P(\uparrow_A|\uparrow_B)=0$$

2. You CAN NOT assigne a state to A or B = **system as a whole**

$$\rho_A=\mathrm{Tr}_B\rho_{AB}=\frac{1}{2}|\uparrow\rangle\langle\uparrow|+\frac{1}{2}|\downarrow\rangle\langle\downarrow|$$

Statistical mixture

3. You CAN NOT “dis-entangled” by a **LOCAL** rotation

Non Local Quantum correlations

Classification of entangled states

Bipartite $\mathcal{H}_{AB} = \mathcal{H}_A \otimes \mathcal{H}_B$

Pure state $|\phi_{AB}\rangle \neq |\phi_A\rangle \otimes |\phi_B\rangle$

Mixed state $\rho \neq \sum p_i |a_i\rangle\langle a_i| \otimes | b_i\rangle\langle b_i| \quad (p_i \geq 0)$

Multipartite $\mathcal{H}_{AB} = \mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_C \otimes ...$

Pure state $|\phi_{ABC...}\rangle \neq |\phi_A\rangle \otimes |\phi_B\rangle \otimes |\phi_C\rangle \otimes ...$

Bi-separable $|\phi_{ABC...}\rangle = \underbrace{|\phi_1\rangle}_{n} \otimes \underbrace{|\phi_2\rangle}_{N-n}$

Fully entangled: not bi-separable w/r to any bi-partition of the system

Examples of inequivalent entangled states

GHZ-state $$|\mathrm{GHZ}\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow,\uparrow,\uparrow\rangle + |\downarrow,\downarrow,\downarrow\rangle\right)$$

W-state $$|\mathrm{W}\rangle = \frac{1}{\sqrt{3}}\left(|\uparrow,\downarrow,\downarrow\rangle + |\downarrow,\uparrow,\downarrow\rangle + |\downarrow,\downarrow,\uparrow\rangle\right)$$

No LOCAL rotation can transform GHZ state into W state

$\Longrightarrow$ defines 2 **inequivalent classes** of entanglement

2 systems	1 classe	
3 systems	2 classes	PRA **62**, 062314 (2000)
4 systems	9 classes	PRA **65**, 052112 (2002)

Beyond …?

Preparation of entangled states (1)

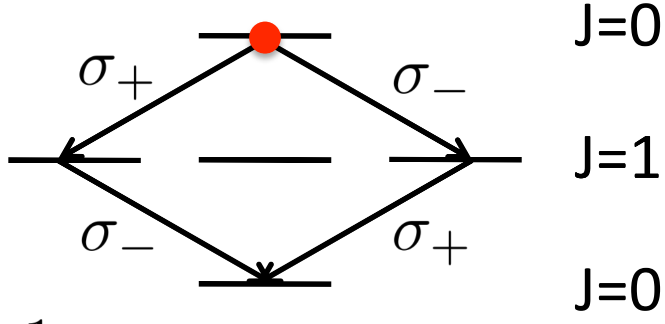
1. Conservation laws

Radio-active decay

$$^{238}_{92}\mathrm{U} \rightarrow \, ^{234}_{90}\mathrm{Th} + \, ^{4}_{2}\mathrm{He}$$

$$\frac{1}{\sqrt{2}}\left(|p,-p\rangle + |-p,p\rangle\right)$$

Atomic cascade (e.g. Ca)



$$\frac{1}{\sqrt{2}}\left(|\sigma_+,\sigma_-\rangle + |\sigma_-,\sigma_+\rangle\right)$$

2. Some "symmetry": e.g. identical particles

Bosons 1 and 2 in two states ϕ_a and ϕ_b

$$\phi_{\mathrm{boson}}(1,2) = \frac{1}{\sqrt{2}}\left(\phi_a(1)\phi_b(2) + \phi_a(2)\phi_b(1)\right)$$

Fermions: Pauli principle

$$\phi_{\mathrm{fermion}}(1,2) = \frac{1}{\sqrt{2}}\left(\phi_a(1)\phi_b(2) - \phi_a(2)\phi_b(1)\right)$$

Preparation of entangled states (2)

3. Interaction between the sub-parts

$$H = H_A + H_B + H_{\rm int}$$

$$|\psi_A\rangle \otimes |\psi_B\rangle \xrightarrow{\text{evolution}} e^{-i\frac{\hat{H}t}{\hbar}} (|\psi_A\rangle \otimes |\psi_B\rangle) \neq |\psi'_A\rangle \otimes |\psi'_B\rangle$$

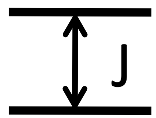
Examples of H_{int}:

Spin orbit coupling $C\vec{L}\cdot\vec{S}$

Hyperfine structure $A\vec{J}\cdot\vec{I}$ (ground state of H)

Magnetic interaction $\dfrac{\vec{\mu}_A\cdot\vec{\mu}_B}{R^3}$

Generic Heisenberg H_{int}: $J\vec{S}_a\cdot\vec{S}_b$ e.g. spin 1/2



$|\uparrow\uparrow\rangle - |\downarrow\downarrow\rangle$

Generation of "cluster" states: $H_{\rm int} = J(1+\sigma_z^{(a)})(1+\sigma_z^{(b)})$ (Ising)

$$(|\uparrow\rangle + |\downarrow\rangle)_A(|\uparrow\rangle + |\downarrow\rangle)_B \longrightarrow |\downarrow\downarrow\rangle + |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle + e^{i\varphi}|\uparrow\uparrow\rangle \quad \varphi = \frac{4J}{\hbar}t$$

Preparation of entangled states (3)

4. Projective measurement

Prepare $|\uparrow\uparrow\uparrow\uparrow\ ...\rangle$

Apply "small" rotation $|\uparrow\rangle \rightarrow |\uparrow\rangle + \epsilon|\downarrow\rangle$ $\quad \epsilon \ll 1$

$$|\uparrow\uparrow\uparrow\uparrow\ ...\rangle \longrightarrow (|\uparrow\rangle + \epsilon|\downarrow\rangle)_A(|\uparrow\rangle + \epsilon|\downarrow\rangle)_B(|\uparrow\rangle + \epsilon|\downarrow\rangle)_C \cdots$$

$$\propto |\uparrow\uparrow\uparrow\uparrow\ ...\rangle + \epsilon\boxed{(|\downarrow\uparrow\uparrow\uparrow\ ...\rangle + |\uparrow\downarrow\uparrow\uparrow\ ...\rangle + |\uparrow\uparrow\downarrow\uparrow\ ...\rangle)} + O(\epsilon^2)$$

Non-destructive measurement of $|\downarrow\rangle$ (e.g. with cavity)

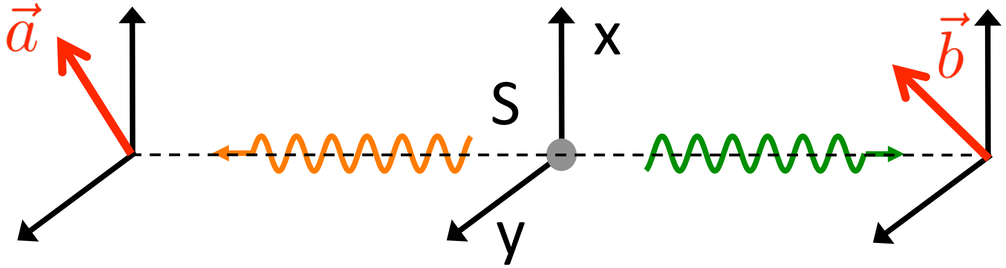
Project onto: $|\mathrm{W}\rangle = \frac{1}{\sqrt{N}}(|\downarrow\uparrow\uparrow\uparrow\ ...\rangle + |\uparrow\downarrow\uparrow\uparrow\ ...\rangle + |\uparrow\uparrow\downarrow\uparrow\ ...\rangle)$

Heralded entanglement (probability ε^2)

Bi-partite entanglement and non-locality

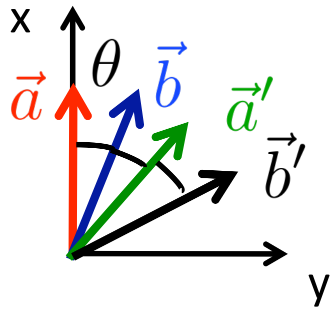
Bell inequality tests (1964)

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|xx\rangle + |yy\rangle\right)$$



$$E(\vec{a},\vec{b}) = \langle\psi|(\vec{\sigma}_A.\vec{a}) \otimes (\vec{\sigma}_B.\vec{b})|\psi\rangle$$
$$= -\cos(\vec{a},\vec{b})$$

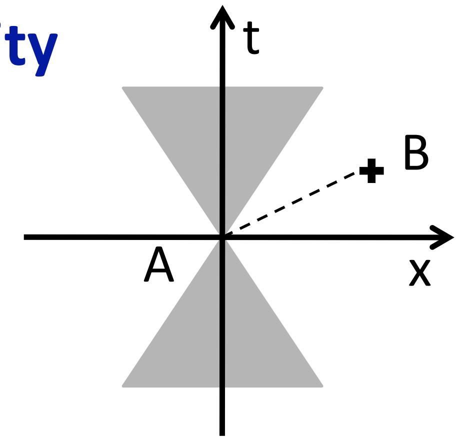
$$S(\theta) = E(\vec{a},\vec{b}) + E(\vec{a}',\vec{b}) + E(\vec{a}',\vec{b}') - E(\vec{a},\vec{b}') = \cos 3\theta - 3\cos\theta$$



Entangled state $\Rightarrow$ $S(\pi/4) = -2\sqrt{2}$

Classical local correlations $\Rightarrow$ $|S(\theta)| \leq 2$

Non-locality



$x_{AB} > ct$ $\Rightarrow$ A and B not causaly related

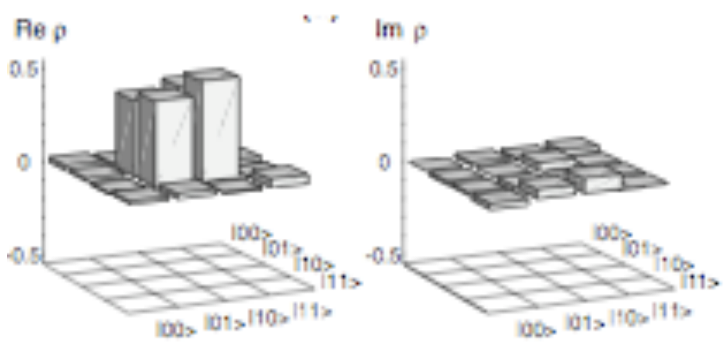
Entanglement between A and B possible

Quantification of entanglement: quantum state tomography

For N atoms $$\hat{\rho} = \sum_{i_A, i_B, i_C \dots} \lambda_{i_A, i_B, i_C \dots} \, \sigma_{i_A} \otimes \sigma_{i_B} \otimes \sigma_{i_C} \cdots$$

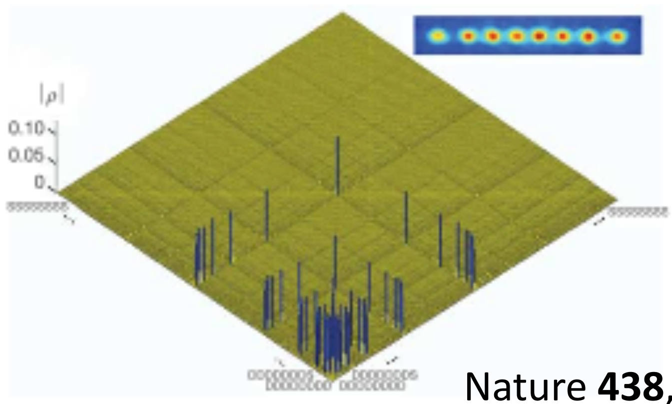
where $\lambda_{i_A, i_B, i_C \dots} = \langle \sigma_{i_A} \otimes \sigma_{i_B} \otimes \sigma_{i_C} \dots \rangle$ (4^N-1 coefficients)

Measure $\langle \sigma_{i_A} \otimes \sigma_{i_B} \otimes \sigma_{i_C} \dots \rangle \Rightarrow$ reconstruct the density matrix



e.g. Bell states of 2 ions
PRL **92**, 220402 (2004)

Le **largest state** characterized:
8 ions W states, 65531 coefficients
10 hours of measurements!
Too hard for “large” systems



Nature **438**, 643 (2005)

Quantification of entanglement (1)

Bipartite ~OK: Schmidt decomposition, entropy, concurrence…

Multipartite

Pure states: generalized Bell inequalities (PRL **104**, 240502 (2010))

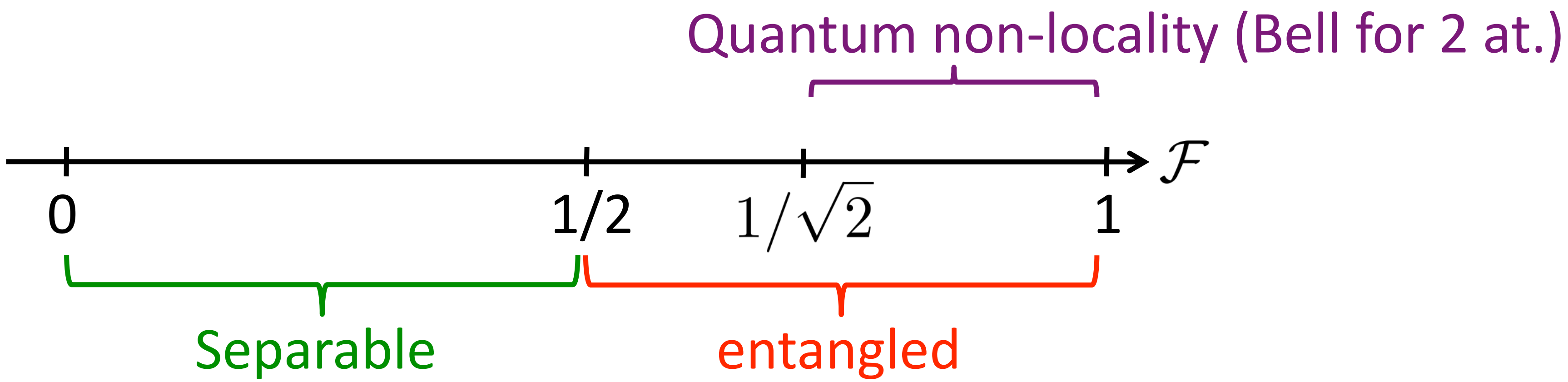
Mixed states: no general criteria (even if you know ρ)

$\Rightarrow$ use weaker criteria

Fidelity: want $|\psi_\mathrm{t}\rangle$, measure ρ_exp

$$\mathcal{F} = \langle \psi_\mathrm{t} | \rho_\mathrm{exp} | \psi_\mathrm{t} \rangle$$

Separable state $\Rightarrow \mathcal{F} \leq \frac{1}{2}$ C.A. Sackett *et al.*, Nature **404**, 256 (2000)



Quantification of entanglement (2)

More generally: entanglement witness operator $\hat{\mathcal{W}}$

separable state $\Rightarrow$ $\langle \hat{\mathcal{W}} \rangle = \mathrm{Tr}(\rho_{\mathrm{exp}} \hat{\mathcal{W}}) \geq 0$

$\langle \hat{\mathcal{W}} \rangle < 0$ $\Rightarrow$ entangled state

Examples:

1. fidelity $\hat{\mathcal{W}} = 1 - 2|\psi_{\mathrm{t}}\rangle\langle\psi_{\mathrm{t}}| \quad \Rightarrow \quad \langle \hat{\mathcal{W}} \rangle = 1 - 2\mathcal{F}$

2. Energy of spin 1/2: $H = -J\vec{S}_a \cdot \vec{S}_b \ (J > 0)$

   Separable: $\langle H \rangle = -J\langle \vec{S}_a \rangle \cdot \langle \vec{S}_b \rangle \quad \Rightarrow -\frac{J}{4} \leq \langle H \rangle \leq \frac{J}{4}$

   Singlet state: $\frac{1}{\sqrt{2}}(|\uparrow,\downarrow\rangle - |\downarrow,\uparrow\rangle) \quad \Rightarrow \quad \langle H \rangle = \frac{3J}{4}$

Choose $\hat{\mathcal{W}} = 1 - \frac{4\hat{H}}{J}$ Useful for macroscopic measurements

Entanglement is fragile…

Atom loss $$|\mathrm{GHZ}_N\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\uparrow\uparrow\ldots\rangle + |\downarrow\downarrow\downarrow\ldots\rangle\right)$$

$$\longrightarrow \rho_{N-1} = \frac{1}{2}|\underbrace{\uparrow\uparrow\uparrow\ldots}_{\text{N-1}}\rangle\langle\uparrow\uparrow\uparrow\ldots| + \frac{1}{2}|\downarrow\downarrow\downarrow\ldots\rangle\langle\downarrow\downarrow\downarrow\ldots|$$

GHZ strongly correlated

While: $|\mathrm{W}_N\rangle \longrightarrow |\mathrm{W}_{N-1}\rangle$ W weakly correlated

Phase fluctuations (fluctuating B, E field)

$$|\mathrm{GHZ}(t)\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\uparrow\uparrow\ldots\rangle + e^{-iN\omega t}|\downarrow\downarrow\downarrow\ldots\rangle\right)$$

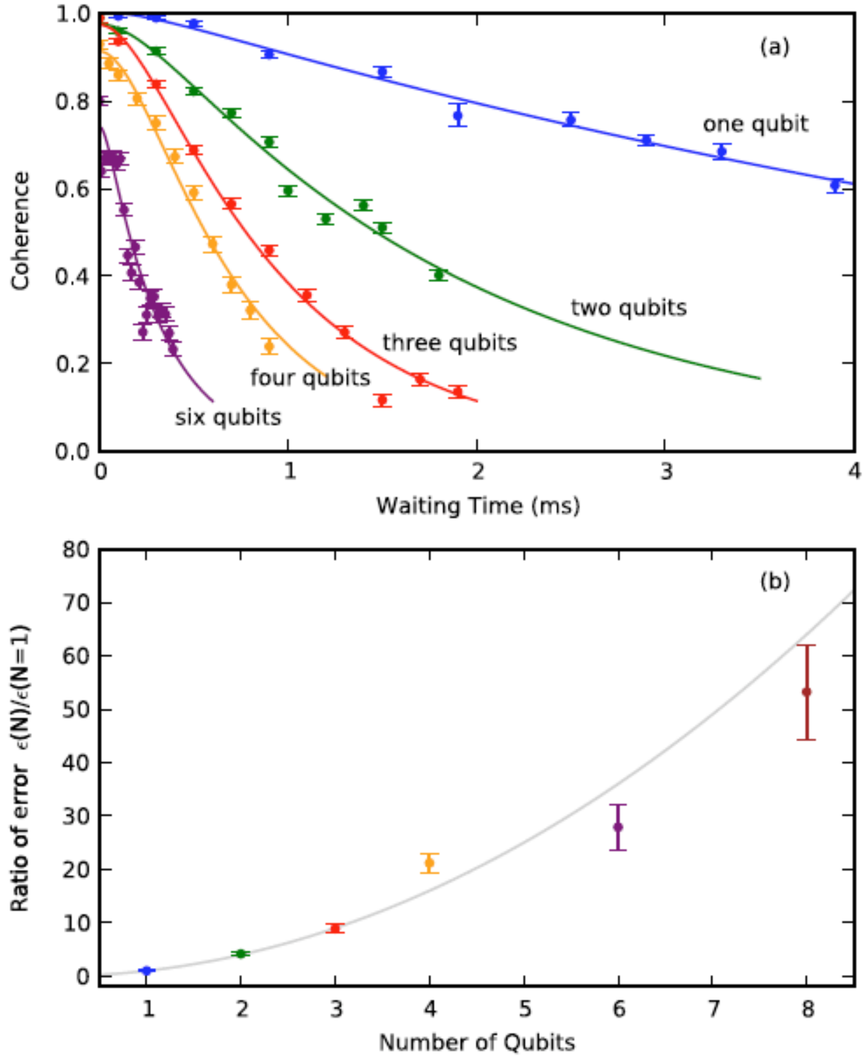
$\rho_{\uparrow\uparrow\ldots,\downarrow\downarrow\ldots} = e^{-iN\omega t}$ and $\langle e^{-iN\omega t}\rangle \propto e^{-N^2\frac{\Delta\omega^2 t^2}{2}} \longrightarrow 0$

Decoherence faster when **N large**

Decoherence of a GHZ state

PRL **106**, 130506 (2011)

N-ion string



But you can protect it...!

Start from $|\mathrm{GHZ}\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\uparrow\uparrow\uparrow\rangle + |\downarrow\downarrow\downarrow\downarrow\rangle\right)$

$$\underbrace{|\uparrow\uparrow\uparrow\uparrow\rangle}_{\text{Energy } 0} \qquad \underbrace{|\downarrow\downarrow\downarrow\downarrow\rangle}_{\text{Energy } 4\hbar\omega_0}$$

Apply local rotation on "atom" B and D
(does not change class of entanglement)

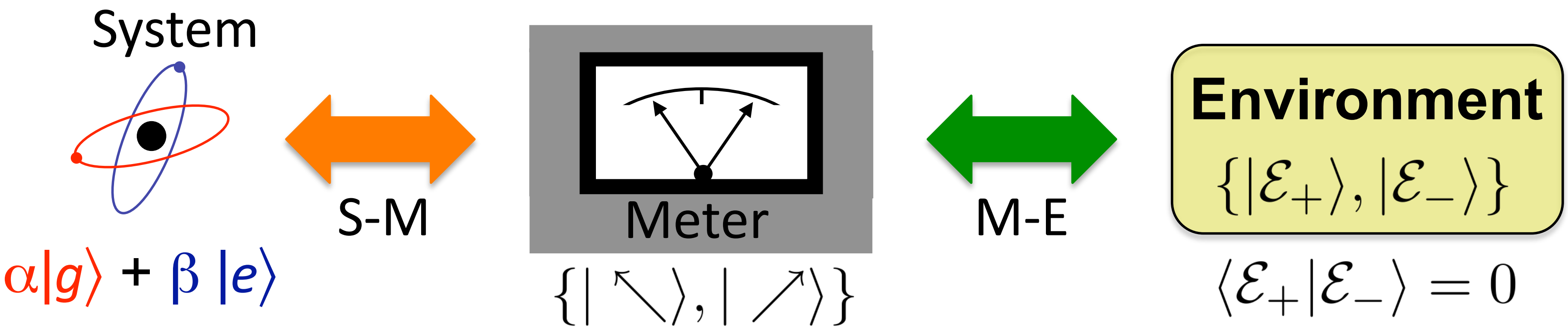
$$\longrightarrow \frac{1}{\sqrt{2}}\left(\underbrace{|\uparrow\downarrow\uparrow\downarrow\rangle}_{2\hbar\omega_0} + \underbrace{|\downarrow\uparrow\downarrow\uparrow\rangle}_{2\hbar\omega_0}\right)$$

$$\xrightarrow{\text{evolution}} \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\uparrow\downarrow\rangle + |\downarrow\uparrow\downarrow\uparrow\rangle\right) e^{-i2\omega_0 t} \Rightarrow$$

insensitive to fluctuation of ω_0

Decoherence free subspace

Entanglement, measurement and classicality



$$(\alpha|g\rangle + \beta|e\rangle)_S \otimes |0\rangle_M \otimes |\mathcal{E}_0\rangle_E \Rightarrow (\alpha|g,\nwarrow\rangle + \beta|e,\nearrow\rangle)_{SM} \otimes |\mathcal{E}_0\rangle_E$$

$$\Rightarrow |\psi\rangle_{SME} = \alpha|g,\nwarrow,\mathcal{E}_+\rangle + \beta|e,\nearrow,\mathcal{E}_-\rangle$$

Entanglement S – M – E

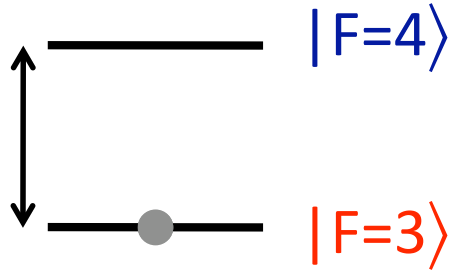
$$\rho_{SM} = \mathrm{Tr}_{\mathcal{E}}(|\psi\rangle\langle\psi|) = |\alpha|^2\,|g,\nwarrow\rangle\langle g,\nwarrow| + |\beta|^2\,|e,\nearrow\rangle\langle e,\nearrow|$$

Statistical mixture: get $|g\rangle$ (proba $|\alpha|^2$) **OR** $|g\rangle$ (proba $|\alpha|^2$)

Entanglement = resource for quantum metrology

Frequency standard

ω_0 = **9 192 631 770 Hz**



^{133}Cs

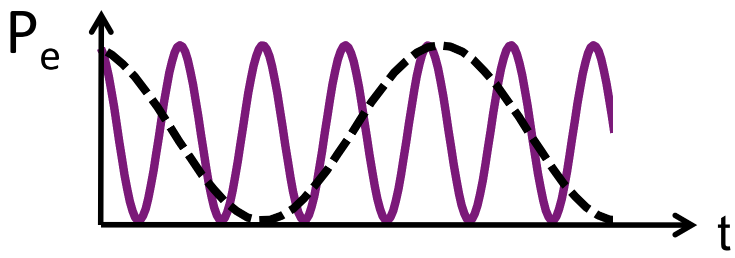
Applications: test of relativity, navigation using GPS…

Today, best atomic clock (ion based) $\Delta\nu/\nu \sim 10^{-17}$

We can do better **using entanglement**!

Prepare N atom in $|ggg...\rangle + |eee...\rangle$

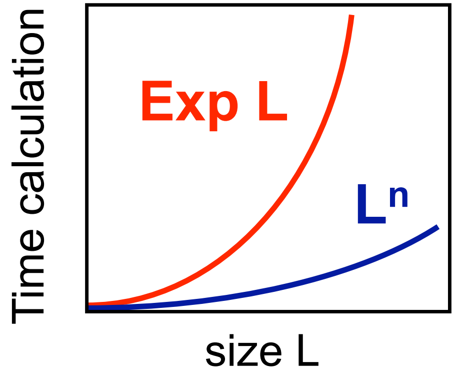
$$\Rightarrow \quad |ggg...\rangle + e^{-iN\omega_0 t}|eee...\rangle$$



$\sqrt{N}$ more sensitive measurement

Entanglement = resource for quantum computation

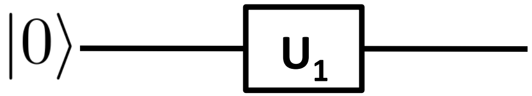
Help solving "hard" problems: factoring (Shor)
searching (Grover)



Encode information on a **qubit**: $|0\rangle$, $|1\rangle$

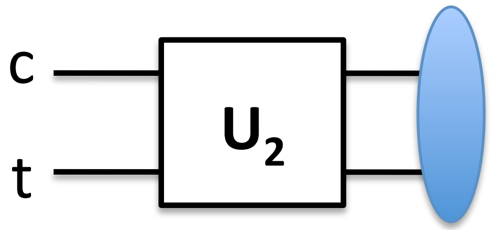
Elementary bricks ("circuit" approach):

One-qubit gate



$\alpha|0\rangle + \beta|1\rangle$

Two-qubit gate

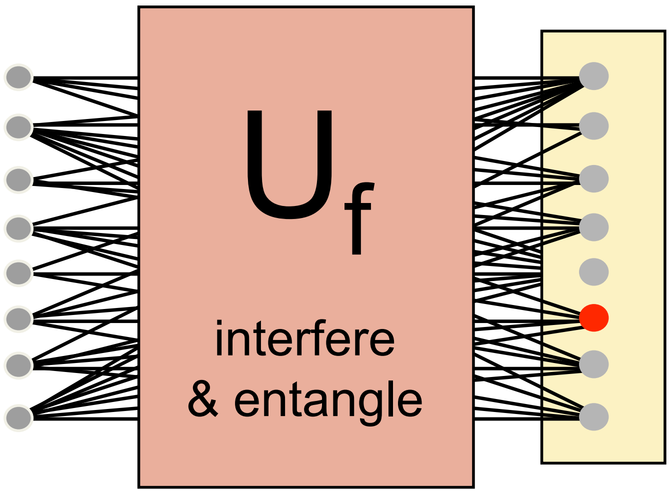


$|0\rangle\langle 0| \otimes \hat{\mathrm{Id}} + |1\rangle\langle 1| \otimes \sigma_x$

C-NOT

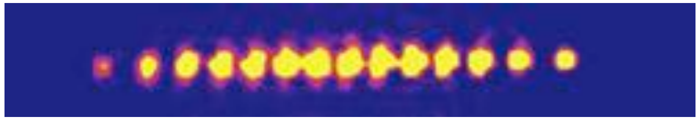
$|0\rangle \otimes |0\rangle \otimes |0\rangle \ldots$

N qubits



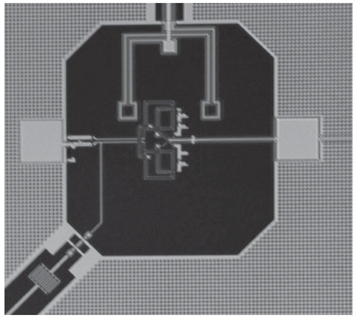
$|\Psi_f\rangle$ Highly entangled

Engineered entangled states: state-of-the art (2013)



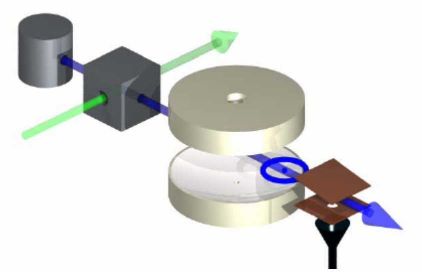
14 ions, GHZ (F=0.51)

PRL **106**, 130506 (2011)



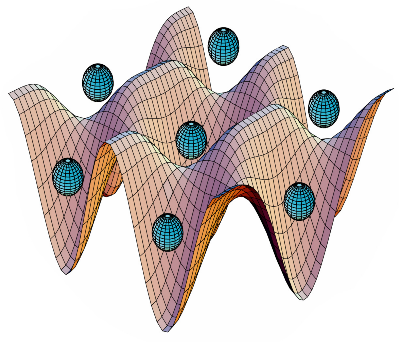
3 supracond. qubits W (F=0.78), GHZ (F=0.62)

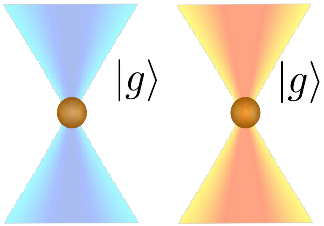
Nature **467**, 570, 574 (2010)



3 Rydberg atom GHZ (F=0.57)

Science **288**, 2024 (2000)





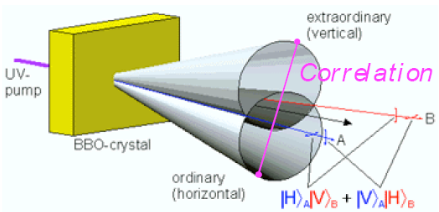
2 ground-state atoms

Bell (F~0.75)

PRL **104,** 010502, 010503 (2010)

~ 50 atoms cluster state

Mandel *et al.*, **425**, 937 (2003)



8 photons (H,V) GHZ (F=0.71)

Nat. Phot. **6**, 227(2012)

A short bibliography on entanglement

Theory (from "easy" to hard…)

"Exploring the quantum", S. Haroche and J-M. Raimond Cambridge

"Quantum computing and entanglement", D. Bruss and C. Machiavello
Les Houches summer school 2009 (Singapore)

"Experimental procedures for entanglement verification",
PRA **75**, 052318 (2007)

"Entanglement in many body system", RMP **80**, 517 (2008)

Experimental papers on quantification of entanglement

Q.A. Turchette *et al.*, PRL **81**, 3631 (1998)
C.A. Sackett *et al.*, Nature **404**, 256 (2000)
D. Leibfried *et al.*, Nature **438**, 639 (2005)

Review on Rydberg and QIP

Saffman RMP **82**, 2313 (2010)

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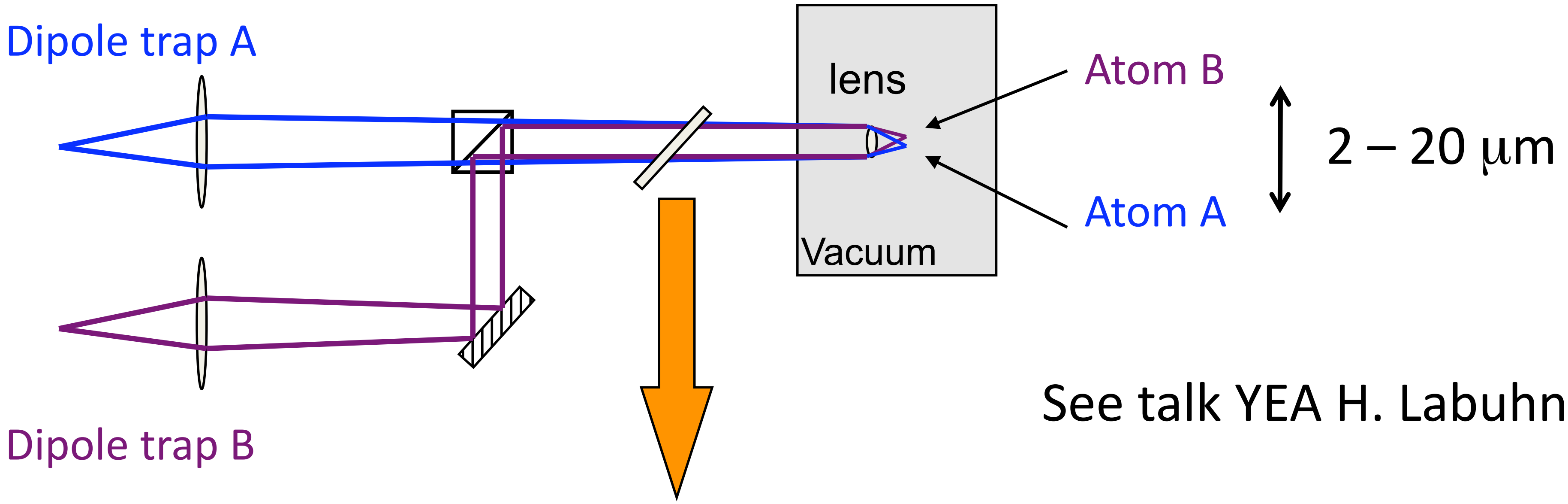
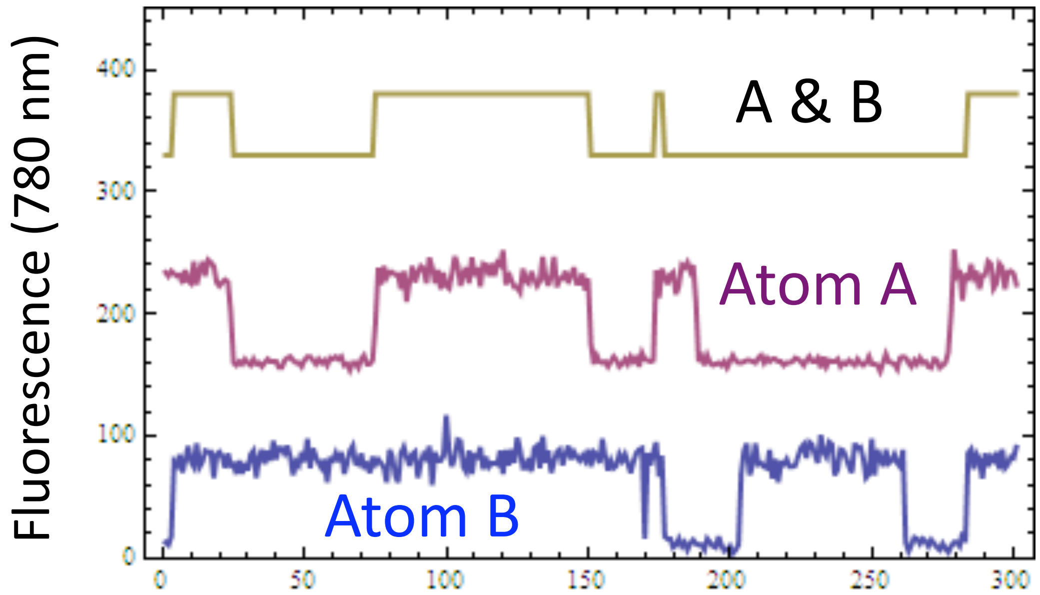
2. Entanglement and 2-atom gate using Rybderg blockade

Trapping two atoms in tweezers: well-defined geometry

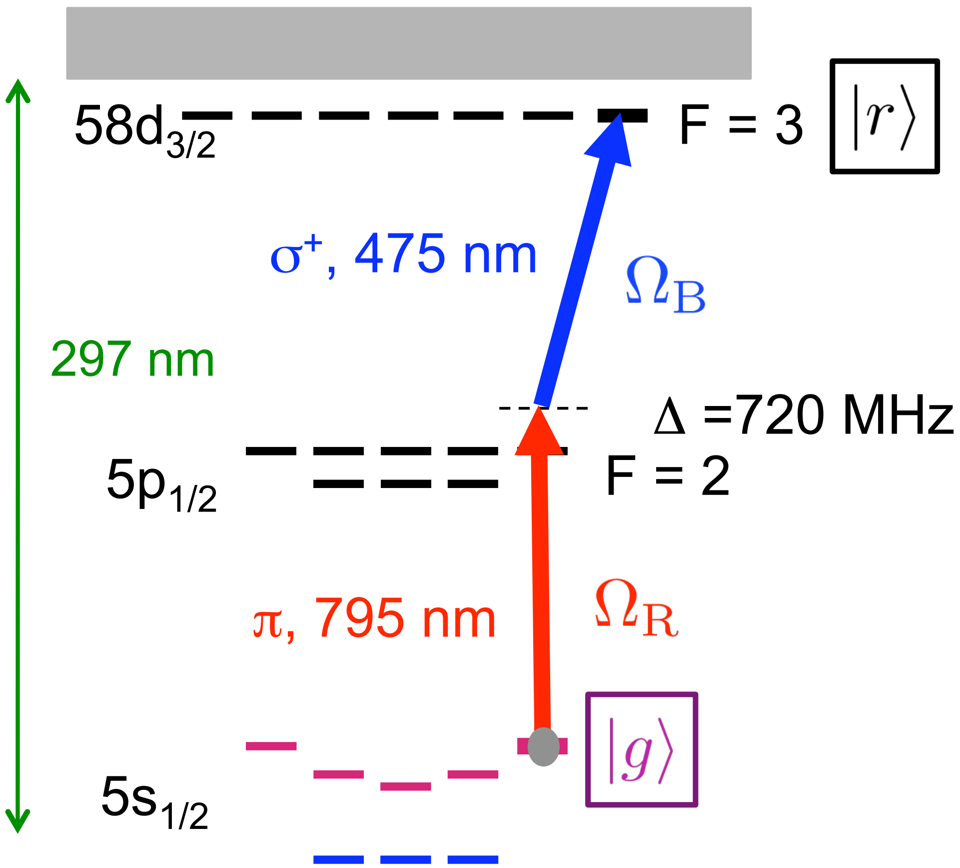


See talk YEA H. Labuhn

One counter / atom



Two-photon Rydberg excitation (^{87}Rb): well-defined states



$$\Omega = \frac{\Omega_R \Omega_B}{2\Delta}$$

See talk YEA H. Labuhn

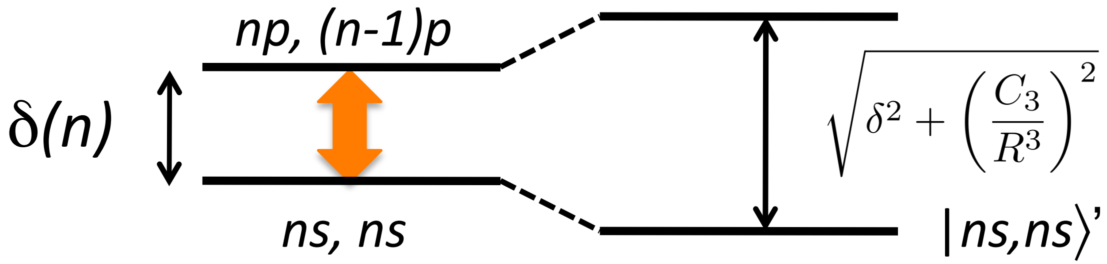
Detect atom loss = **Rydberg not trapped**

Miroshnychenko *et al.*, PRA **82**, 023623 (2010)

vdW interaction between 2 atoms

2 atoms A and B with states *ns*, *(n-1)p* and *np*; distance *R*

$$\hat{V} = -\frac{1}{4\pi\epsilon_0 R^3}\left(2\hat{\mathbf{d}}_z^A\hat{\mathbf{d}}_z^B + \hat{\mathbf{d}}_+^A\hat{\mathbf{d}}_-^B + \hat{\mathbf{d}}_-^A\hat{\mathbf{d}}_+^B\right)$$



$$\hat{H} = \begin{pmatrix} 0 & \frac{C_3}{R^3} \\ \frac{C_3}{R^3} & \delta \end{pmatrix}$$

$$\langle nsjm; nsjm|\hat{V}|npj'm'; (n-1)pj''m''\rangle = \frac{C_3}{R^3}$$

Ex.: $n = 60$
δ= 5-10 GHz,
$C_3/R^3 \approx 100$ MHz

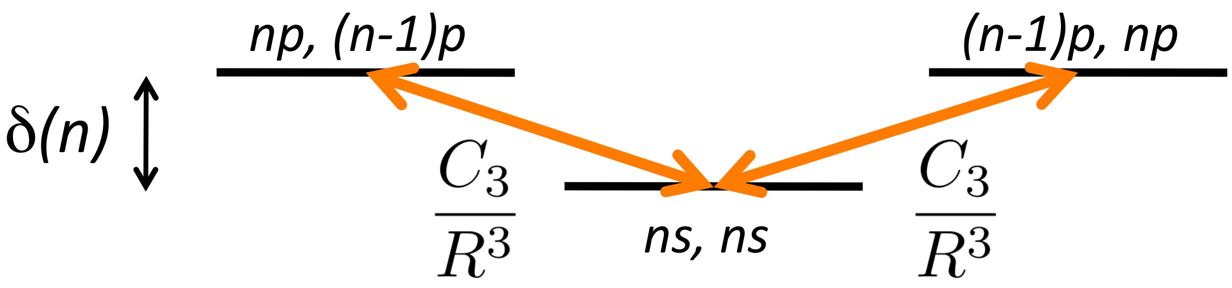
$$\Rightarrow \quad \Delta E_{ns,ns} = \frac{1}{2\delta}\left(\frac{C_3}{R^3}\right)^2 = \frac{C_6}{R^6}$$

$$|ns, ns\rangle' = |ns, ns\rangle + \frac{1}{\delta}\frac{C_3}{R^3}|np, (n-1)p\rangle \approx |ns, ns\rangle$$

In the **vdW approximation**: shift of $|r,r\rangle$ only…

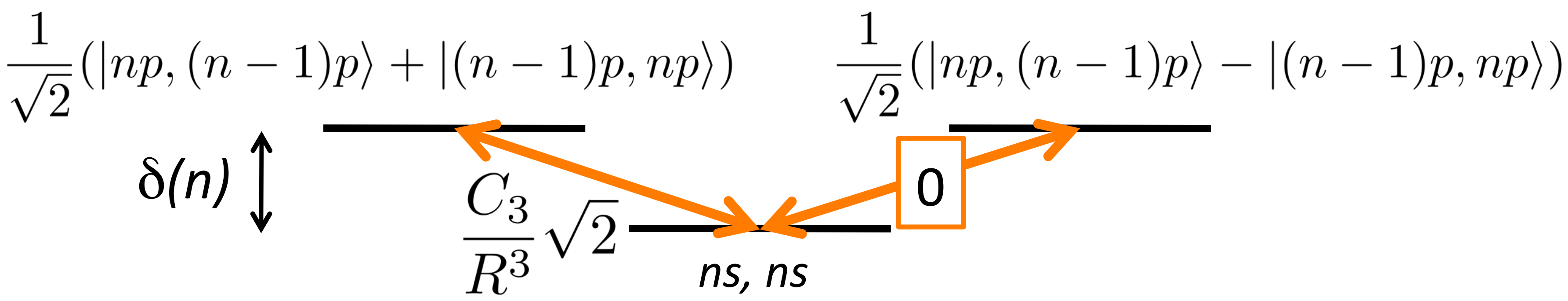
A note on vdW interaction between 2 atoms: role of symetry

There are 3 two-atom states!



A note on vdW interaction between 2 atoms: role of symetry

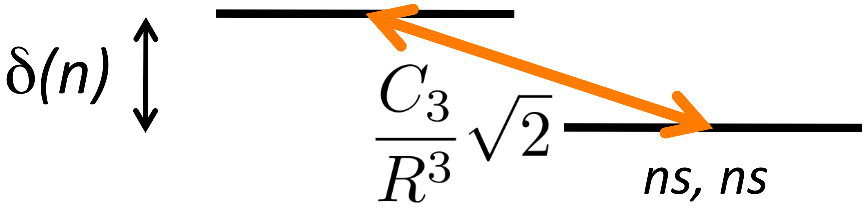
There are 3 two-atom states!



A note on vdW interaction between 2 atoms: role of symetry

There are 3 two-atom states!

$$\frac{1}{\sqrt{2}}(|np,(n-1)p\rangle + |(n-1)p,np\rangle)$$



Still two-state model, but: $\frac{C_3}{R^3} \rightarrow \frac{C_3}{R^3}\sqrt{2}$

Typical interaction strength:

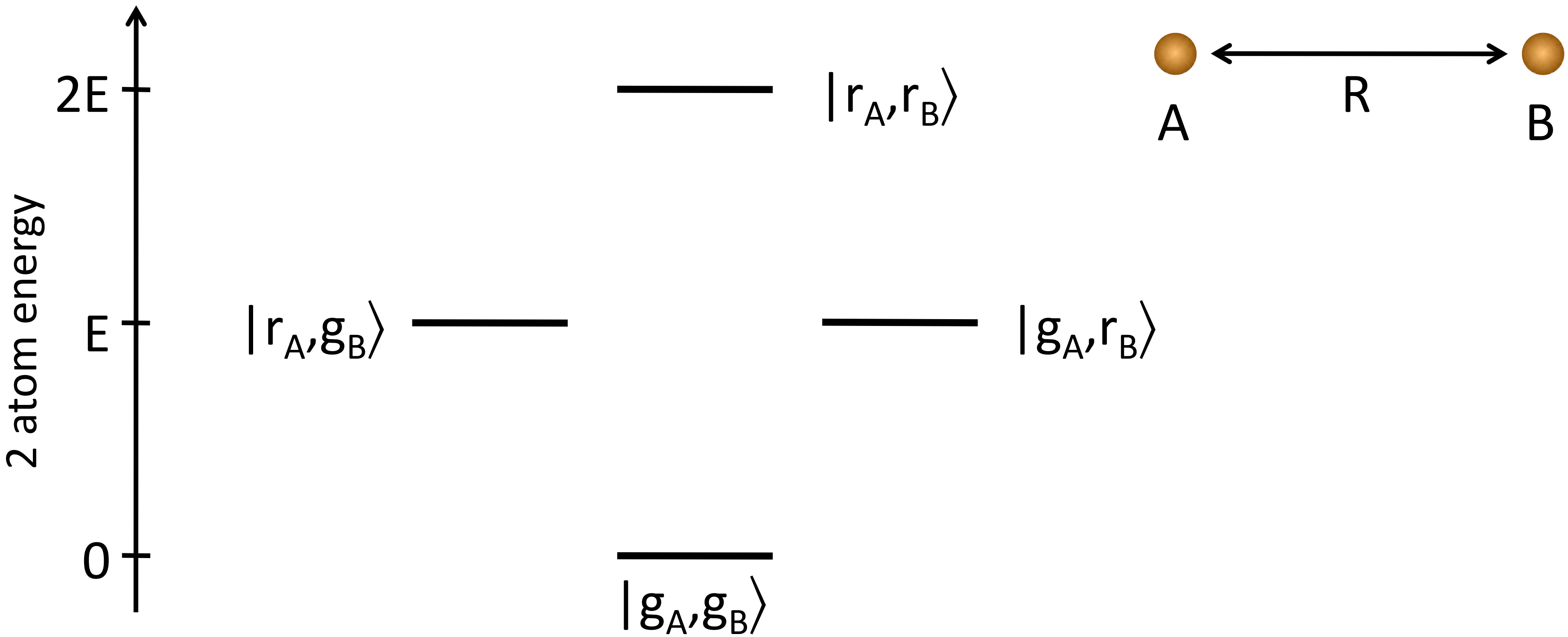
$C_6(43s) \approx -2.4$ GHz.μm^6

$C_6(53d) \approx 15$ GHz.μm^6

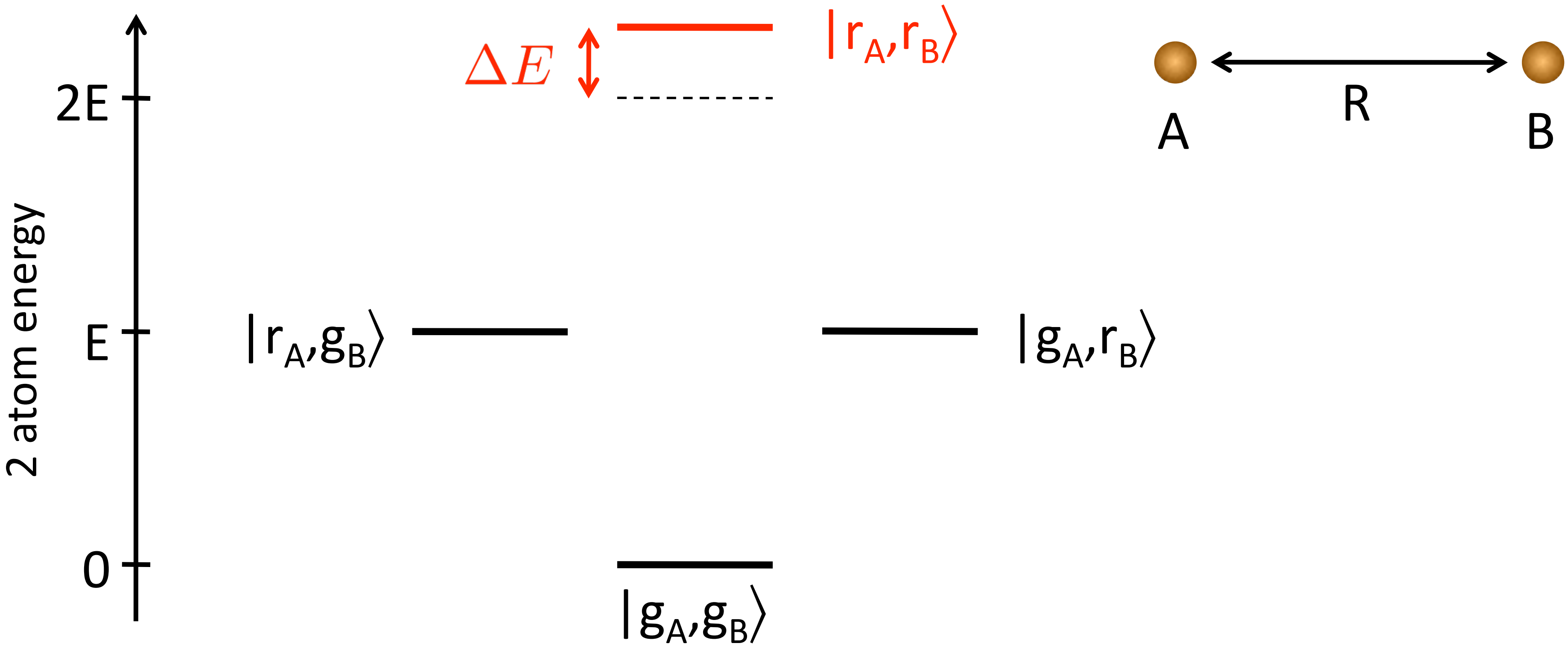
$C_6(82d) \approx 8500$ GHz.μm^6

⇒ V_{vdw} ~ 1 - 100 MHz for R ~ few μm

Rydberg blockade: "addressable" version (U. Wisconsin)



Rydberg blockade: "addressable" version (U. Wisconsin)



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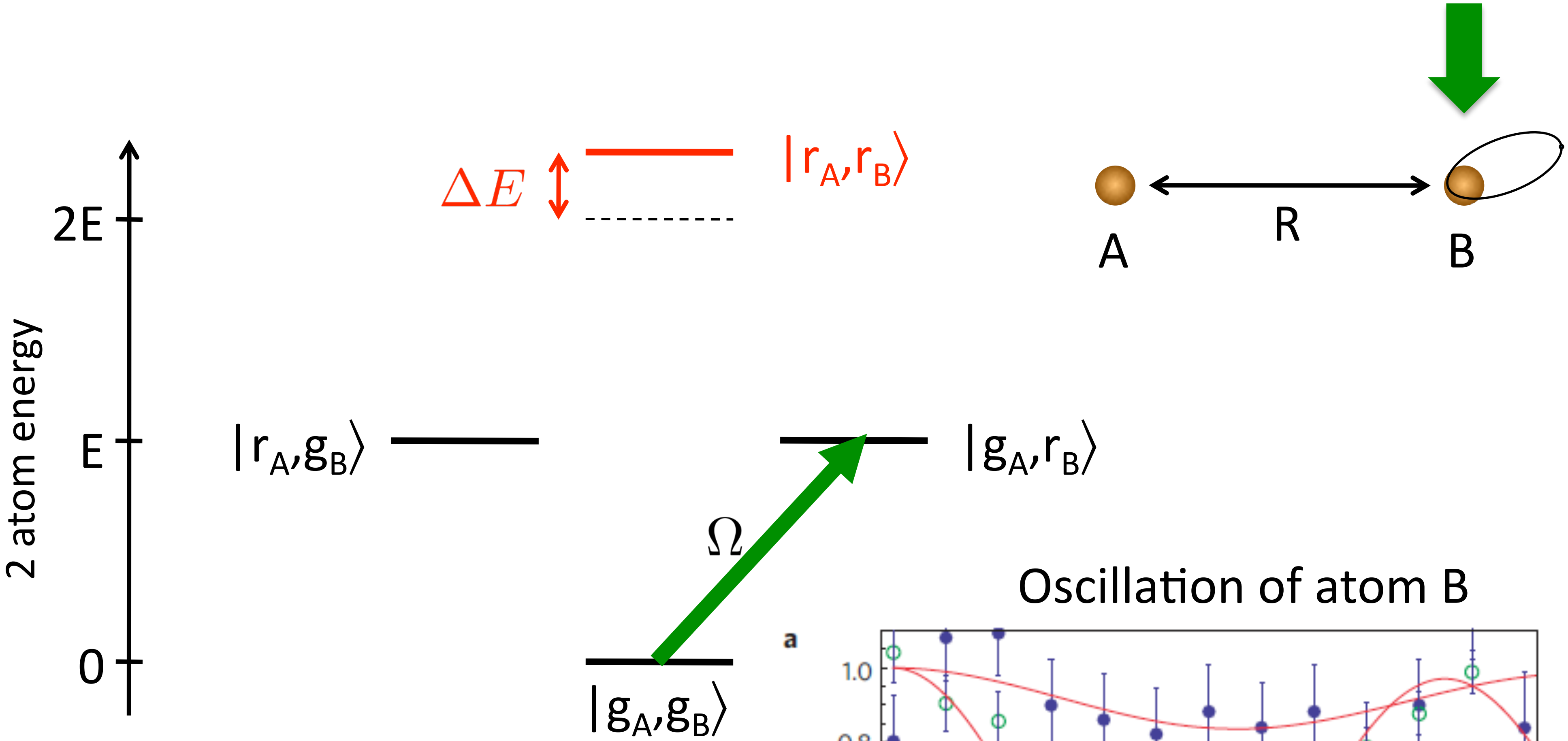
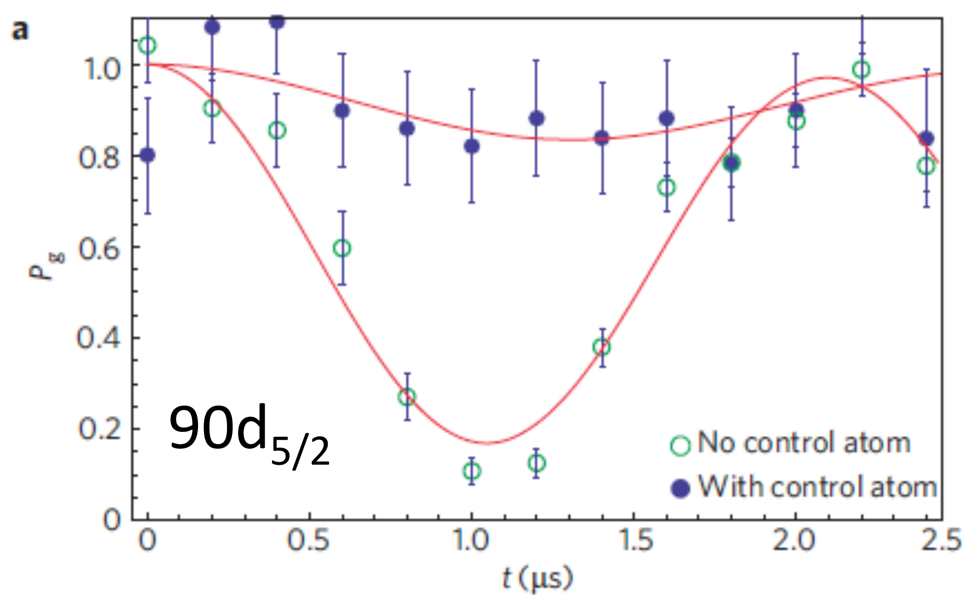
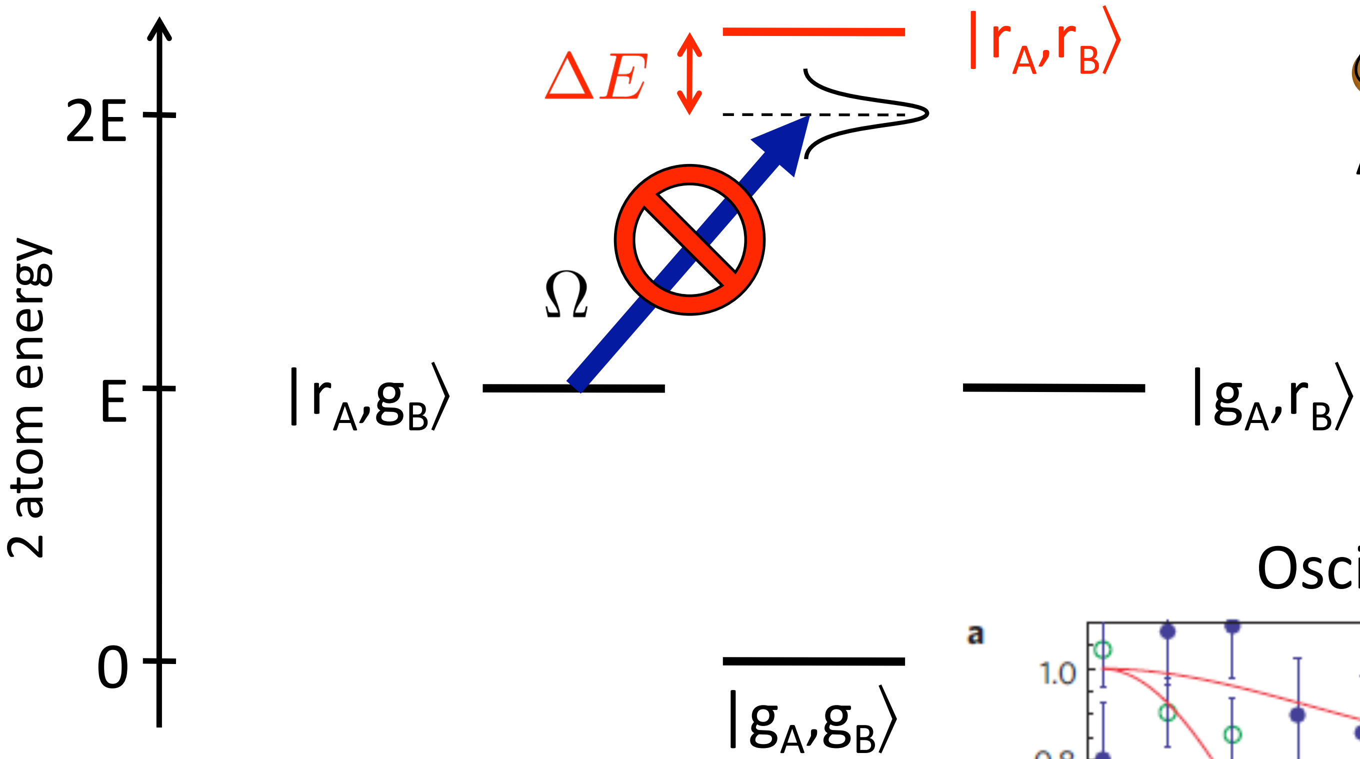
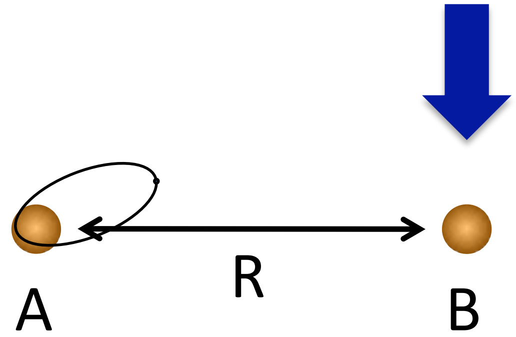




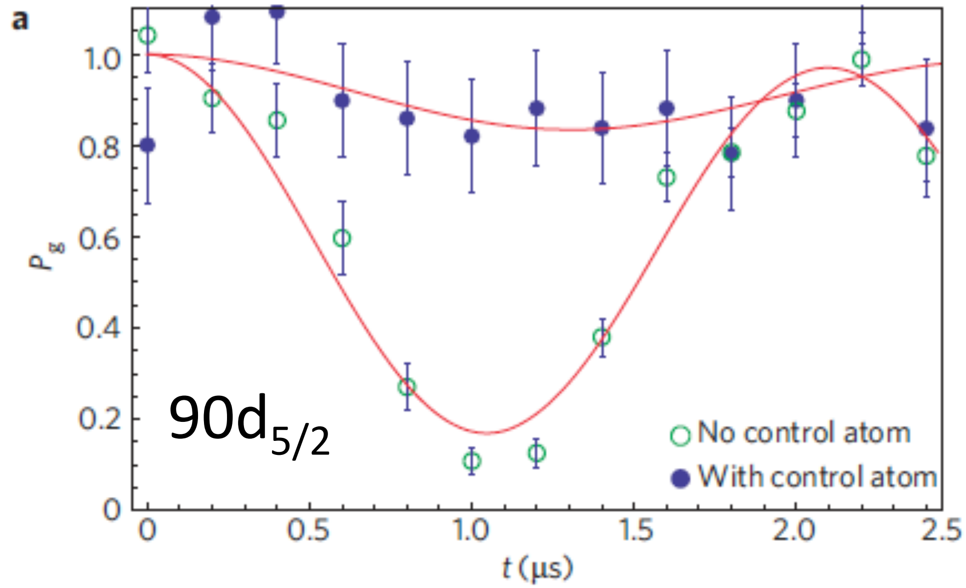
E. Urban *et al.*, *Nat. Phys*. **5**, 110 (2009)

Rydberg blockade: "addressable" version (U. Wisconsin)





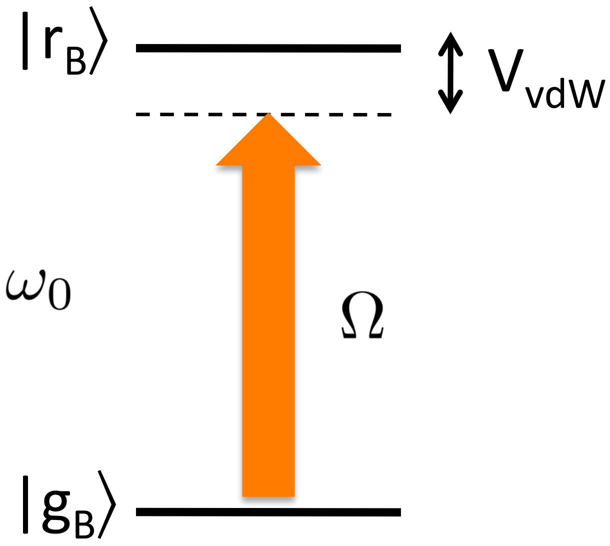
Oscillation of atom B



E. Urban *et al.*, *Nat. Phys*. **5**, 110 (2009)

Rydberg blockade: "addressable" version

The "blockade radius" picture

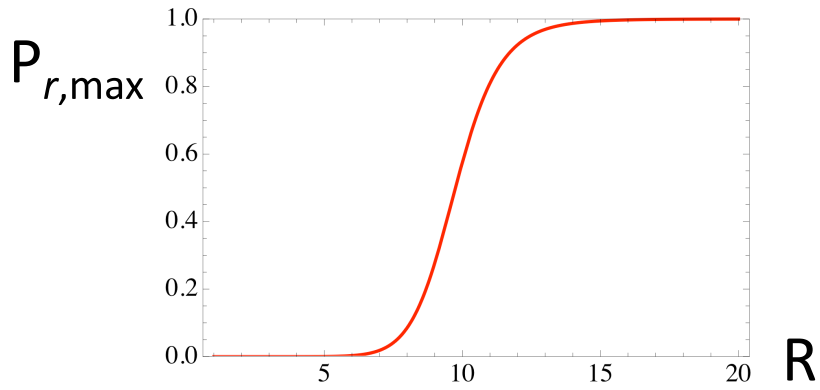


Atome B when A in $|r_A\rangle$

Rabi formula:

$$P_r(t) = \frac{\Omega^2}{\Omega^2 + V_{\mathrm{vdW}}^2} \sin^2 \frac{\sqrt{\Omega^2 + V_{\mathrm{vdW}}^2}}{2} t$$

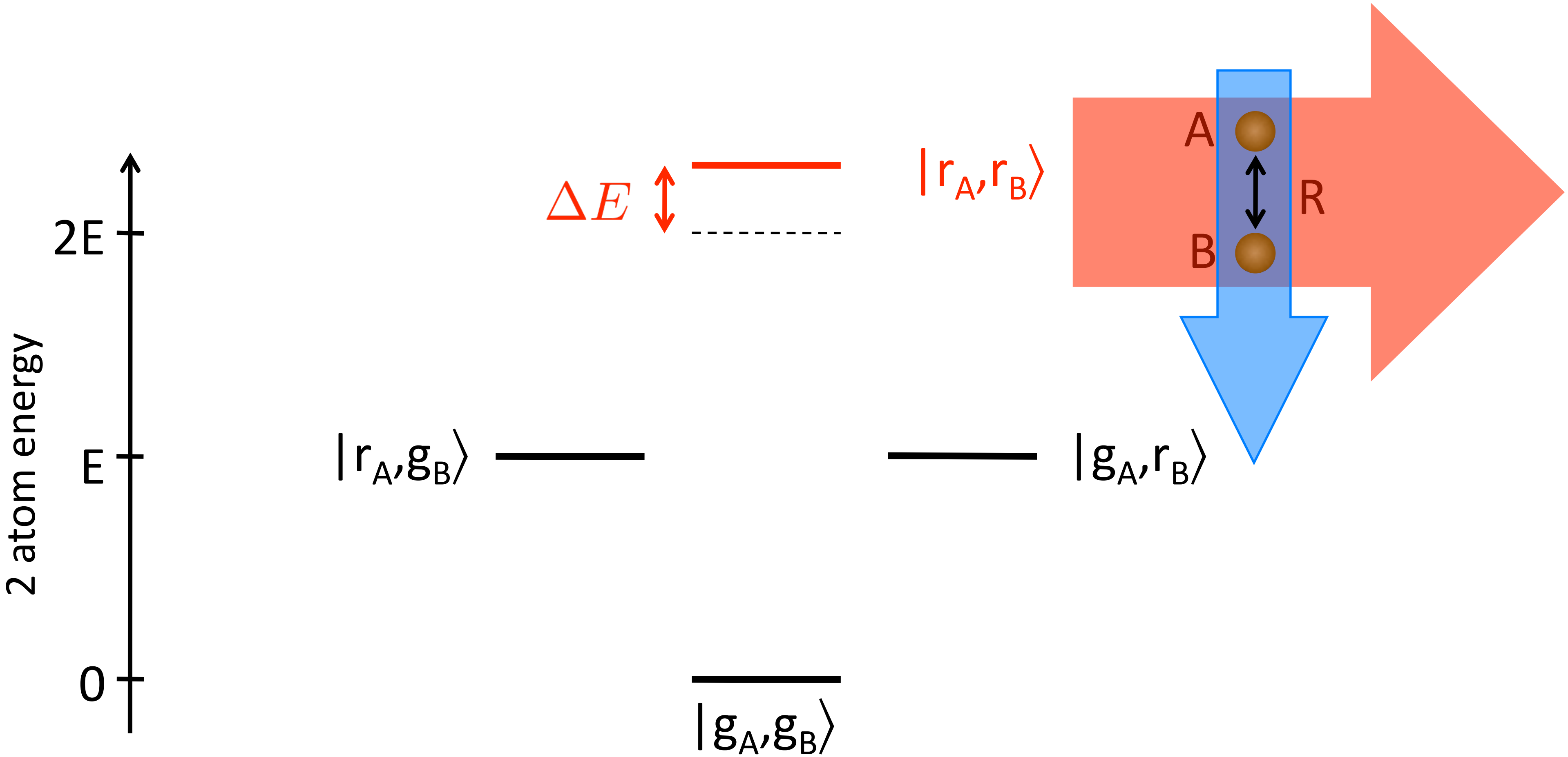
Defines R_b: $\hbar\Omega = V_{\mathrm{vdW}}(R_{\mathrm{b}})$



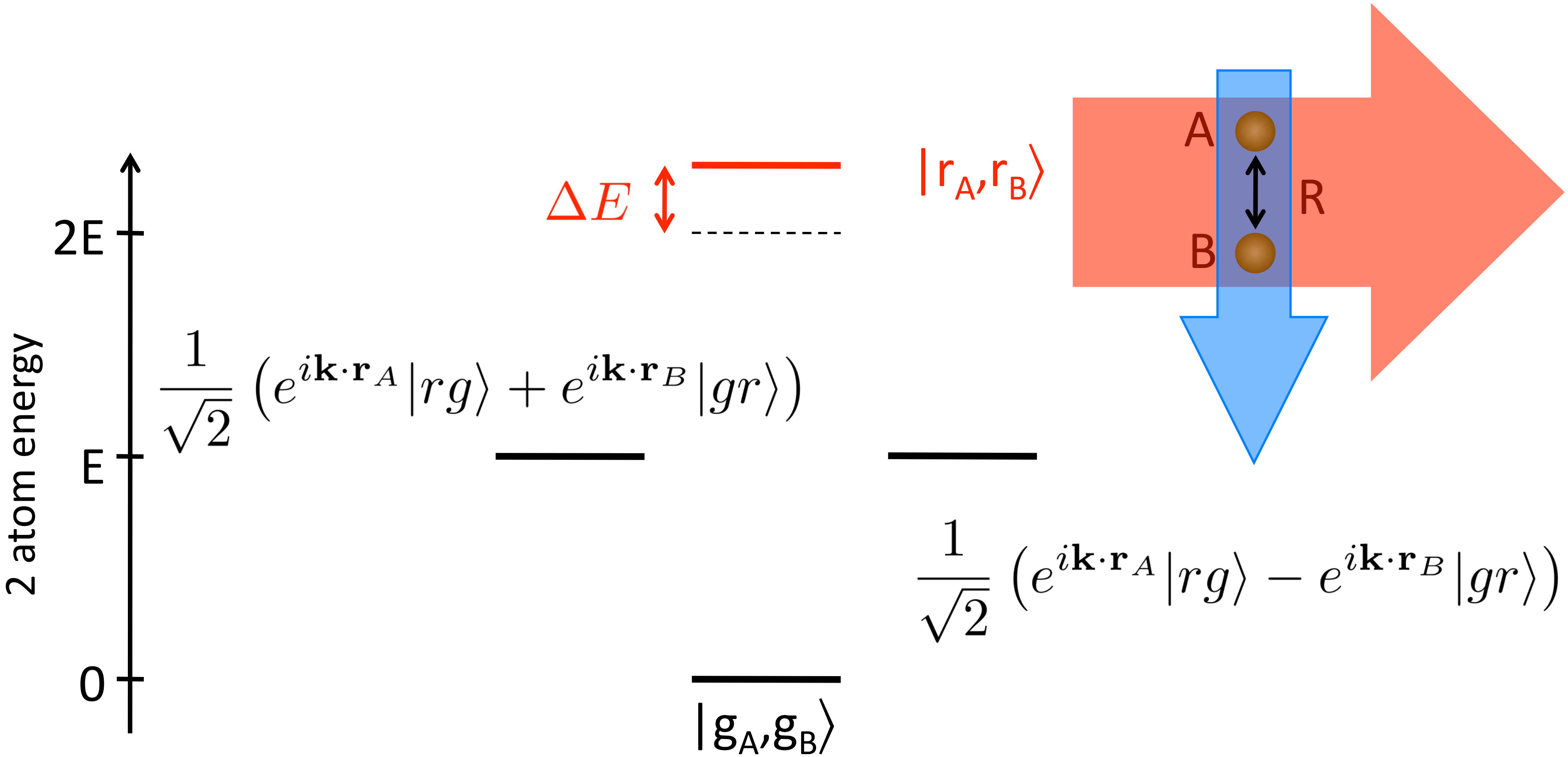
Blockade alone **does not** provide entanglement : need indistinguishability w/r excitation

$\Rightarrow$ **collective excitation**

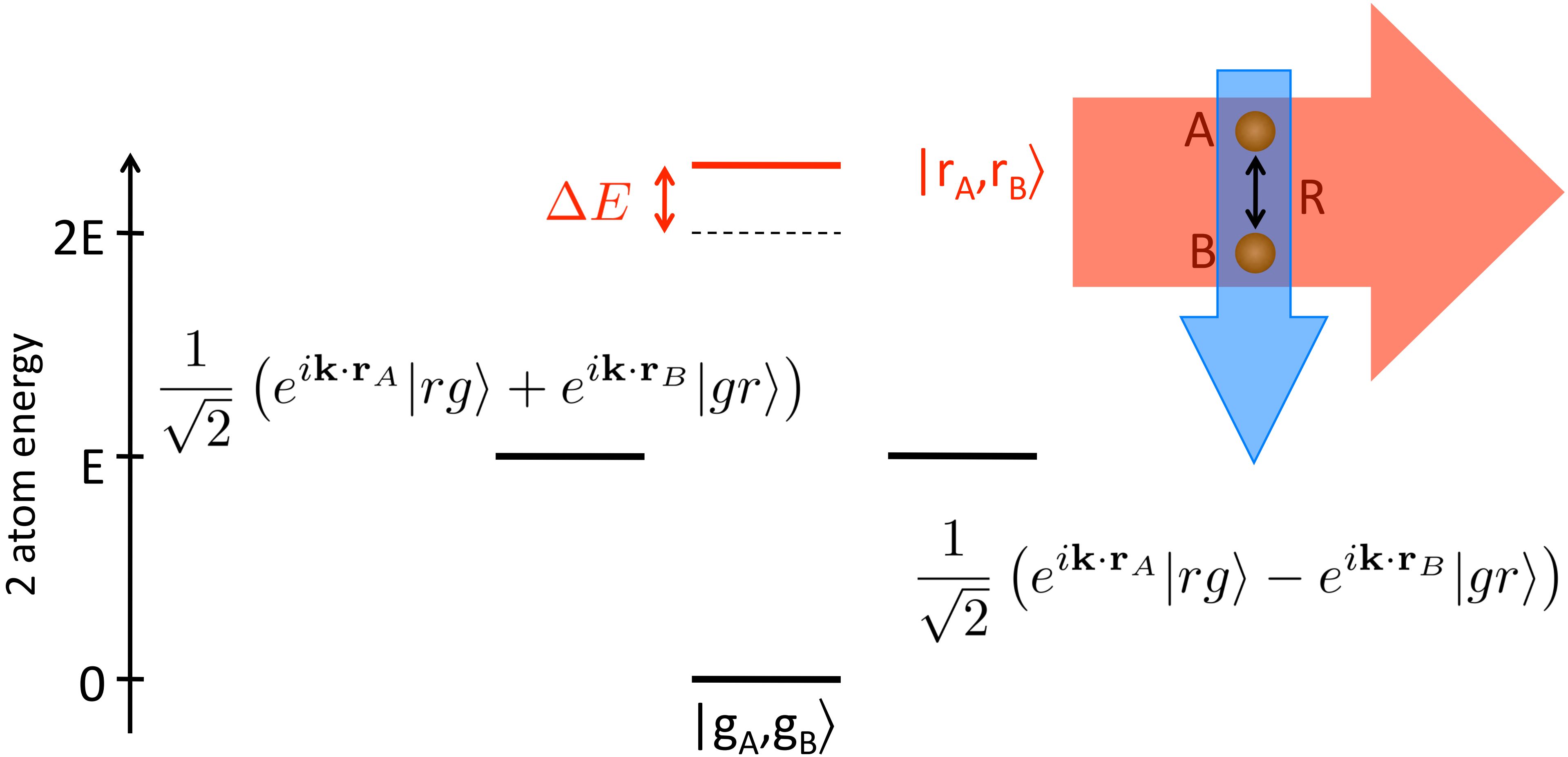
Rydberg blockade: collective excitation (IO Palaiseau)



Rydberg blockade: collective excitation (IO Palaiseau)



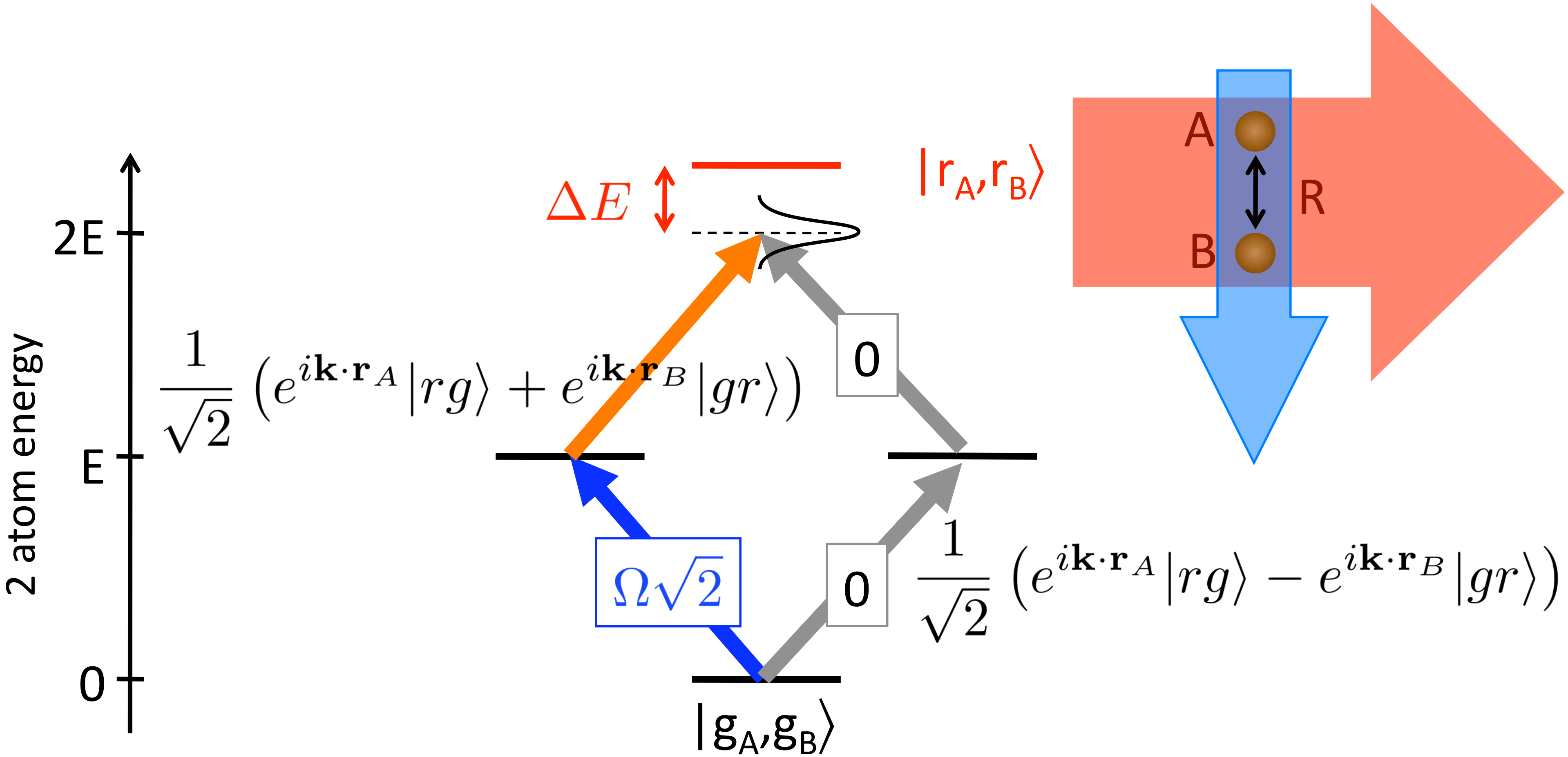
Rydberg blockade: collective excitation (IO Palaiseau)



Recall (RW approx.): $H^j_{\mathrm{at-L}} = \frac{\hbar\Omega}{2}\left(|r\rangle\langle g|\, e^{i\mathbf{k}\cdot\mathbf{r_j}} + |g\rangle\langle r|\, e^{-i\mathbf{k}\cdot\mathbf{r_j}}\right)$

$$H_{\mathrm{at-L}} = H^A_{\mathrm{at-L}} \otimes \hat{\mathrm{Id}}_B + \hat{\mathrm{Id}}_A \otimes H^B_{\mathrm{at-L}}$$

Rydberg blockade: collective excitation (IO Palaiseau)



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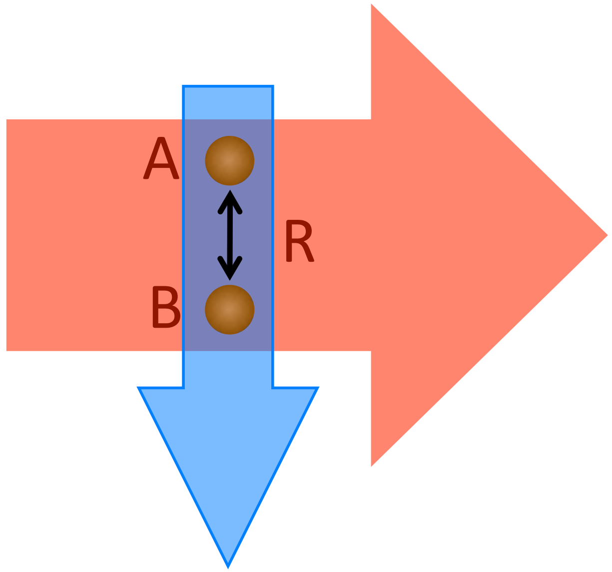
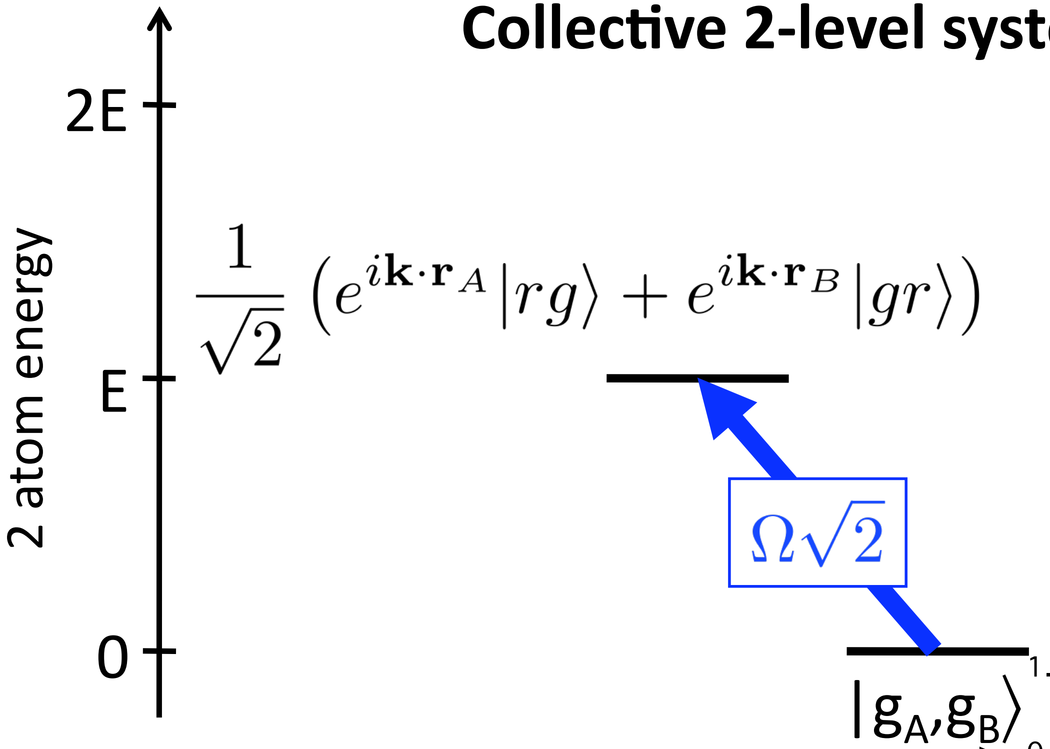
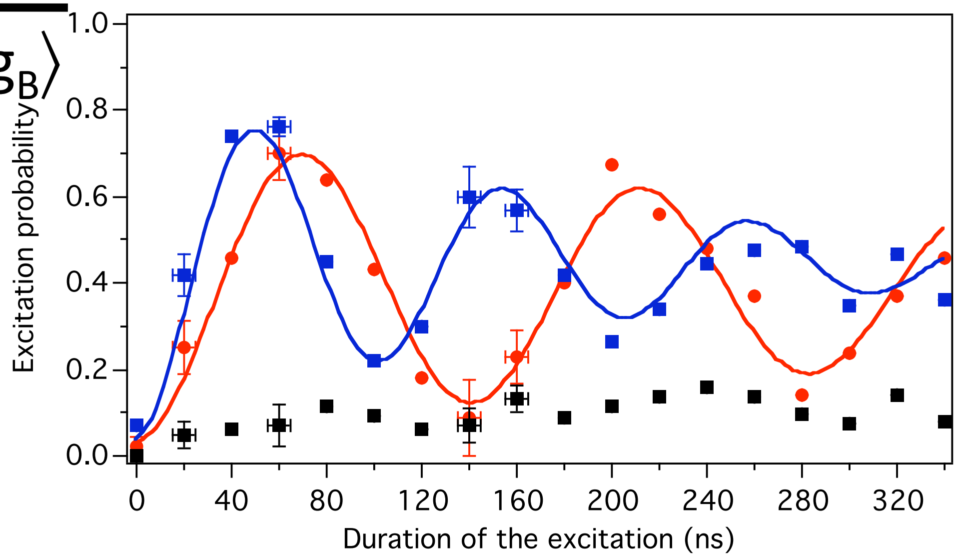
Rydberg blockade: collective excitation (IO Palaiseau)



Collective 2-level system

$$\frac{1}{\sqrt{2}}\left(e^{i\mathbf{k}\cdot\mathbf{r}_A}|rg\rangle + e^{i\mathbf{k}\cdot\mathbf{r}_B}|gr\rangle\right)$$

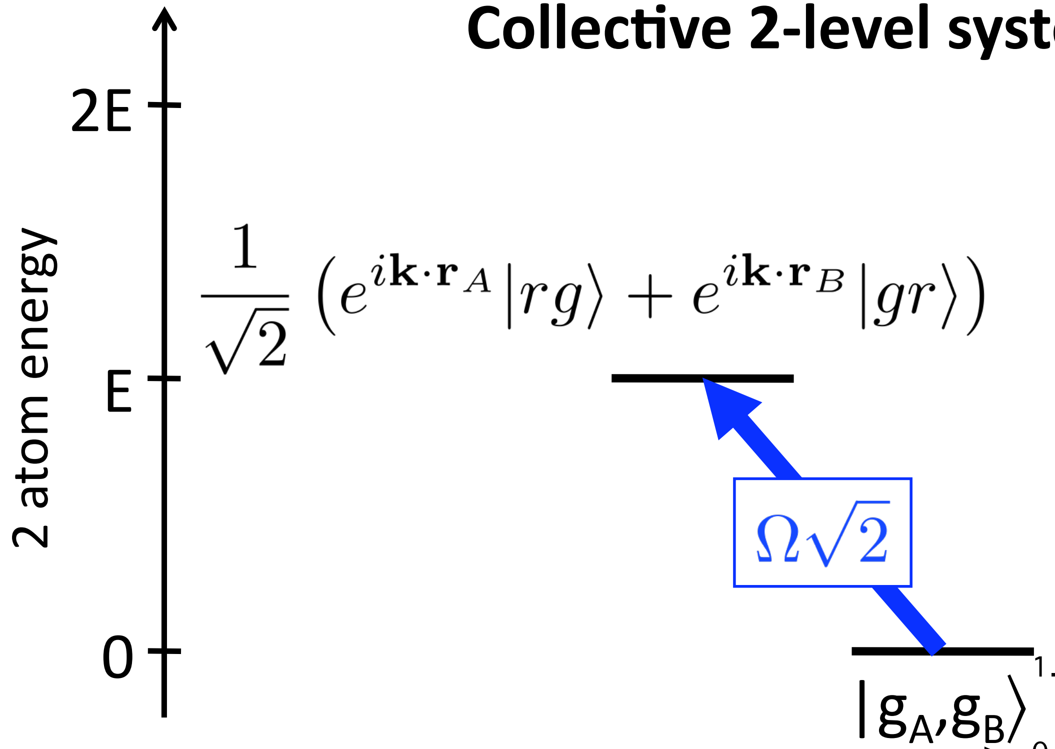


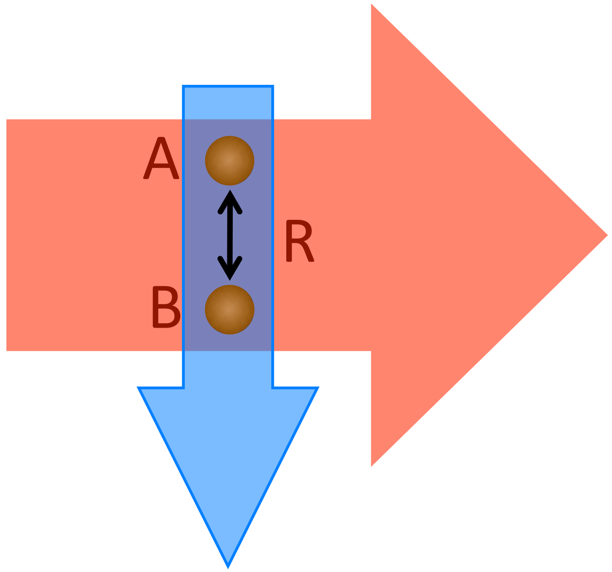


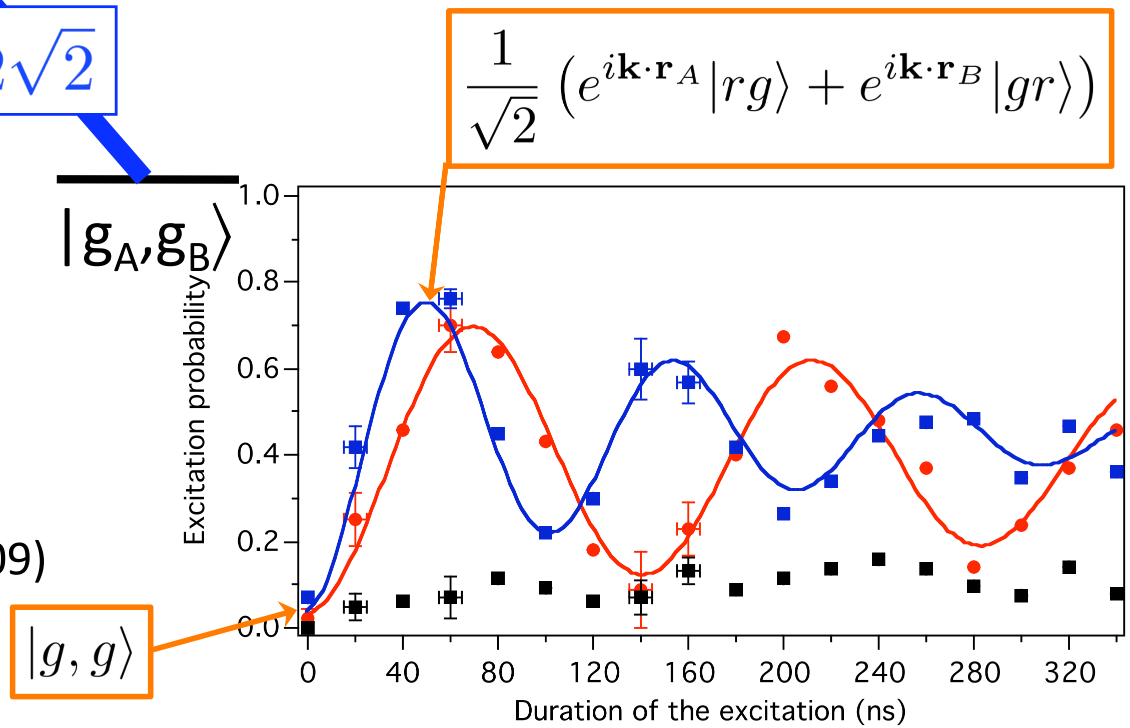
A. Gaëtan *et al.*, *Nat. Phys.* **5**, 115 (2009)

Rydberg blockade: collective excitation (IO Palaiseau)

Collective 2-level system



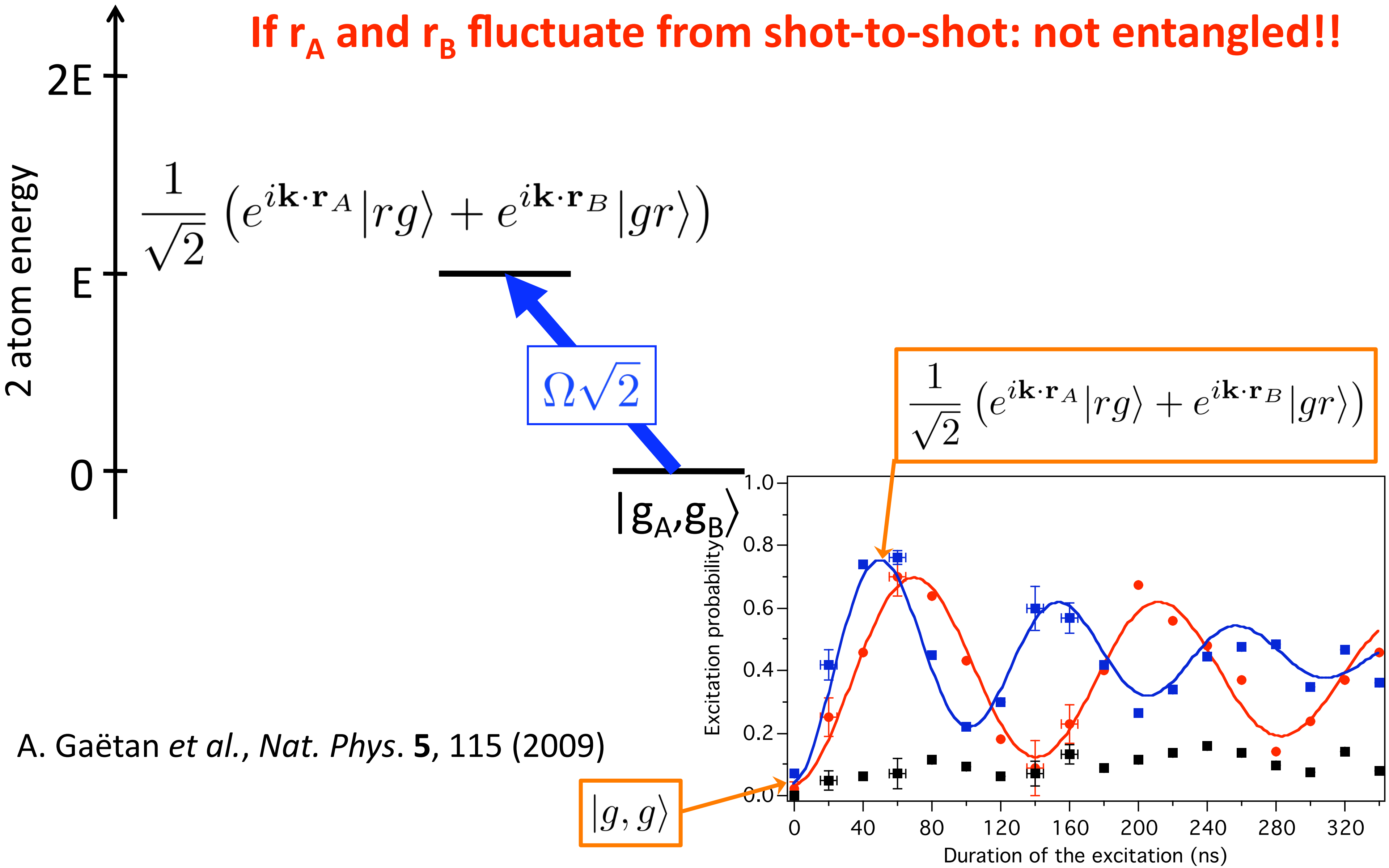




A. Gaëtan *et al.*, *Nat. Phys.* **5**, 115 (2009)

Rydberg blockade: collective excitation (IO Palaiseau)

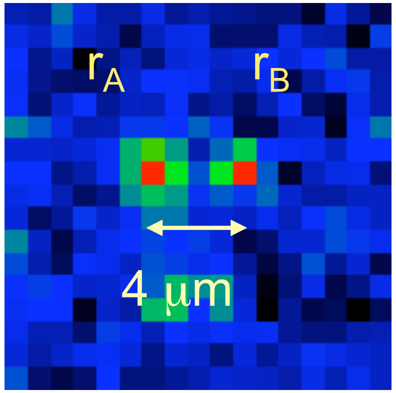
If r_A and r_B fluctuate from shot-to-shot: not entangled!!

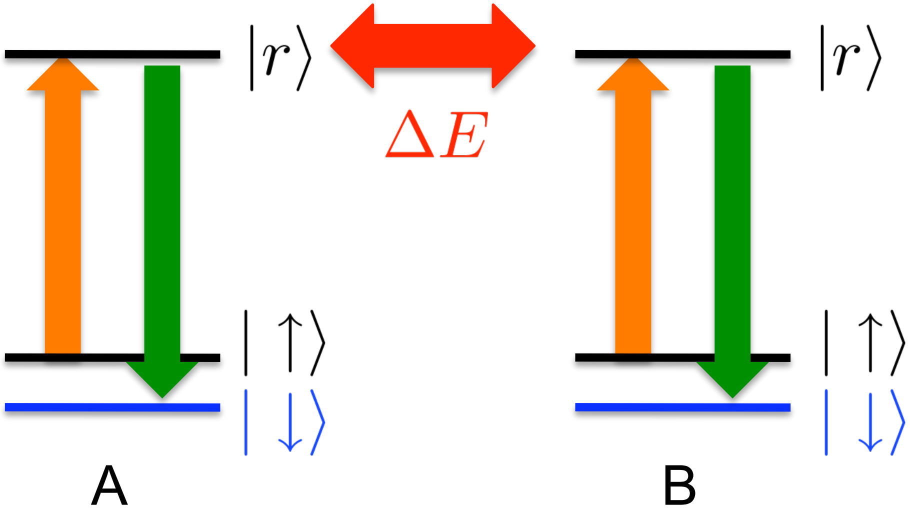


A. Gaëtan *et al.*, *Nat. Phys.* **5**, 115 (2009)

Entangled of two atoms using the Rydberg blockade

Collective excitation version





$$|\uparrow,\uparrow\rangle \;\Rightarrow\; \frac{1}{\sqrt{2}}\left(e^{i\mathbf{k}\cdot\mathbf{r}_A}|r,\uparrow\rangle + e^{i\mathbf{k}\cdot\mathbf{r}_B}|\uparrow,r\rangle\right) \quad k = k_{\mathrm{R}} + k_{\mathrm{B}}$$

$$\Rightarrow \frac{1}{\sqrt{2}}\left(e^{i(\mathbf{k}\cdot\mathbf{r}_A - \mathbf{k}'\cdot\mathbf{r}'_A)}|\downarrow,\uparrow\rangle + e^{i(\mathbf{k}\cdot\mathbf{r}_B - \mathbf{k}'\cdot\mathbf{r}'_B)}|\uparrow,\downarrow\rangle\right)$$

$$\Rightarrow \frac{1}{\sqrt{2}}(|\downarrow,\uparrow\rangle + e^{i\phi}|\uparrow,\downarrow\rangle) \text{ with } \phi = \mathbf{k}\cdot(\delta\mathbf{r}_A - \delta\mathbf{r}_B) \quad (k \approx k')$$

If atomic motion frozen $\Rightarrow \delta r_A \approx \delta r_B \approx 0$

Analyzing entanglement

$$|\psi_+\rangle = \frac{1}{\sqrt{2}}(|\downarrow,\uparrow\rangle + |\uparrow,\downarrow\rangle)$$

Measure the density matrix

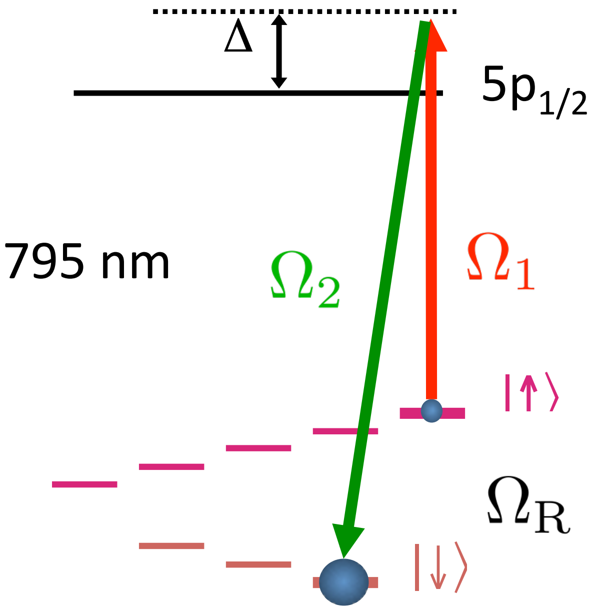
$$\hat{\rho} = \begin{pmatrix} \boxed{P_{\downarrow\downarrow}} & a & b & c \\ a^* & \boxed{P_{\downarrow\uparrow}} & \boxed{\rho_{\uparrow\downarrow,\downarrow\uparrow}} & d \\ b^* & \boxed{\rho^*_{\uparrow\downarrow,\downarrow\uparrow}} & \boxed{P_{\uparrow\downarrow}} & e \\ c^* & d^* & e^* & \boxed{P_{\uparrow\uparrow}} \end{pmatrix}_{|\downarrow\downarrow\rangle,|\downarrow\uparrow\rangle,|\uparrow\downarrow\rangle|,\uparrow\uparrow\rangle}$$

Extract the fidelity

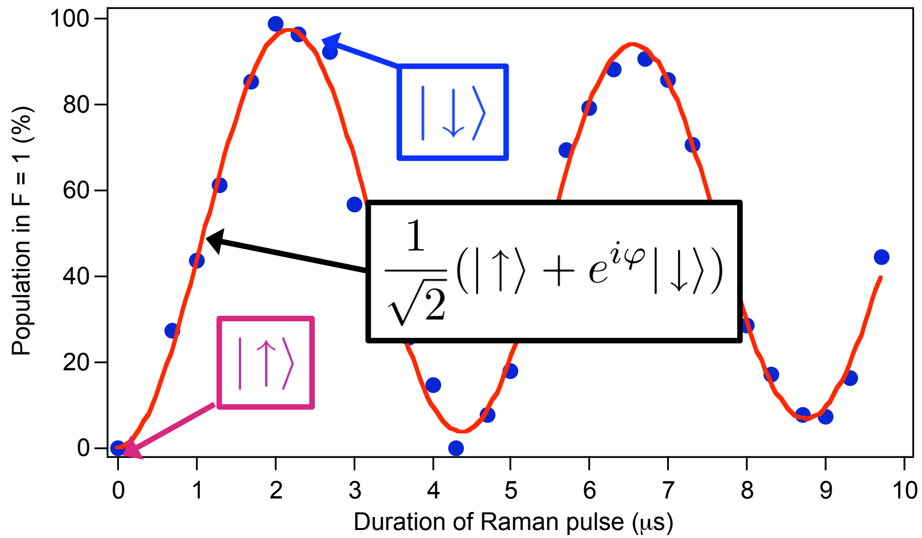
$$\boxed{F = \langle\psi_+|\hat{\rho}|\psi_+\rangle}$$

$$F = \frac{1}{2}(P_{\downarrow\uparrow} + P_{\uparrow\downarrow} + 2\,\Re(\rho_{\downarrow\uparrow,\uparrow\downarrow}))$$

Raman rotation: check coherence of the superposition



$$\Omega_R = \frac{\Omega_1 \Omega_2^*}{2\Delta}$$



Global rotation on the two atoms: $R_{A,B}(\theta,\varphi) = R_A(\theta,\varphi) \otimes R_B(\theta,\varphi)$

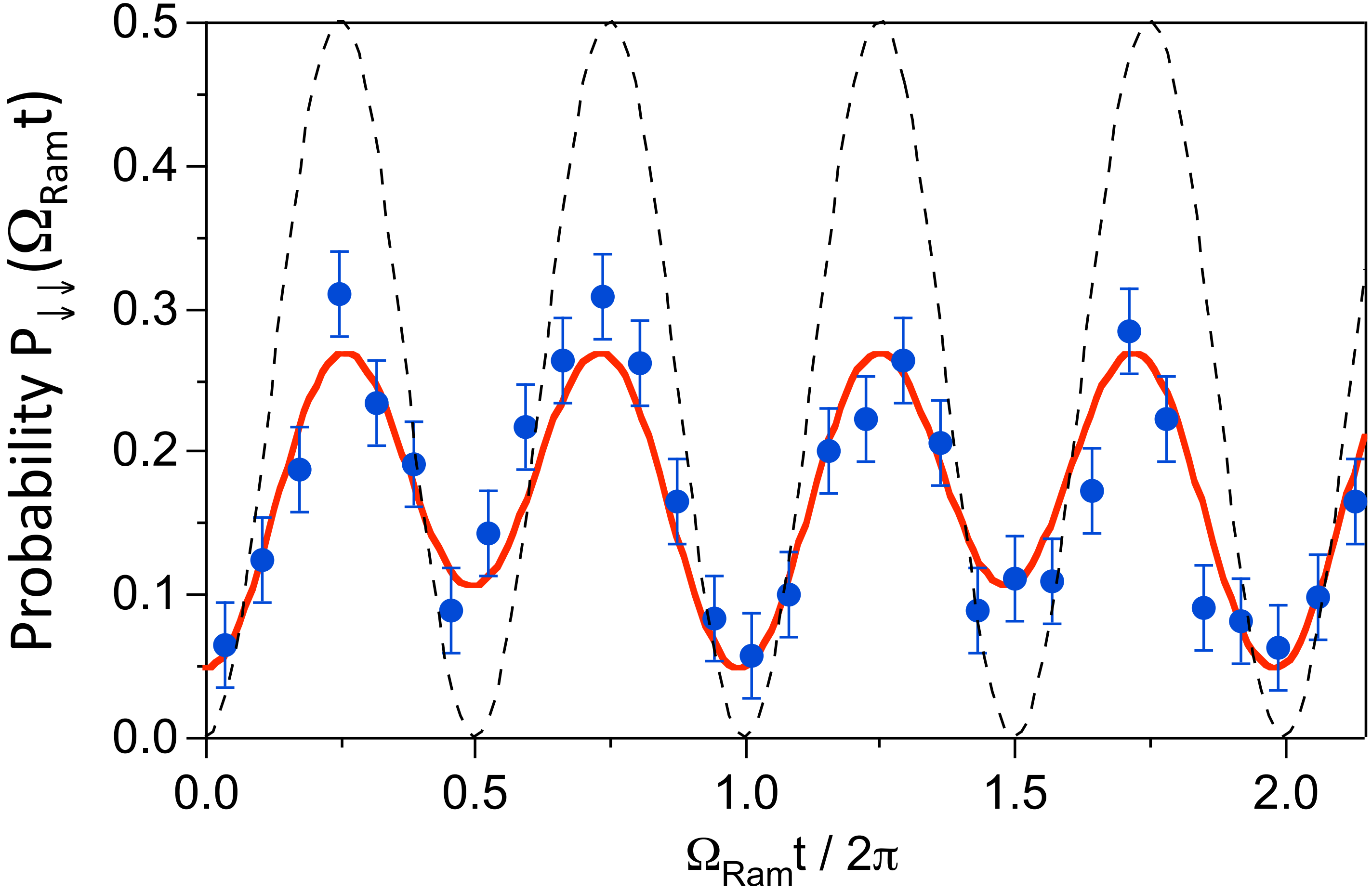
$$|\psi_+\rangle = \frac{1}{\sqrt{2}}(|\downarrow,\uparrow\rangle + |\uparrow,\downarrow\rangle) \Longleftrightarrow \frac{1}{\sqrt{2}}(|\downarrow,\downarrow\rangle - |\uparrow,\uparrow\rangle)$$

$2 \times \Omega_{Raman}$

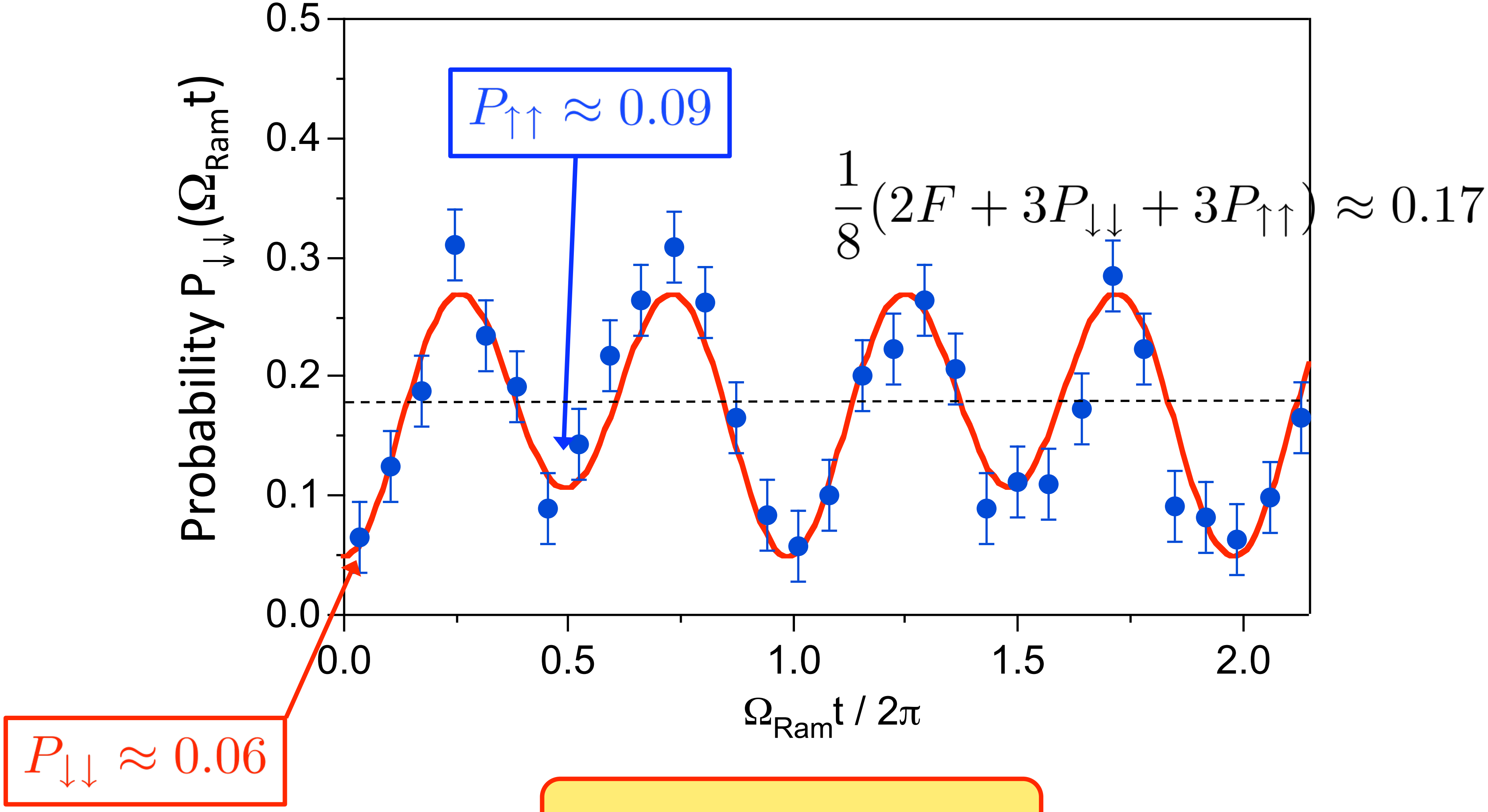
$$\Rightarrow \rho(\theta,\varphi) = R^{-1}_{A,B}(\theta,\varphi) \cdot \rho \cdot R_{A,B}(\theta,\varphi)$$

Rotation $\hat{\rho} \longrightarrow \hat{\rho}_{\mathrm{rot}}$ $\Rightarrow$ transformes **coherence** into **population**

Measurement of entanglement



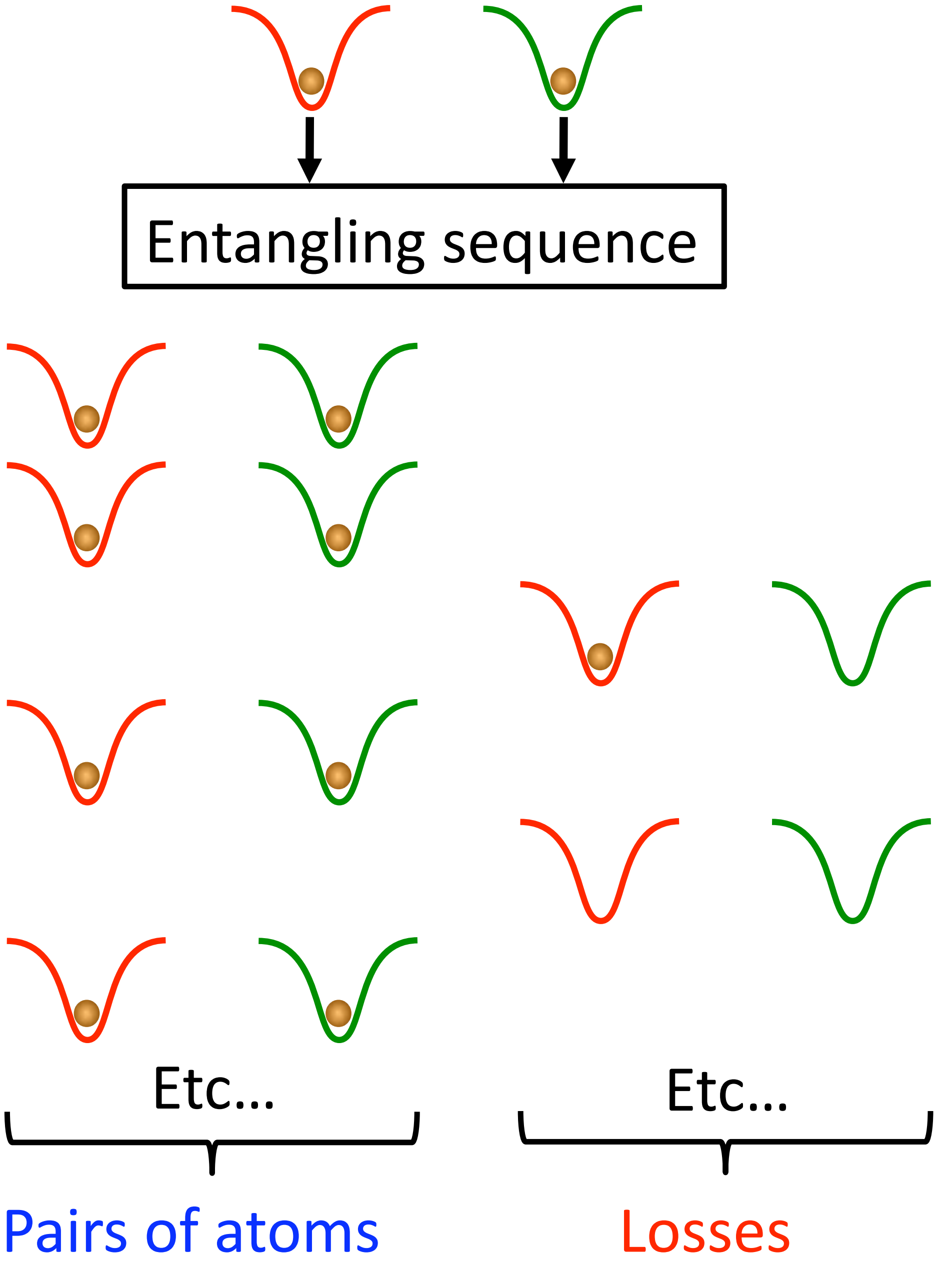
Measurement of entanglement

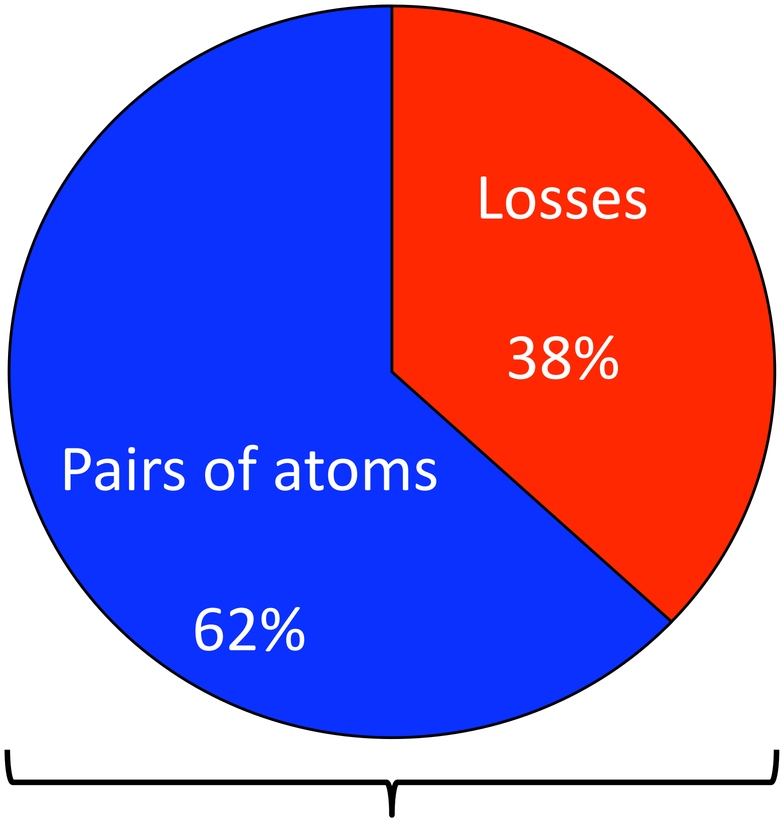


$$\Rightarrow F \approx 0.46 < 0.5$$

Details in Gaëtan *et al.*, NJP **12**, 065040 (2010)

Atom losses during the entangling sequence





$F_{total} = 0.46$

$F_{pairs} = F_{tot} / 0.62$

$F_{pairs} = 0.75 \pm 0.07$

Partial state reconstruction and error budget

State prepared: $\rho_{exp} = 0.46|\psi_+\rangle\langle\psi_+| + \rho_{\text{junk}}$

After loss correction: $\rho'_{exp} = 0.75|\psi_+\rangle\langle\psi_+| + \rho'_{\text{junk}}$

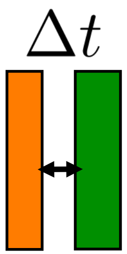
$$\rho'_{\text{exp}} = \begin{pmatrix} 0.1 & a & b & c \\ a^* & 0.38 & 0.38 & d \\ b^* & 0.38 & 0.38 & e \\ c^* & d^* & e^* & 0.12 \end{pmatrix}_{|\downarrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\uparrow\downarrow\rangle|, \uparrow\uparrow\rangle}$$

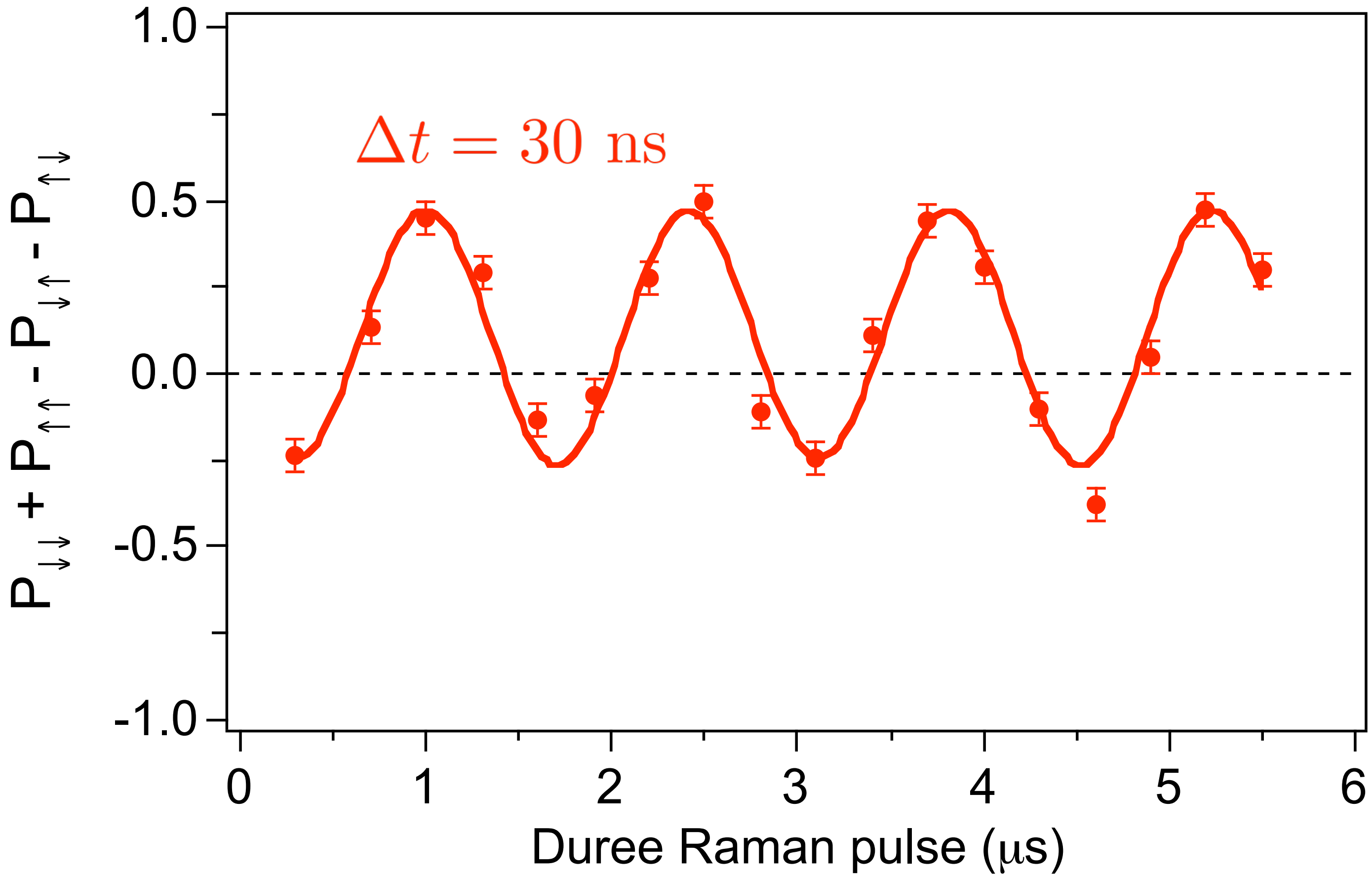
Error budget F_{pairs} = 0.75:

Imperfect blockade	**0.12**
Technical (laser stability, sp. emission)	0.1
Motion of the atoms	0.03

Wilk *et al.*, PRL **104,** 010502 (2010)

Influence of the motion of the atoms: how much frozen?





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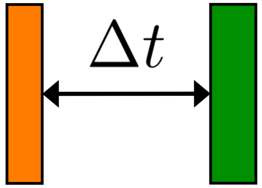
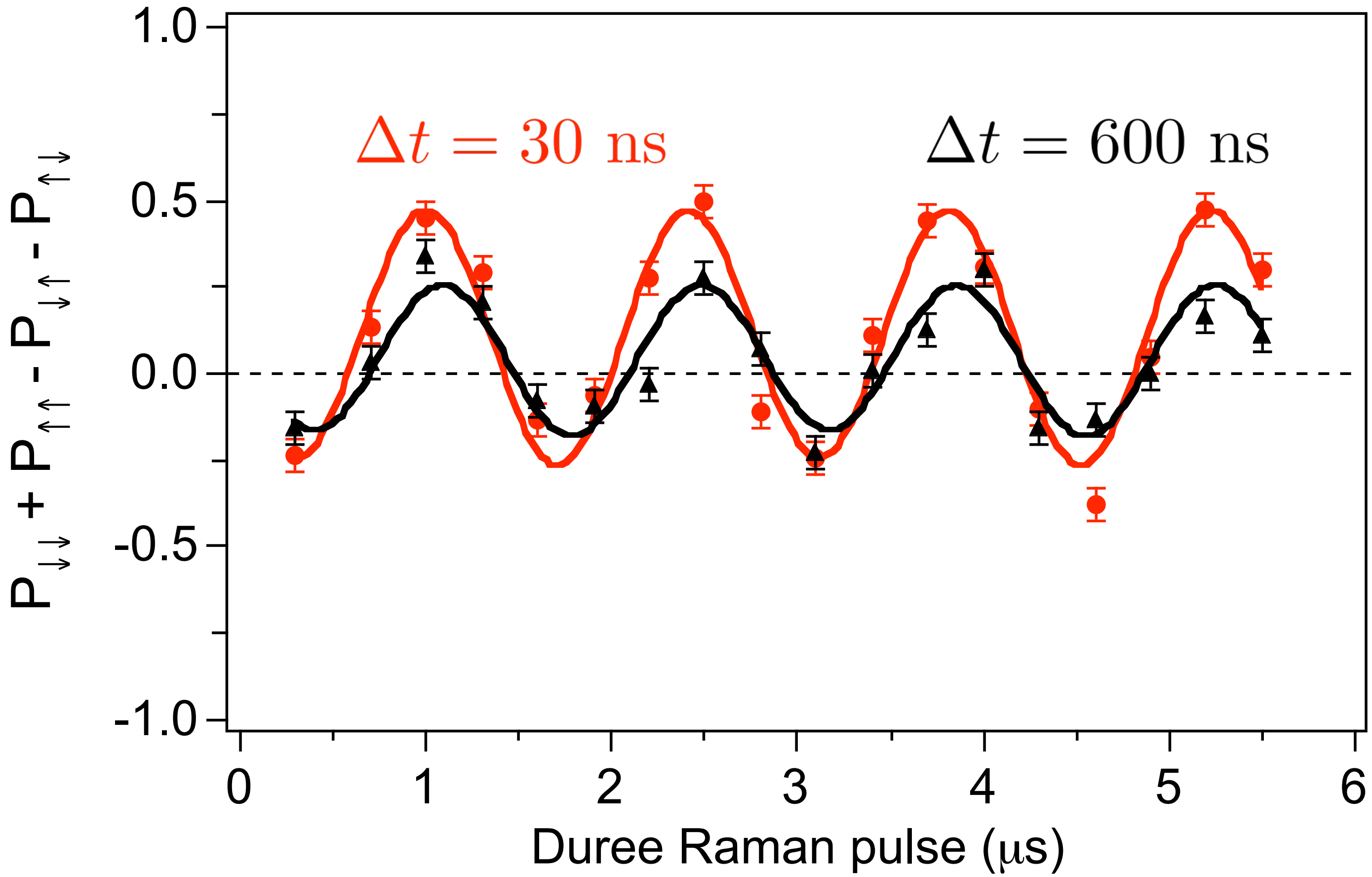


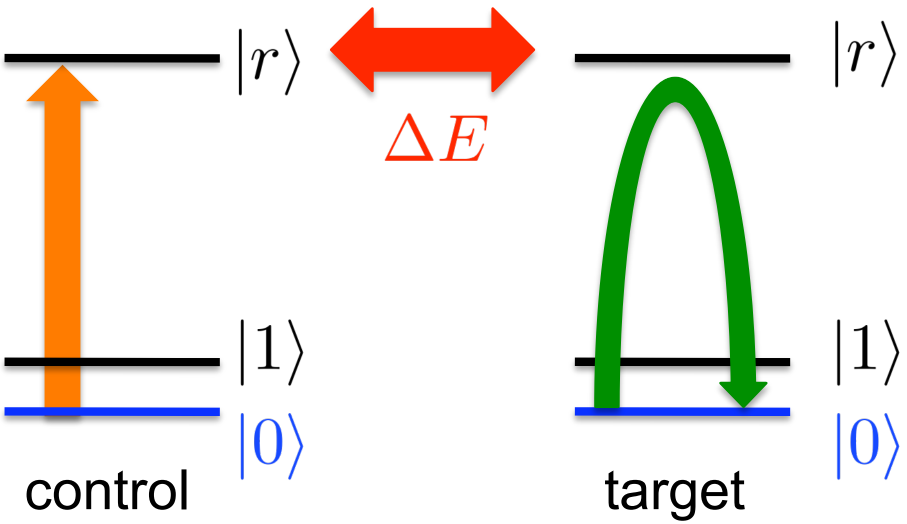


Coherence $\Re(\hat{\rho}_{\downarrow\uparrow,\uparrow\downarrow})$ reduced by 0.5

Theory: $\langle e^{ik(v_A - v_B)\Delta t} \rangle = e^{-\frac{k_B T}{M} k^2 \Delta t^2} \approx 0.45$

Quantum gate using Rydberg blockade

D. Jacksch *et al.*, PRL **85**, 2208 (2000)



Sequence:

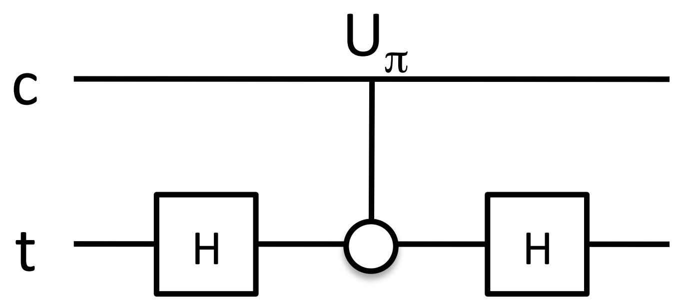
$\pi_A - 2\pi_B - \pi_A$

Table of truth:

11	→	11	→	11	→	11
10	→	1r	→	1r	→	-10
01	→	r0	→	r0	→	-01
00	→	r0	→	r0	→	-00

Blockade

From π-gate to CNOT



11	→	10
10	→	11
01	→	01
00	→	00

The CNOT gate at uni. Wisconsin (1)

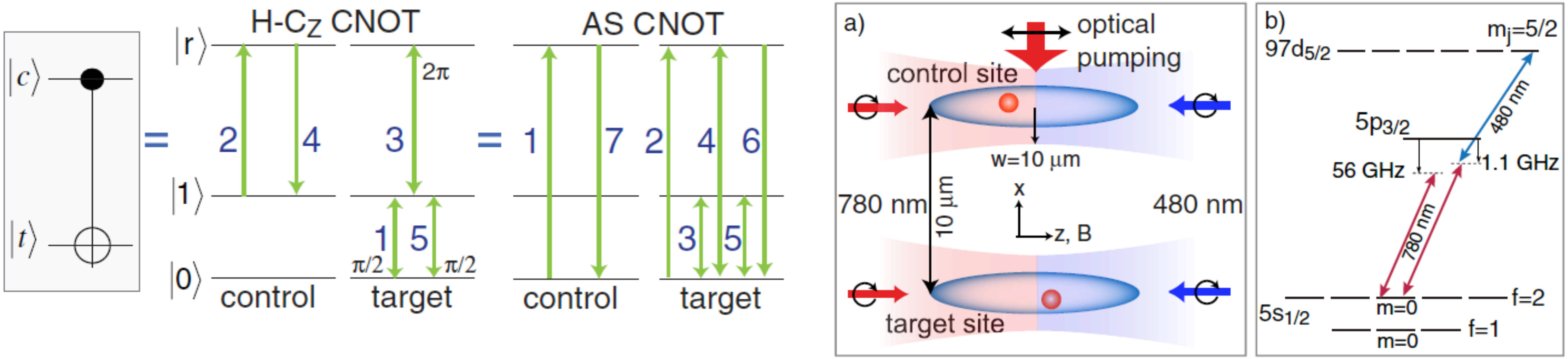
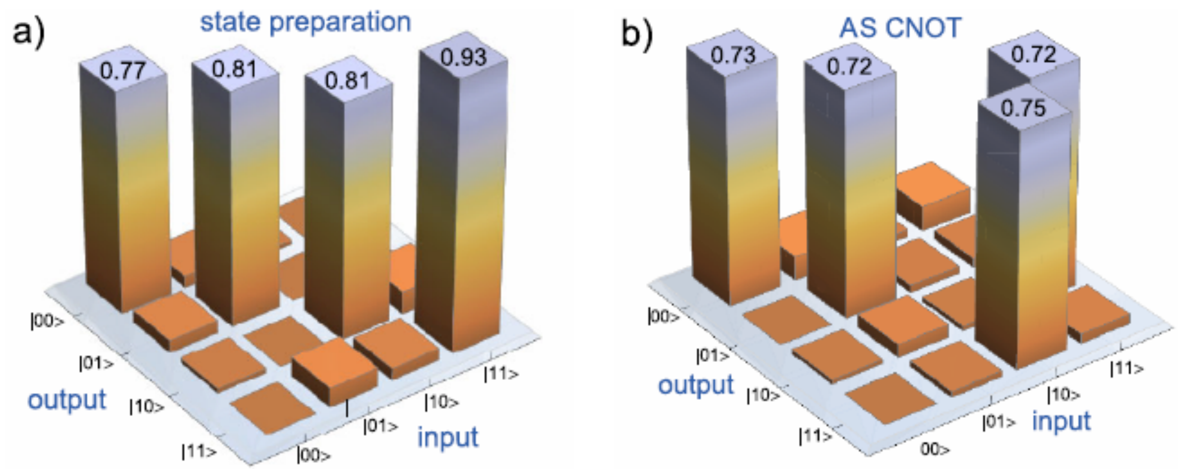


Table of truth: check blockade



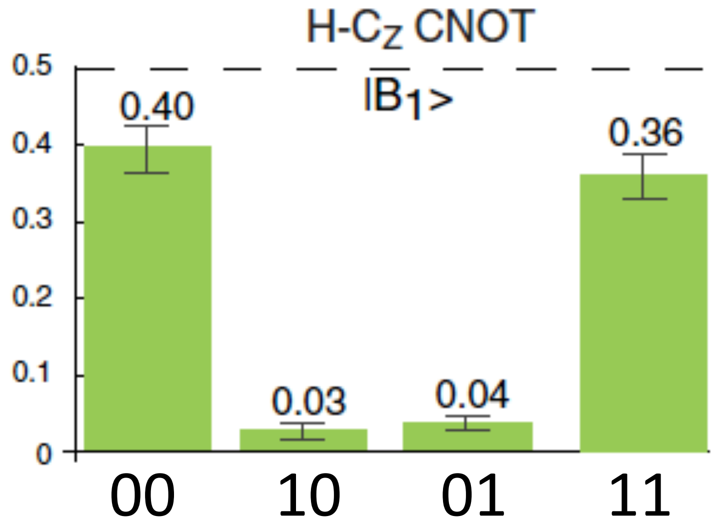
F = 0.73

Isenhower *et al.*, PRL **104**, 010503 (2010)

The CNOT gate at uni. Wisconsin (2)

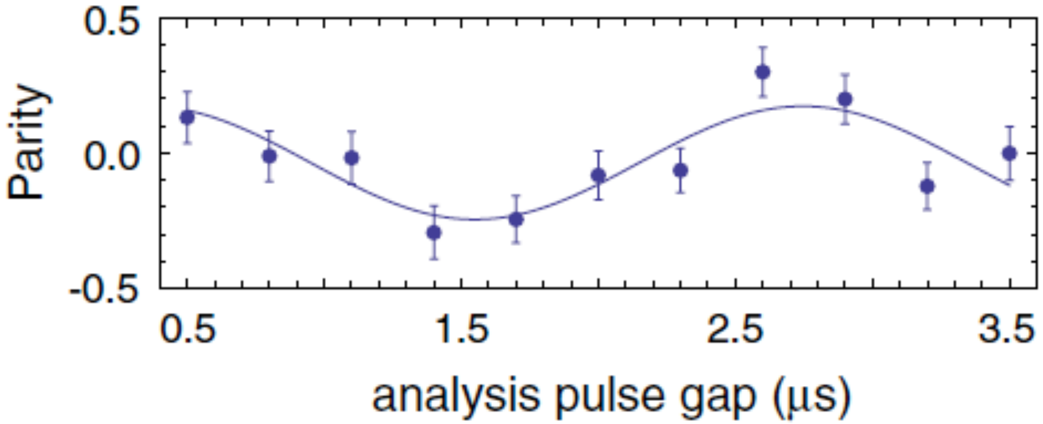
Prepare entangled states

Prepare $(|0\rangle+|1\rangle)|0\rangle$ → gate



Check coherence: global Raman rotation

$$\Pi = P_{00} + P_{11} - P_{01} - P_{10}$$



Amplitude

$\Rightarrow \Re(\rho_{0011})$

$$\mathcal{F} = \frac{1}{2}(P_{00} + P_{11}) + \Re(\rho_{0011})$$

F = 0.58

(with loss correction)

See also: Zhang *et al.*, PRA **82**, 030306(R) (2010)

Conclusion on the gate and entanglement

Proof-of-principle: OK

But needs to improve fidelity!!

And careful with the phases…