Hydrodynamic conservation laws and turbulent friction in atmospheric circulation models

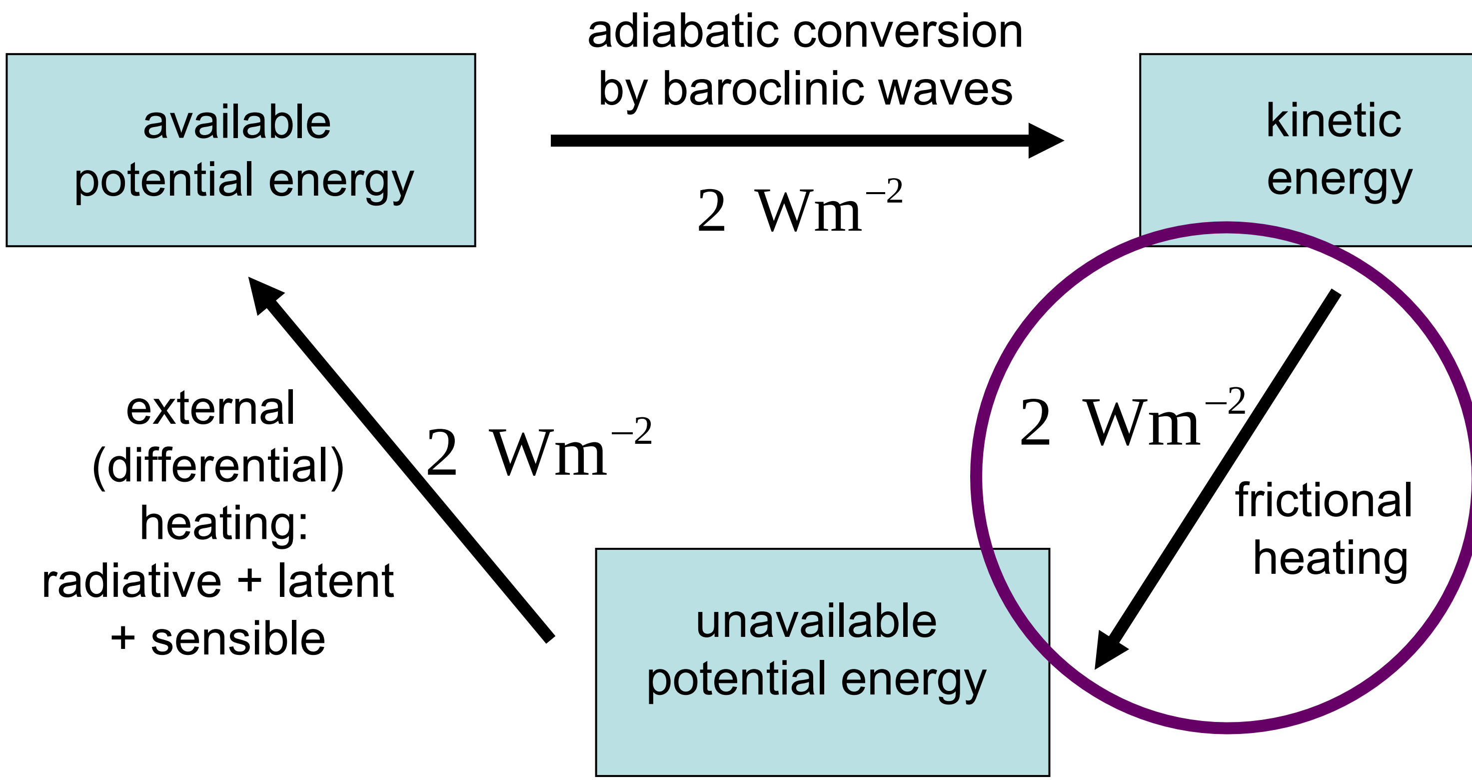
Erich Becker
Leibniz-Institute of Atmospheric Physics, Kühlungsborn, Germany

Including contributions from

Ulrike Burkhardt
DLR-Institute of Atmospheric Physics, Oberpfaffenhofen, Germany

Tiffany Shaw
University of Toronto, Toronto, Canada

Closed Lorenz energy cycle



- frictional heating essential for entropy and energy budgets:
  net diabatic heating of the atmosphere = frictional heating (Lorenz, 1967)

- irreversibility of baroclinic waves associated with gravity-wave generation

Outline

Recapitulate parameterizations of turbulent friction and frictional heating that ensure consistency with the conservation laws:

- **Diagnostic turbulence model (quasi-stationary TKE budget)**

- **Prognostic turbulence model based on the full TKE budget**

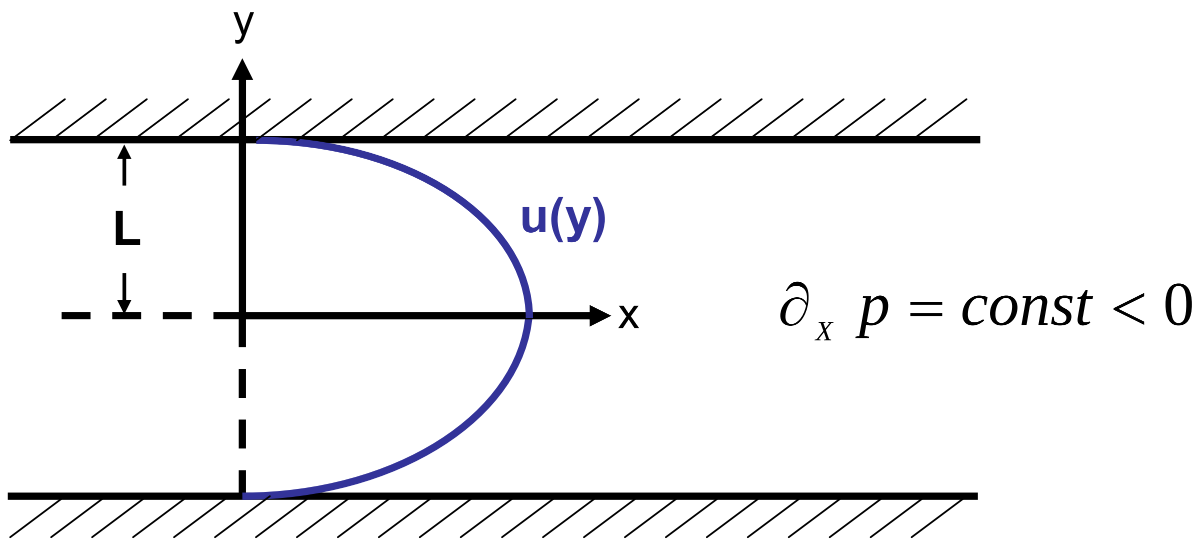
First-principle constraints on any (diagnostic) parameterization of turbulent friction

frictional force
= divergence of a symmetric Reynolds stress tensor
(equivalent to angular momentum conservation)

frictional heating
= shear production
= (stress tensor times gradient operator) • velocity field
(consequence of energy conservation)

frictional heating $\geq$ 0
(guarantees validity of the second law under all circumstances)

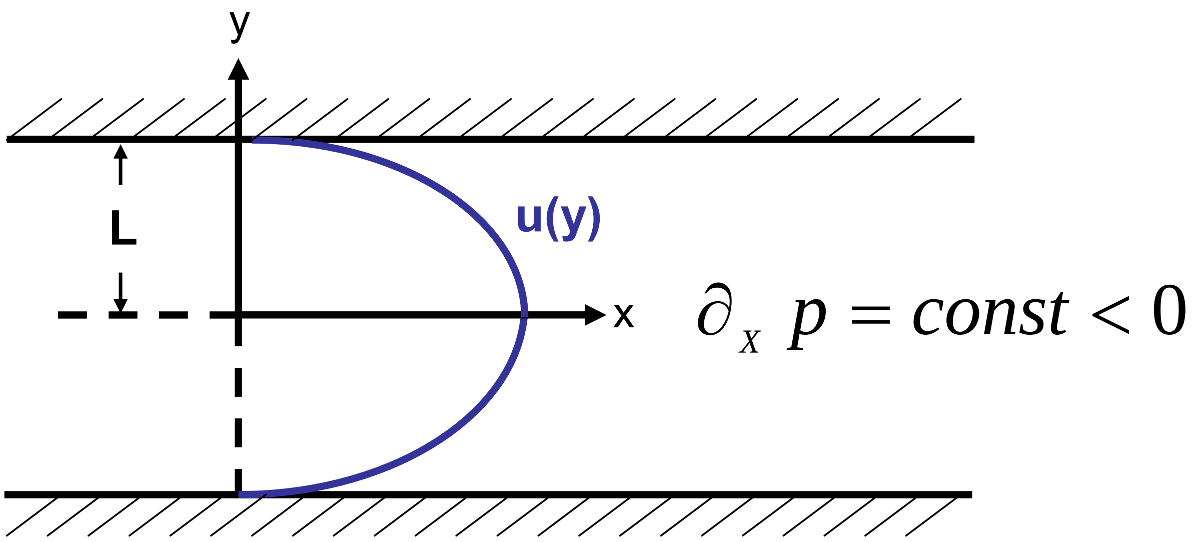
Example for violation of the second law due to hyperdiffusion: Poiseuille flow in a channel



$$
\begin{aligned}
\text{momentum budget:} \quad \partial_x p &= \partial_y \tau_{xy} \\
\text{no-slip condition:} \quad 0 &= u(y = \pm L) \\
\text{momentum diffusion:} \quad \partial_y \tau_{xy} &= (-)^{\frac{n}{2}-1} \rho\, K\, L^{n-2}\, \partial_y^n u(y) \\
\text{solution:} \quad u(y) &= A\,(1 - y^n/L^n)\,, \quad A = (-)^{\frac{n}{2}} \frac{L^2\,\partial_x p}{n!\,K\,\rho}
\end{aligned}
$$

- sign convention: momentum diffusion $\propto (-)^{\frac{n}{2}-1}\,\nabla^n \mathbf{v}$

  $\Rightarrow$ All wave motions are damped, but $u(y)$ is in the direction of $-\partial_x p$ only for $n = 2, 6, 10, \ldots$



frictional heating: $$\tau_{xy}\,\partial_y u = (-)^{\frac{n}{2}+1}\,\frac{(\partial_x p)^2\,L^2}{(n-1)!\,K\,\rho}\,\frac{y^n}{L^n}$$

≥ 0 only for n = 2, 6, 10, …

Conventional GCM with a diagnostic turbulence model

$$d_t\mathbf{v} = -f\,\mathbf{e}_z \times \mathbf{v} - \frac{\nabla p}{\rho} + \rho^{-1}\,\partial_z\,(\rho\,K_z\,\partial_z\mathbf{v}\,) + \{\text{hyperdiffusion}\}$$

$$d_t h = \frac{d_t p}{\rho} + Q_{rad} + Q_{lat} + \underbrace{\frac{c_p}{\rho}\,\partial_z\,(\rho\,\frac{T}{\Theta}K_z\,\partial_z\Theta\,)}_{Q_{sens}} - \mathbf{v}\cdot\frac{\partial_z\,(\rho\,K_z\,\partial_z\mathbf{v}\,)}{\rho}, \quad h = c_p\,T$$

$$K_z = \left(\frac{1}{\kappa z} + \frac{1}{l_a}\right)^{-1}\,|\,\partial_z\mathbf{v}\,|\;F(R_i)$$

dynamic boundary conditions (L denotes the lowest model layer):

$$\begin{aligned}(\,\rho\,K_z\,\partial_z\mathbf{v}\,)_{surface} &= \rho_L\,C_D\,|\mathbf{v}_L|\;\mathbf{v}_L \\ (\,\rho\,K_z\,\partial_z\Theta\,)_{surface} &= \rho_L\,C_D\,|\mathbf{v}_L|\left(\Theta_L - \Theta_{surface}\right)\end{aligned}$$

problematic:

- $\{\text{hyperdiffusion}\}$ is not derived from a stress tensor
- $-\,\mathbf{v}\cdot\frac{\partial_z\,(\rho\,K_z\,\partial_z\mathbf{v}\,)}{\rho}$ is flawed and not complete

Consistent formulation

$$d_t\mathbf{v} = -f\,\mathbf{e}_z \times \mathbf{v} - \frac{\nabla p}{\rho} + \rho^{-1}\,\partial_z\,(\rho\,K_z\,\partial_z\mathbf{v}) + \rho^{-1}\,\nabla\,(\rho\,K_h\,\mathsf{S}_h)$$

$$d_t h = \frac{d_t p}{\rho} + Q_{rad} + Q_{lat} + \frac{c_p}{\rho}\,\partial_z\,(\rho\,\frac{T}{\Theta}K_z\,\partial_z\Theta\,) + \underbrace{K_z\,(\partial_z\mathbf{v})^2}_{\geq\,0} + K_h\,(\,\mathsf{S}_h\nabla\,)\cdot\mathbf{v}$$

- K_z and dynamic boundary conditions as before
- $\mathsf{S}_h = \mathsf{S}_h^T$, $K_h\,(\,\mathsf{S}_h\nabla\,)\cdot\mathbf{v} \geq 0$, and $\mathsf{S}_h\mathbf{e}_z = 0$ (Becker, 2001, JAS)
- energy conserving discretization of vertical momentum diffusion and shear production (Burchardt, 2002, OM; Becker, 2003, MWR):

$$\sum_{l=1}^{L} \Delta p_l \left(\mathbf{v}_l \cdot \left(\frac{\partial_z\,(\rho\,K_z\,\partial_z\mathbf{v}\,)}{\rho}\right)_l + \left(K_z\,(\partial_z\mathbf{v})^2\right)_l\right) \equiv 0$$

$\longrightarrow$ finite-difference analogue of the no-slip condition

Some details of the horizontal diffusion and shear production

(Becker & Burkhardt, 2007, MWR; Becker, 2009, JAS)

$$\rho K_h \mathsf{S}_h = \rho K_h \left(\{ (\nabla + \mathbf{e}_z/a) \circ \mathbf{v} \} + \{ (\nabla + \mathbf{e}_z/a) \circ \mathbf{v} \}^T \right)$$

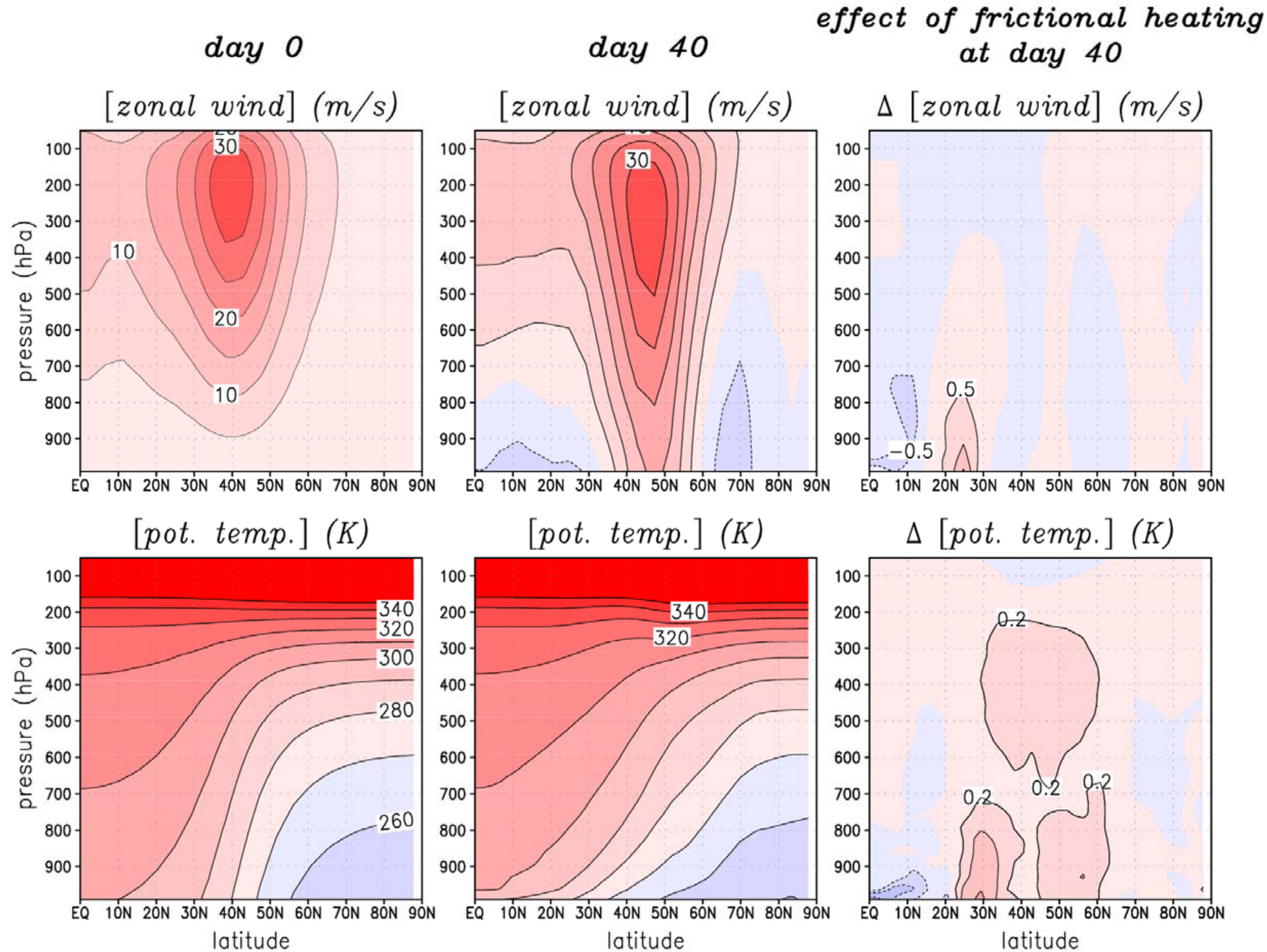
$$K_h = l_h^2 \sqrt{|\mathsf{S}_h|^2 + S_0^2} \qquad \text{(Smagorinsky-type)}$$

$$K_h = l_h^2 \sqrt{|\mathsf{S}_h|^2 + S_0^2} \left(1 + \alpha F(Ri)\right) \qquad \text{(with resolved GWs)}$$

$$K_h (\mathsf{S}_h \nabla) \cdot \mathbf{v} = K_h |\mathsf{S}_h|^2 \geq 0$$

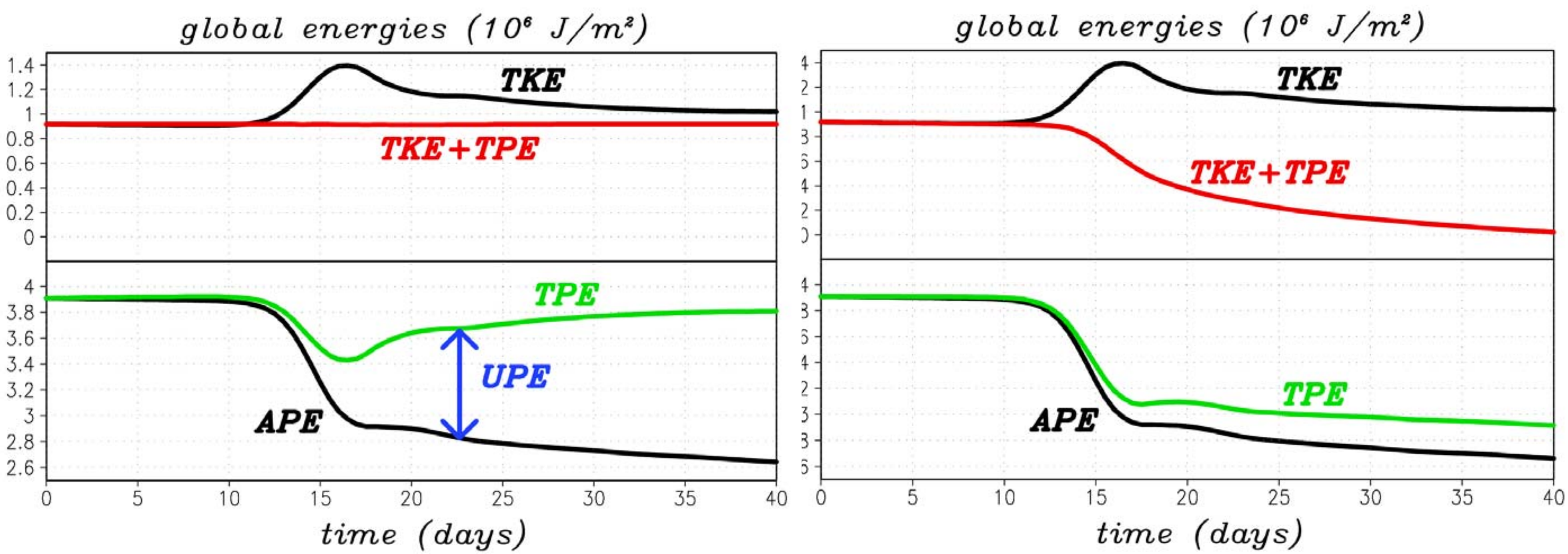
Smagorinsky's generalized mixing-length concept corresponds to a nonlinear, but harmonic diffusion.

Life cycle of a baroclinic wave in a thermally and mechanically isolated model atmosphere (mechanistic GCM,T42L30)



(Simmons and Hoskins, 1978, JAS; with Smagorinsky-type horiz. diffusion and frictional heating: Becker and Burkardt, 2007, MWR)

WITH frictional heating WITHOUT frictional heating



TKE = total kinetic energy

TPE = total potential energy (offset: APE(t=0)-TPE(t=0))

TKE + TPE = total energy (offset: -TPE(t=0))

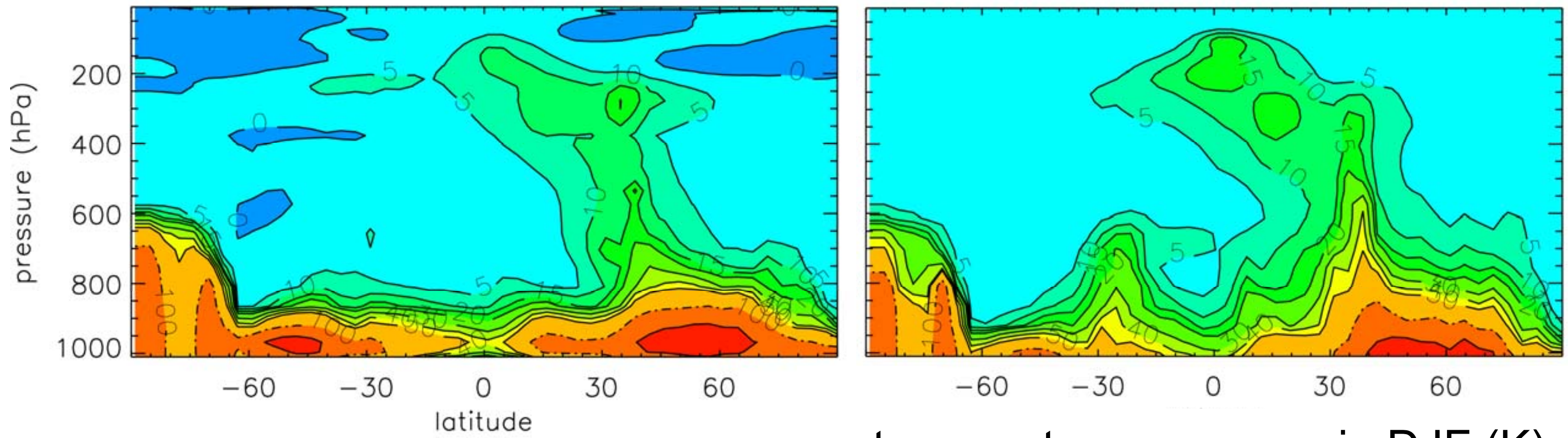
APE = available potential energy

UPE = TPE - APE = unavailable potential energy

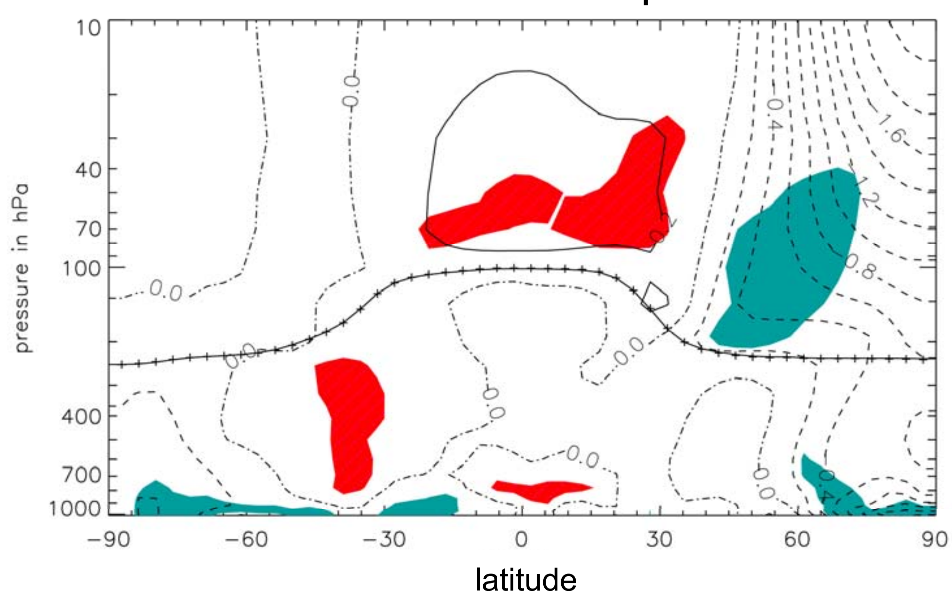
(Becker & Burkhardt, 2007, MWR)

ECHAM4/L39DLR (T42) with old and new formulation of the shear production due to vertical momentum diffusion (DJF)

$-\mathbf{v}\cdot\frac{\partial_z(\rho K_z \partial_z \mathbf{v})}{\rho}$ $(10^{-3}$ K/d$)$ $\qquad$ $K_z(\partial_z \mathbf{v})^2$ $(10^{-3}$ K/d$)$



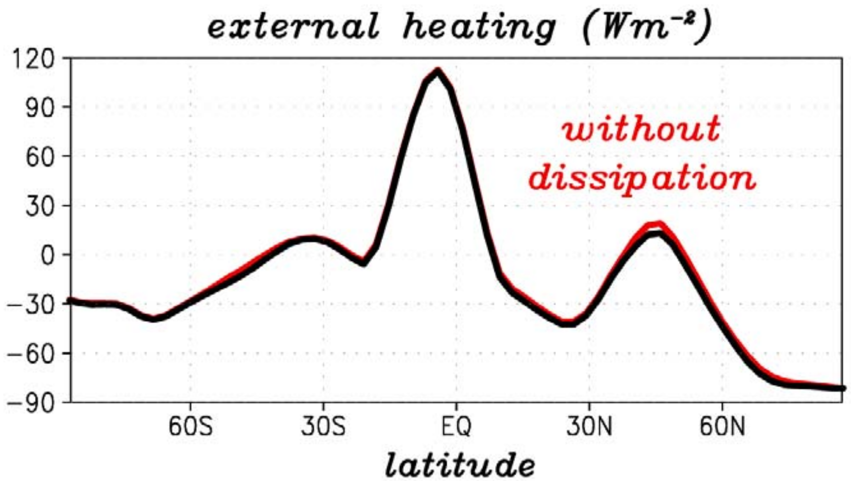
temperature response in DJF (K):
new – old vert. shear production

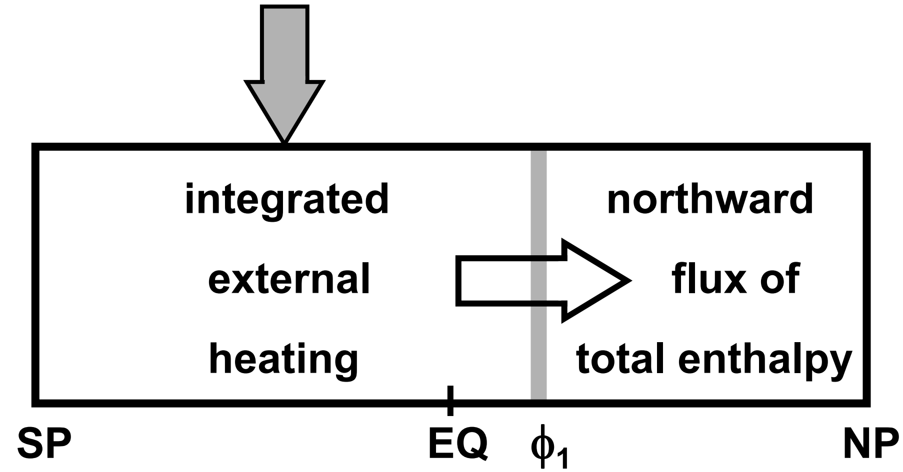


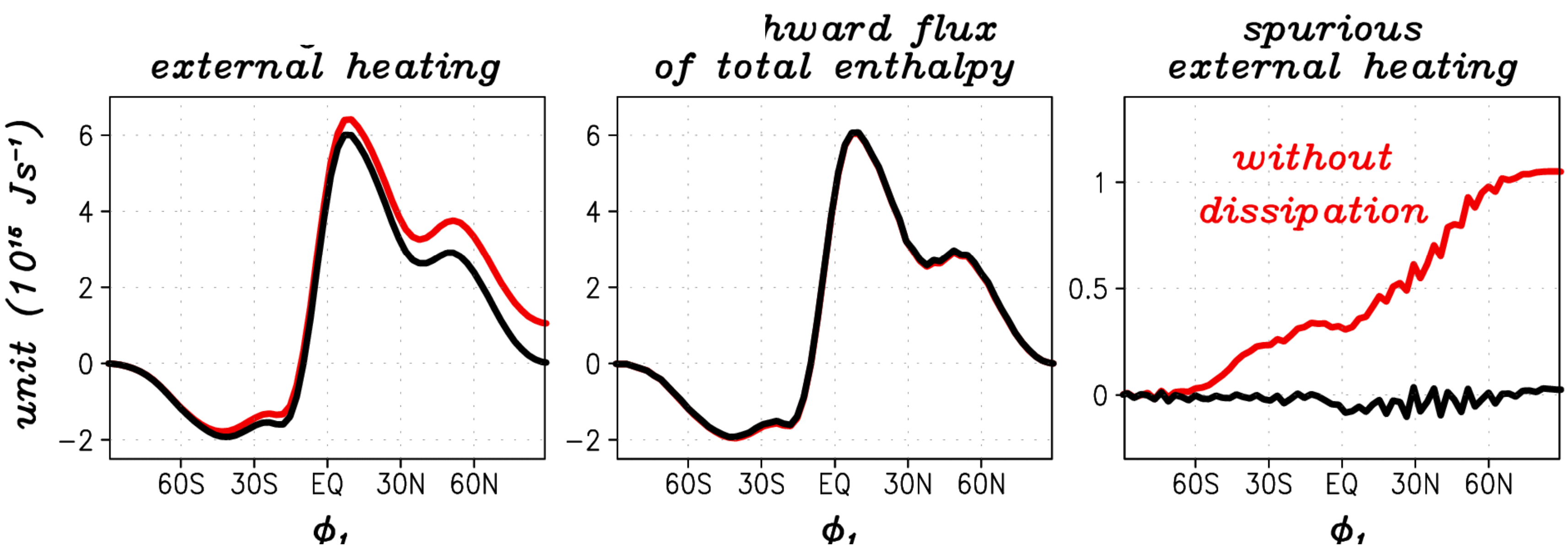
reduced thermal forcing of quasi-stationary Rossby waves
→
cooling of the NH winter stratosphere due to weakening of the residual circulation

(Burkhardt & Becker, 2006, MWR)

Energy conservation in the climatological global mean: Mechanistic GCM simulations (T42L30)







(Becker, 2003, MWR)

Mechanistic model results based on the

Kühlungsborn Mechanistic general Circulation Model (KMCM)

- **standard spectral dynamical core**

- **“mechanistic“ because**

$$Q_{rad} = -c_p\,(T - T_e(\phi, p))\,/\,\tau$$

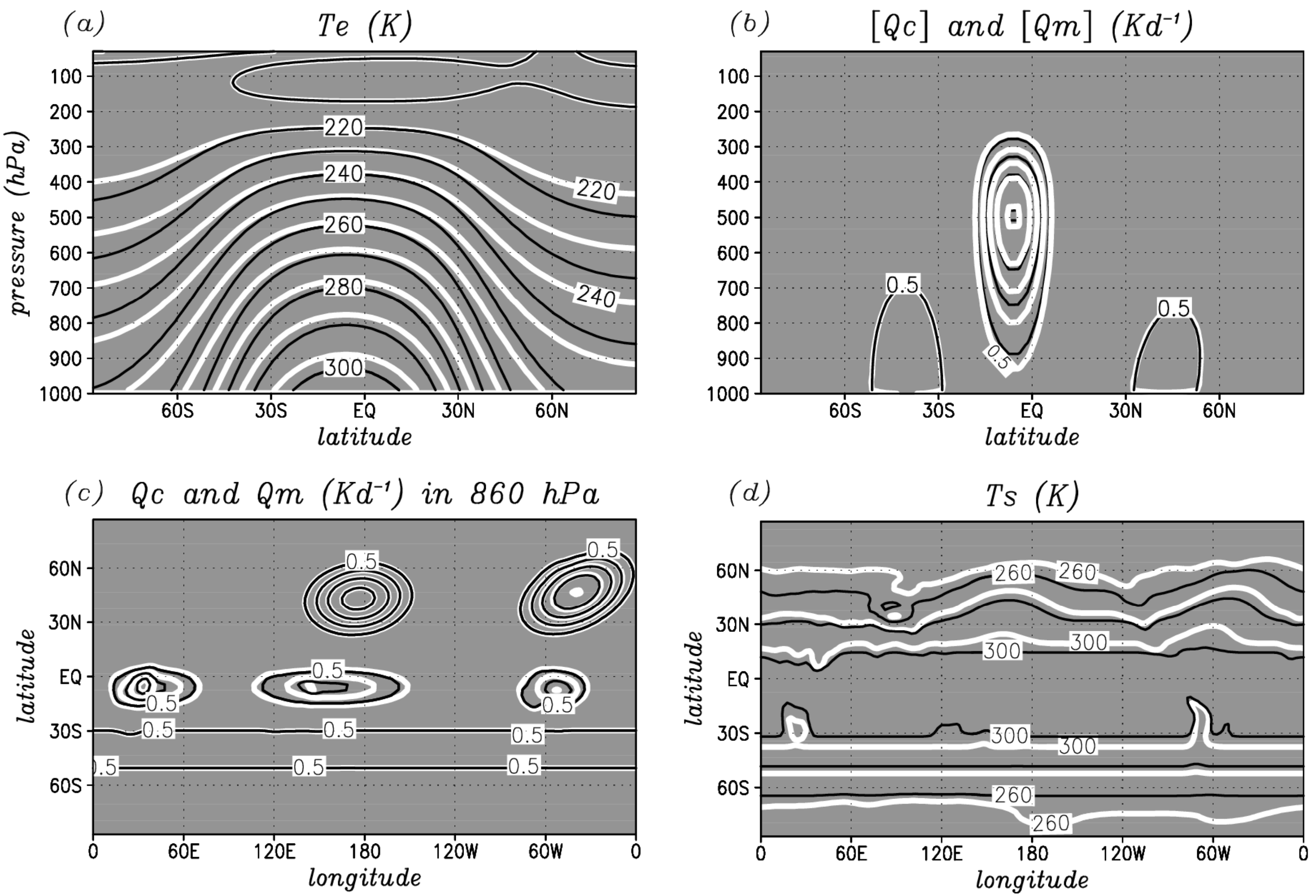
$$Q_{lat} = Q_c(\lambda, \phi, p) + Q_m(\lambda, \phi, p)\,h(-\omega)\,|\,\omega\,|\,/\,\omega_0$$

$$T_s = (\,T_e + 0.4\,\tau\,(Q_c + Q_m)\,)_{surface}$$

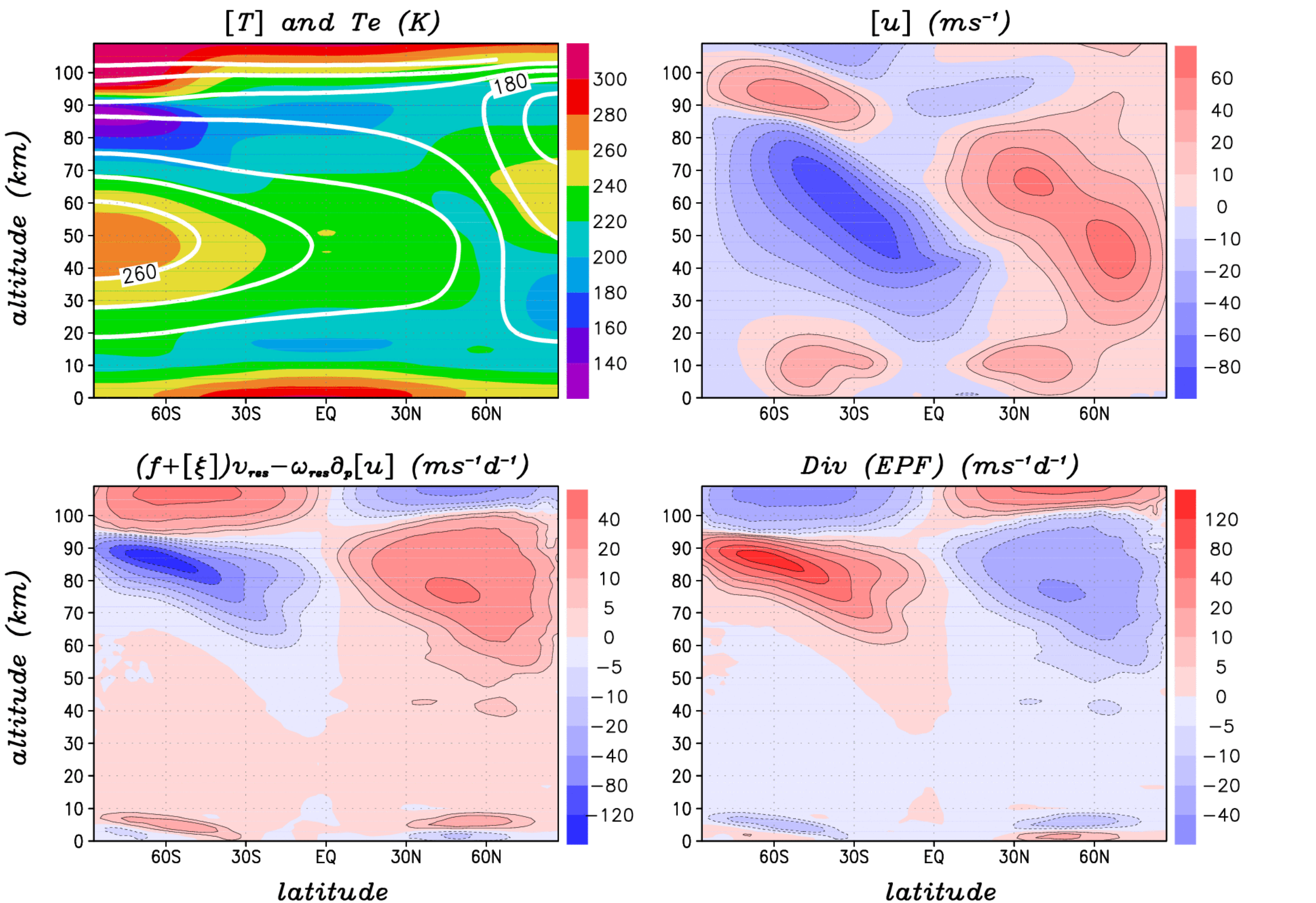
- **diagnostic turbulent diffusion formulated and implemented in line with the conservation laws**

- **both the vertical and the Smagorinsky-type horizontal diffusion coefficients are scaled by the Richardson criterion for dynamic instability in order simulate gravity waves explicitly for high spatial resolution (T120L190, T210L190) up to the mesopause region** (Becker, 2009, JAS)

CONTROL and PERTURBATION simulation

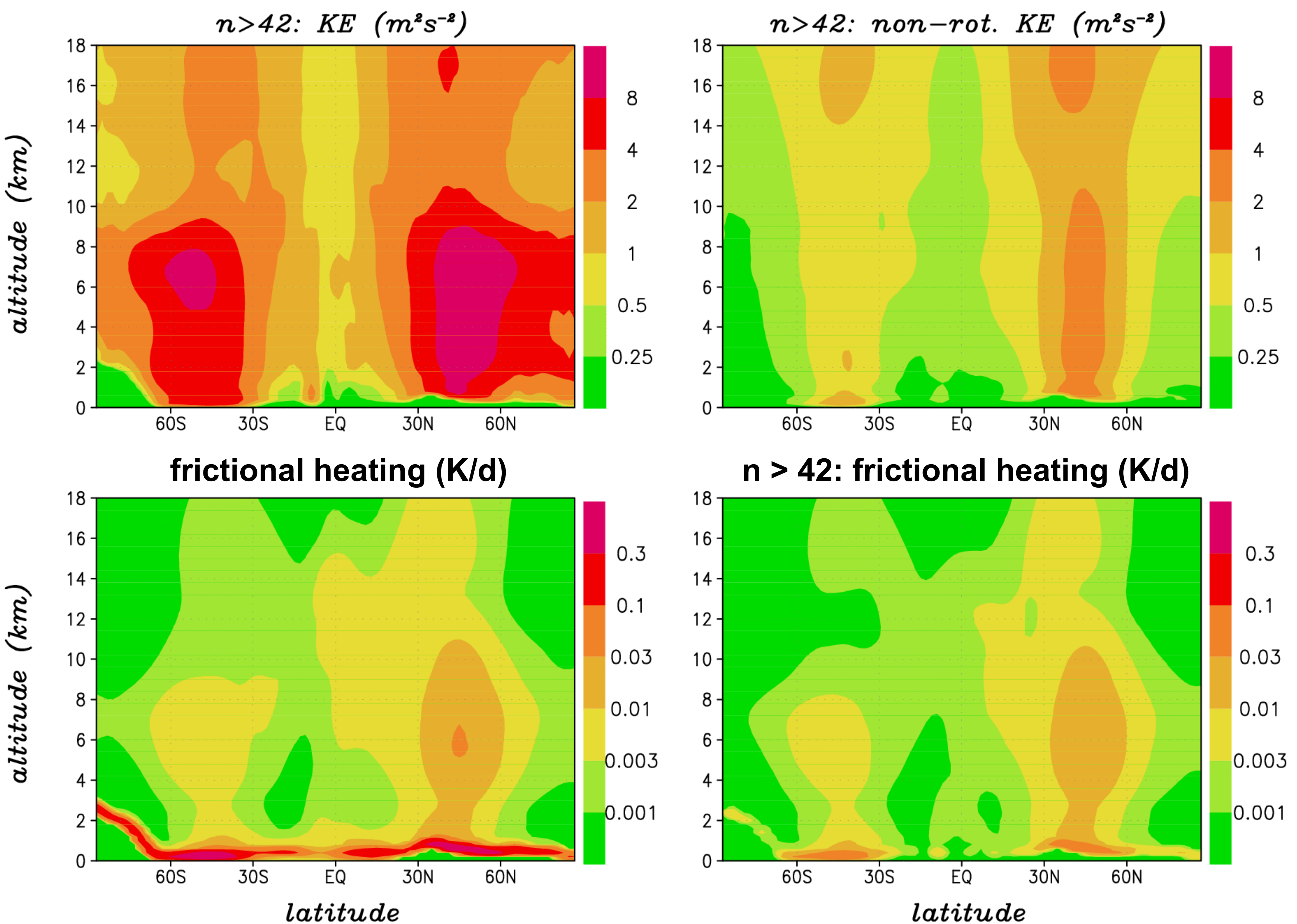


January climatology in the zonal mean (CONTROL)



→ **reasonable representation of the general circulation and wave-mean flow interaction up to the mesopause region**

Mesoscale kinetic energy (KE) and dissipation in the troposphere (CONTROL)

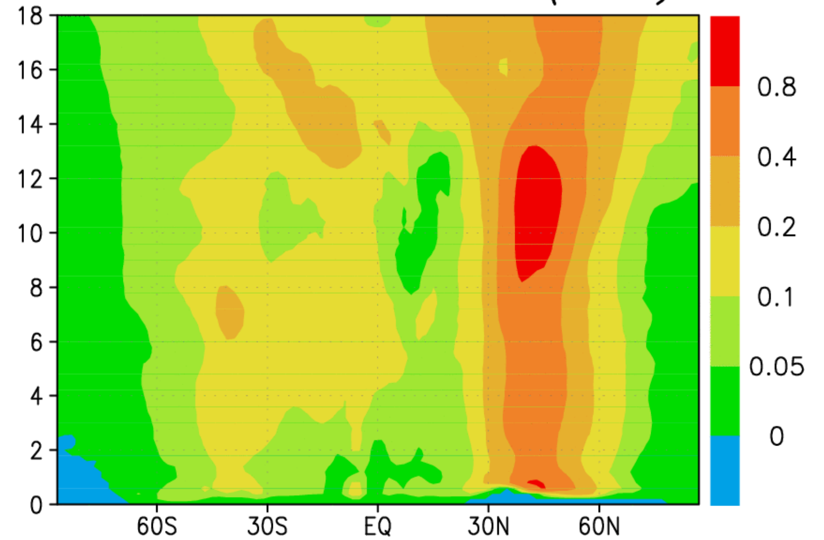


Mesoscale KE and dissipation in the troposphere: Model response (PERTURBATION minus CONTROL)

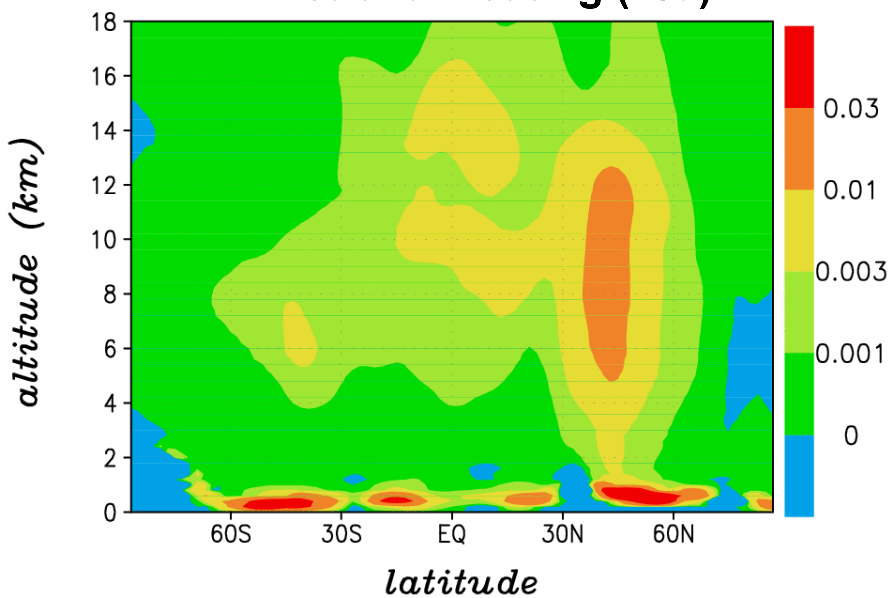
enhanced energy cycle (from 2.3 to 2.7 W/m²)

AND

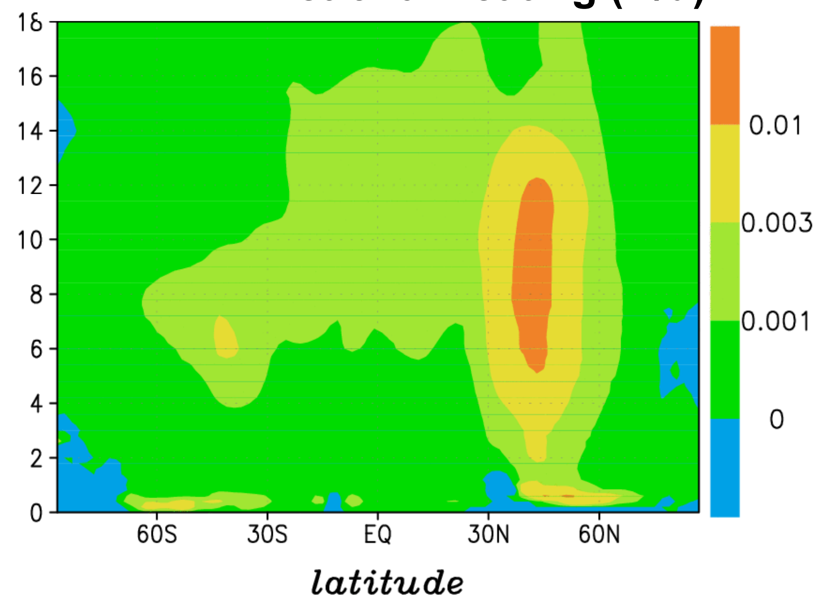
stronger GW sources



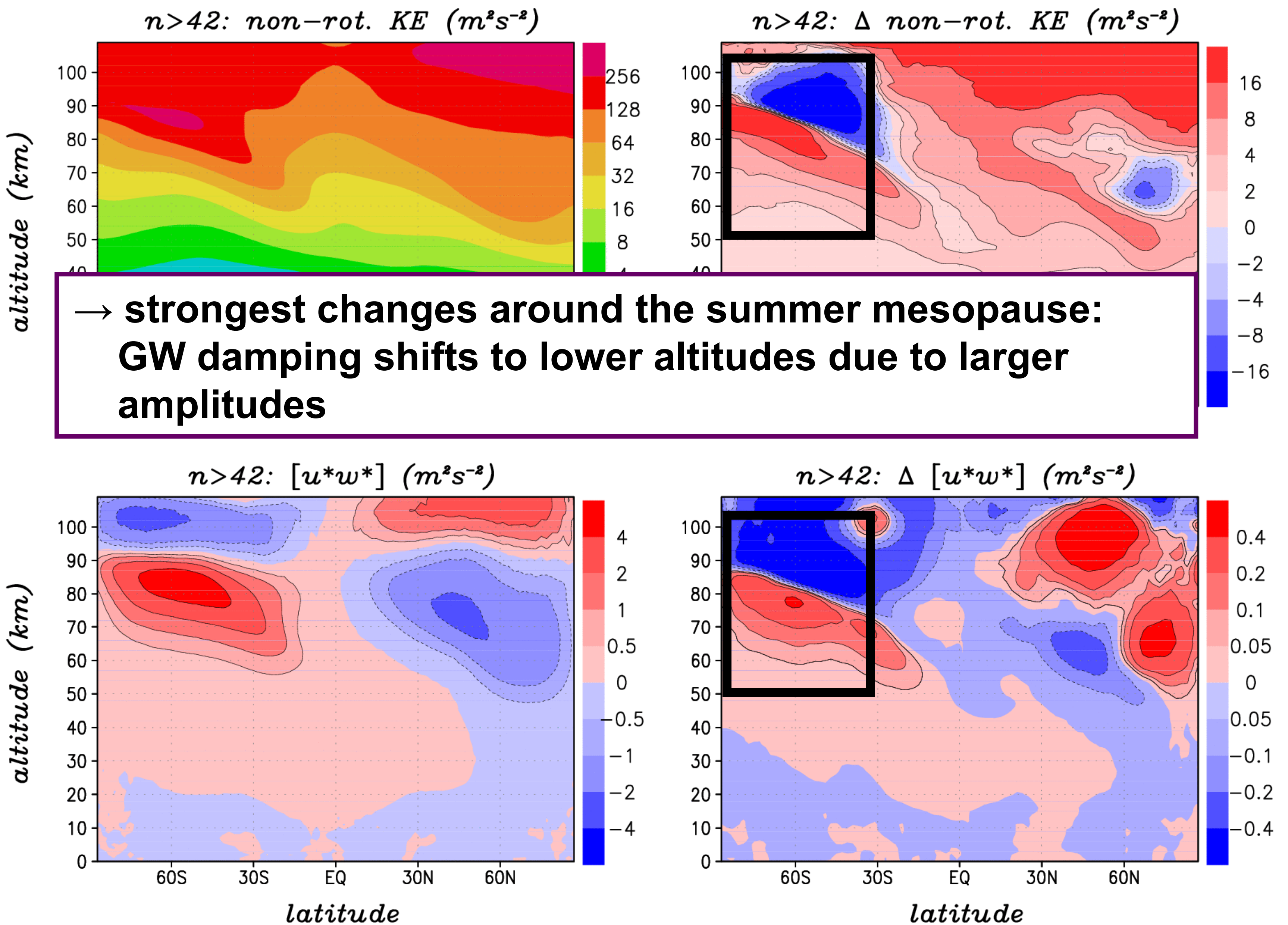
Δ frictional heating (K/d)



n > 42: Δ frictional heating (K/d)

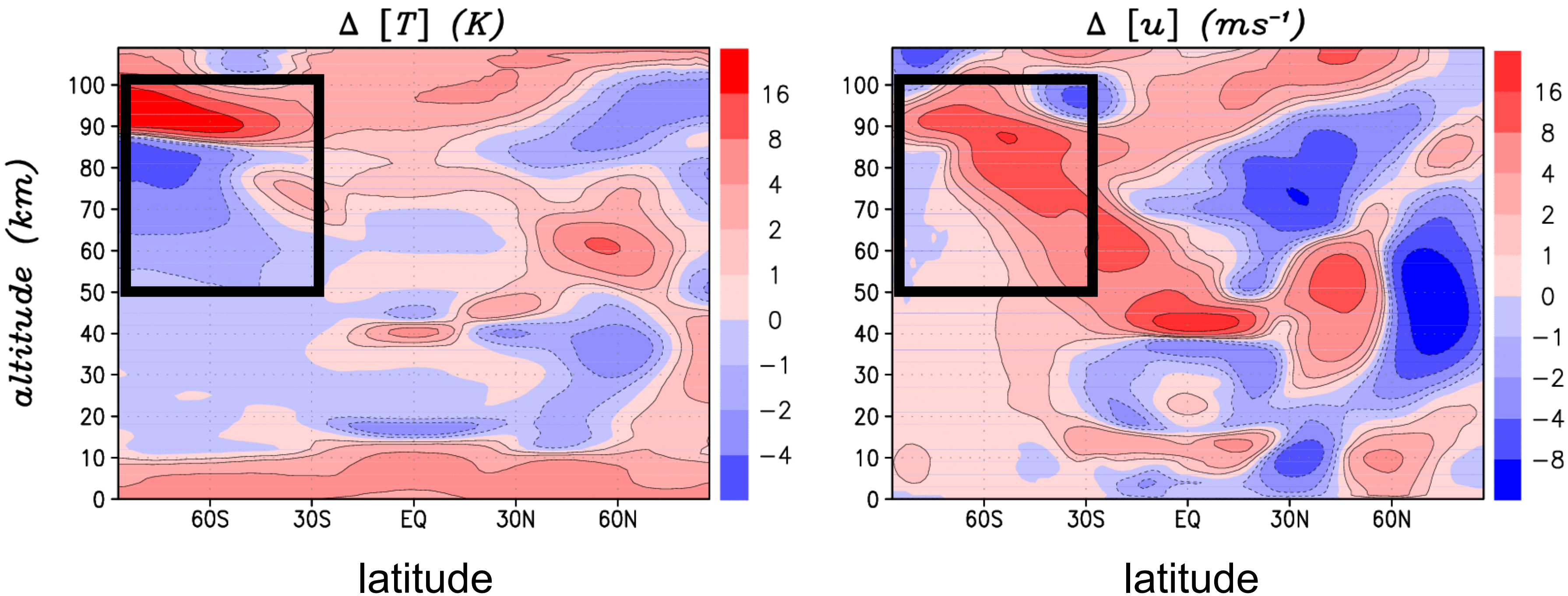


Non-rotational mesoscale KE and vertical flux of zonal momentum (CONTROL and model response)



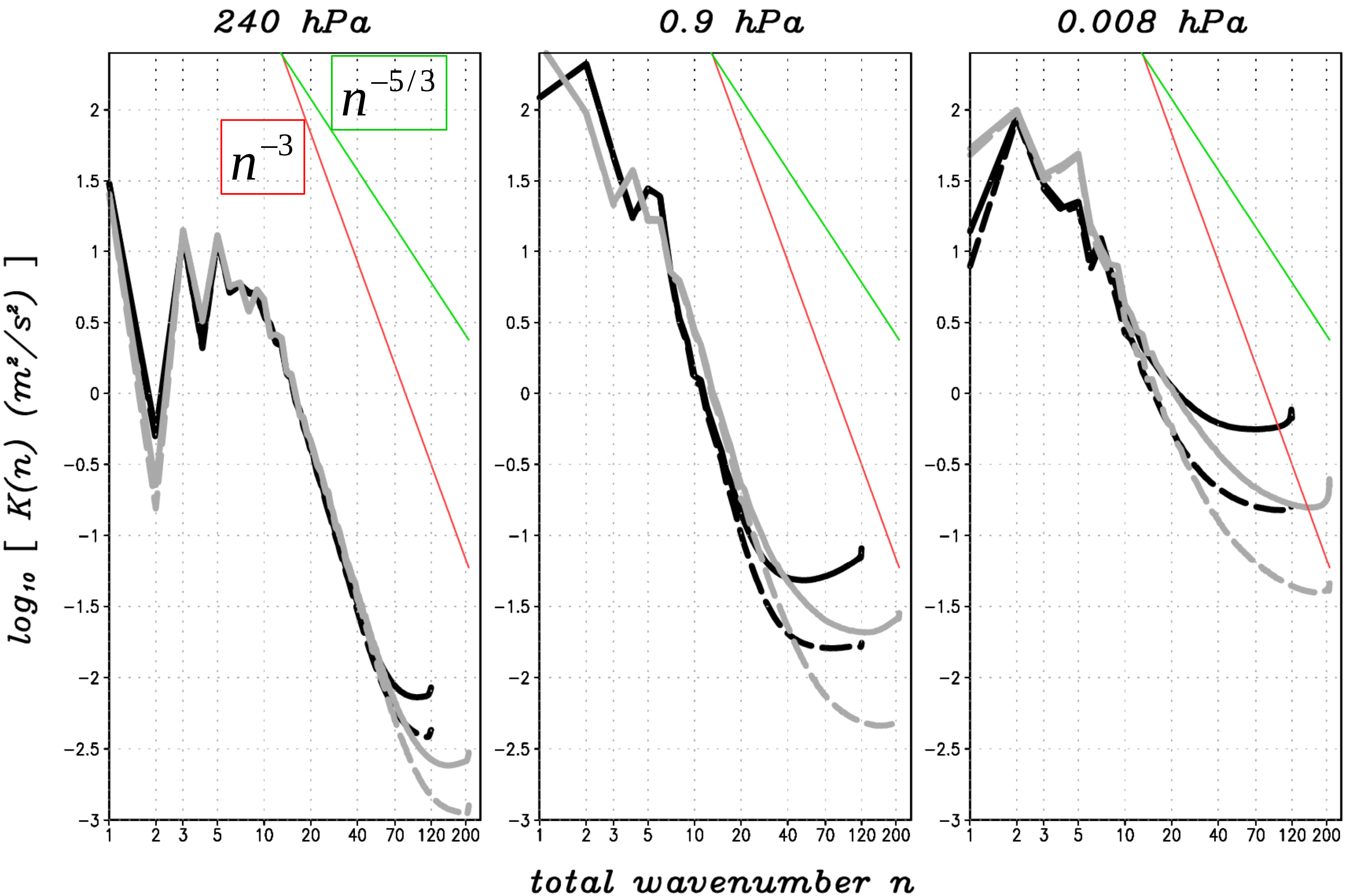
→ **strongest changes around the summer mesopause: GW damping shifts to lower altitudes due to larger amplitudes**

Zonal-mean model response of temperature and zonal wind



→ **strongest changes around the summer mesopause: GW damping shifts to lower altitudes due to larger amplitudes**

Deficiency of the turbulent friction parameterization: Unrealistic high-wavenumber end of the horizontal kinetic energy spectrum



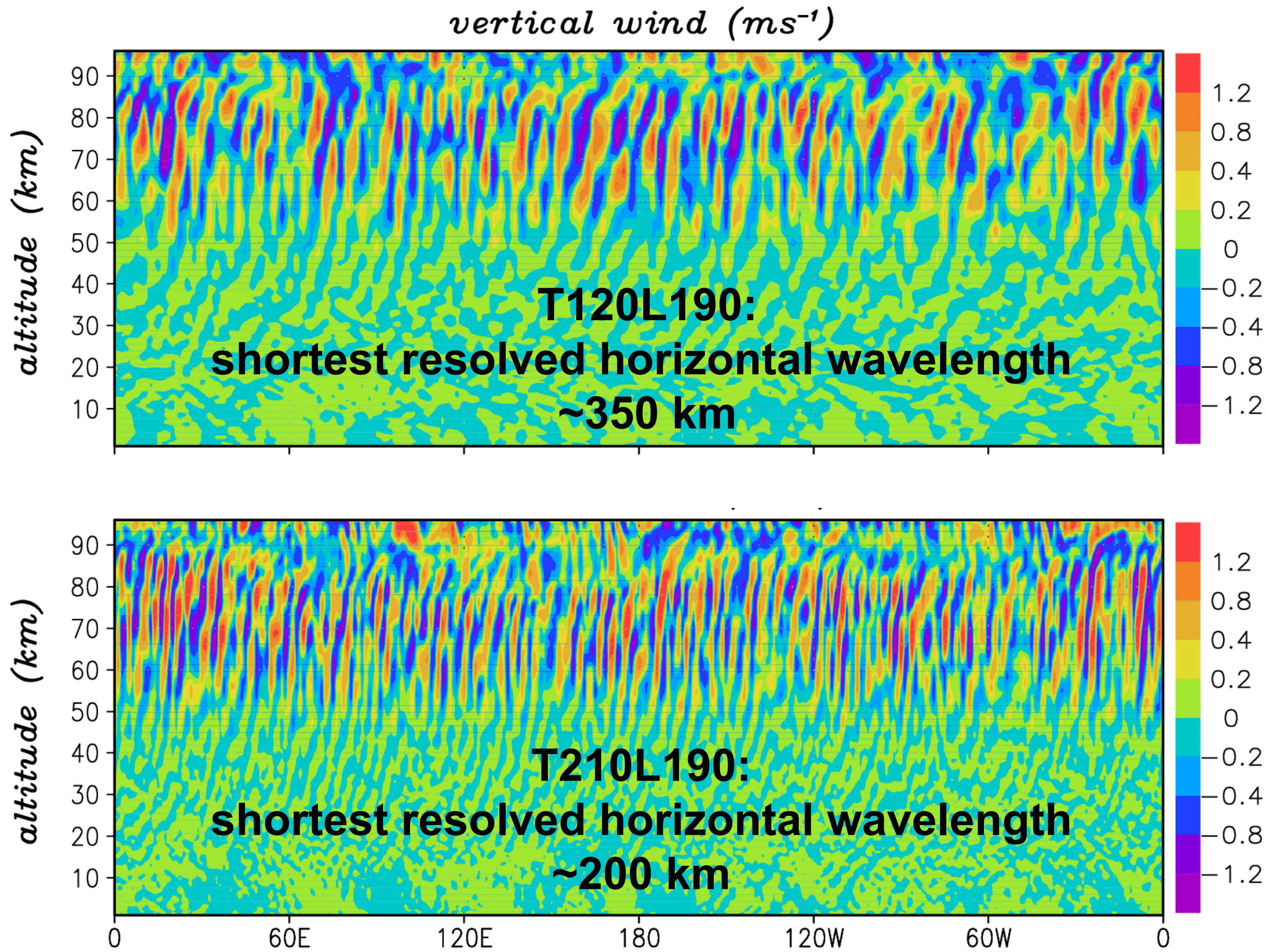
black: T120L190 (Smagorinsky)

grey: T210L190 (Smagorinsky)

—— rotational + non-rotational flow

- - - rotational flow only

Snapshots of the vertical wind in the summer hemisphere



→ **For higher resolution, the simulated gravity waves have smaller spatial scales (and higher frequencies).**

Extending the Smagorinsky scheme by an additional spectrally filtered linear harmonic diffusion (SFHD)

$$\rho^{-1}\nabla\left(\rho K_h \mathsf{S}_h\right) \longrightarrow \rho^{-1}\nabla\left(\rho K_h \mathsf{S}_h + \rho K_{h0}\mathsf{S}_{hf}\right)$$

$$\mathsf{S}_h = \{(\nabla + \mathbf{e}_z/a)\circ\mathbf{v}\} + \{(\nabla + \mathbf{e}_z/a)\circ\mathbf{v}\}^T$$

$$K_h = l_h^2\sqrt{|\mathsf{S}_h|^2 + S_0^2} \quad \text{(nonlinear, Smagorinsky-type)}$$

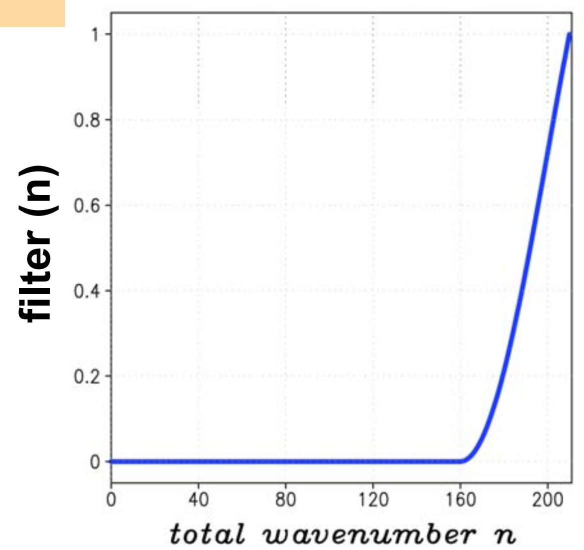
$$\mathbf{v} = \sum_{n=1}^{N}\sum_{m=-n}^{+n}\left(\psi_{nm}(\mathbf{e}_z\times\nabla Y_{nm}) + \chi_{nm}\nabla Y_{nm}\right)$$

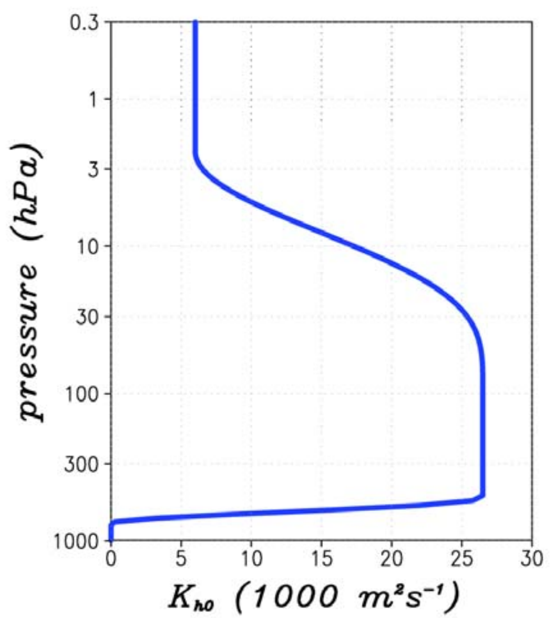
$$\mathsf{S}_{hf} = \{(\nabla + \mathbf{e}_z/a)\circ\mathbf{v}_f\} + \{(\nabla + \mathbf{e}_z/a)\circ\mathbf{v}_f\}^T$$

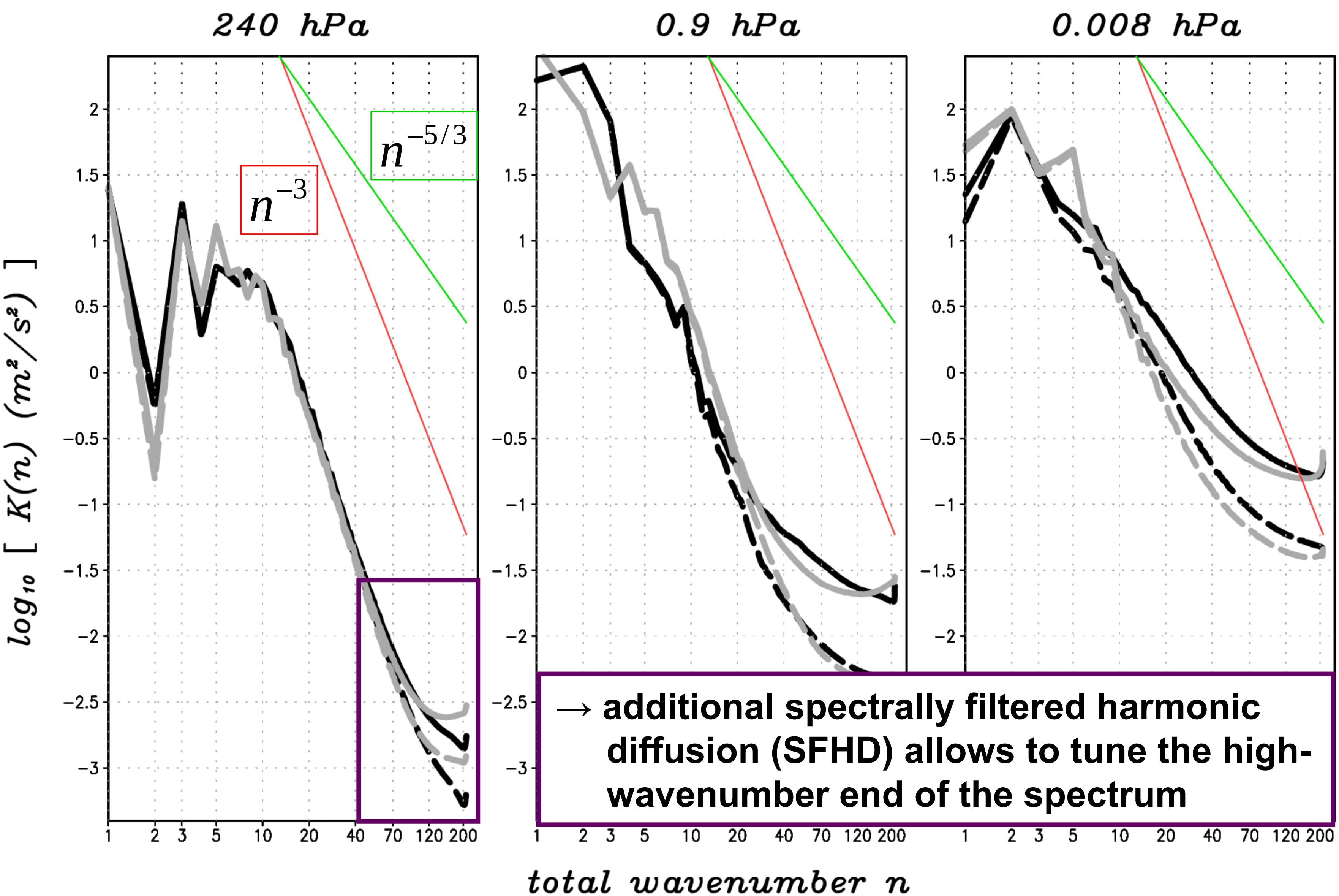
$$K_{h0} = \text{const (vertical coordinate)}$$

$$\mathbf{v}_f = \sum_{n=1}^{N}\text{filter}(n)\sum_{m=-n}^{+n}\left(\psi_{nm}(\mathbf{e}_z\times\nabla Y_{nm}) + \chi_{nm}\nabla Y_{nm}\right)$$

$$K_h(\mathsf{S}_h\nabla)\cdot\mathbf{v} \longrightarrow \underbrace{K_h(\mathsf{S}_h\nabla)\cdot\mathbf{v}}_{\geq 0} + \underbrace{K_{h0}(\mathsf{S}_{hf}\nabla)\cdot\mathbf{v}}_{\text{arbitrary sign}} \geq 0$$







→ **additional spectrally filtered harmonic diffusion (SFHD) allows to tune the high-wavenumber end of the spectrum**

black: T210L190 (Smagorinsky+SFHD)

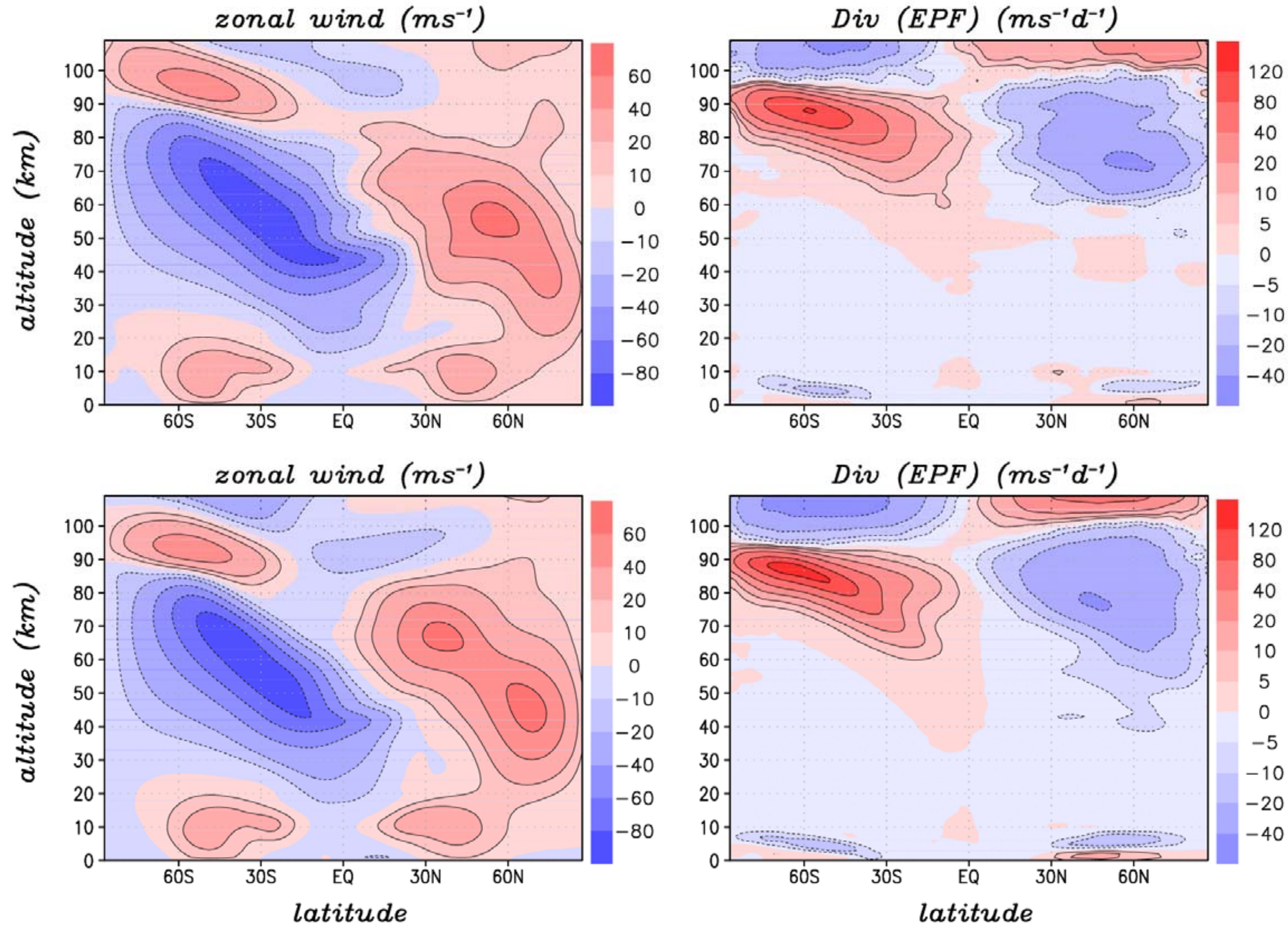
grey: T210L190 (Smagorinsky)

——— rotational + non-rot. flow

- - - rotational flow only

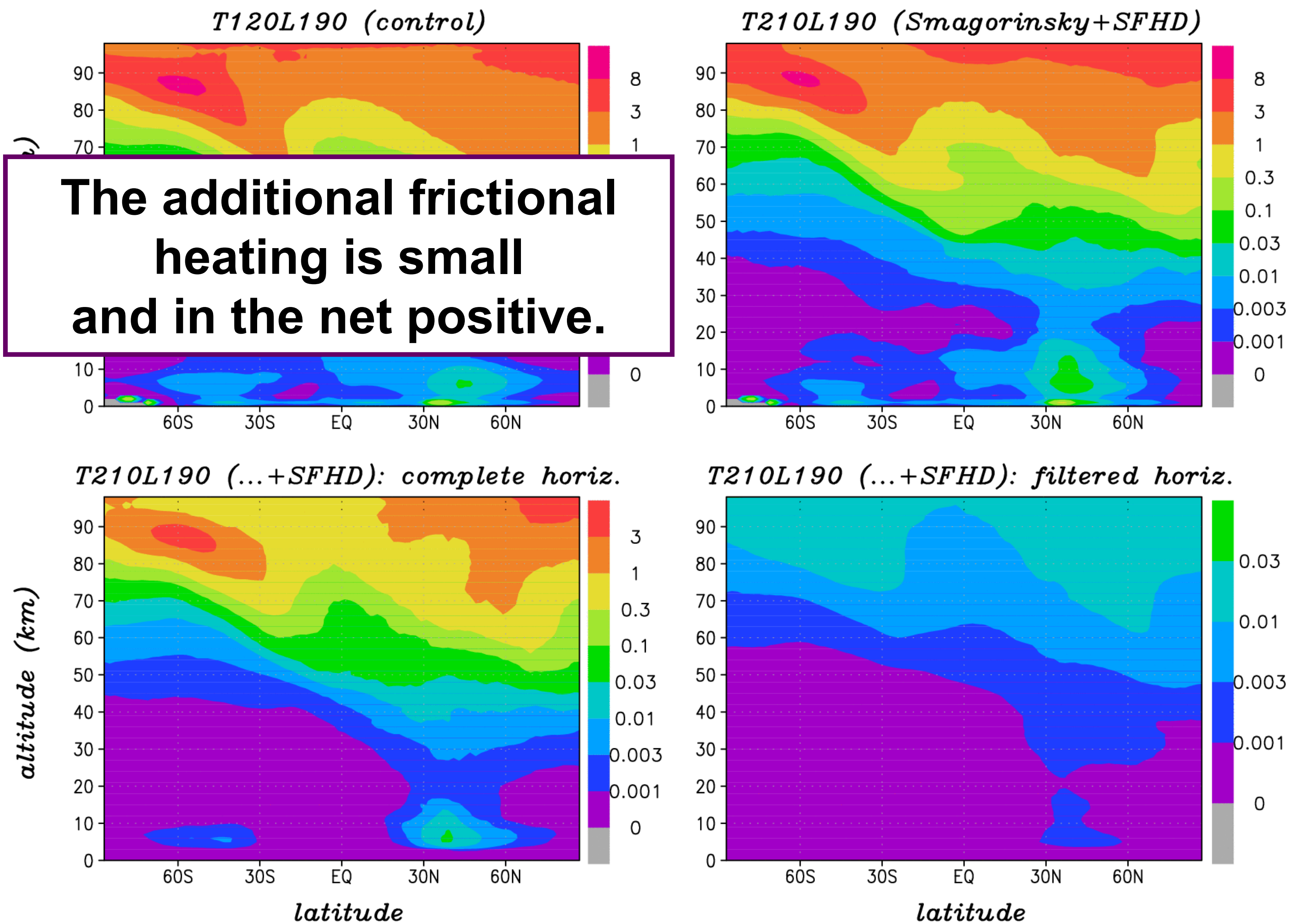
T210L190
with
additional
SFHD

T120L190
with pure
Smagorinsky
scheme



… while maintaining the resolved GW drag in the middle atmosphere

frictional heating (K/d)



The additional frictional heating is small and in the net positive.

Outline

Recapitulate parameterizations of turbulent friction and frictional heating (shear production) that ensure consistency with the conservation laws:

- Diagnostic turbulence model
  (quasi-stationary TKE budget)

- **Prognostic turbulence model based on the full TKE budget**

Prognostic turbulence model: Consistent formulation including also horizontal diffusion in the TKE-budget
(Shaw, Becker, and McFarlane, in preparation)

$$d_t\mathbf{v} = -f\,\mathbf{e}_z \times \mathbf{v} - \frac{\nabla p}{\rho} + \frac{\partial_z(\rho K_z \partial_z \mathbf{v})}{\rho} + \frac{\nabla(\rho K_h \mathsf{S}_h)}{\rho}$$

$$d_t h = \frac{d_t p}{\rho} + Q_{rad} + Q_{lat} + \frac{c_p}{\rho}\partial_z(\rho \frac{T}{\Theta} K_z \partial_z \Theta) + K_z N^2 + \epsilon_z + \epsilon_h$$

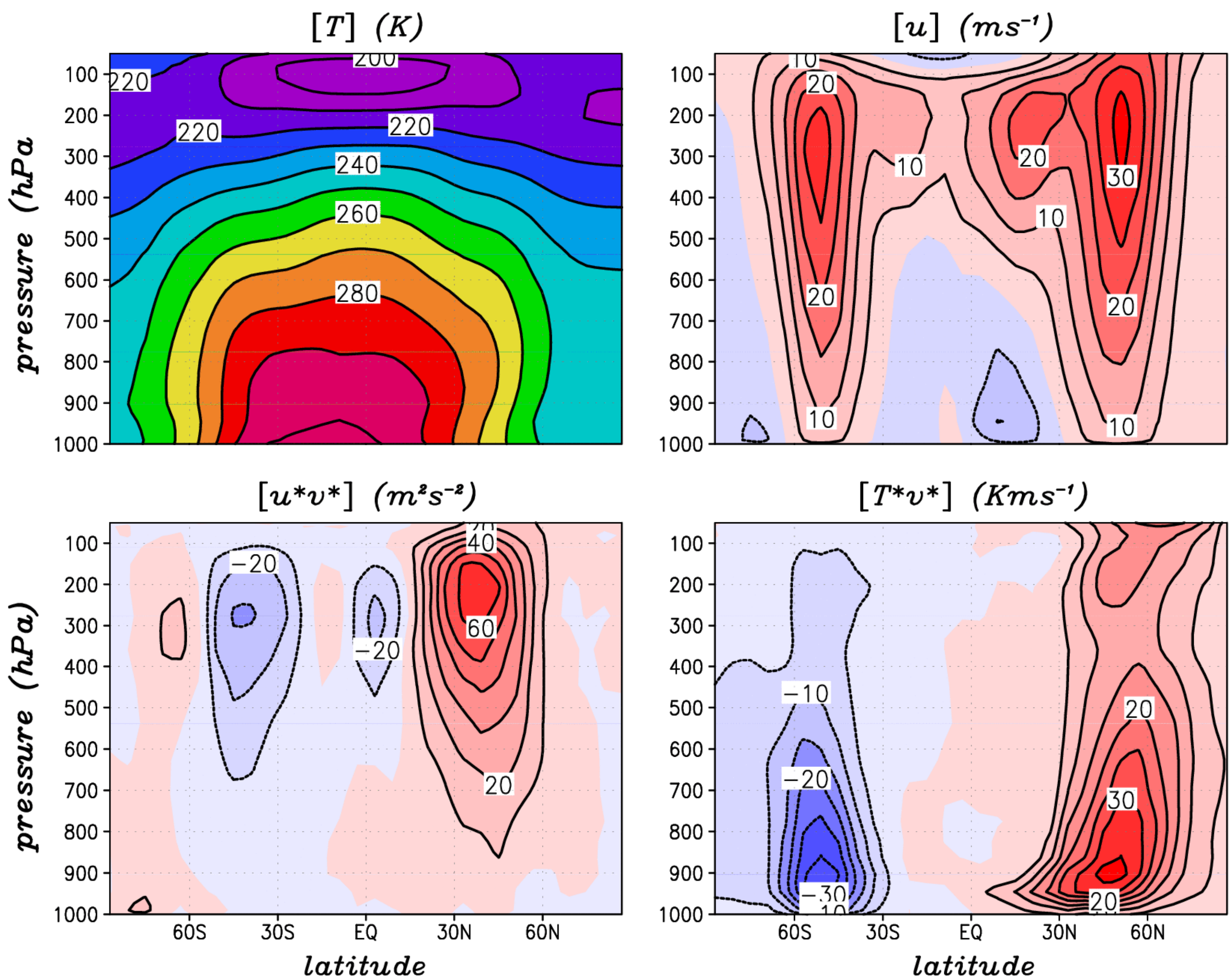
$$d_t k = \frac{\partial_z(\rho K_z \partial_z k)}{\rho} + \frac{\nabla\cdot(\rho K_h \nabla k)}{\rho} + K_z(\partial_z \mathbf{v})^2 + K_h(\mathsf{S}_h \nabla)\cdot\mathbf{v}$$
$$- K_z N^2 - (\epsilon_z + \epsilon_h) \quad \text{(at full levels)}$$

$$K_z \propto l_z \sqrt{k}\,, \quad \epsilon_z \propto l_z^{-1} k^{3/2}\,, \quad l_z = \left(\frac{1}{\kappa z} + \frac{1}{l_a}\right)^{-1} \sqrt{F(R_i)}$$

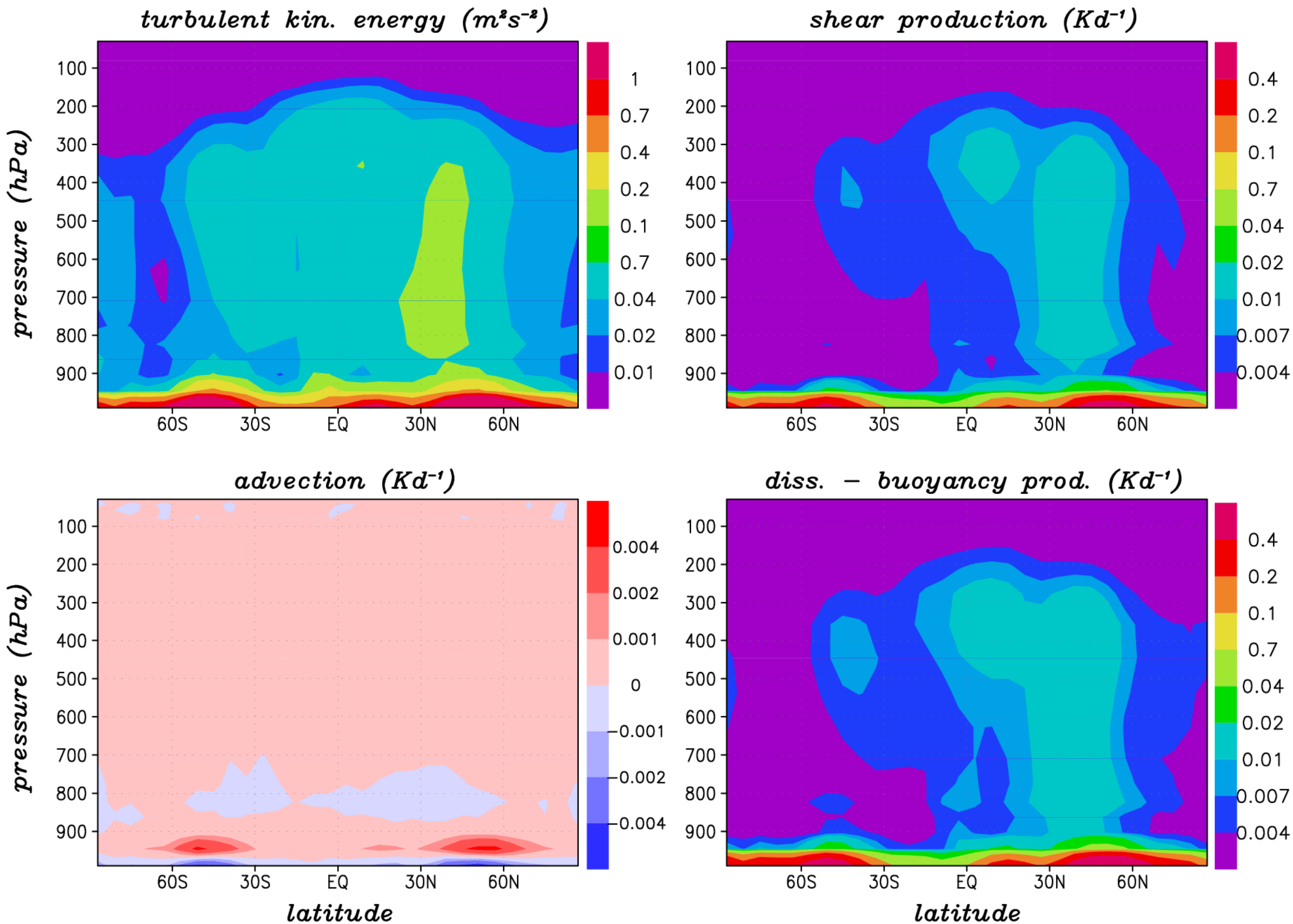
$$K_h = l_h \sqrt{k}\,, \quad \epsilon_h \propto l_h^{-1} k^{3/2}\,, \quad l_h = l_h(z)\sqrt{1 + \alpha F(R_i)}$$

- dynamic boundary condition for k: $(\rho K_z \partial_z k)_{surf} = 0$
- dyn. boundary conditions for $\mathbf{v}$ and h, as well as shear production rates and S_h like in the prognostic case

Test simulations with T31L30 (January)



Test simulations with T31L30 (January)



SUMMARY

A consistent simulation of the Lorenz energy cycle requires all contributions from turbulent friction to be formulated in accordance with the conservation laws.

The Lorenz energy is likely to intensify due to tropospheric climate change. A mechanistic GCM with high spatial resolution and a Ri-dependent Smagorinsky-type turbulence parameterization describes the resulting change in gravity-wave driving of the mesosphere.

At present, a realistic high-wavenumber end of the simulated energy spectrum can only be obtained by some empirical spectral damping in addition to the Smagorinsky scheme.

Also a fully prognostic scheme for vertical and horizontal diffusion may not solve this problem.

Some basically new parameterization of unresolved dynamical scales is required that provides an analytical filtering of the equations of motion.