Statistical significance tests for climate networks

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Summary

Complex networks present a promising novel tool for climate data analysis. We introduce significance tests to quantify the robustness of measured network properties to uncertainties and illustrate them for the betweenness field of a surface air temperature (SAT) network.

Network construction

(i) Start with climatological field of *N* time series.
(ii) Calculate correlation or mutual information matrix.
(iii) Identify:

- **Vertices** *v* with grid points (regions),
- **Edges** (*v*, *w*) with strongly and significantly interrelated pairs of time series (thresholding).

Network surrogates

(i) **Configuration model**: Random networks with prescribed degree field of empirical network (Fig. 1a).
(ii) **Surrogate data set:** Networks constructed from surrogate data sets with each time series replaced by one of its twin surrogates (Fig. 1b).

Betweenness

Betweenness measures the centrality of a region *v* by counting the number of shortest paths passing it (Fig. 2).

$$BC_v = \sum_{i,j\neq v}^{N} \frac{\sigma_{ij}(v)}{\sigma_{ij}}$$

σ_{ij} gives the total number of shortest paths from *i* to *j*, and $\sigma_{ij}(v)$ the number of such shortest paths including *v*.

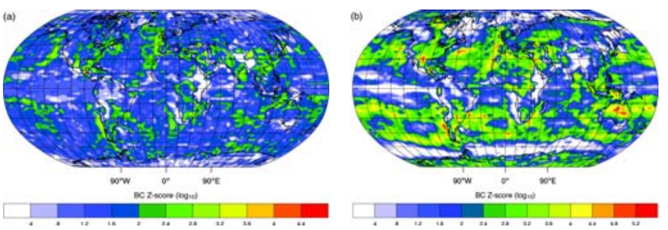

Figure 1. Betweenness z-Score for a) configuration model and b) twin surrogate networks.

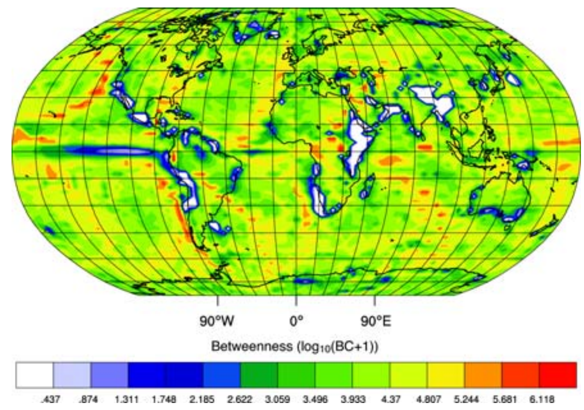

Figure 2. Betweenness field for HadCM3 SAT data set.

Significance testing

(i) Generate ensemble of $M = 100$ network surrogates.
(ii) Calculate betweenness for all ensemble members.
(iii) Obtain betweenness z-Score field for empirical network with respect to ensemble distribution.
(iv) Regions with high z-Score have a significant betweenness with respect to surrogate model (Fig. 1).

Conclusions

High betweenness structures in SAT network (Fig. 2) are found to be significant (high z-Score in Fig. 1).

Improved null hypotheses need to be developed to allow for more powerful significance tests.

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References

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