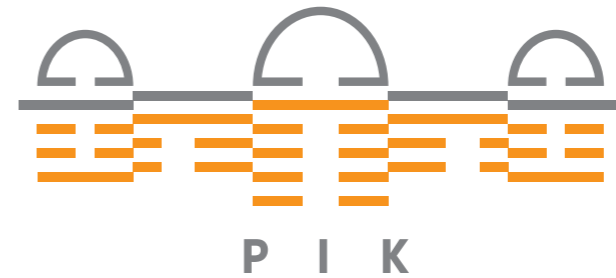


# Complex network approach for recurrence analysis of time series



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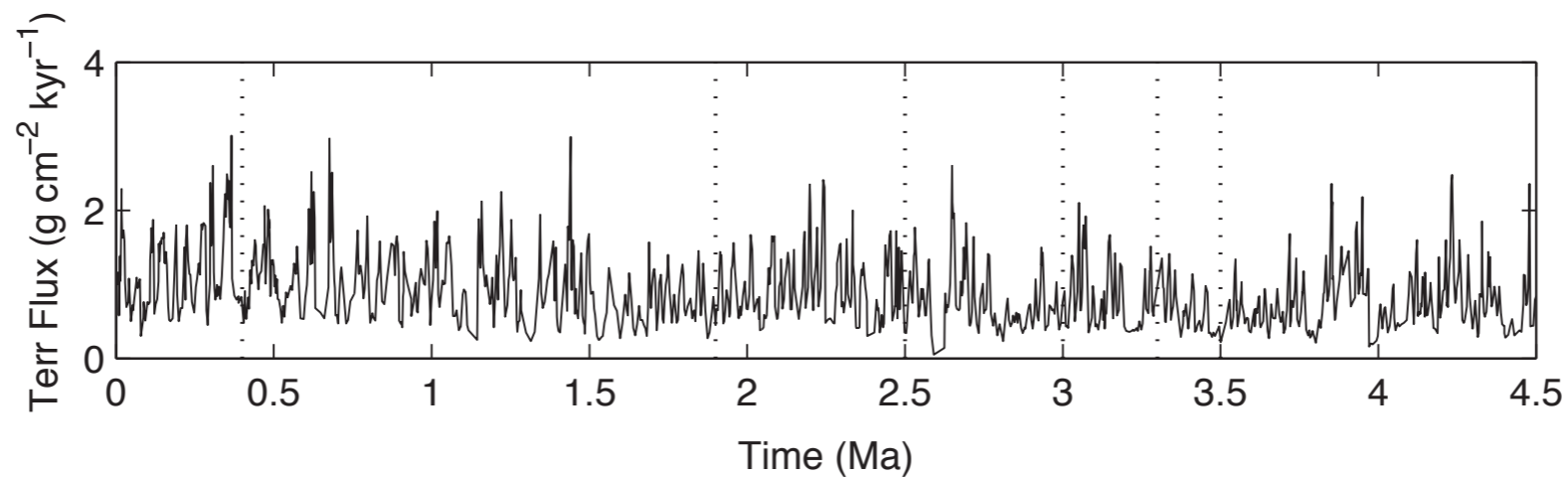
# Outline

- Introduction
- Conceptual foundation
- Quantifying recurrence networks
- Application to climatological time series
- Conclusions & Outlook

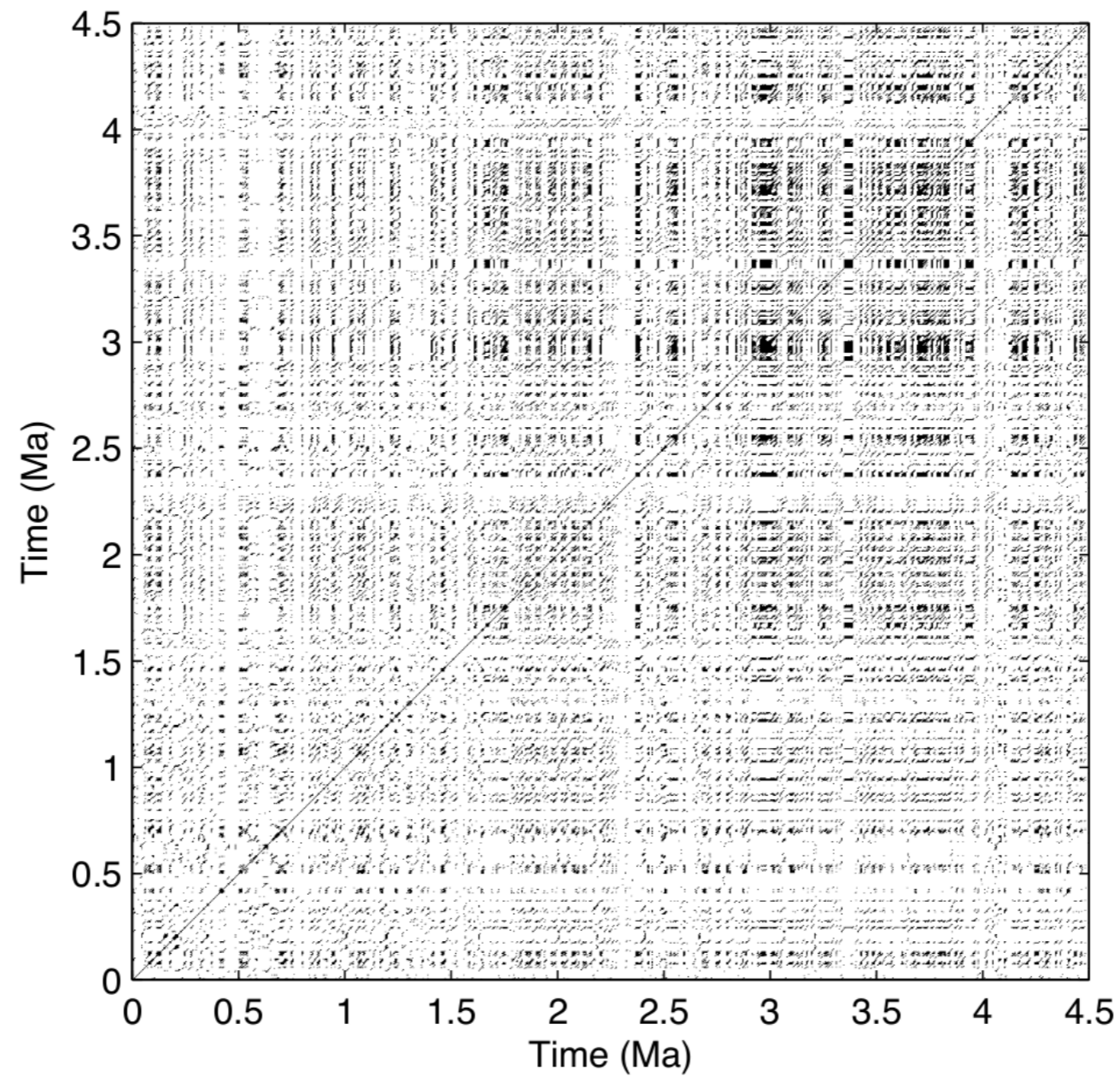
# Introduction

# What do we start with?

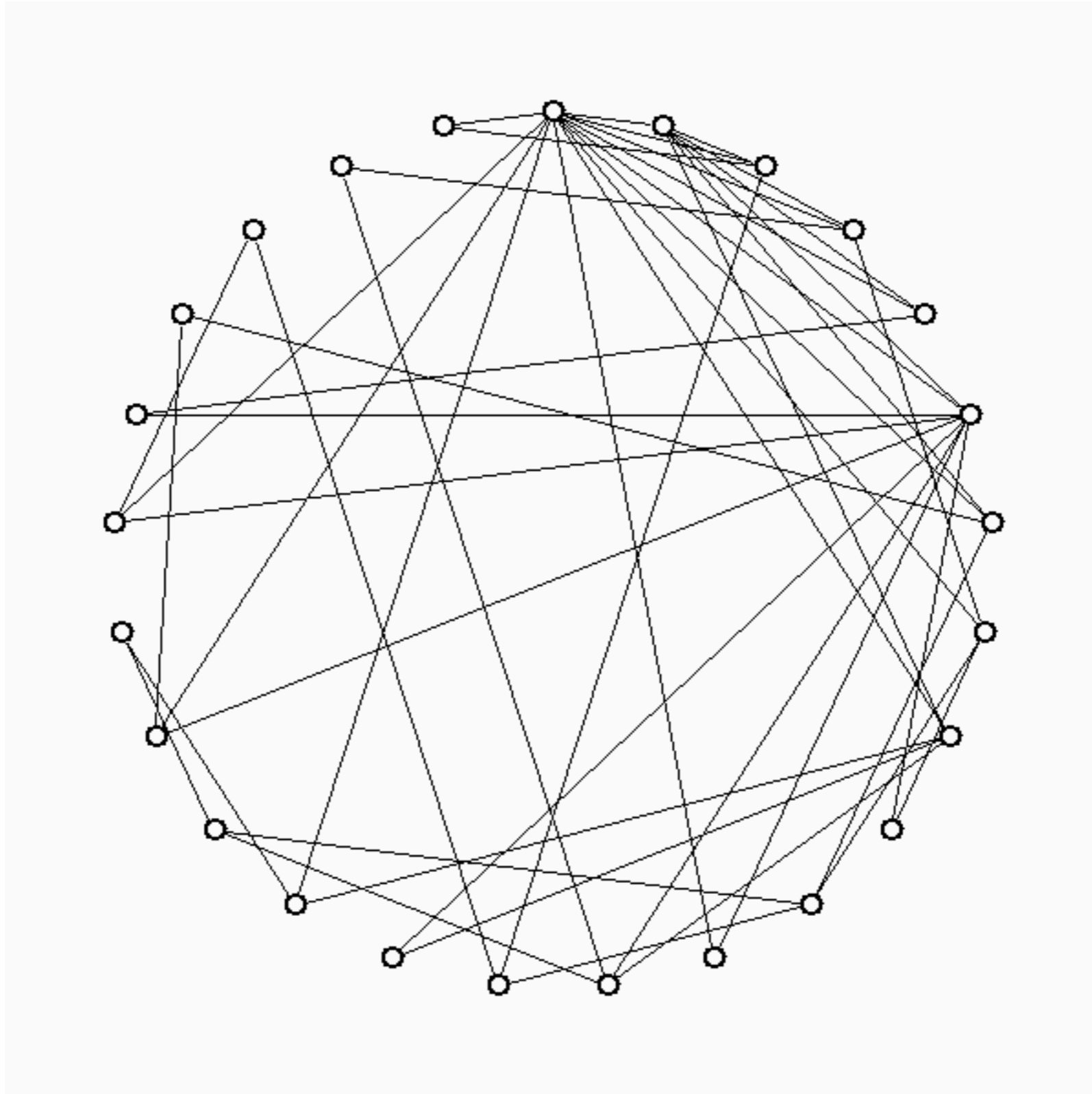
Time series  $x(t)$



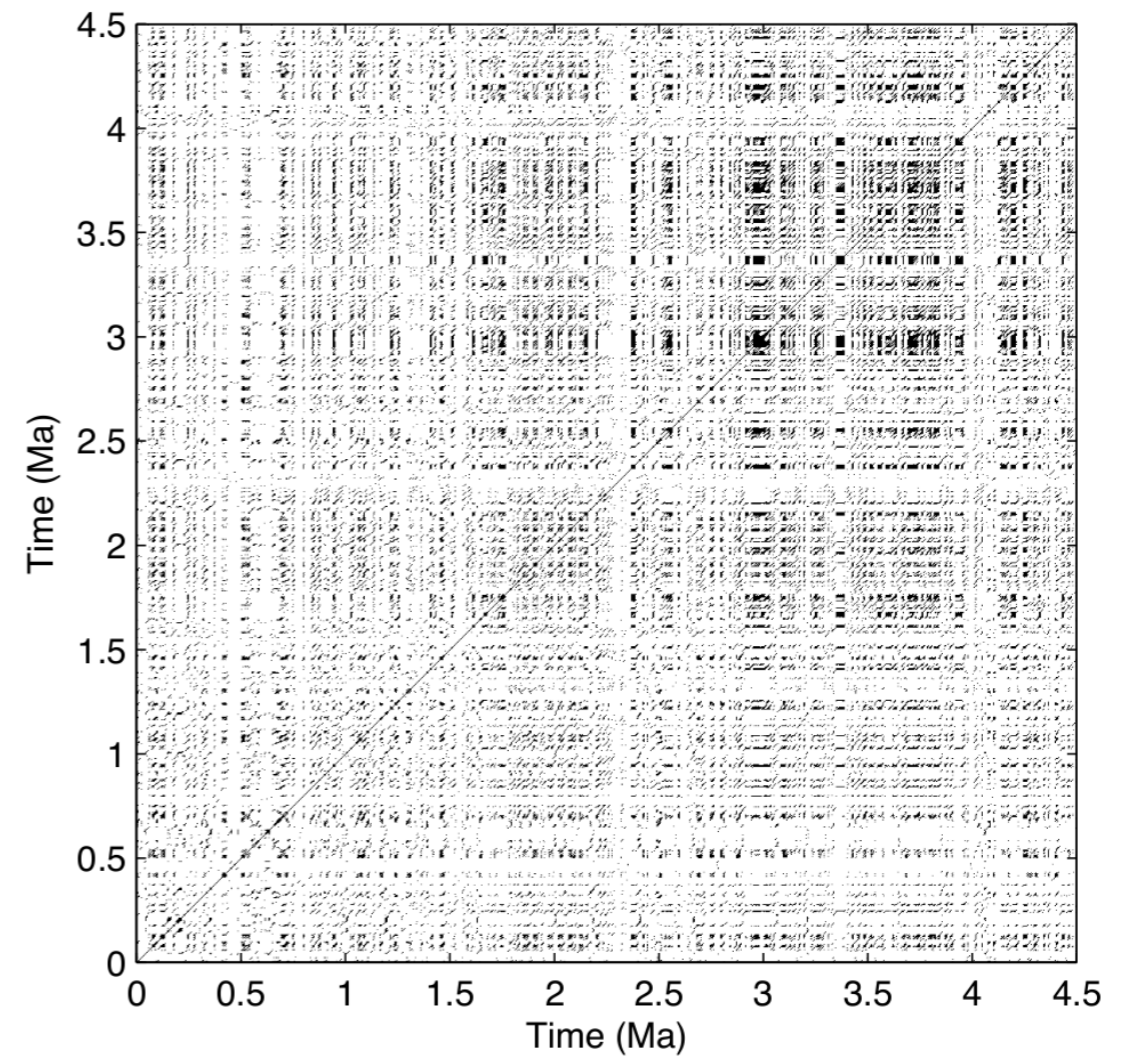
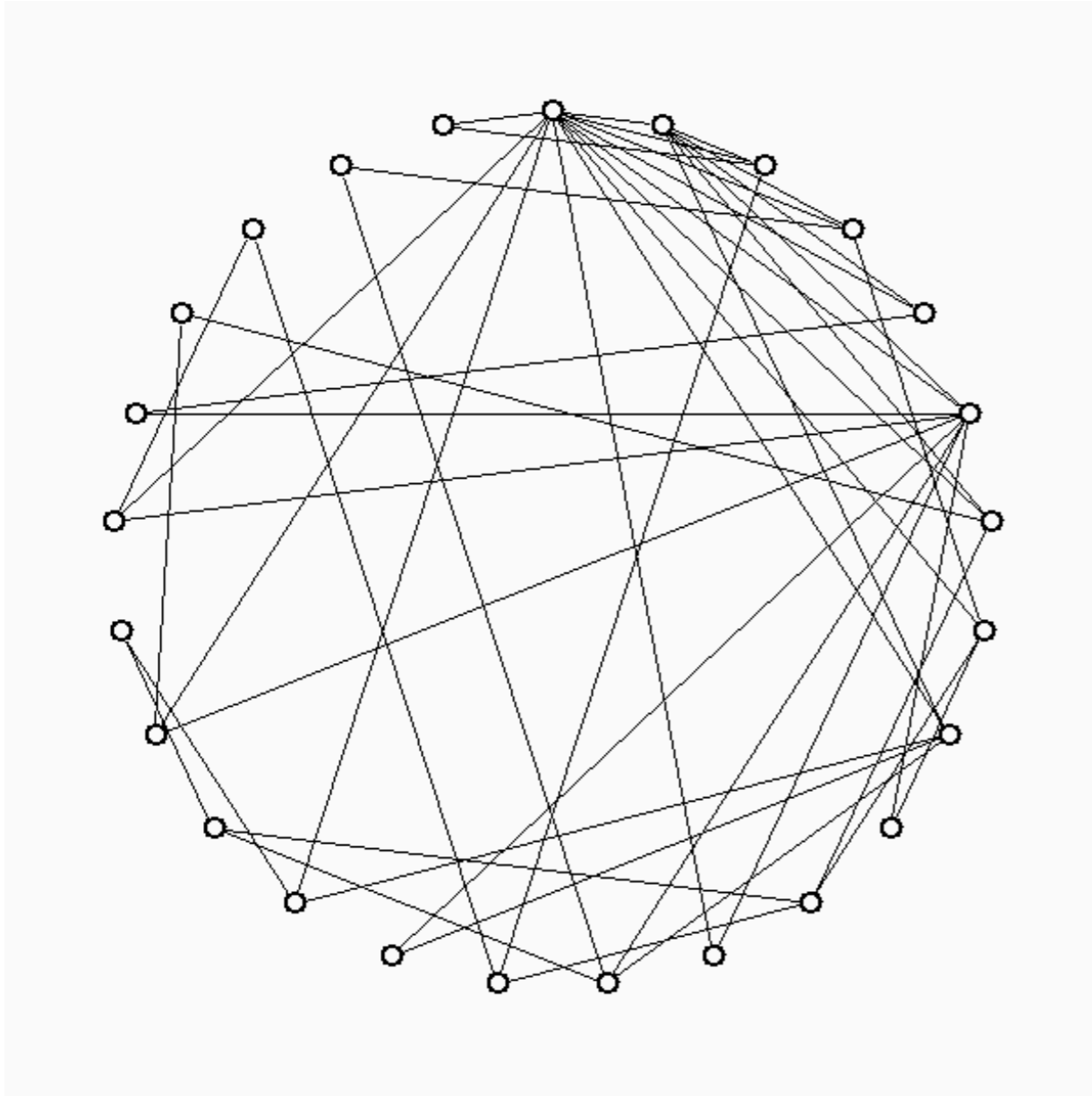
# Recurrence plot



# Complex network



# Duality ?



# Analogies

Recurrence plot	Complex network
Recurrence matrix <b><i>R</i></b>	Adjacency matrix <b><i>A</i></b>
RQA	Network statistics



# Recurrence network

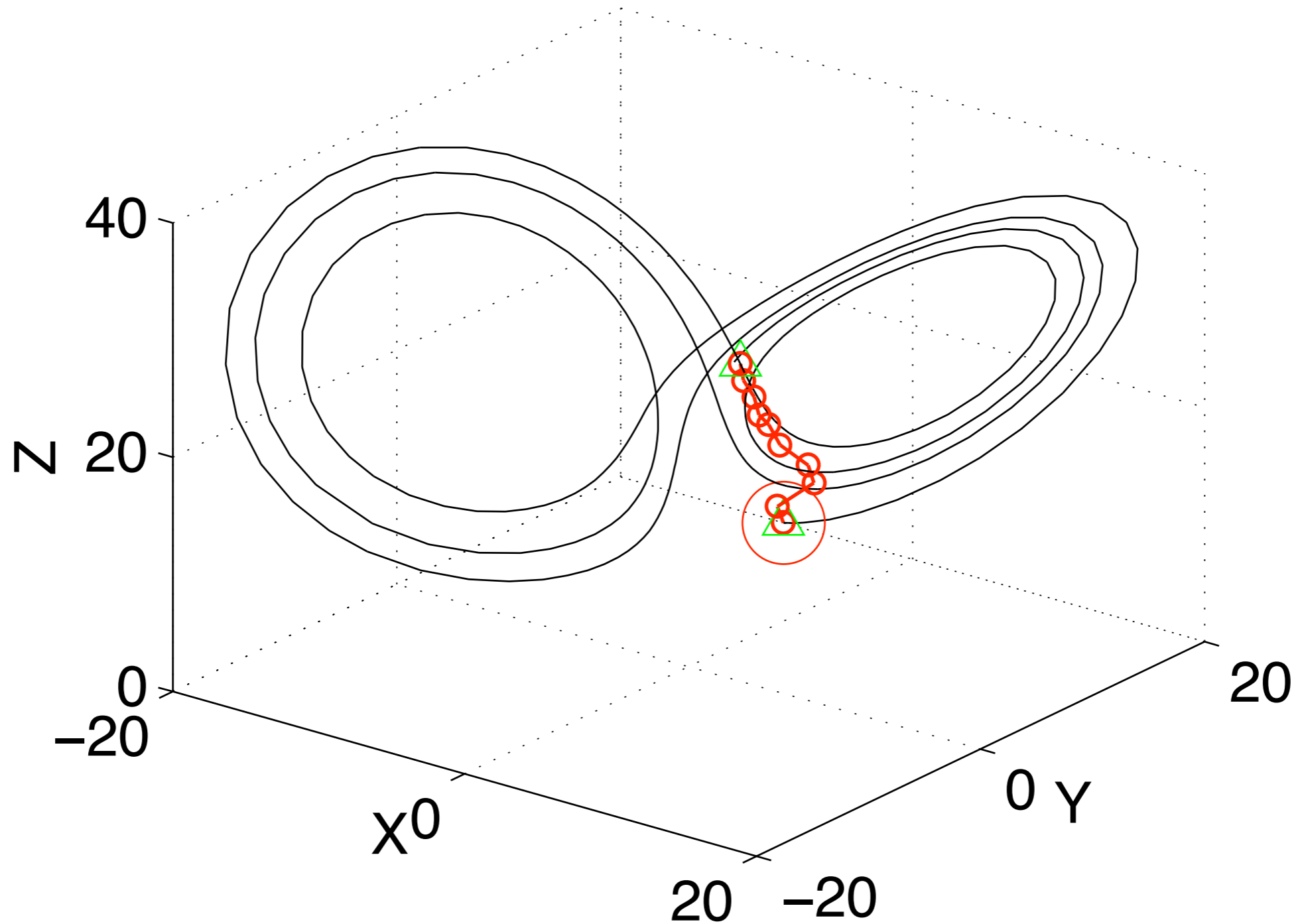
$$A = R - I$$

**Conceptual foundation**

# Equivalence I

Phase space	Recurrence network
State $x(i)$	Vertex $i$
Recurrence $\ x(i) - x(j)\  < \varepsilon$	Edge $(i, j)$

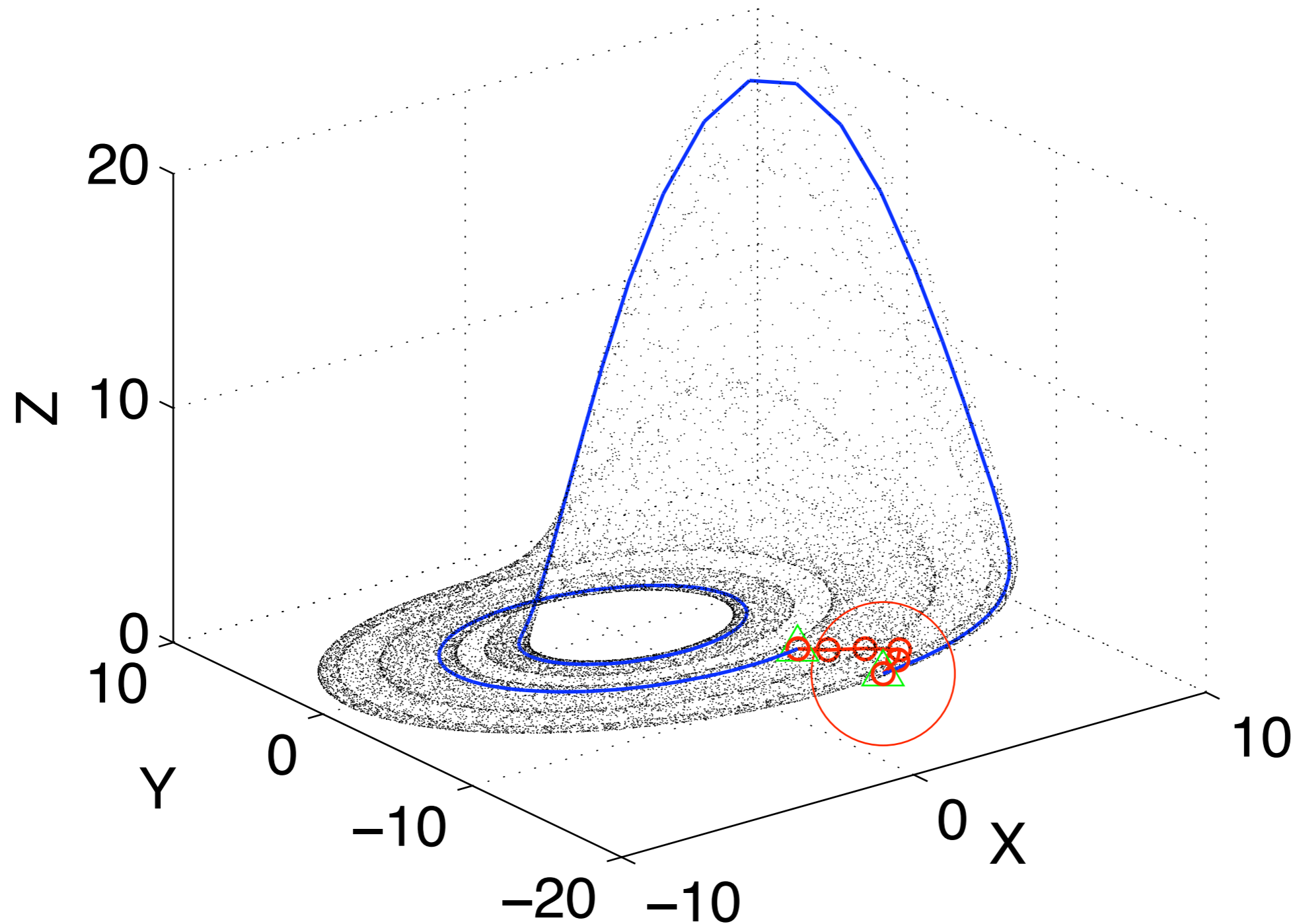
# Phase space (Lorenz)



# Equivalence II

Phase space	Recurrence network
$\varepsilon$ - ball sequence	Path

# Phase space (Rössler)



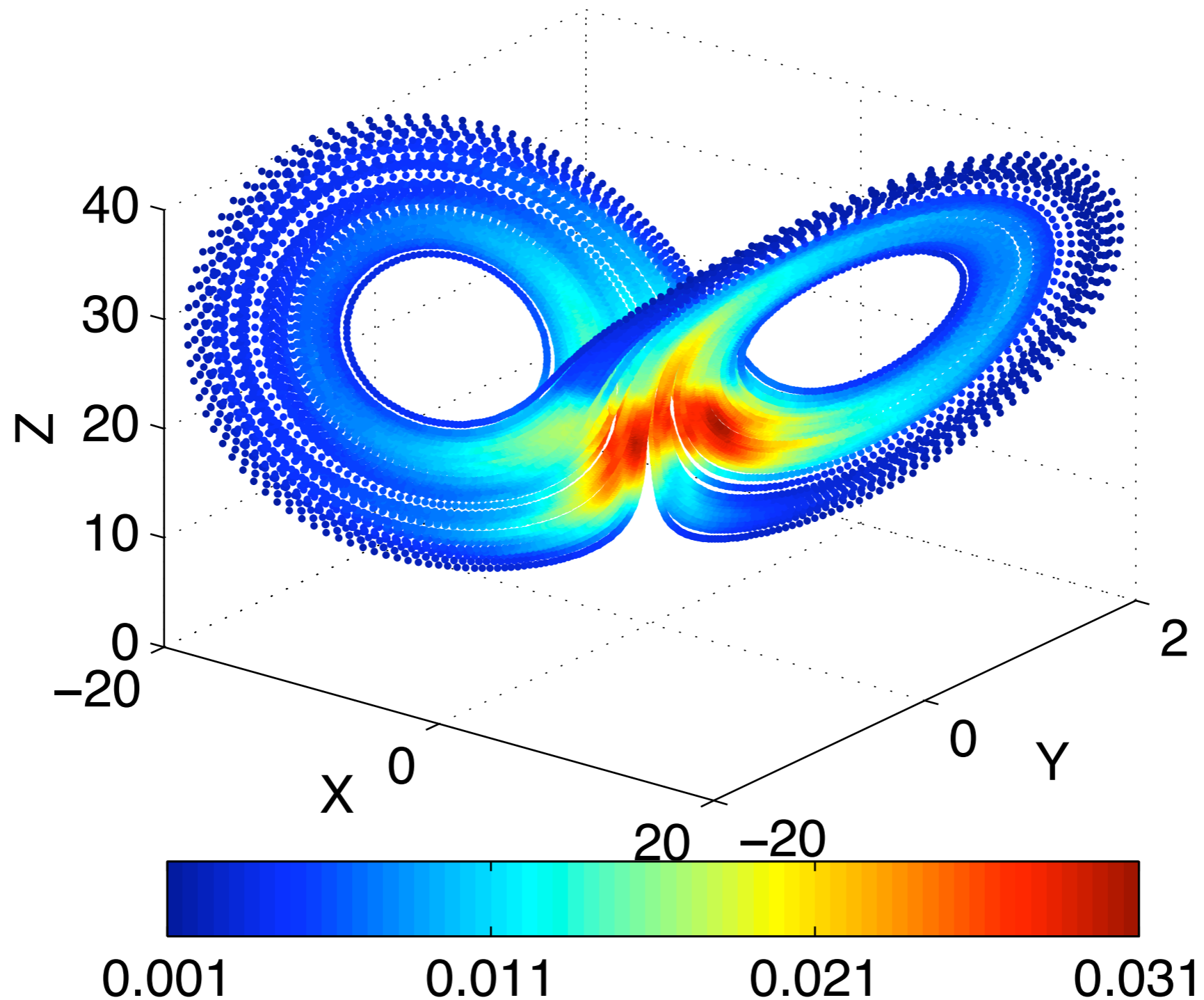
# Crucial properties

- Time ordering is lost - Arbitrary vertex labeling
- Strong spacial constraints
- Recurrence network reflects attractor geometry

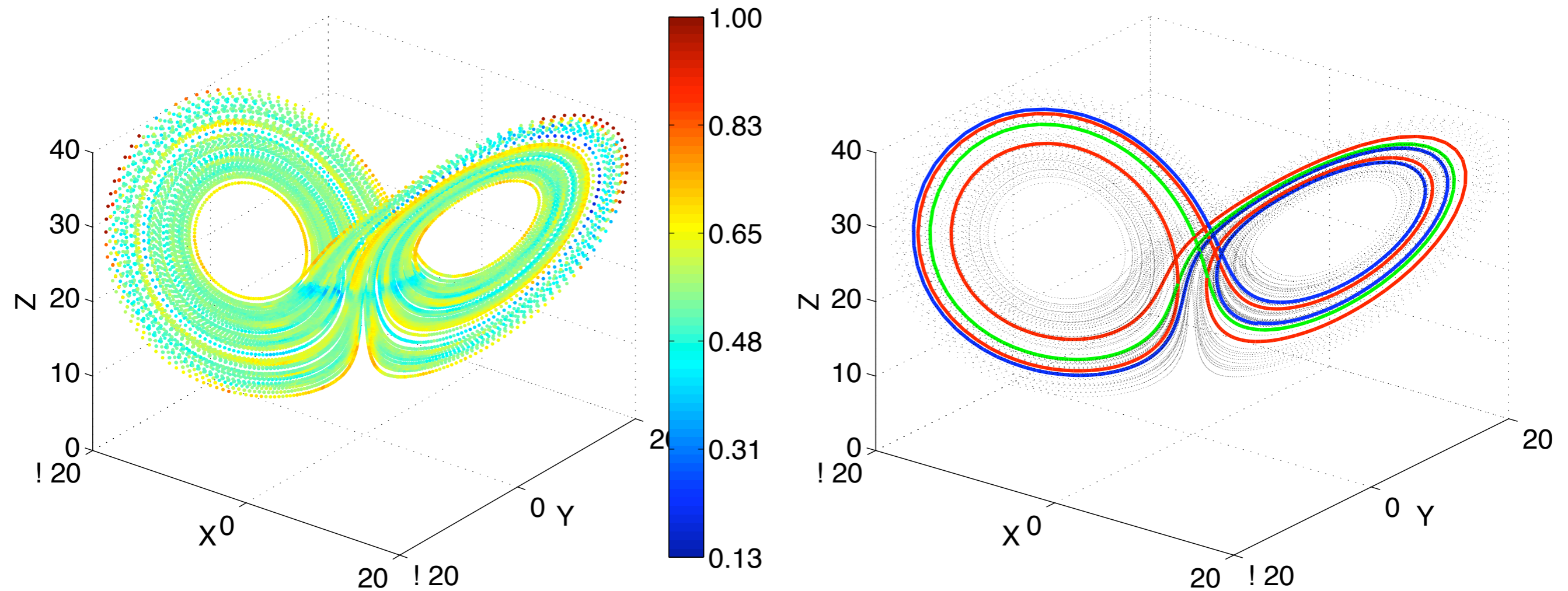
# Quantifying recurrence networks



# Degree centrality



# Clustering coefficient

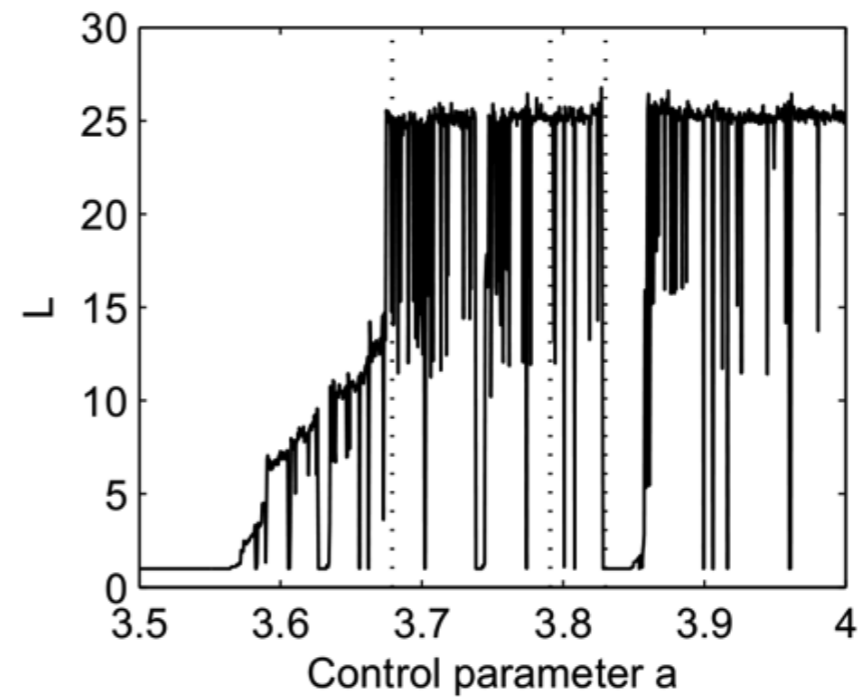
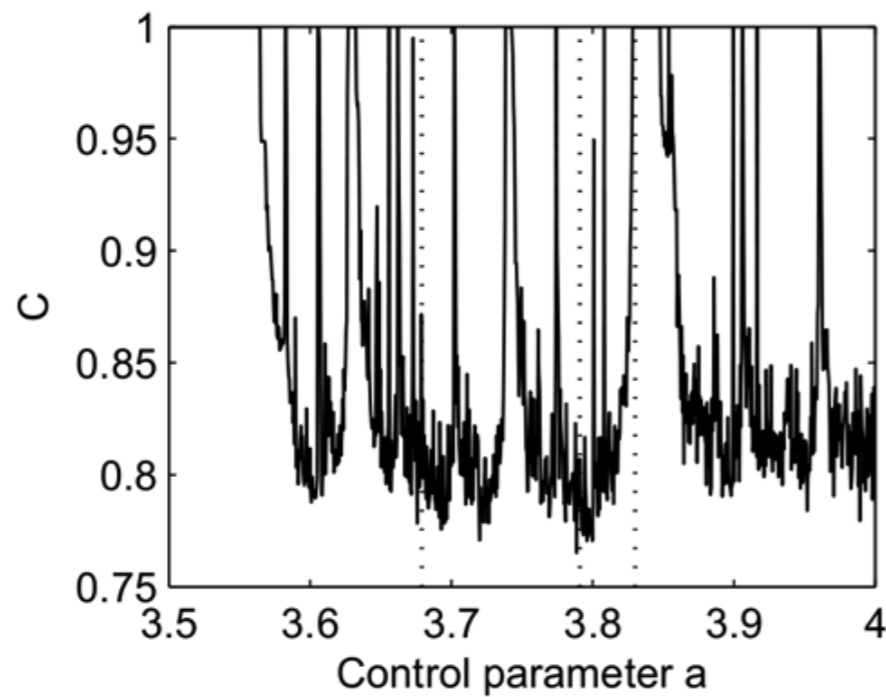
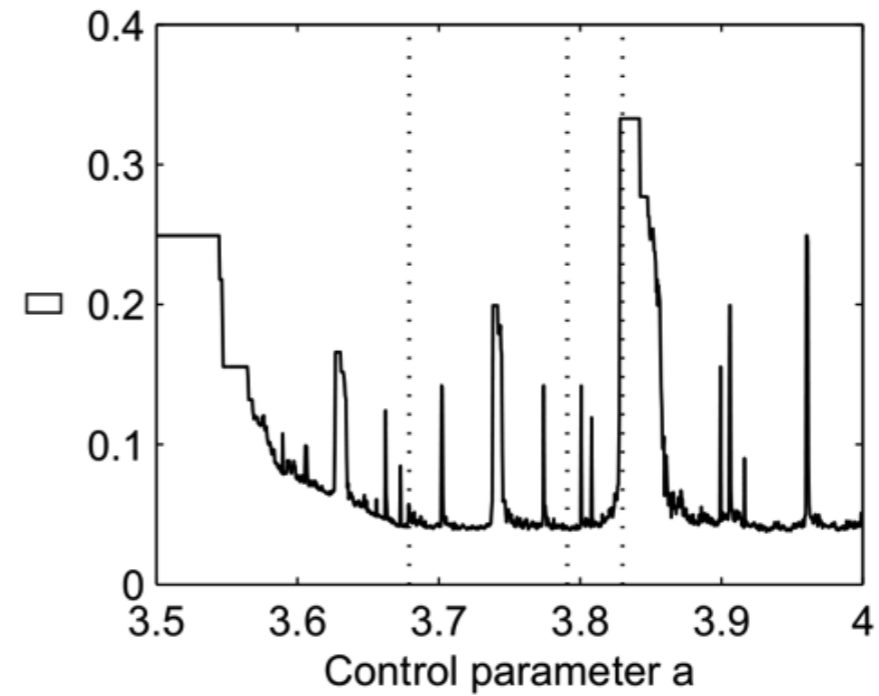
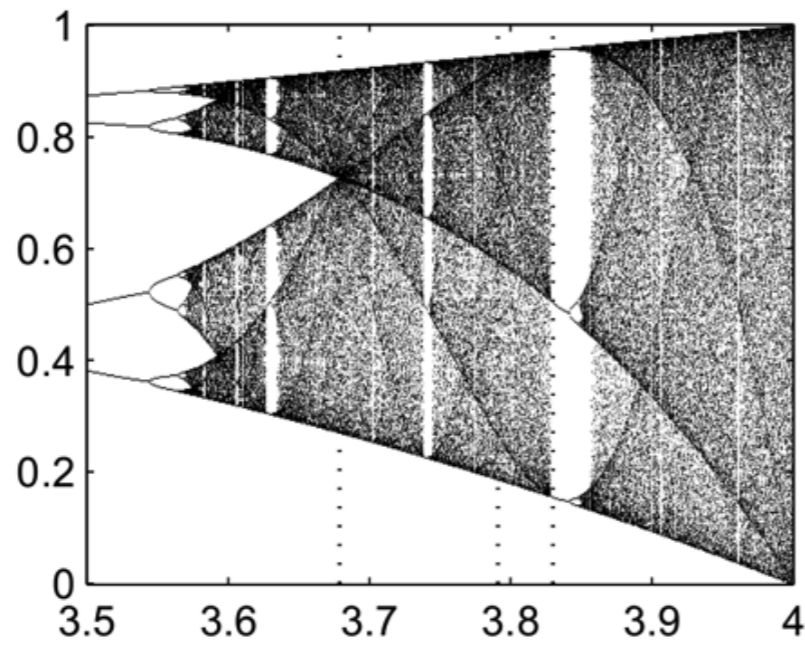


# Scalar measures

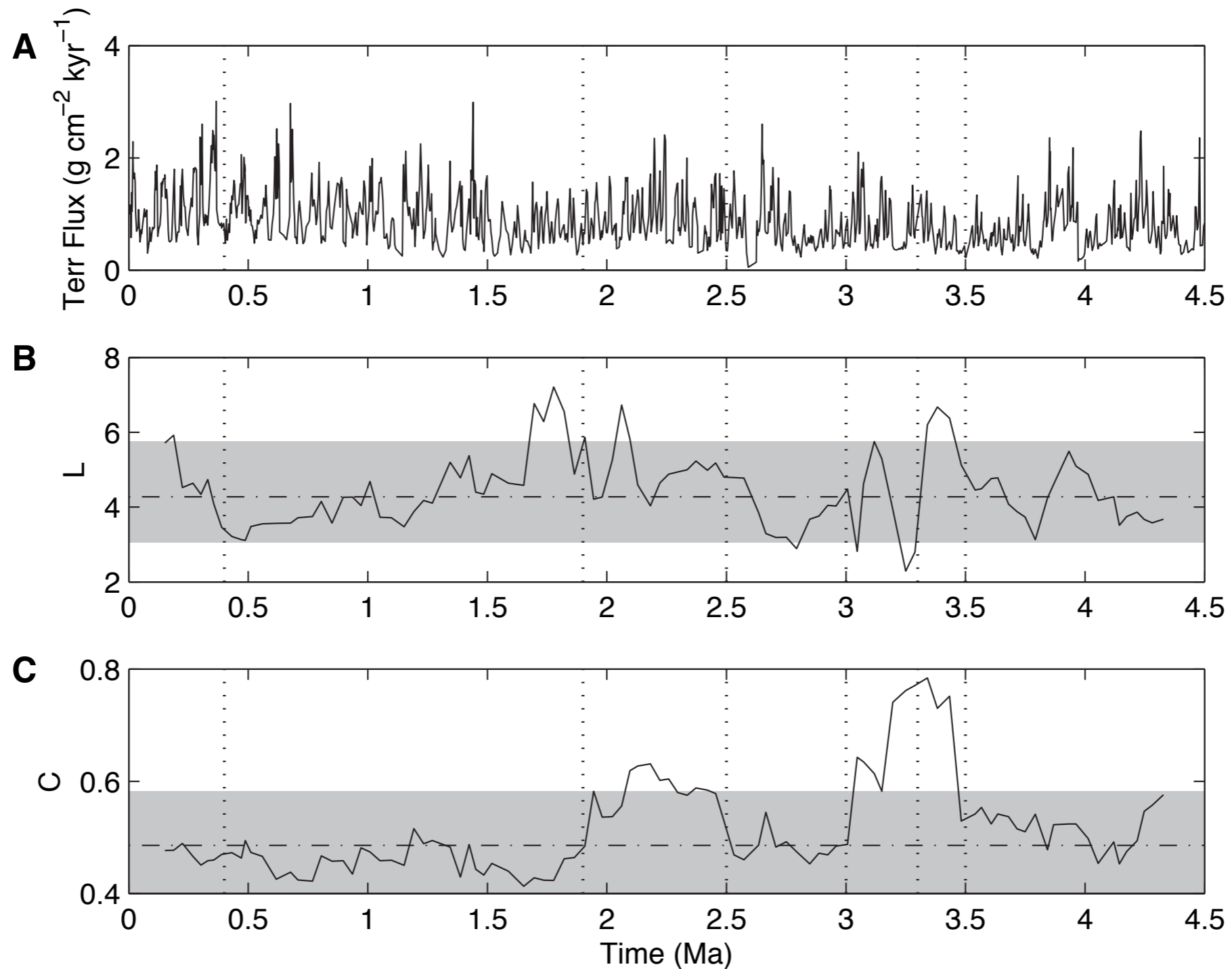
Recurrence network	Phase space
Edge density	Recurrence rate
Clustering	Higher order density
Average path length	Mean separation

# Applications

# Logistic map



# Terrigenous dust flux



# Lessons learned

- Clustering coefficient is sensitive to periodicity (longer time scales)
- Average path length sensitive to transition periods (shorter time scales)

# Conclusions & Outlook



# Advantages of recurrence networks

- Method does NOT require equidistant time scale
- Applicable to univariate and multivariate time series
- Simple significance testing

# Take home messages

- Toolbox of complex network theory available to time series analysis
- RNs allow an intuitive and natural interpretation of results
- Complementary to established methods of time series analysis (linear, nonlinear, RQA)

# Related publications

N. Marwan, J.F. Donges, Y. Zou, R.V. Donner and J. Kurths, Complex network approach for recurrence analysis of time series (2009), arXiv:0907.3368v1 [nlin.CD],

R.V. Donner, Y. Zou, J.F. Donges, N. Marwan and J. Kurths, Recurrence networks - A novel paradigm for nonlinear time series analysis (2009), in preparation.

**Thank you!**