



THE NELSON INSTITUTE  
FOR ENVIRONMENTAL STUDIES  
University of Wisconsin-Madison

*Where Environmental Leadership Begins*



# **Assessing Large-Scale Climate Feedback from the Observation ---- The GEFA Approach (Generalized Equilibrium Feedback Analysis)**

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# Tropical Ocean-Atmos. Interaction

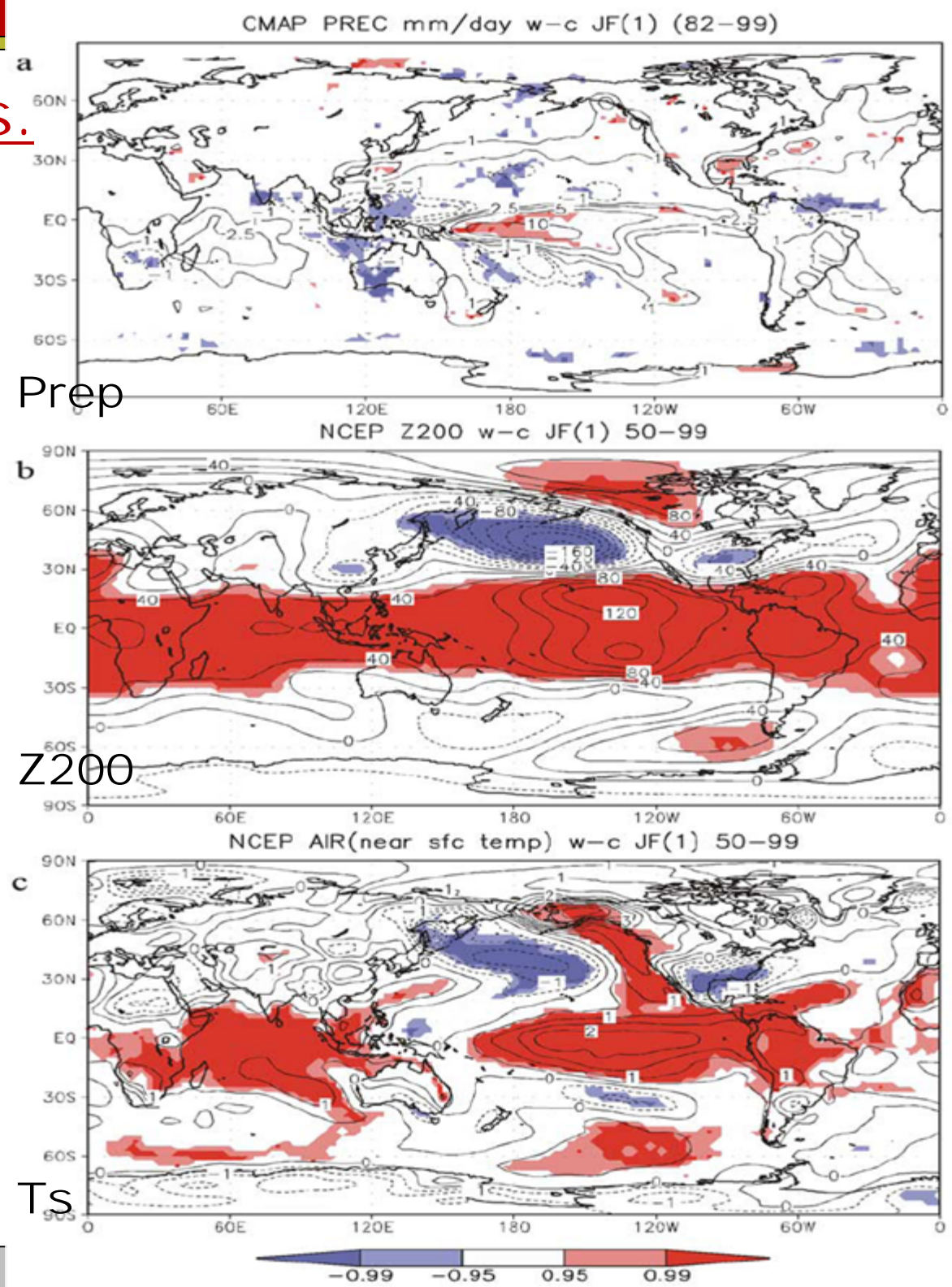
## ENSO Impact !

Atm      SST      Noise

$$X(t) = BT(t) + N(t)$$

Strong Signal

$$BT \gg N$$



# Motivation I: Extratropical Ocean-Atmos. Interaction

NP/NA Impact ?

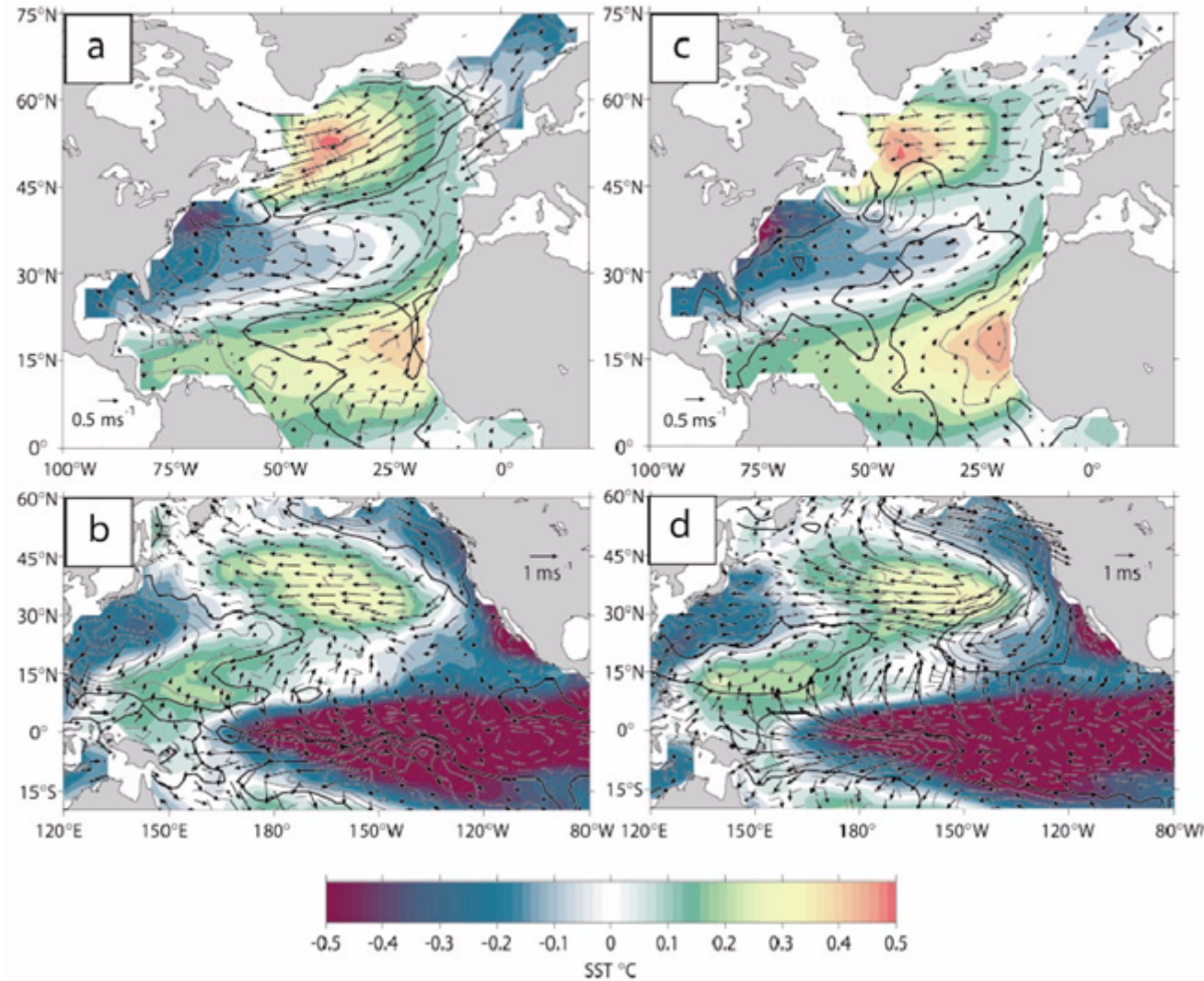
$$X(t) = BT(t) + N(t)$$

Weak Signal

$$BT \ll N$$

Regression/SVD (lag=0)

Modeling



# Modeling?

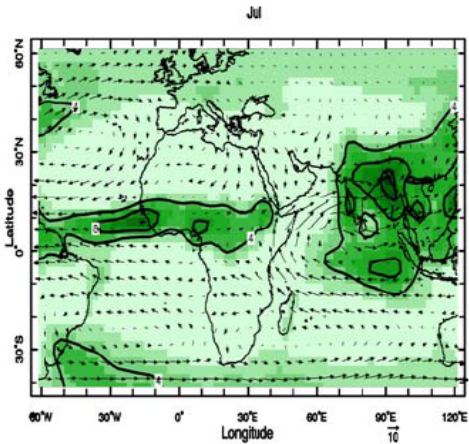
<i>Reference</i>	<i>Exp.</i>	<i>Resoln.</i>	<i>Response</i>
Palmer and Sun (1985)	AMIP	350km*15	EqBt high
Pitcher et al. (1988)	AMIP	R15*9	EqBt low (W and C)
Kushnir and Lau (1992)	AMIP	R15*9	EqBt low (W and C)
Ferranti et al. (1994)	AMIP	R63*15	EqBt high
Peng et al. (1995)	AMIP	T42*21	EqBt high (Nov.) <u>Baroclinic low (Jan.)</u>
Kushnir and Held (1996)	AMIP	R15*9	Barclinic low
Latif and Barnett (1995,96)	AMIP	T42*19	EqBt high
Peng et al. (1997)	AMIP	T42*19	EqBt high (Feb.) <u>Baroclinic low (Jan.)</u>

Yulaeva et al. (2001)	FHXE	T42*19+slab	Baroclinic low
Sutton and Mathieu (2002)	FHXE	T42*19+slab	Baroclinic low

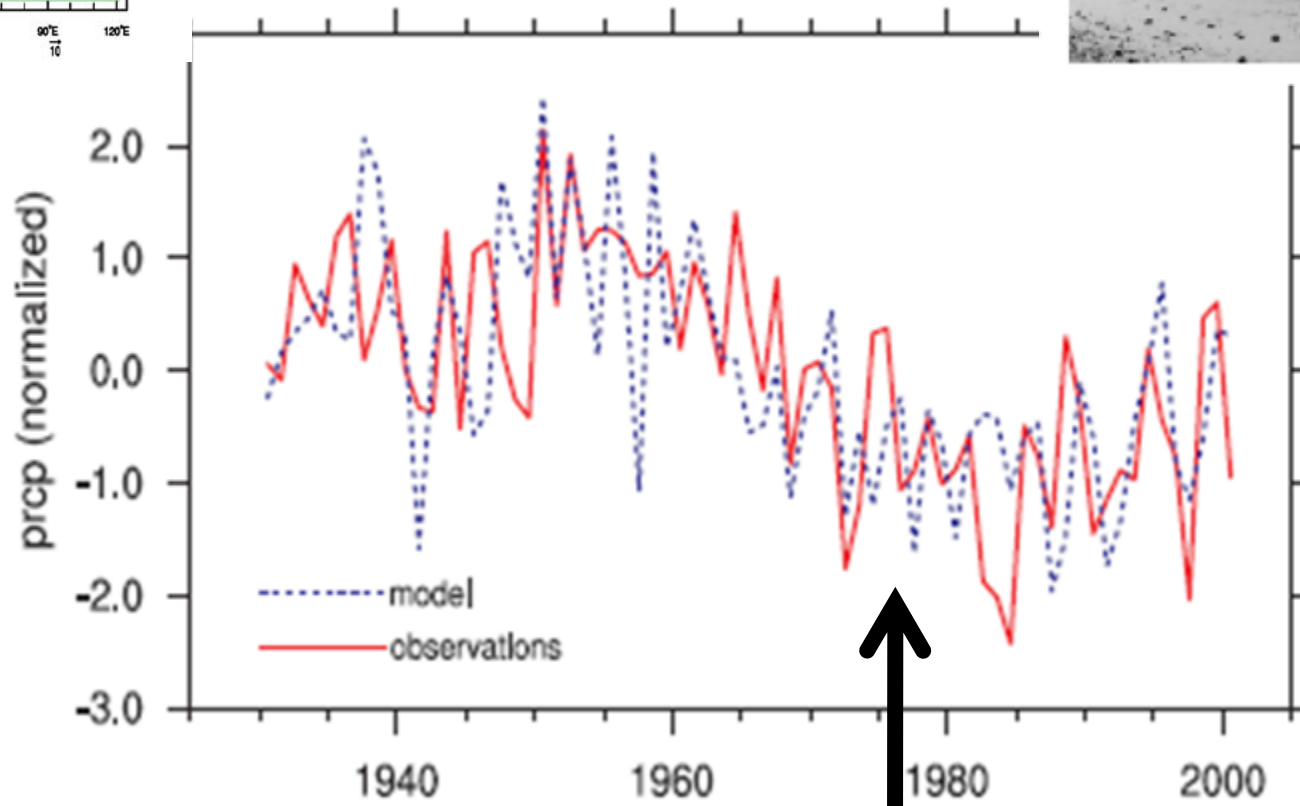


# Motivation II: Land-Atmosphere Interaction

## Sahel Rainfall



Sahel precipitation - July-September



Charney

Giannini et al., 2003: Science

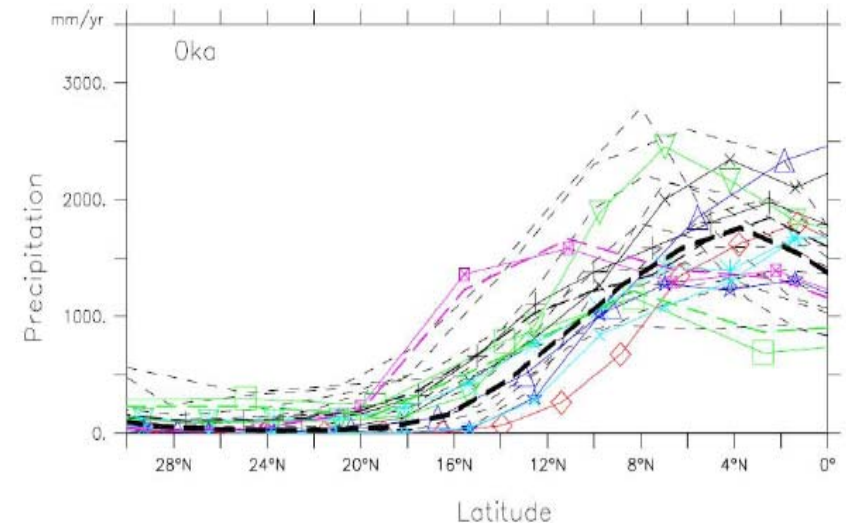
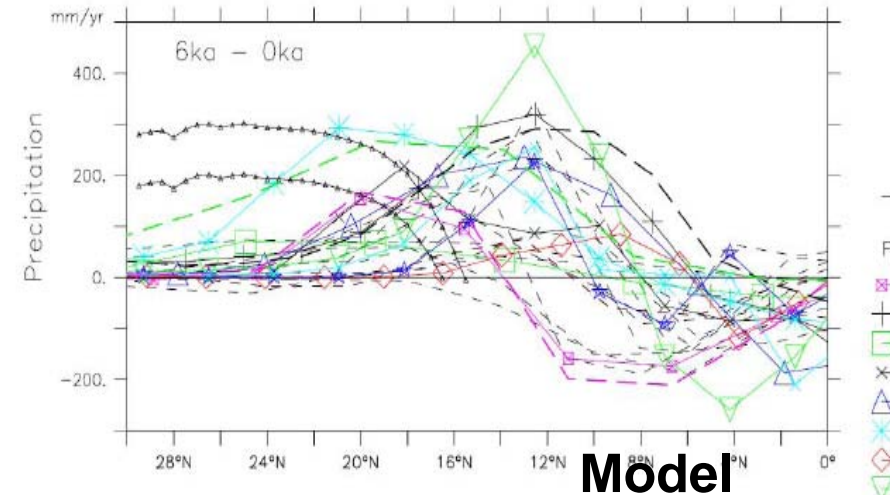
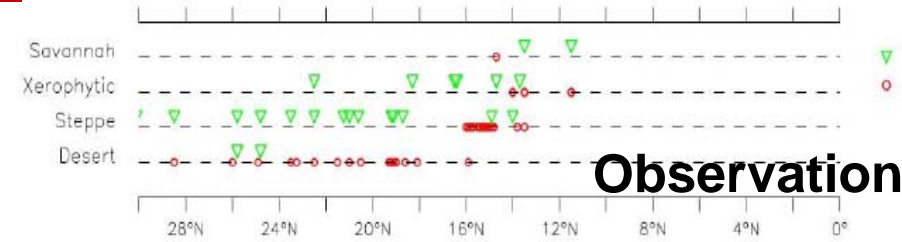
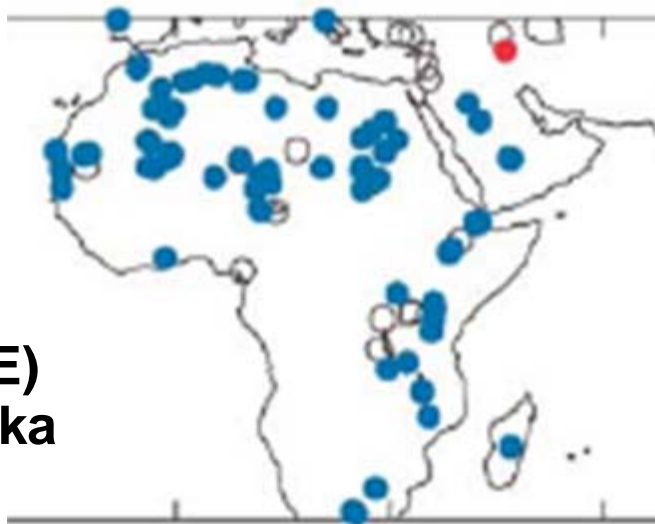
# N. Africa Vegetation Impact ? 6000 years ago

$$X(t) = BL(t) + N(t)$$

Weak Signal

$$BL \ll N$$

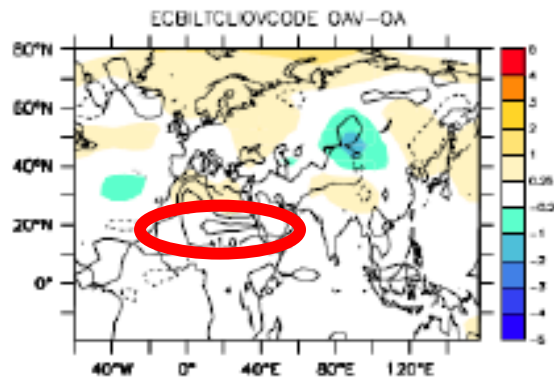
$\Delta(P-E)$   
6ka-0ka



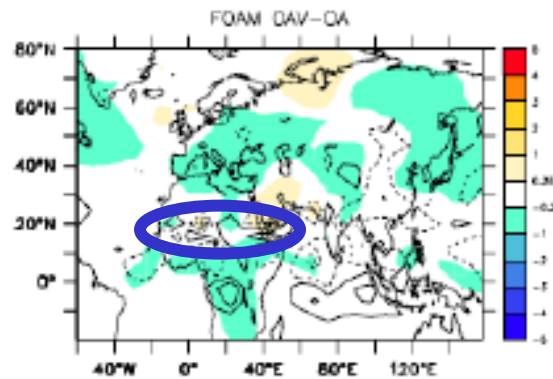
# PMIP II: Vegetation Feedback

## (OAV-OA)

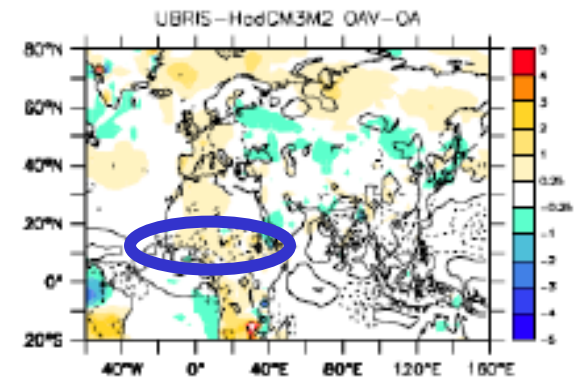
6ka



**Positive FB**



**Negative FB**



**Negative FB**

0ka

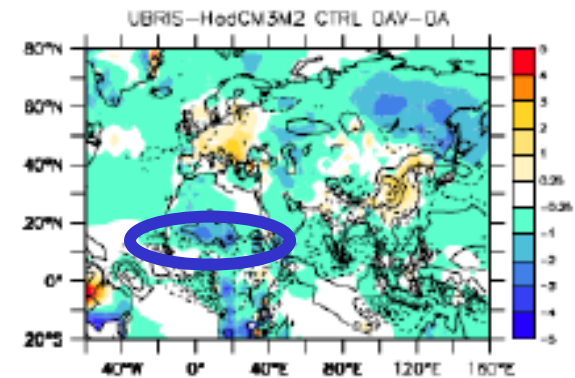
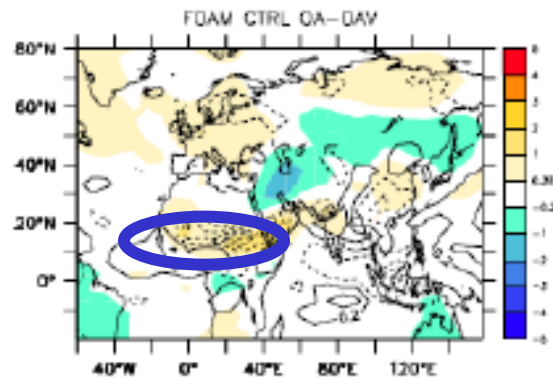
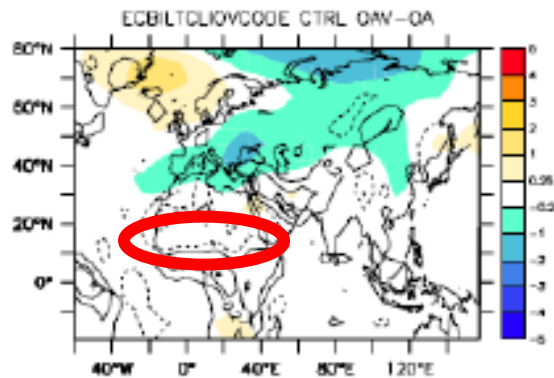


Fig. 5. MH JJAS difference in 2m air temperature ( $^{\circ}\text{C}$ , color map) and precipitation (mm/d, isolines) for the three models for which OA and OAV simulations are available in the PMIP2 database. The first two lines represent the MH differences simulated by the OA and the OAV simulations respectively, the third line is the difference between the first two, and the last line represents the differences between OAV and OA for the preindustrial simulations.



# The Question ?

Climate impact of ocean/land on the atmosphere

in the observation?







EFA: Equilibrium Feedback Analysis  
---- Univariate Statistical Assessment

GEFA: Generalized Equilibrium Feedback Analysis  
---- Multivariate Statistical Assessment

Comparison with LIM/FDT





EFA: Equilibrium Feedback Analysis  
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# Equilibrium Feedback

$x$ : atmosphere (fast),  $T$ : ocean/land (slow)

$$\frac{dx}{dt} = ax + bT + n$$

$$\frac{dT}{dt} = cx + dT + N_T$$

Time average the atmosphere equation (>adjustment time of  $x$ )

$$0 \approx \frac{d\bar{x}}{dt} = a\bar{x} + b\bar{T} + \bar{n}$$

Quasi Equilibrium Feedback Response

$$\bar{x}(t) = B\bar{T}(t) + N(t)$$

Feedback parameter

$$B = -\frac{b}{a}$$





# EFA (Equilibrium Feedback Analysis)

Frankignoul et al., 1998, JPO

$$x(t) = BT(t) + N(t)$$

Since  $\langle N(t), T(t - \tau) \rangle = 0$

We have  $\langle x(t), T(t - \tau) \rangle = B \langle T(t), T(t - \tau) \rangle + \langle N(t), T(t - \tau) \rangle$

$$B = \frac{\langle x(t), T(t - \tau) \rangle}{\langle T(t), T(t - \tau) \rangle} = \frac{C_{xT}(\tau)}{C_{TT}(\tau)}$$

EFA  
response

$$= \frac{C_{xT}(\tau)}{C_{TT}(0)} / \frac{C_{TT}(\tau)}{C_{TT}(0)}$$

Alternative (AMIP exp.)

$\langle x(t) \rangle = B \langle T(t) \rangle + \langle N(t) \rangle$

$$B = \langle x(t) \rangle / \langle T(t) \rangle$$

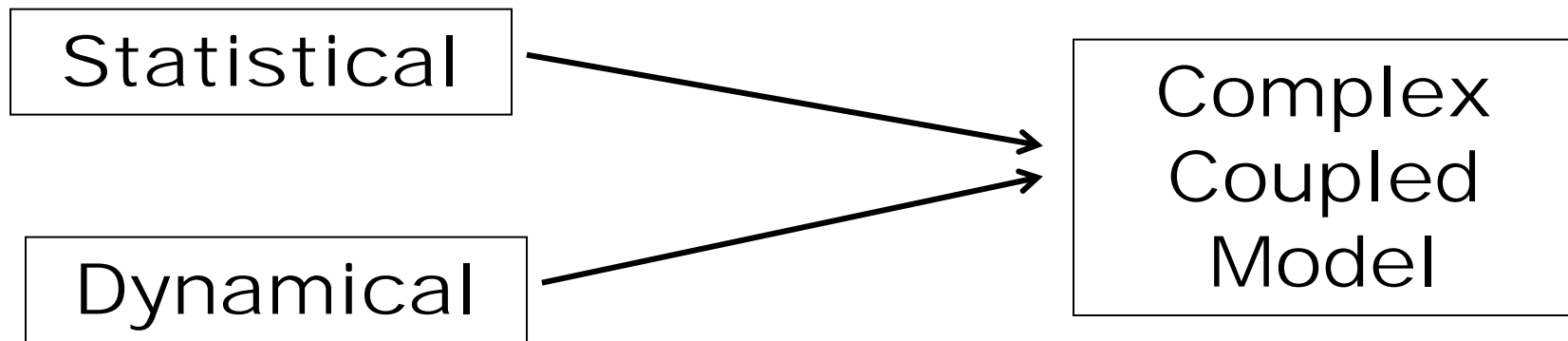
Dynamic  
Assessment



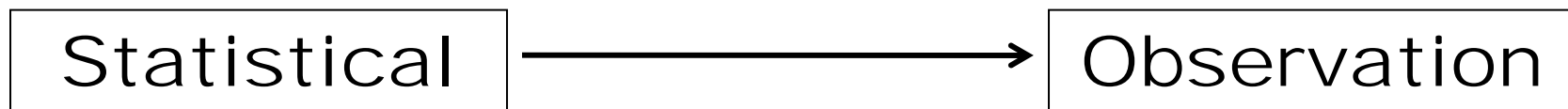


# Combined Statistical-Dynamical Strategy

Step I: Validating Statistical Method  
and Understanding Mechanism



Step II: Statistical Assessment in Observation

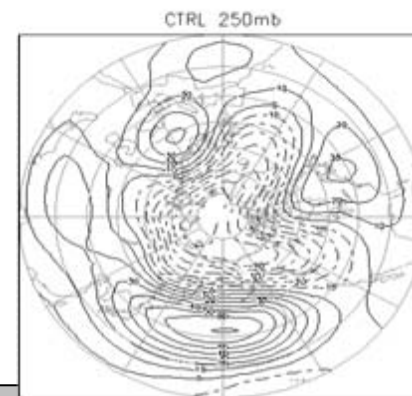
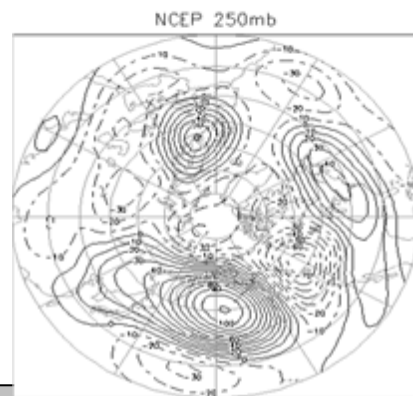
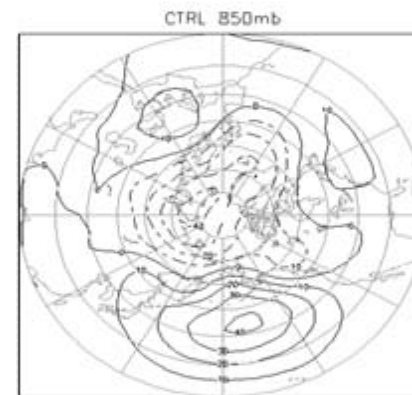
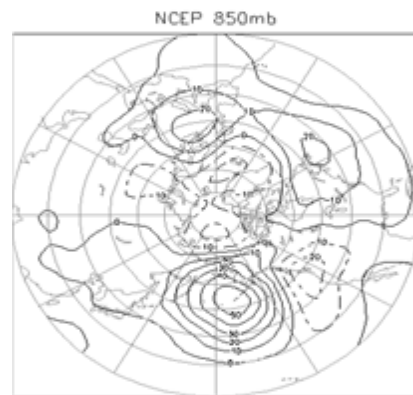
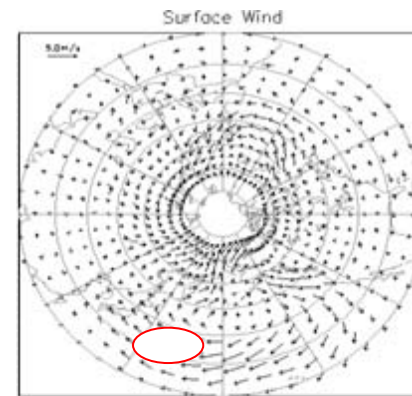
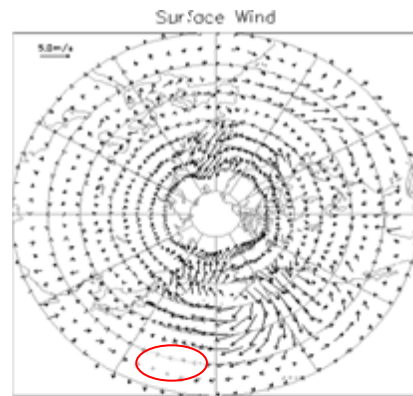


# EFA Example I: Atmos. Response to KOE SST



Obs.

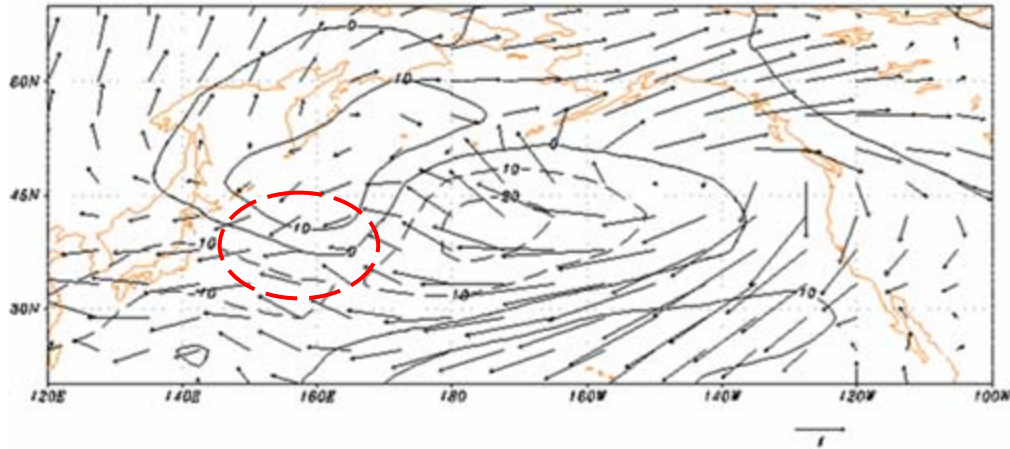
Model (FOAM)



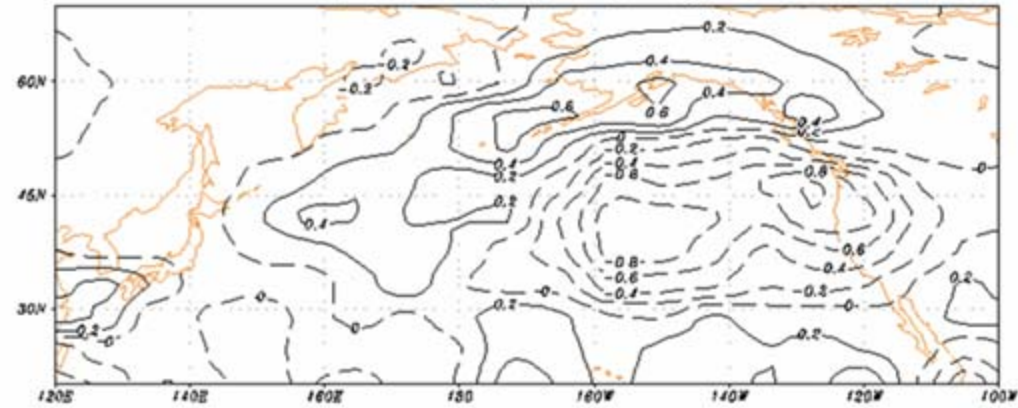
# Fully Coupled Response (Dec)



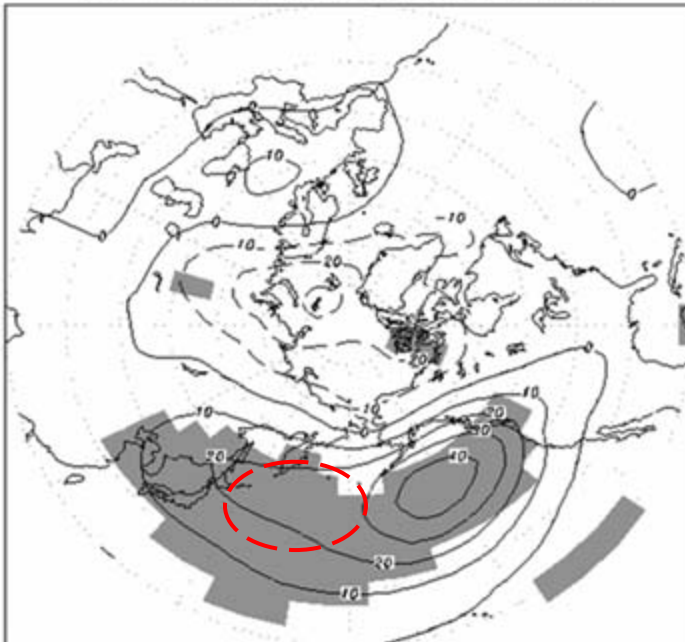
(a) FCE surface wind and turbulent heat flux



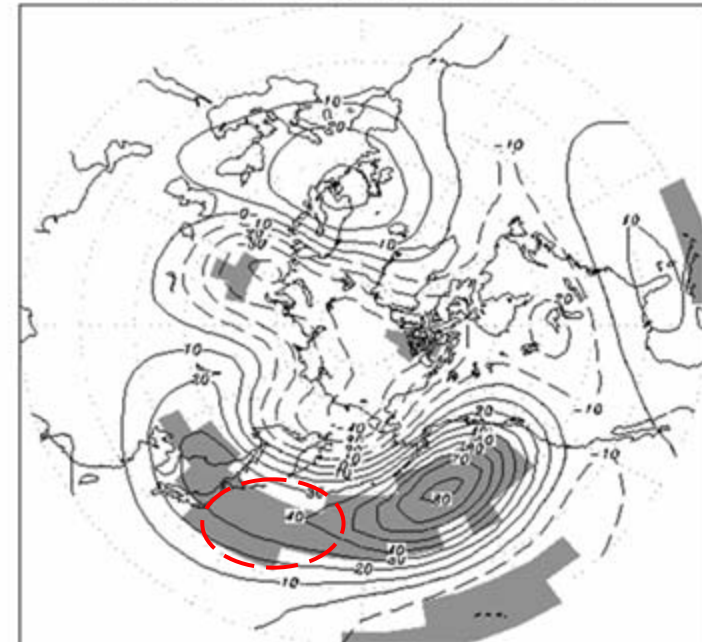
(b) Precip. (mm/day)



(c) 850mb Geopotential Height



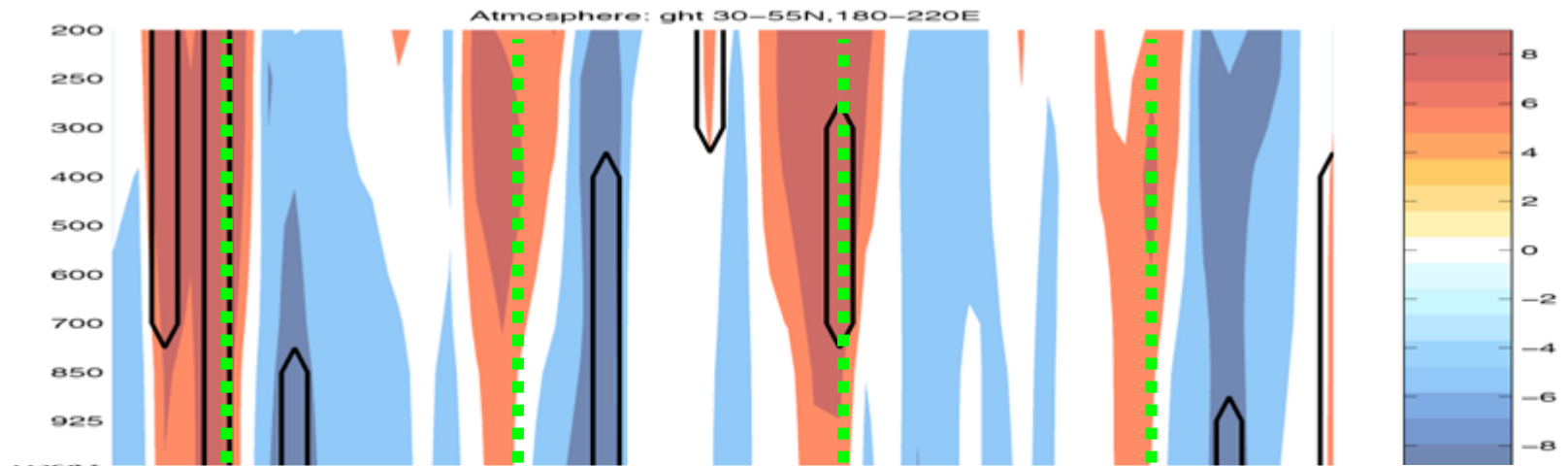
(d) 250mb Geopotential Height



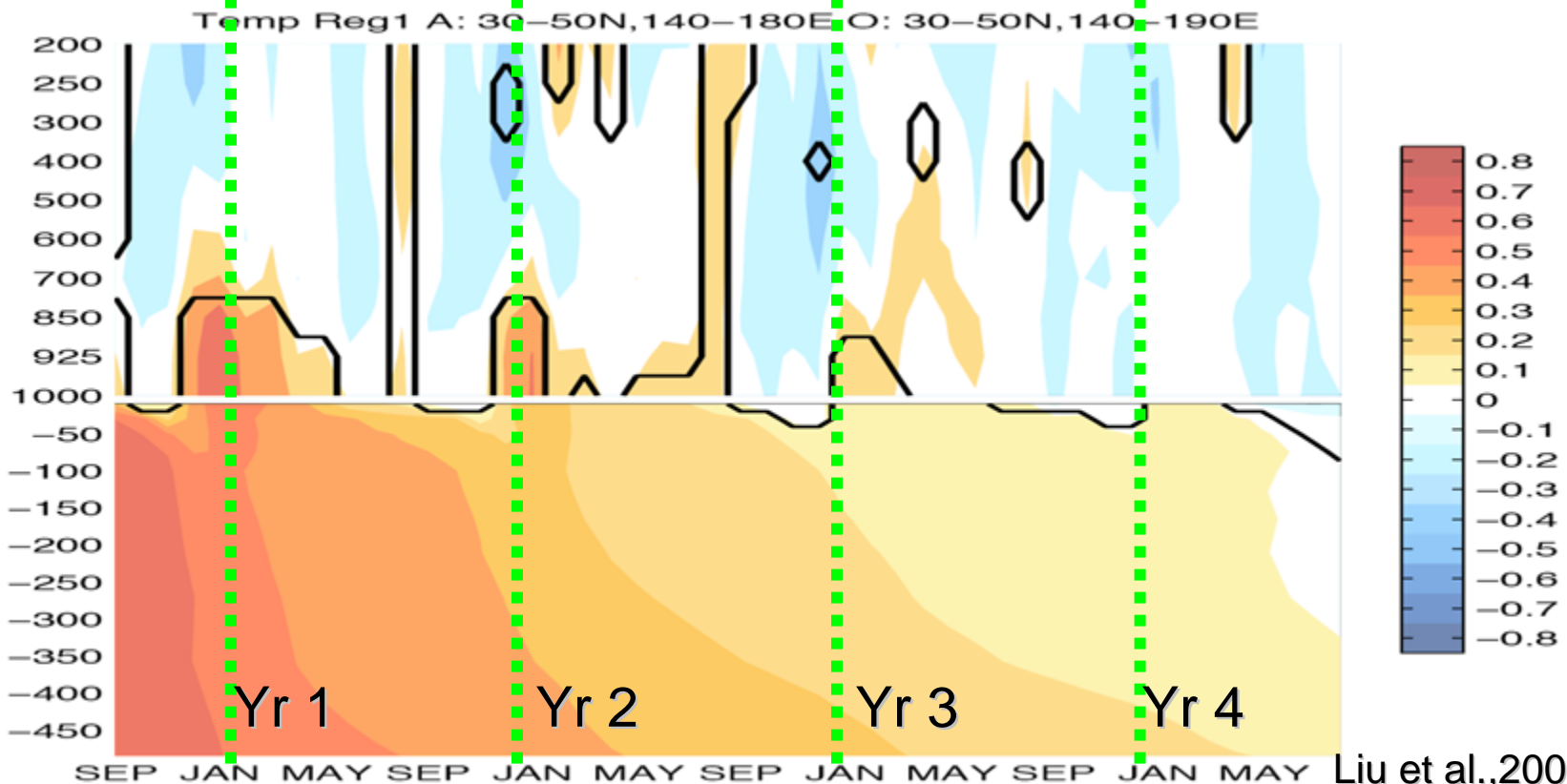
# Long Term Response: The Role of Ocean Memory



Atmos Z



Atmos T



Ocean T



SEP JAN MAY SEP JAN MAY SEP JAN MAY Liu et al., 2007, J. C



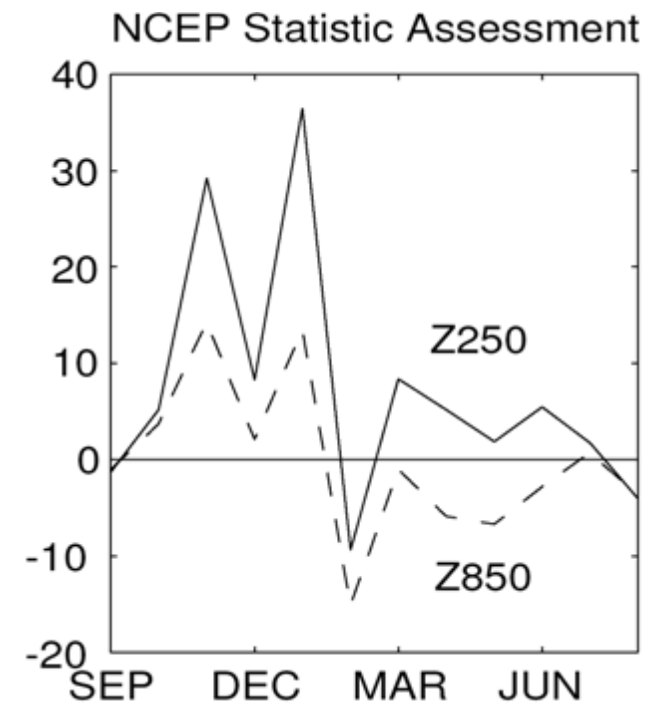
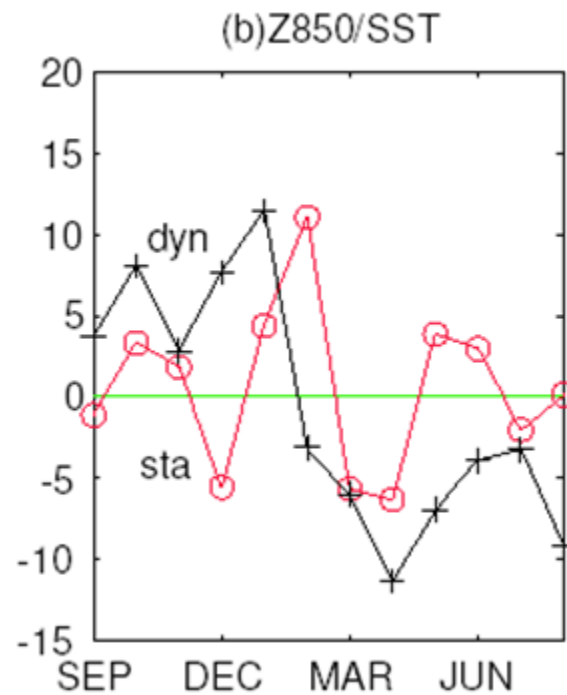
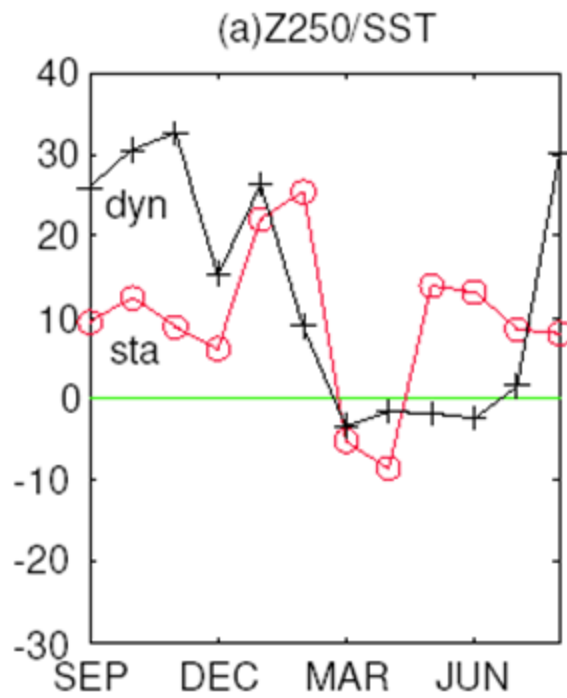
# Atmospheric Response to KOE SST

## GEFA vs Dyn.



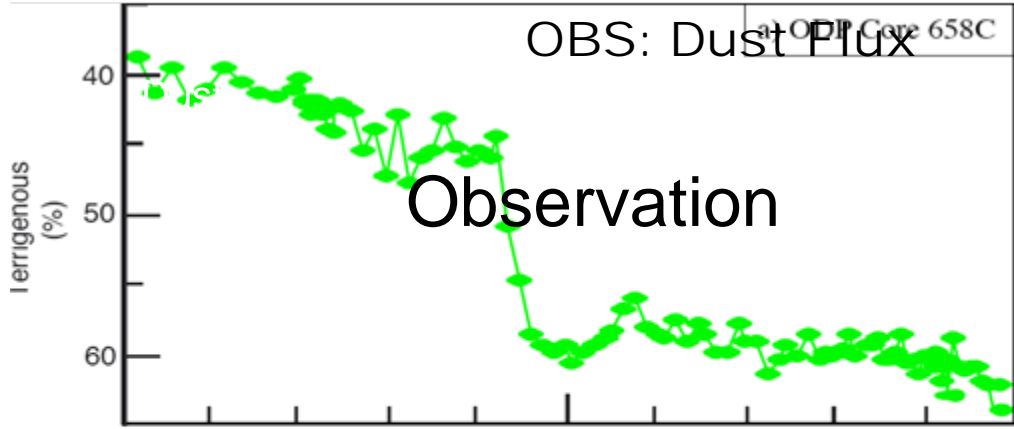
FOAM

Obs

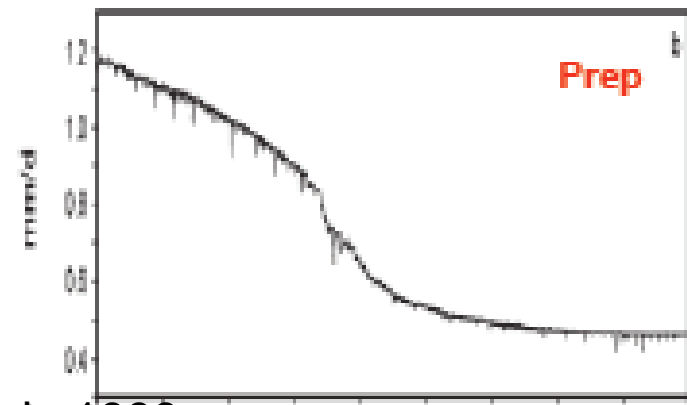
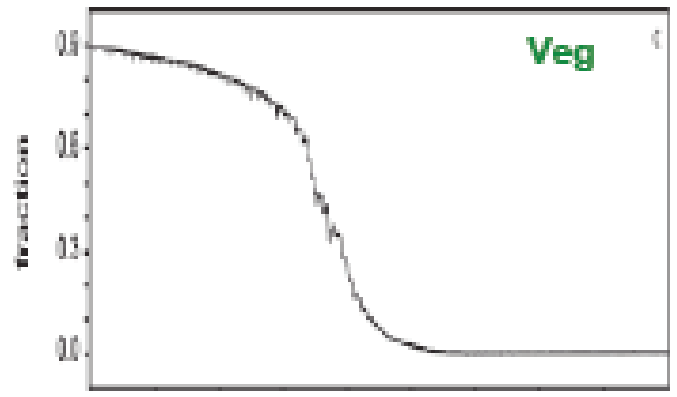




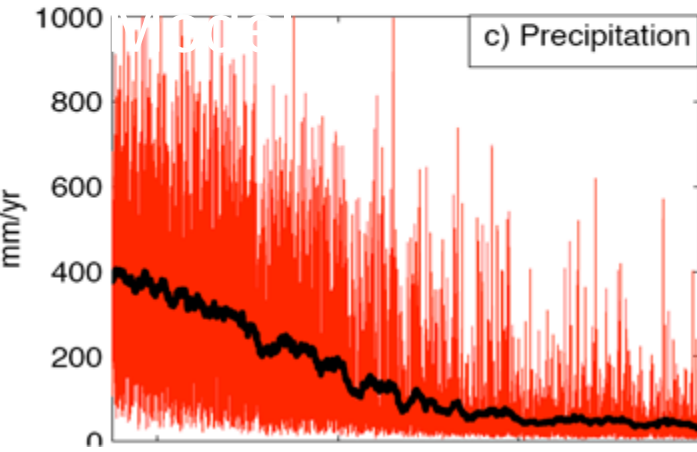
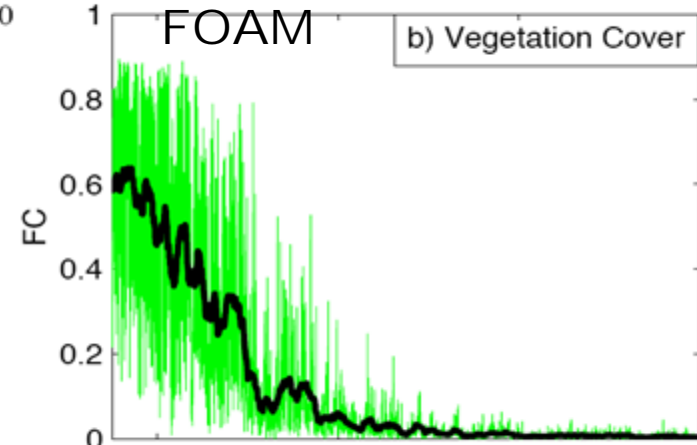
# EFA Example II: N. Africa Abrupt Climate Change



CLIMBER-2



Claussin et al., 1999

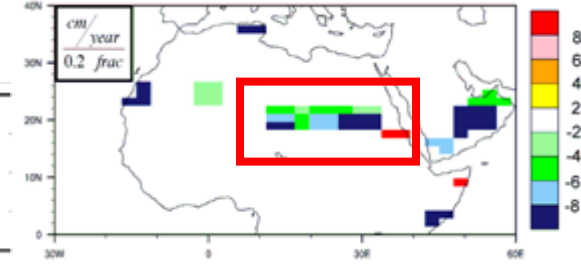
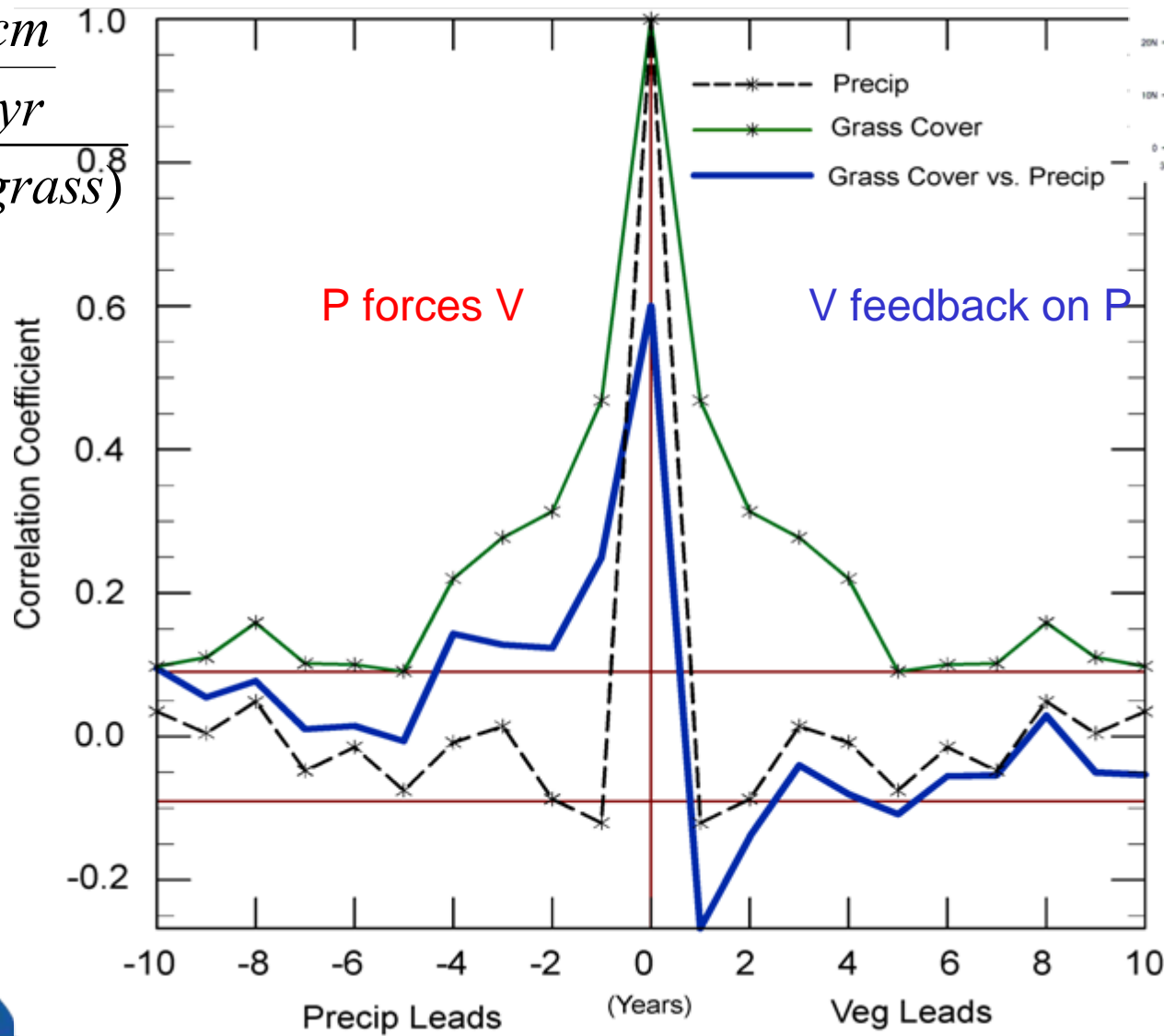


Liu et al., 2006, GRL



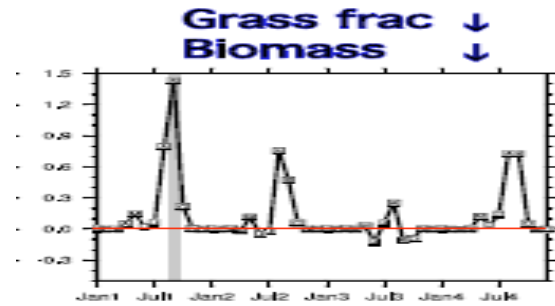
# Corr<grass cover, annual rainfall>

$$\frac{-9.2 \frac{cm}{yr}}{0.2 fpc(grass)}$$

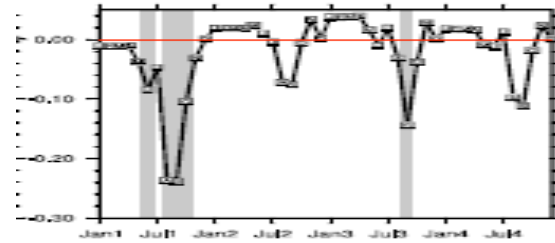




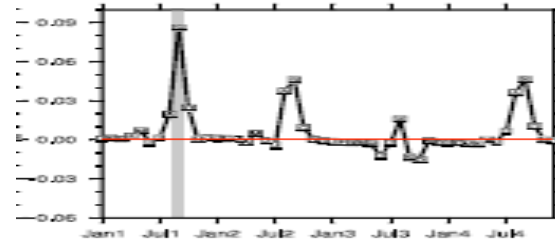
# Dynamic Assessment



Prep

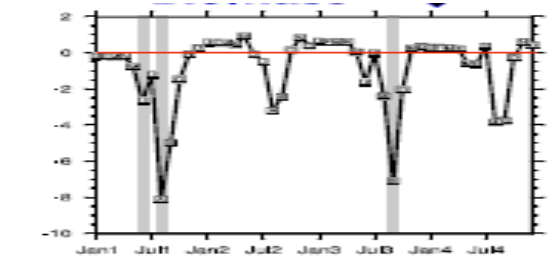


Leaf Area Index



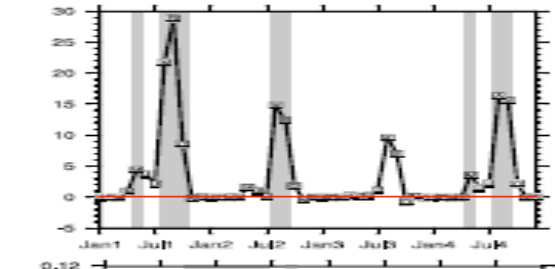
Top Soil Moisture

-10



Transpiration

+30



BareGround Evap



Notaro et al., 2008,  
Glb Change Biology





## Statistical and Dynamical Response in Annual Variables to a Decrease in North African Grass Cover Fraction by 0.2

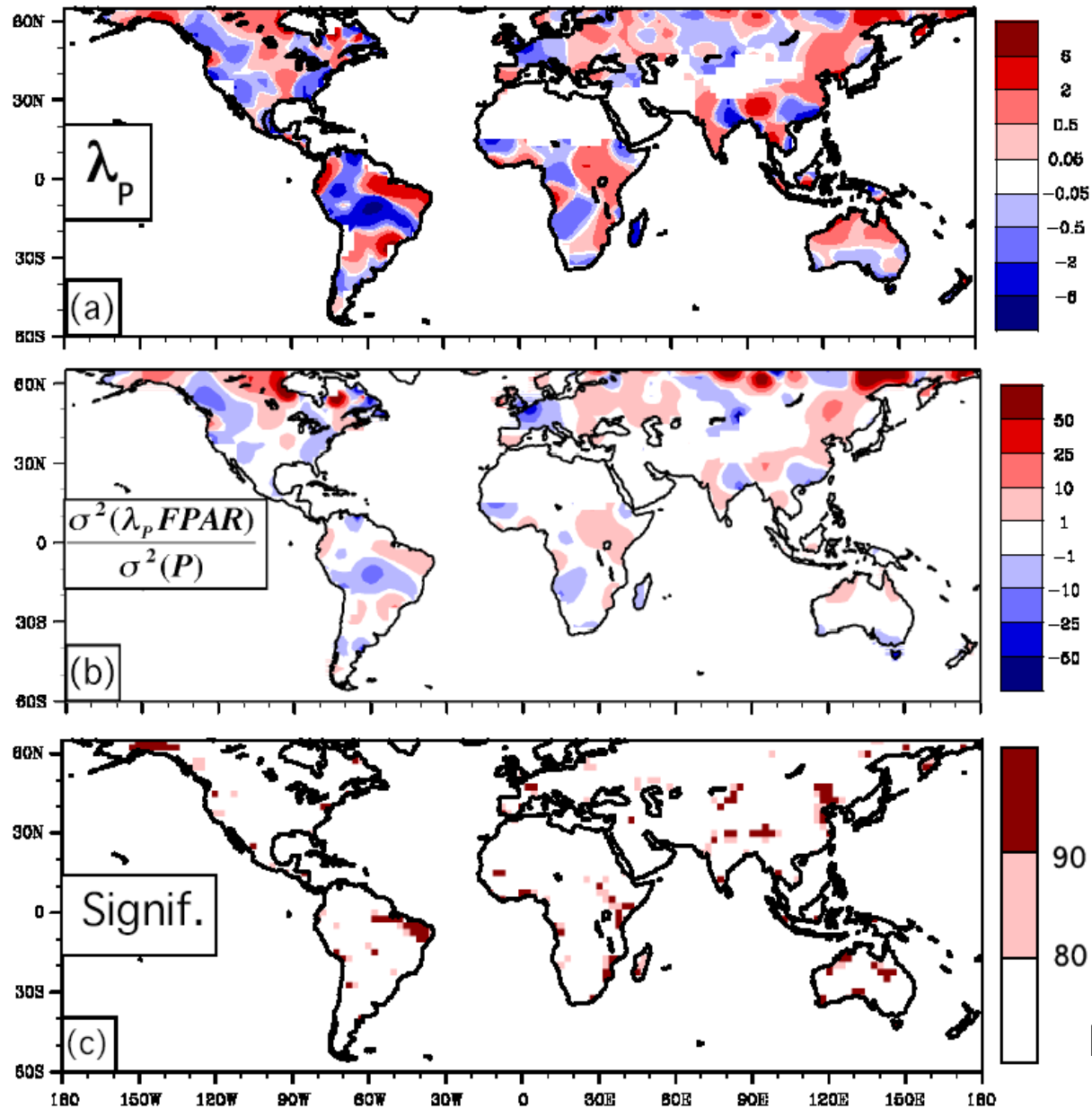
	Units	Statistical (FOAM)	Dynamical (FOAM)	Statistical (CCSM2)
Precipitation	cm/yr	+9.2	+9.7	+3.3
Evaporation	cm/yr	+4.7	+6.1	+0.7
Bare soil evaporation	cm/yr	+10.2	+8.6	+8.7
Transpiration	cm/yr	-5.5	-2.5	-8.0
Sfc specific humidity	g/kg	+0.19	+0.27	+0.08





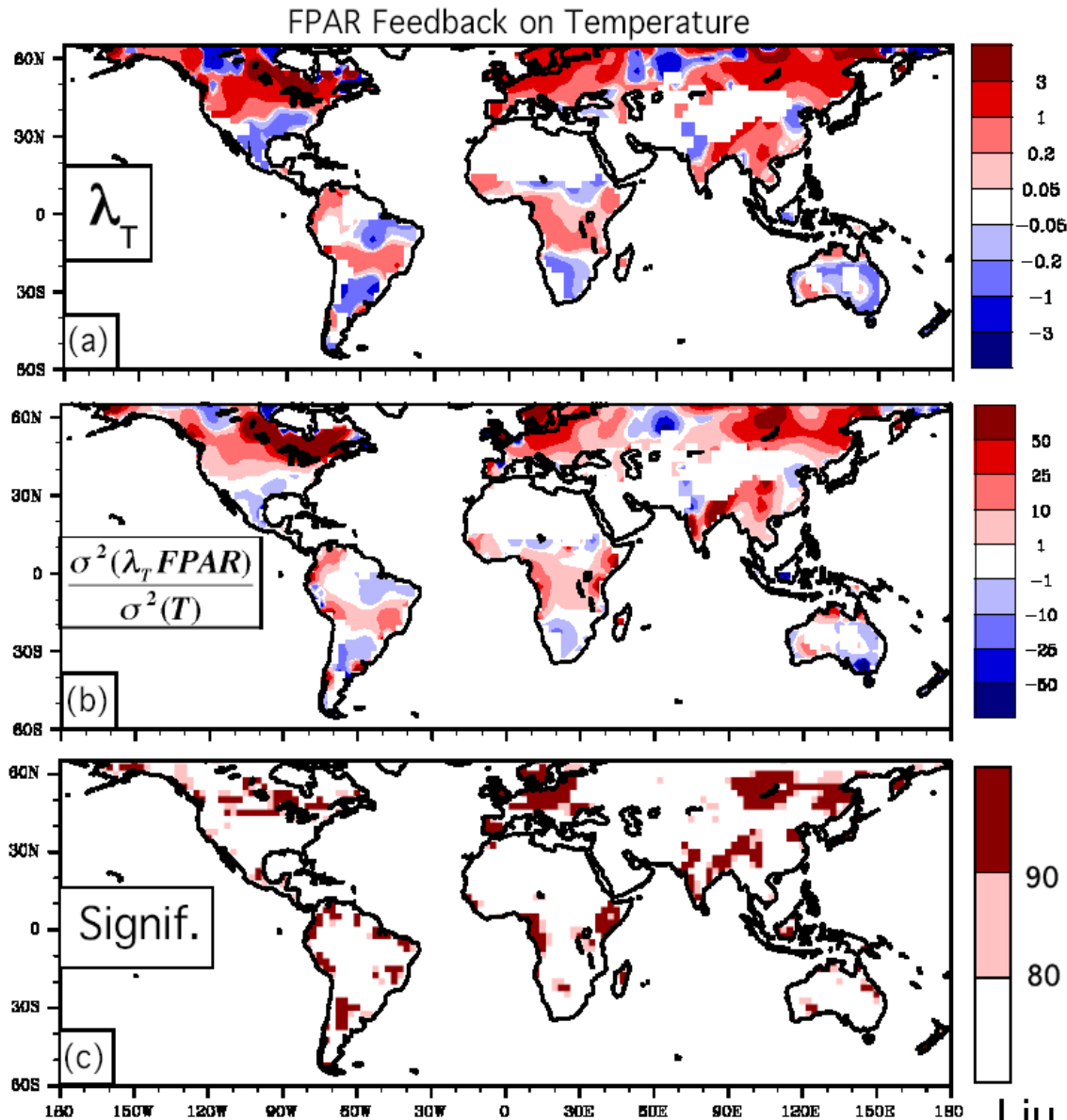
# EFA Vegetation feedback on Prep Observation (1980-2000)

FPAR Feedback on Precipitation





# EFA Vegetation feedback on temperature Observation (1980-2000)





EFA: Equilibrium Feedback Analysis  
---- Univariate Statistical Assessment

GEFA: Generalized Equilibrium Feedback Analysis  
---- Multivariate Statistical Assessment

Comparison with LIM/FDT





# GEFA (Generalized EFA)

Liu et al., 2008, J. Climate

$$X(t) = \begin{bmatrix} b_1 & b_2 & \dots & b_J \end{bmatrix} \begin{bmatrix} T_1(t) \\ T_2(t) \\ \dots \\ T_J(t) \end{bmatrix} + N(t) \equiv \mathbf{B} T(t) + N(t)$$

GEFA response

$$\mathbf{B} = \mathbf{C}_{xT}(\tau) \mathbf{C}_{TT}^{-1}(\tau)$$







# Testing GEFA:

## Ocean feedback in the Advective Hasselmann Model

Advective Hasselmann Model

$$\frac{\partial T_a}{\partial t} + U \frac{\partial T_a}{\partial x} = \lambda(T - T_a) + N(t, x)$$

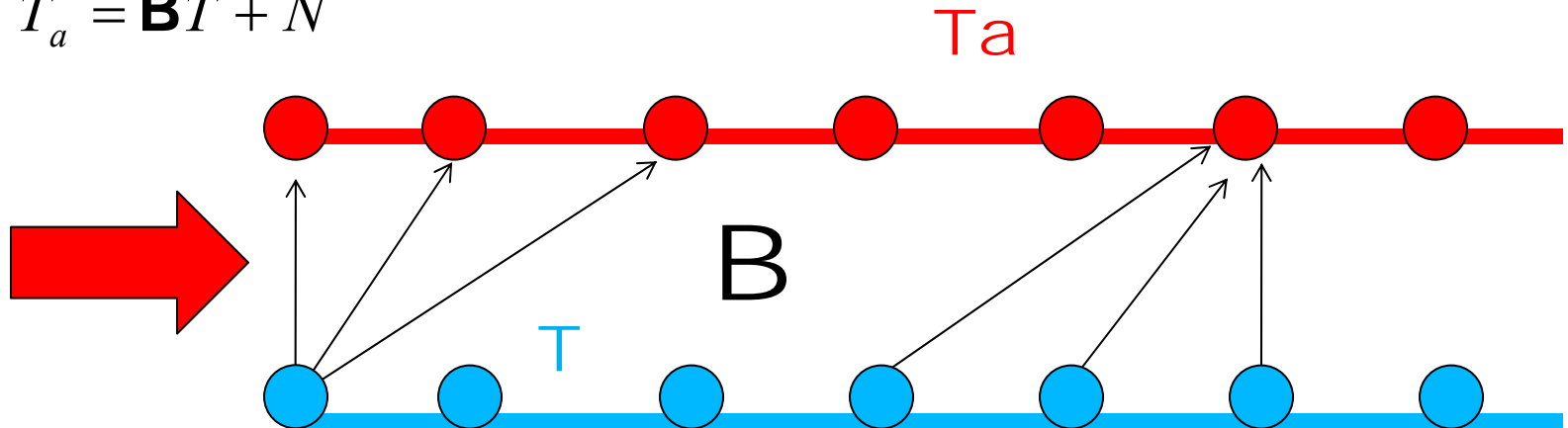
$$\frac{dT}{dt} = \lambda(T_a - T) - dT$$

Equilibrium Response

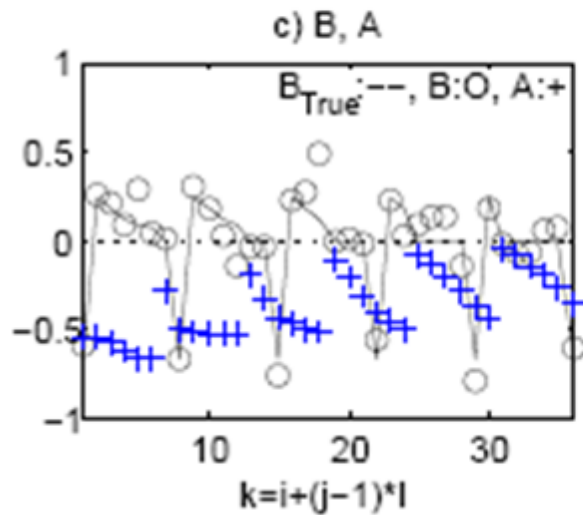
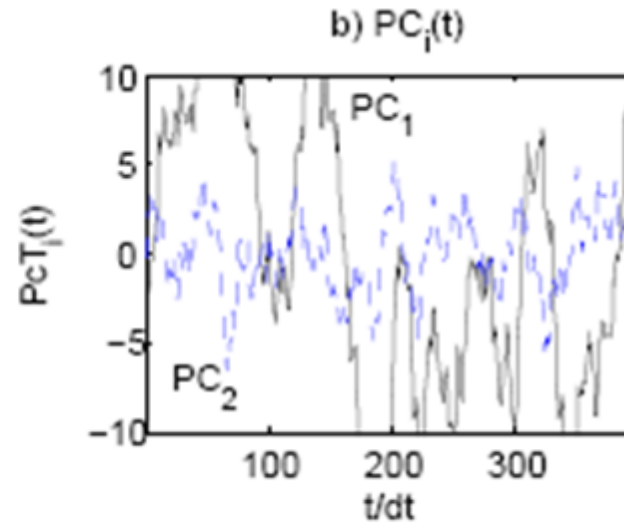
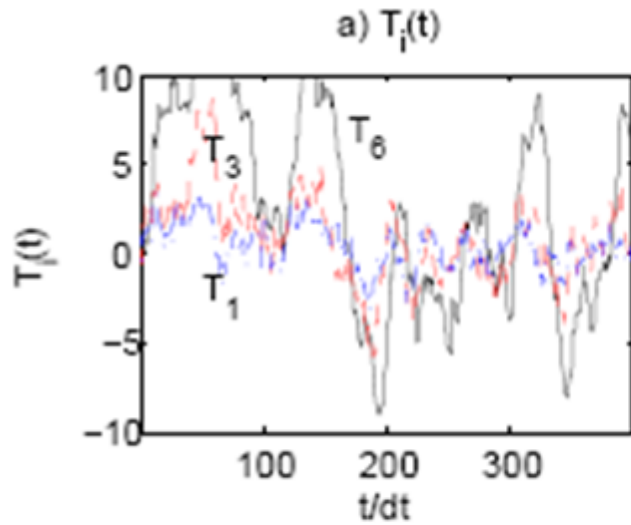
$$U \frac{\partial T_a}{\partial x} = \lambda(T - T_a) + N(t, x)$$

Finite difference

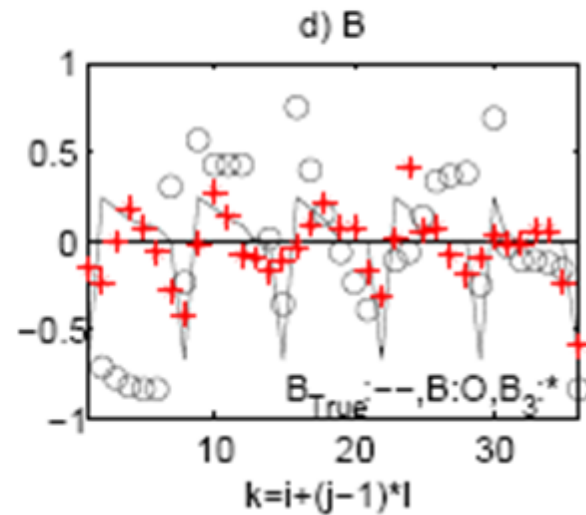
$$T_a = \mathbf{B}T + N$$



# Ocean feedback in Advective Hasselman Model (6 points)



B and A

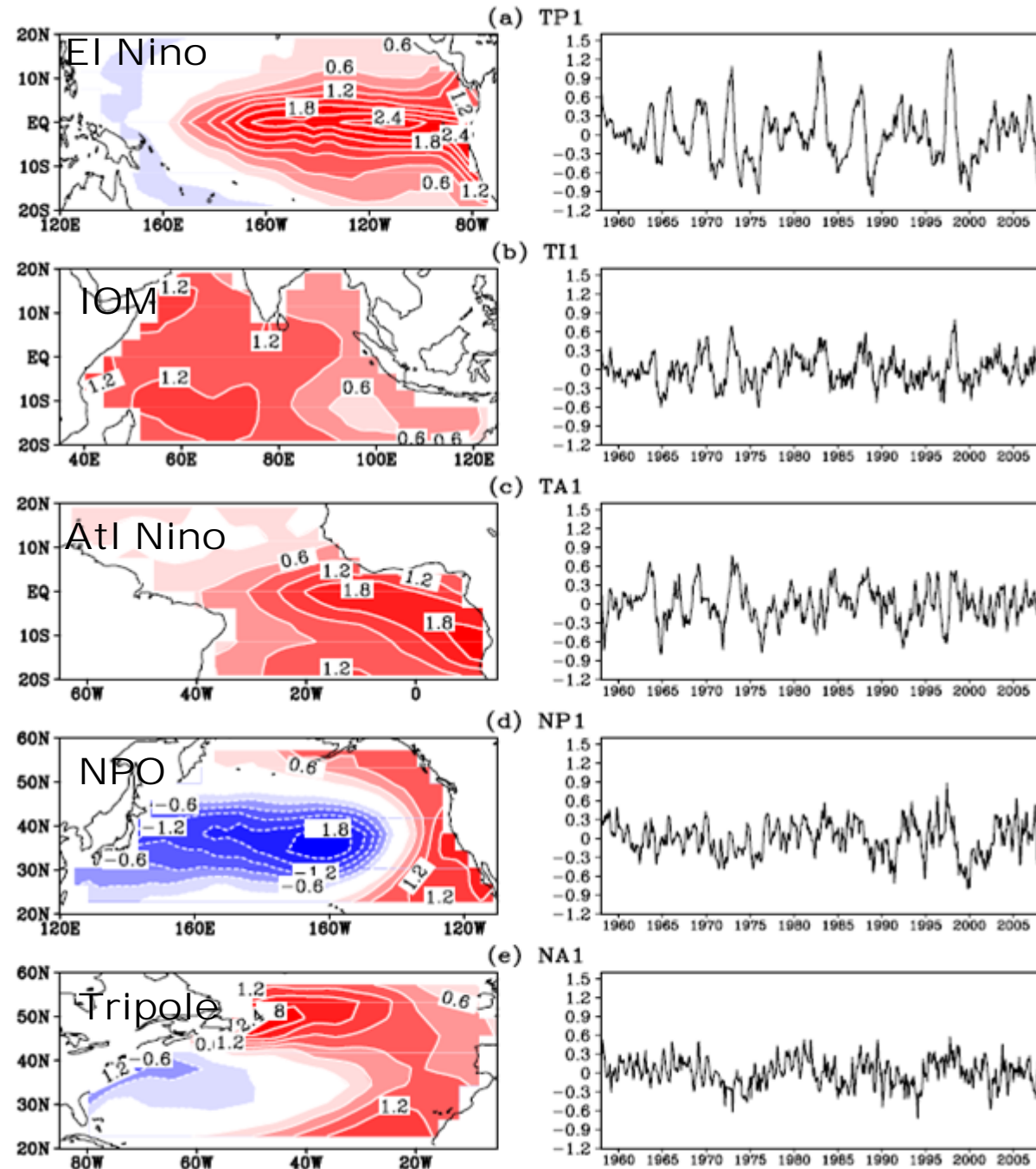


B and B3 (EOF)





# Forcing: Regional SST EOF1



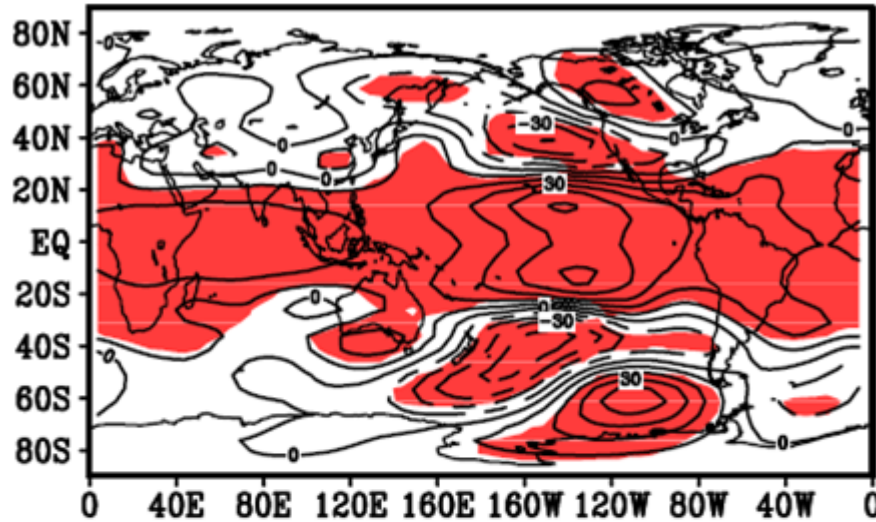


# GEFA Response to ENSO

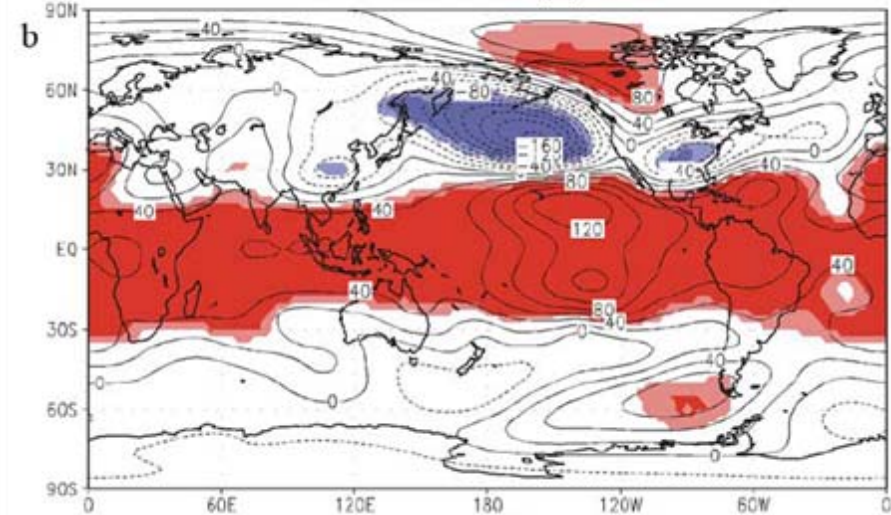
## Tropical Pacific (3) x North Pacific (3)

Composite

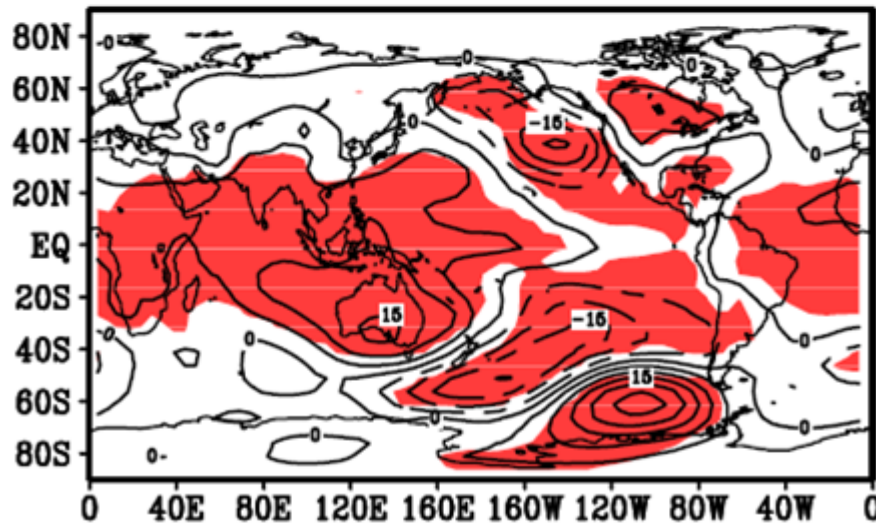
a) GEFA Z250 Rsp to TP1,  $\sigma(\text{TP1})=0.42^\circ\text{C}$



NCEP Z200 w-c JF(1) 50-99



b) GEFA Z850 Rsp to TP1,  $\sigma(\text{TP1})=0.42^\circ\text{C}$



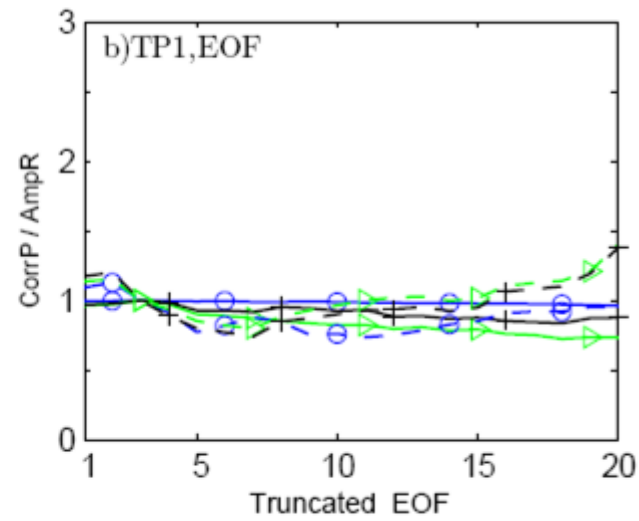
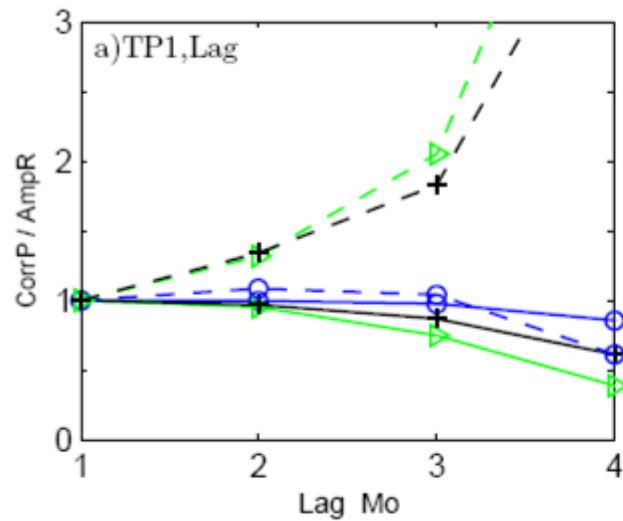


# Sensitivity to lag and EOF truncation

Tp1

Lag

EOFs







# GEFA Response to TP1 and NP1

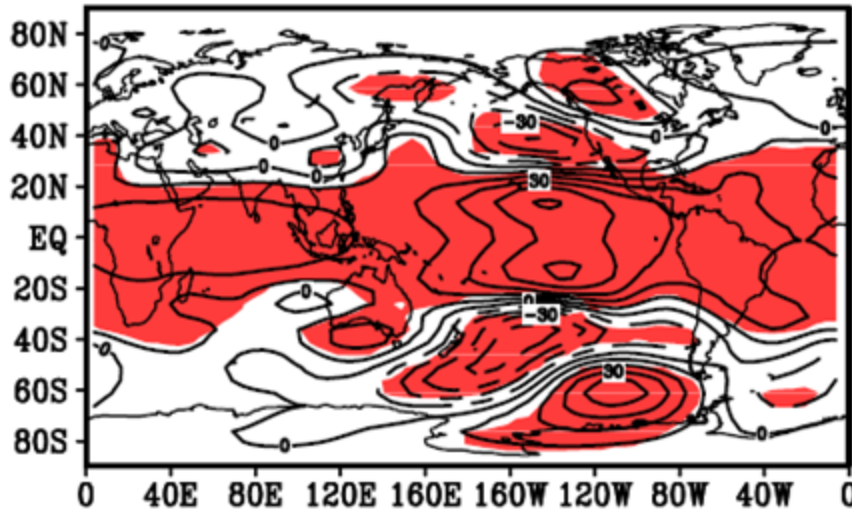
Trop Pac (3) x N. Pac (3)

TP1

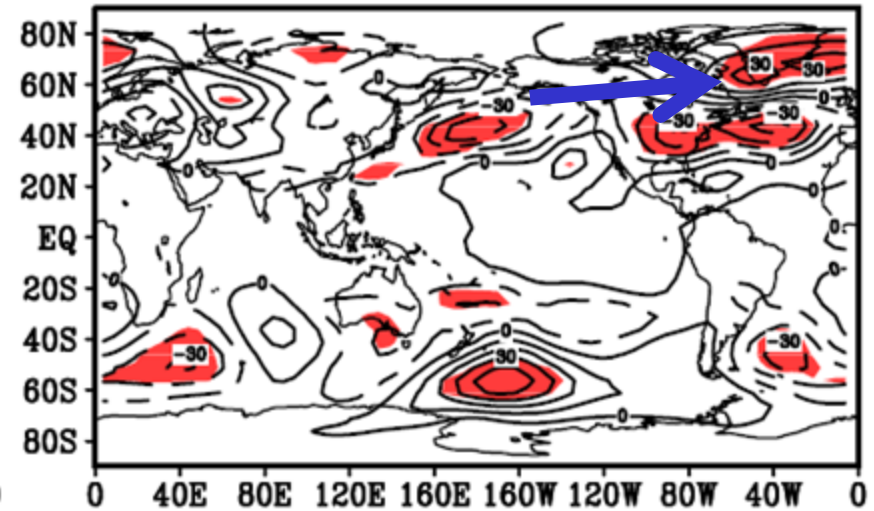
NP1

AL-IL Seesaw

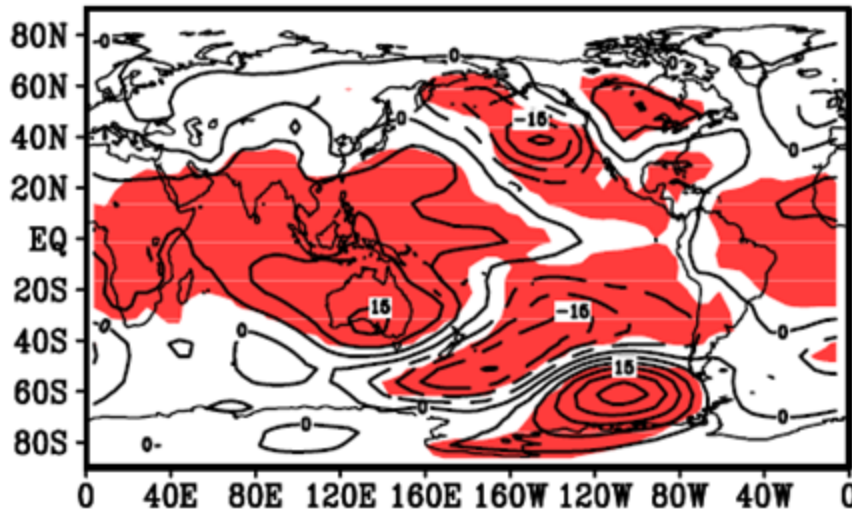
a) GEFA Z250 Rsp to TP1,  $\sigma(\text{TP1})=0.42^\circ\text{C}$



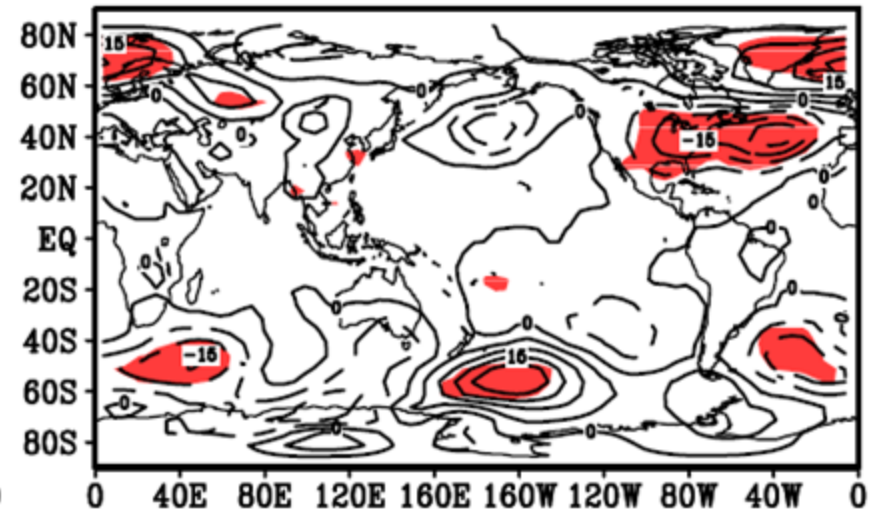
c) GEFA Z250 Rsp to NP1,  $\sigma(\text{NP1})=0.26^\circ\text{C}$



b) GEFA Z850 Rsp to TP1,  $\sigma(\text{TP1})=0.42^\circ\text{C}$



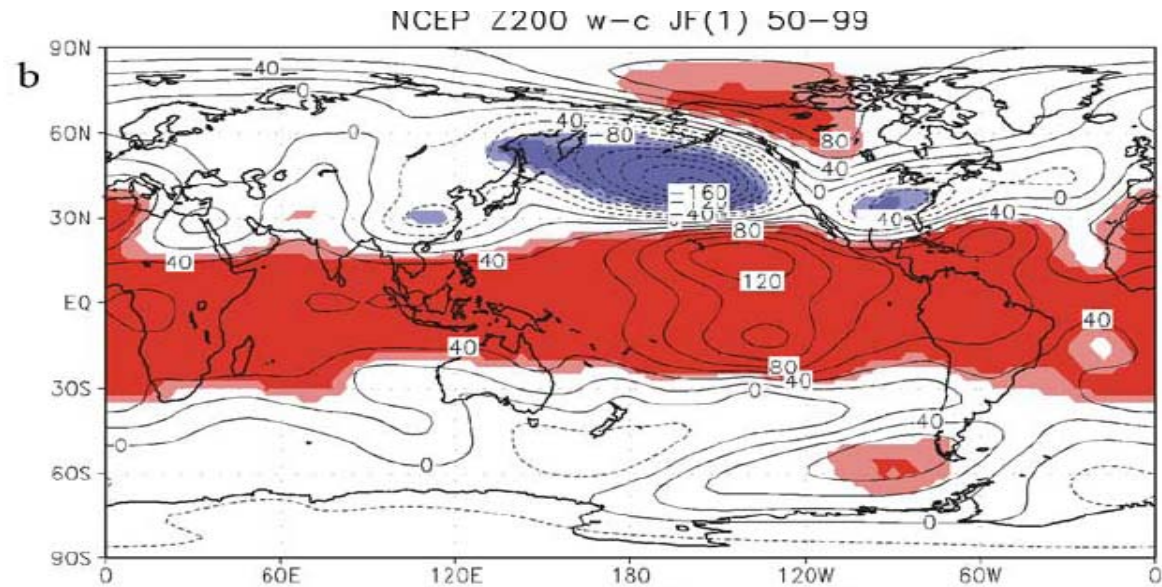
d) GEFA Z850 Rsp to NP1,  $\sigma(\text{NP1})=0.26^\circ\text{C}$



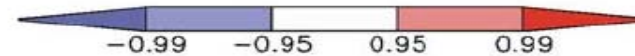
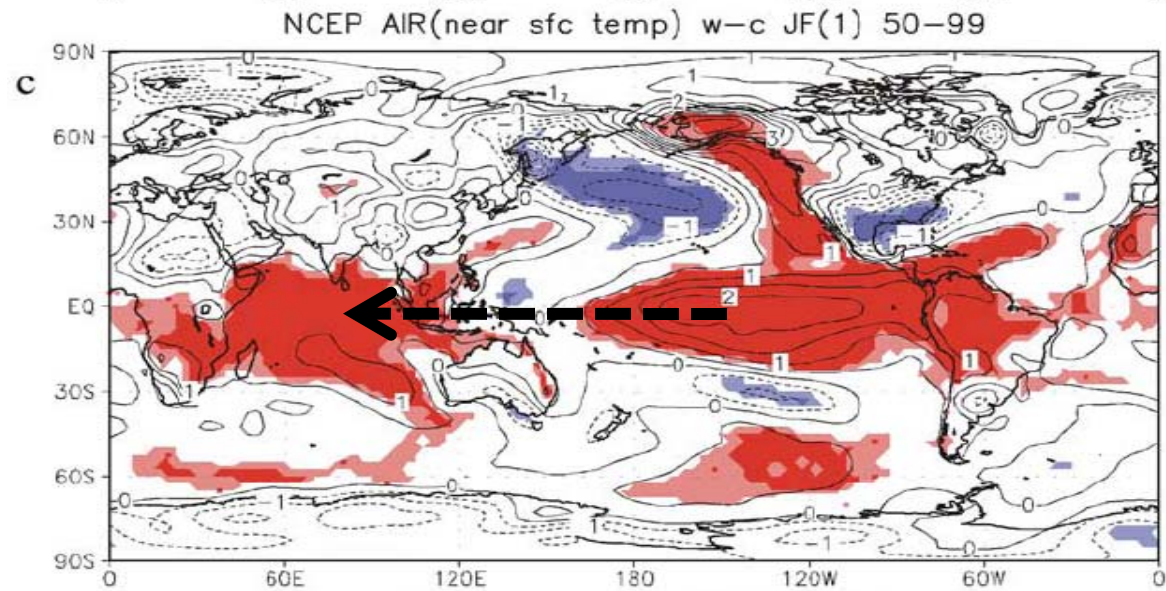
# ENSO forcing? or Indian Ocean forcing?

Composite

Z200



Ts





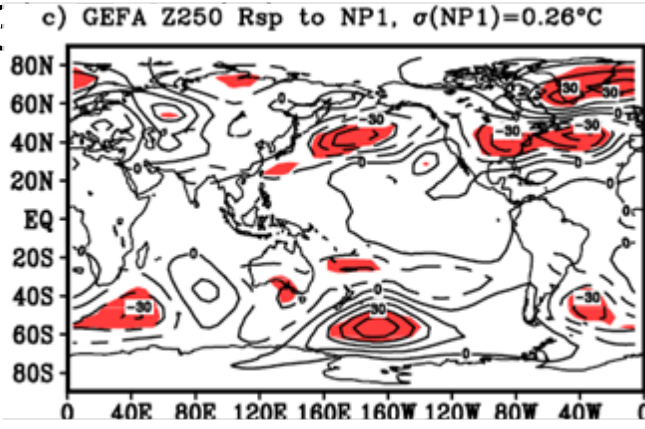


# GEFA Z250 Response

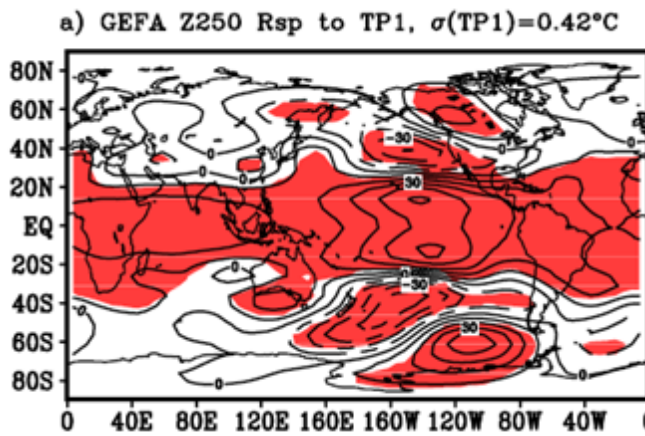
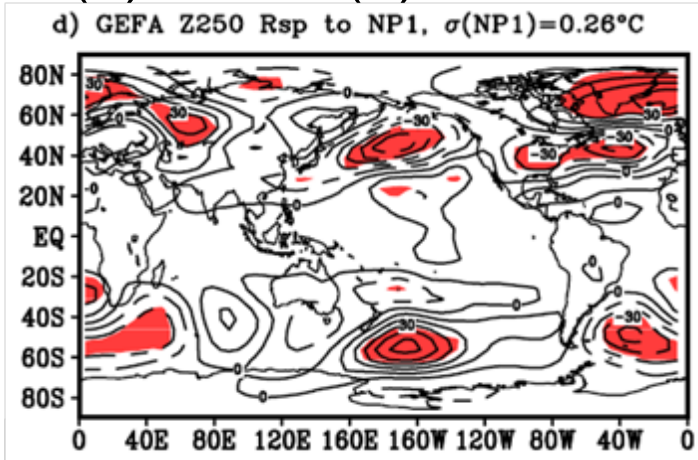
T. Pac(3)xN. Pac(3)

T. Pac(3)xN. Pac(3)xT. Ind(3)xN.

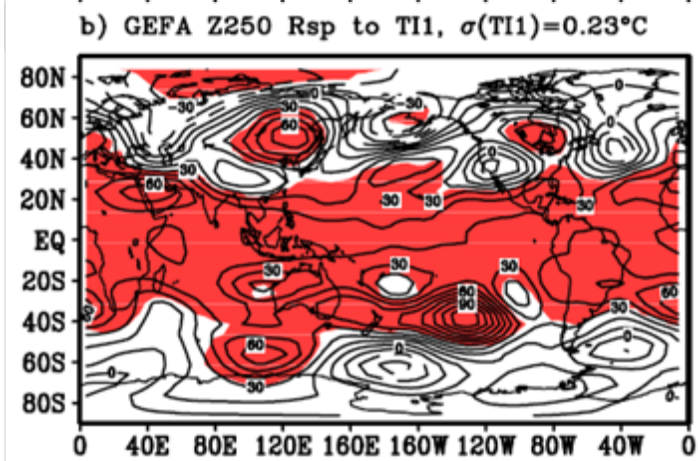
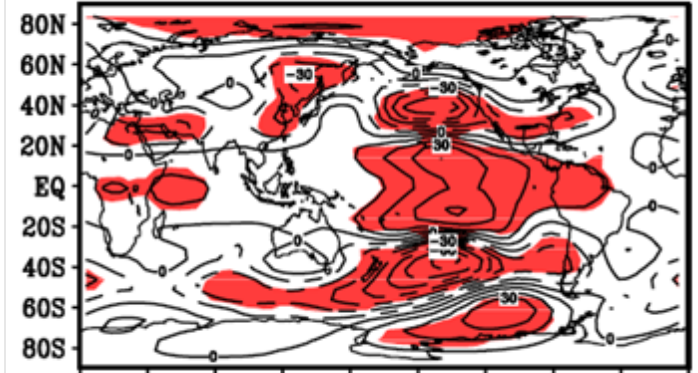
Alt(3)



NP1



TP1



TI1



# Lead-Lag Composite to ENSO forcing

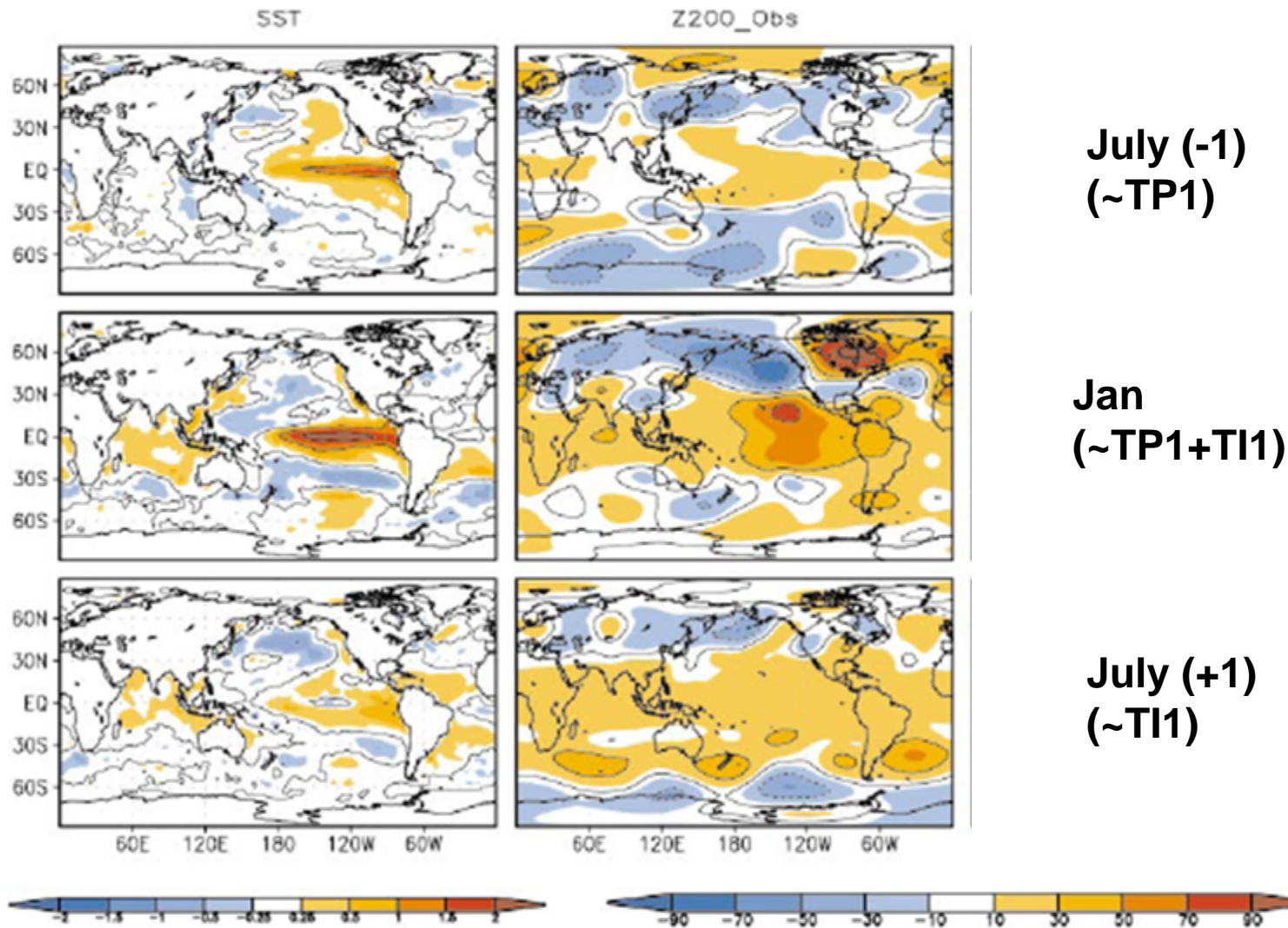


FIG. 5. The spatial maps of warm ENSO event composite anomalies for (left column) SST ( $^{\circ}\text{C}$ ), (middle column) observed 200-mb height (m), and (right column) CCM3 simulated 200-mb height (m). Warm events defined as in Fig. 3. The top (bottom) row shows Jul composites preceding (following) the Jan base month of the Niño-3.4 index. The middle row shows simultaneous composites for Jan. Negative anomalies are dashed.





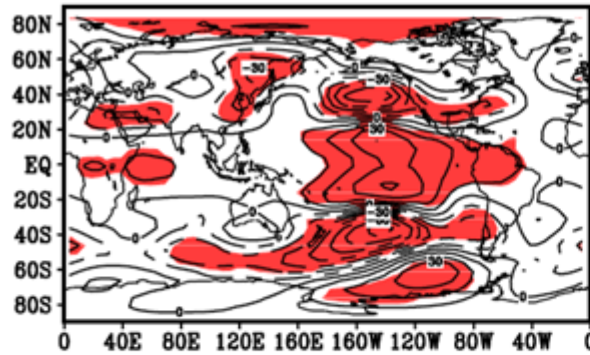


# GEFA Z250 Response to EOF1s

T. Pac(3)xN. Pac(3)xT. Ind(3)xN. Atl(3)xT. Atl (3)

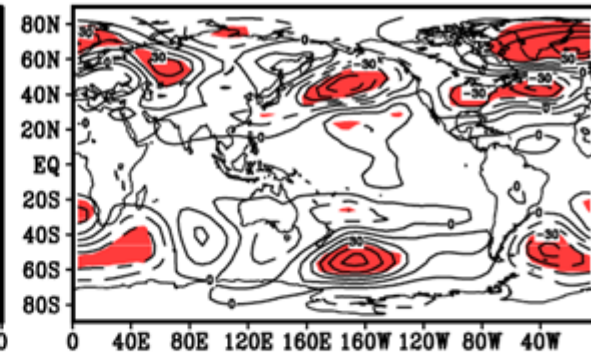
TP1

a) GEFA Z250 Rsp to TP1,  $\sigma(\text{TP1})=0.42^\circ\text{C}$



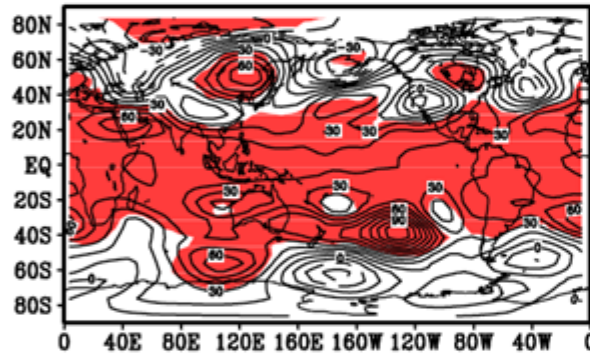
NP1

d) GEFA Z250 Rsp to NP1,  $\sigma(\text{NP1})=0.26^\circ\text{C}$



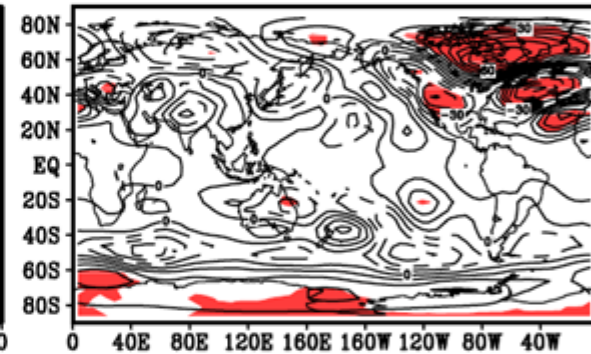
TI1

b) GEFA Z250 Rsp to TI1,  $\sigma(\text{TI1})=0.23^\circ\text{C}$

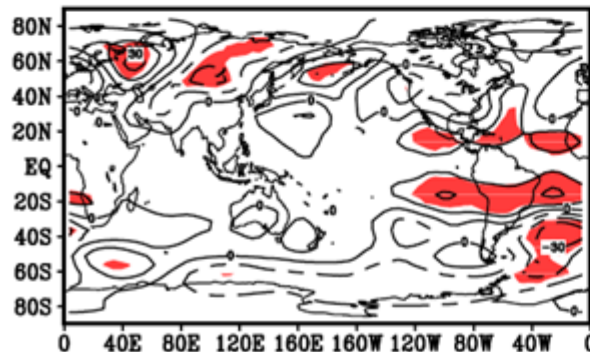


NA1

e) GEFA Z250 Rsp to NA1,  $\sigma(\text{NA1})=0.22^\circ\text{C}$



c) GEFA Z250 Rsp to TA1,  $\sigma(\text{TA1})=0.28^\circ\text{C}$



TA1







EFA: Equilibrium Feedback Analysis  
---- Univariate Statistical Assessment

GEFA: Generalized Equilibrium Feedback Analysis  
---- Multivariate Statistical Assessment

Comparison with LIM/FDT





# Linear Inverse Modeling (LIM)/ Fluctuation-Dissipation Theorem(FDT)

Dynamic system  
(N white, Gaussian)

$$\frac{dZ}{dt} = \mathbf{G}Z + N(t)$$

Most probable solution

$$\hat{Z}(t + \tau) = \exp(\mathbf{G}\tau)Z(t)$$

Matrix estimator

$$\mathbf{G} = \tau^{-1} \ln\{\mathbf{C}(\tau)\mathbf{C}(0)^{-1}\}, \quad LIM$$

$$-\mathbf{G}^{-1} = \int_0^{\infty} \mathbf{C}(\tau)\mathbf{C}(0)^{-1} d\tau, \quad FDT$$

$$\text{where } \mathbf{C}(\tau) = \langle Z(t + \tau), Z^T(t) \rangle$$

Forecast/prediction

$$Z(t + \tau_0) = \exp(\mathbf{G}\tau_0)Z(t) + \varepsilon$$





# LIM/FDT

$X$  : atmosphere (fast),  $T$  : ocean/land (slow)

$$\frac{dX}{dt} = aX + bT + N_X$$

$$\frac{dT}{dt} = cX + dT + N_T$$

GEFA

$$0 = \frac{dX}{dt} = aX + bT + N_X$$

$$X = \mathbf{B} T + N, \quad \mathbf{B} = -a/b$$

$$Z = \begin{bmatrix} X \\ T \end{bmatrix}, \quad \mathbf{G} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \quad N = \begin{bmatrix} N_X \\ N_T \end{bmatrix};$$

$$\frac{dZ}{dt} = \mathbf{G}Z + N(t) \quad \Rightarrow \quad \mathbf{B} = -a/b$$





## Summary

- GEFA provides a potentially effective approach to **assess and understand** atmospheric response to SST/Land surface forcing, **systematically**.
- There is a modest but significant atmospheric response to extratropical SST anomaly
- There is no observational evidence of significant positive vegetation feedback on rainfall in N. Africa, yet.

## Future Work

- GEFA modifications/applications
- GEFA + Dynamic assessment
- Prediction: GEFA(Atm)+LIM/FDT(SST)  
FDT(Atm+SST)





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