Adaptive stochastic modeling using data assimilation

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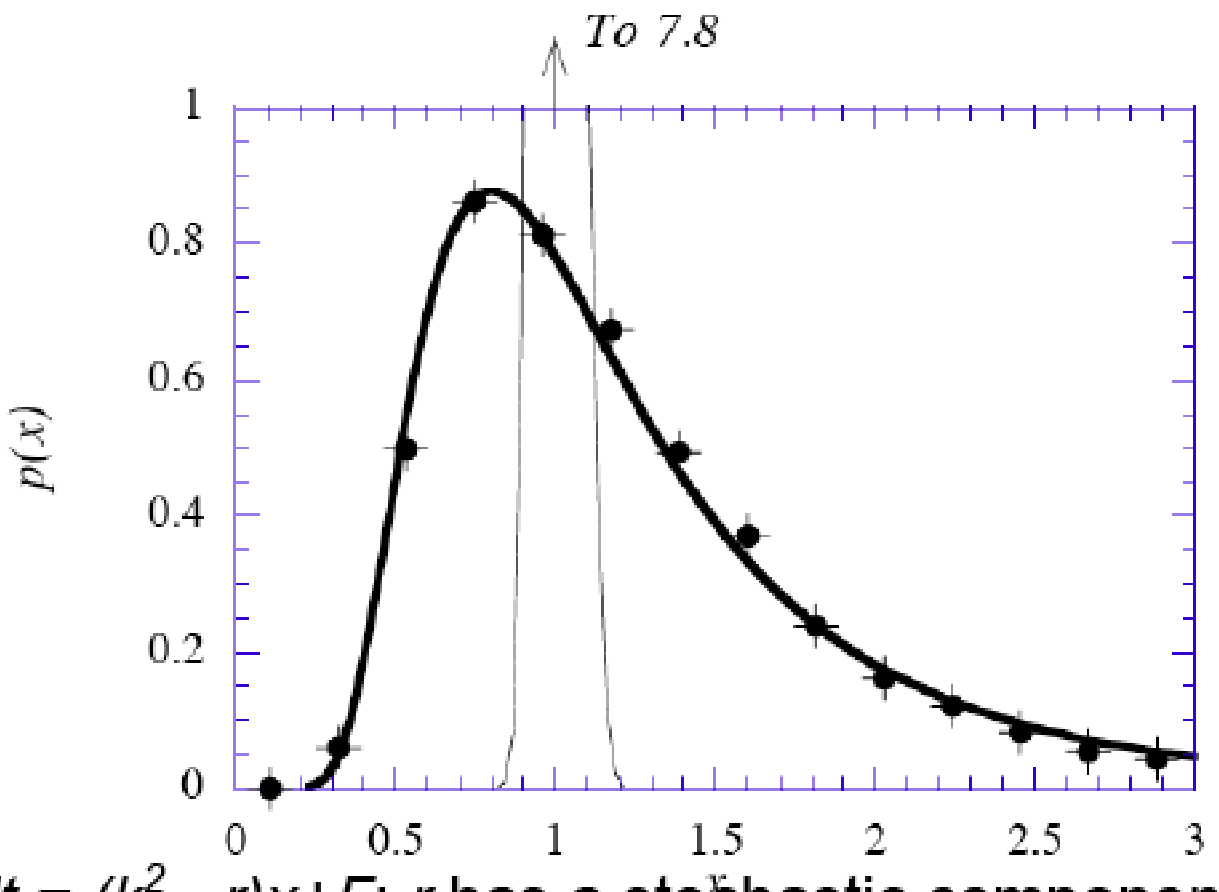
Approach

- Model inadequacy can be reduced through an iterative approach to parameter estimation.
  - Use data assimilation to estimate statistics of stochastic parameters
  - Collaboration between process scientists and those specializing in the science of prediction.
- Biases due to insufficient resolution can be reduced through an assimilation-like process during the forecast cycle.
  - How do we use these distributions?
  - Can we get around having to rewrite the model to include stochastic parameterizations?

Ewald, Penland and Temam (2004)



dx/dt = (*k*^{2} - *r*)*x*+*F*; *r* has a stochastic component.

Moral: Can't just add random numbers to a code.

Proof-of-concept experiments
(Hansen and Penland *Physica D* 2008)

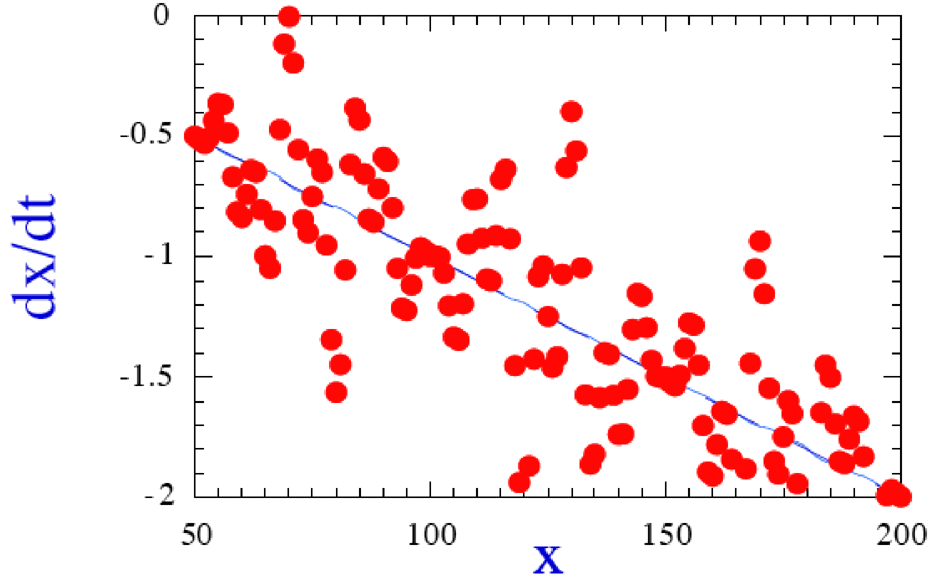
- Experiment where system is SDE and model is DDE
  - Can we uncover correct stochasticity? (YES, with proper scaling)
  - Can we successfully use the parametric distribution in our DDE to improve forecasts? (YES, if applied as estimated)
- Experiment where system is SDE.

Quick Review of the Central Limit Theorem

$$d\mathbf{x}/dt = \varepsilon^2 \mathbf{G}(\mathbf{x},t) + \varepsilon \mathbf{F}(\mathbf{x},t)$$

$\varepsilon^2 \mathbf{G}(\mathbf{x},t)$ is slow

$\varepsilon \mathbf{F}(\mathbf{x},t)$ is fast



Papanicolaou and Kohler (1974)

Choose a scaling $s = \varepsilon^2 t$:

$$\frac{d\mathbf{x}}{ds} = \mathbf{G}(\mathbf{x}, s/\varepsilon^2) + \frac{1}{\varepsilon}\boldsymbol{F}(\mathbf{x}, s/\varepsilon^2) \quad (*)$$

For simplicity, say

$$F_i(\mathbf{x}, s/\varepsilon^2) = \sum_k F_i^k(\mathbf{x}, s)\eta_k(s/\varepsilon^2)$$

and

$$C_{km} = \int_{-\infty}^{\infty} < \eta_k(t)\eta_m(t'+t) > dt' \equiv (\boldsymbol{\phi}\boldsymbol{\phi}^T)_{km}$$

$$\lim_{\substack{t\to\infty \\ \varepsilon\to 0}} (*) \rightarrow \quad d\mathbf{x} = \mathbf{G}(\mathbf{x}, s)\, ds + \sum_{k,\alpha} \boldsymbol{F}^k(\mathbf{x}, s)\phi_{k\alpha} \bullet dW_\alpha$$

(W is a Brownian motion; $dW \in \boldsymbol{N}(0, dt)$).

System is SDE, Model is DDE

- System equations

$$dx = -\alpha_0 (x - y)dt$$
$$- \alpha_s (x - y) \circ dW$$
$$dy = (rx - xz - y)dt$$
$$dz = (xy - bz)dt$$

- Model equations

$$\dot{x} = -\tilde{\alpha}(x - y)$$
$$\dot{y} = rx - xz - y$$
$$\dot{z} = xy - bz$$
$$\dot{\tilde{\alpha}} = 0$$

$$\tilde{\mathbf{x}} = [x \ y \ z \ \tilde{\alpha}]^T$$

1) Get "truth" by integrating using stochastic RK4 with Δ_s = 0.00025, α_0 = 10; α_S = 0.1

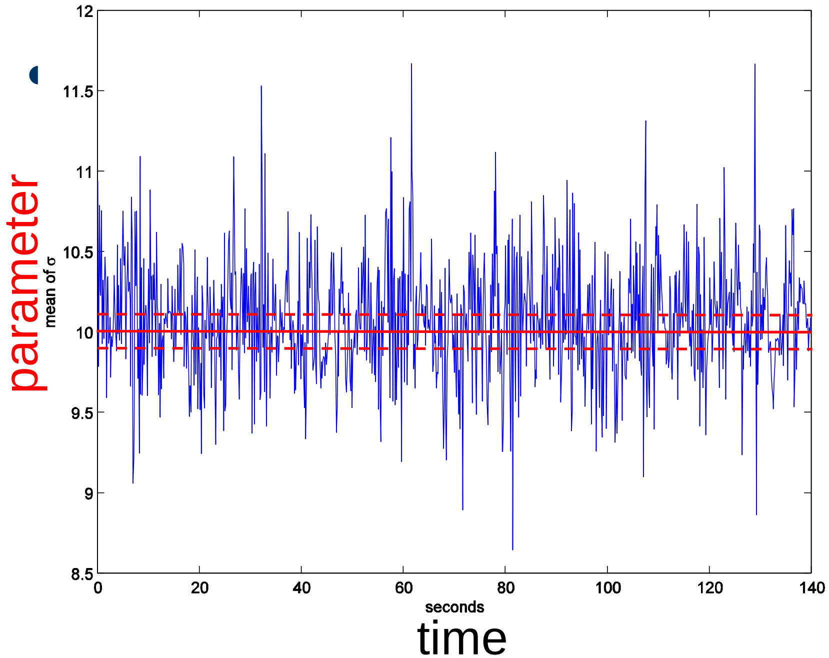
2) Use the ensemble Kalman filter or 4-D Var to assimilate $\{x,y,z,\alpha\}$ for N_{obs}=1000 observations, each separated by an interval of τ_{obs}=0.05.

3) Assimilation model is the deterministic Lorenz model with a timestep Δ_D = 0.01.

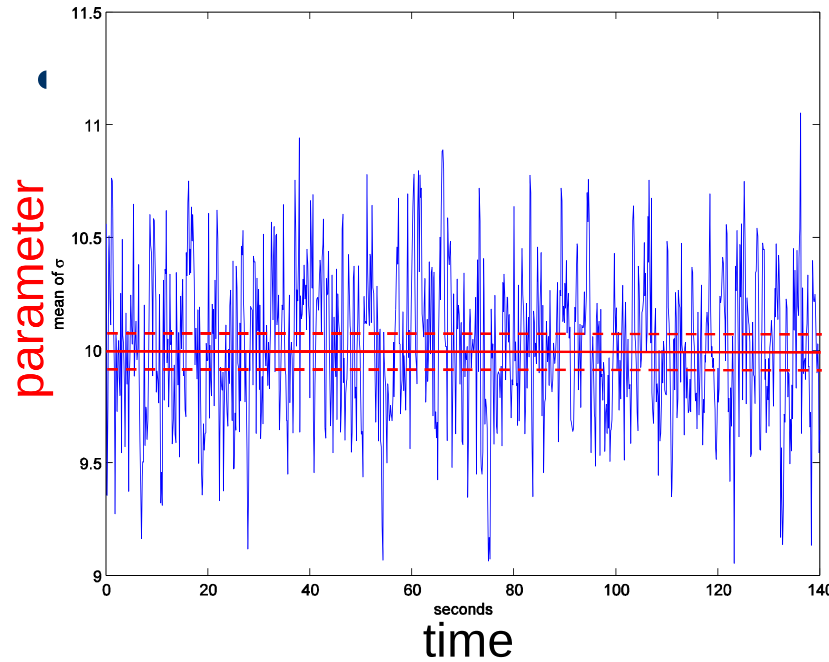
4) Size of ensemble: N_{ens}=250.

5) Initial guess: $\tilde{\alpha}$ = 11±0.7

Can DA uncover the correct form of the stochasticity? - IF DONE RIGHT



$\bar{\alpha} = 10.08, \; std(\alpha) = 0.36$



$\bar{\alpha} = 10.02, \; std(\alpha) = 0.44$

$$\alpha_0 = 10, \; \alpha_s = 0.1$$

Scale results with the CLT

- Note that DA doesn't estimate α_s but rather $\alpha_s \left\| \frac{dW}{dt} \right\|$

$$\frac{dx}{dt} = -\alpha_0 (x-y) - \alpha_s (x-y) \circ \frac{dW}{dt}$$

- dW/dt has statistics $N(0, \frac{1}{\Delta t})$

- In the DA case, $\Delta t = \tau_{obs}$ so scale DA's est. a_s by $\sqrt{\tau_{obs}}$

$$a_s \left\| \frac{dW}{dt} \right\| \sqrt{\tau_{obs}} = 0.44\sqrt{0.05} = 0.098$$

$$\alpha_0 = 10, \ \alpha_s = 0.1$$

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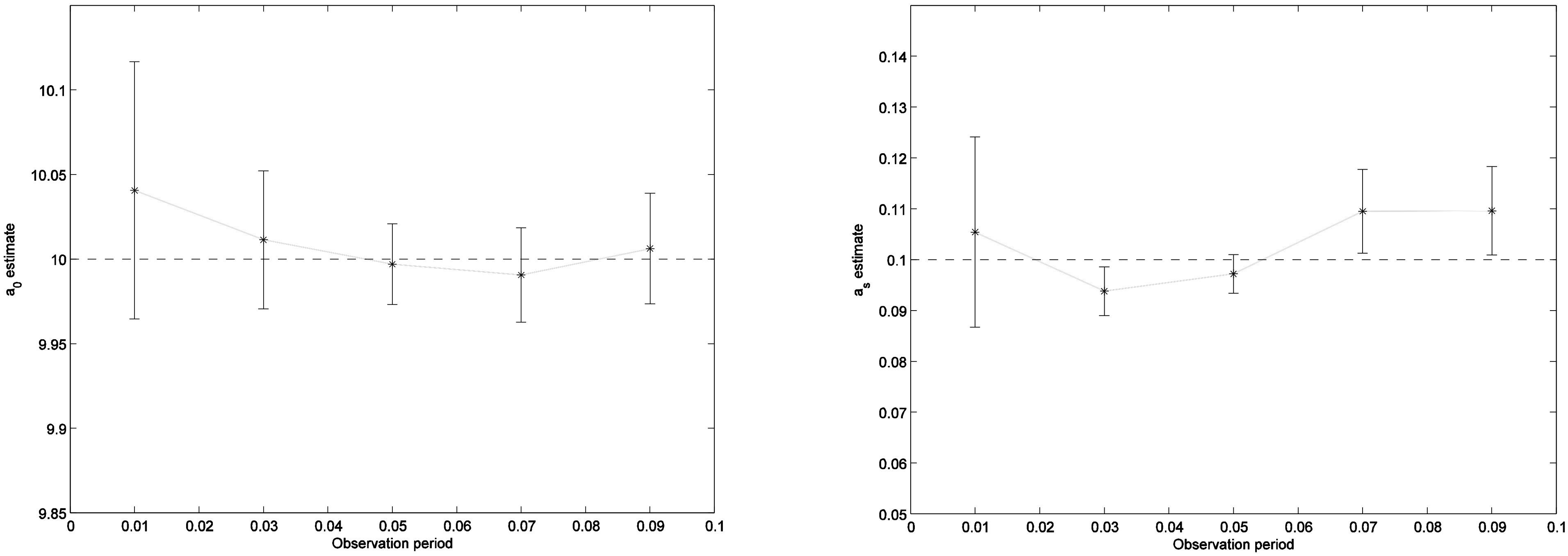
$$\lim_{\substack{t\to\infty \\ \varepsilon\to 0}} (*) \rightarrow \; d\mathbf{x} = \mathbf{G}(\mathbf{x}, s)\, ds + \sum_{k,\alpha} \boldsymbol{F}^k(\mathbf{x}, s)\phi_{k\alpha} \bullet dW_\alpha$$

(W is a Brownian motion; $dW \in \boldsymbol{N}(0, dt)$).

Test of CLT scaling argument

Estimate of α_0 | Estimate of α_s



How does this help us with the integration problem?

Back to stochastic Lorenz system: We have a trajectory; let's try to simulate it.

- Untuned deterministic Lorenz model using first-guess value of $\alpha_0 = 11$; $\alpha_S = 0$. $\Delta_D = 0.01$.

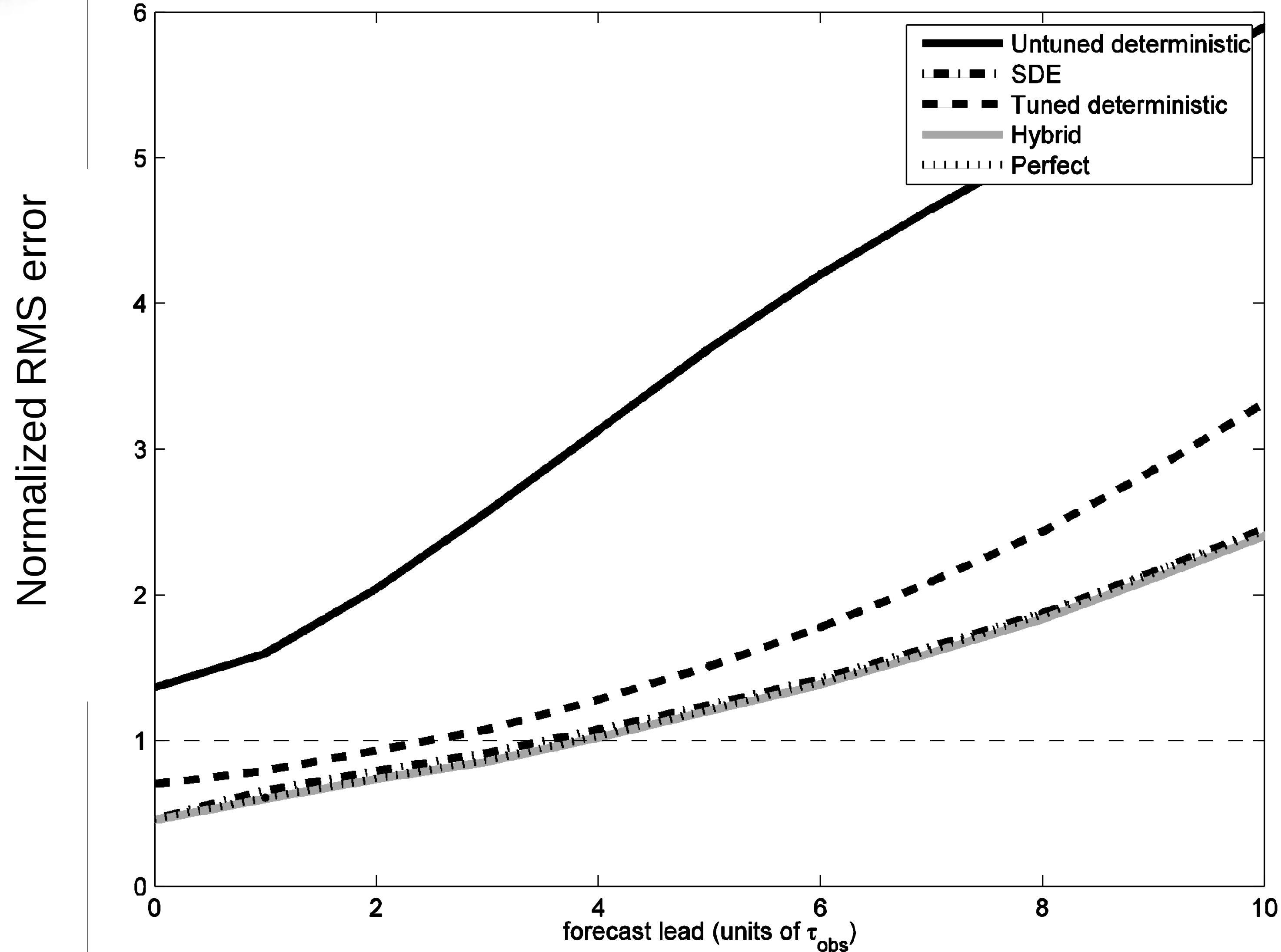
- Tuned deterministic Lorenz model using assimilated value of $\alpha_0 = 10.2$; $\alpha_S = 0$. $\Delta_D = 0.01$.

- SDE with assimilated values of $\alpha_0 = 10.2$; $\alpha_S = 0.098$. $\Delta_s = 0.00025$.

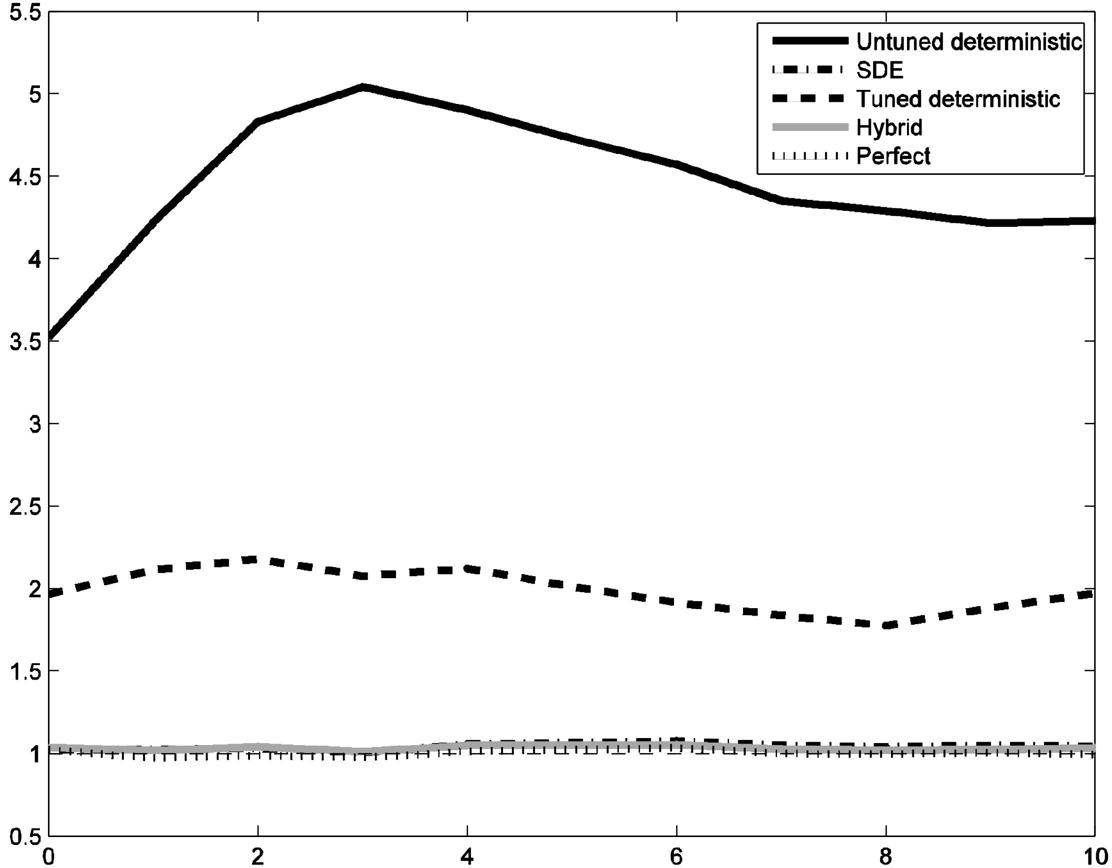
- Perfect model (SDE with $\alpha_0 = 10.$; $\alpha_S = 0.1$). $\Delta_s = 0.00025$.

- Hybrid model: Piecewise deterministic; $\alpha_0 = 10.2$; $\alpha_S = 0.44$; perturbation held constant over interval $\tau_{obs}=0.05$. $\Delta_D = 0.01$.

Median of ensemble mean forecast distributions



Must assess probabilistically!



std(err/ens_std)

Forecast lead (units of τ_{obs})

Conclusions

- Parameter estimation efforts can be brought to bear on the structural model inadequacy problem, particularly resolution problems.
- Such efforts to reduce model inadequacy ultimately lead to a sensible way to account for model inadequacy stochastically.
- Synoptic time-scale “stochasticity” via parameter estimation does a great job accounting for model inadequacy during forecasting
  - A LOT of research necessary (e.g., one-sided distributions?)
  - Efforts underway to apply to an NWP model.