

# The correction of forecast errors based on MOS: The impact of initial condition and model errors

S. Vannitsem

Institut Royal Météorologique de Belgique

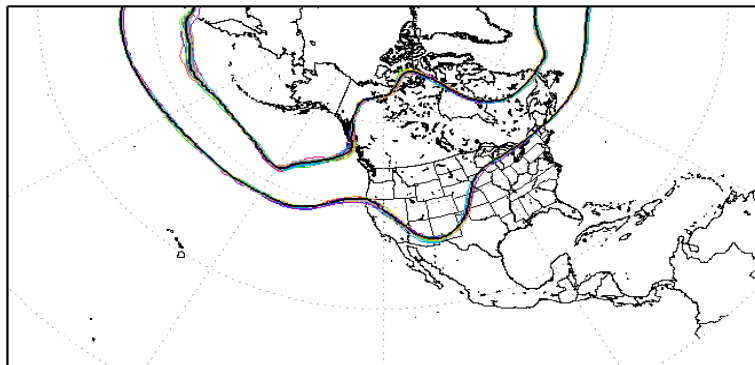
Dresden, July 30, 2009

## Outline

1. Introduction
2. The MOS technique
3. The role of initial and model errors: a dynamical view
4. Analysis of operational forecasts
5. A unified MOS scheme for deterministic AND ensemble forecasts:  
EVMOS
6. Conclusions and perspectives

LEGEND:      eps p01      eps p05      eps p09      eps p13  
 ens mean      eps p02      eps p06      eps p10      eps p14  
 oper GFS      eps p03      eps p07      eps p11  
 eps ctl      eps p04      eps p08      eps p12

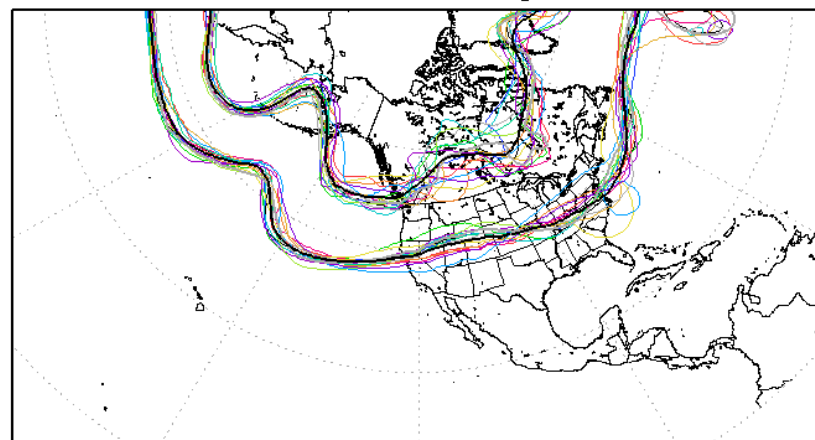
ens run for 00Z 11Dec2006 valid 06Z11DEC2006 522 and 564 height contours at 500-hPa, 1x1 degree resolution



# The quality of atmospheric forecasts decreases as a function of time

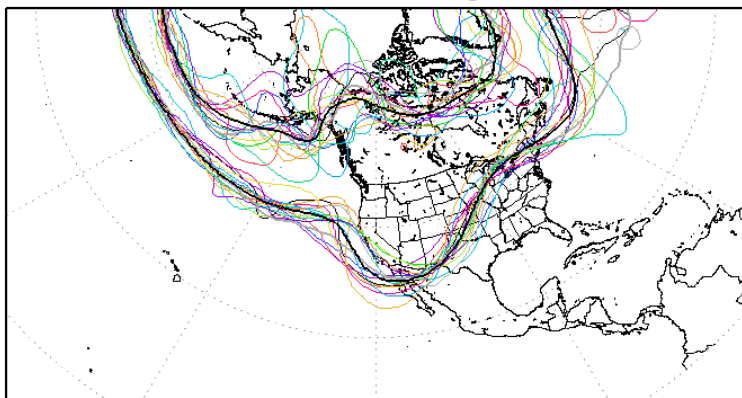
LEGEND:      eps p01      eps p05      eps p09      eps p13  
 ens mean      eps p02      eps p06      eps p10      eps p14  
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 eps ctl      eps p04      eps p08      eps p12

ens run for 00Z 11Dec2006 valid 12Z15DEC2006 522 and 564 height contours at 500-hPa, 1x1 degree resolution



LEGEND:      eps p01      eps p05      eps p09      eps p13  
 ens mean      eps p02      eps p06      eps p10      eps p14  
 oper GFS      eps p03      eps p07      eps p11  
 eps ctl      eps p04      eps p08      eps p12

ens run for 00Z 11Dec2006 valid 12Z18DEC2006 522 and 564 height contours at 500-hPa, 1x1 degree resolution

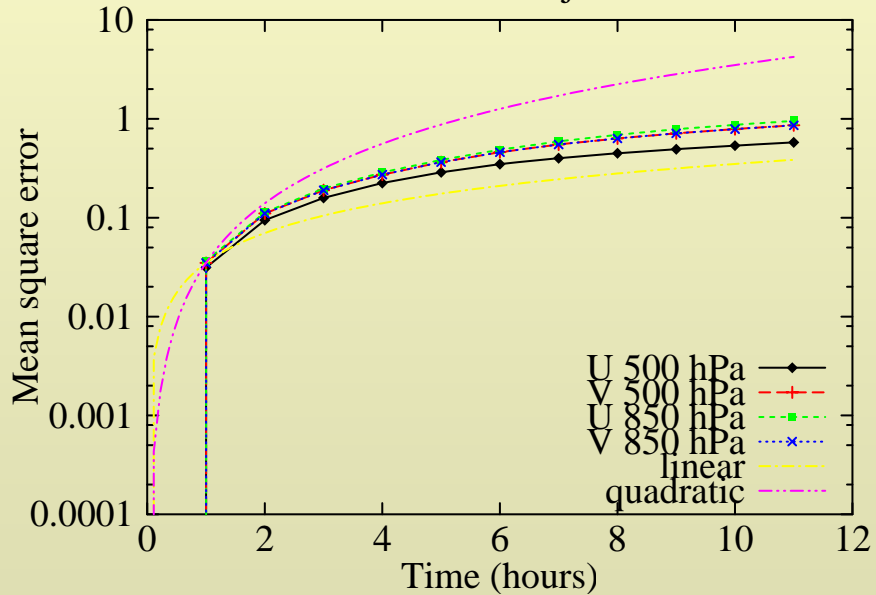


NCEP ensemble forecasts

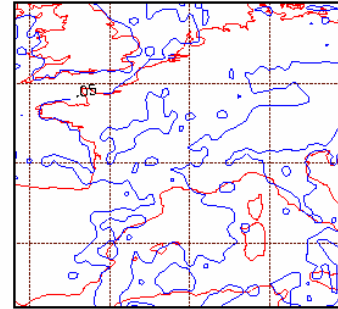
# Model error dynamics

Eta model runs with  
Two different convective  
schemes

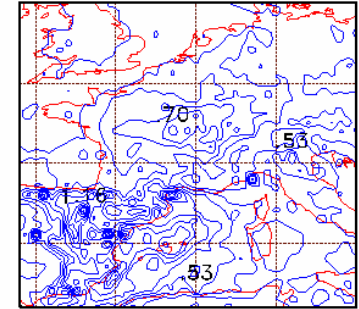
Kain-Fritch vs Betts-Miller-Janjic convective scheme



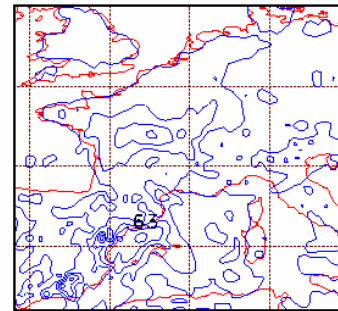
error vs time, T 850 01h



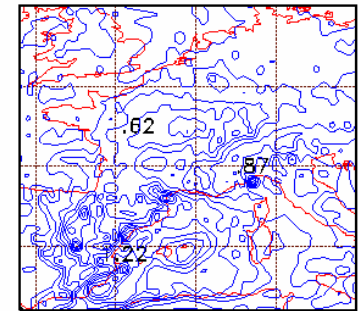
error vs time, T 850 07h



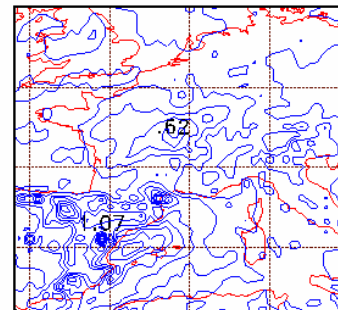
error vs time, T 850 03h



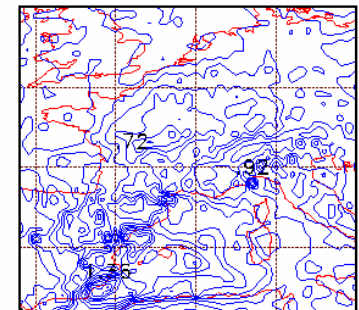
error vs time, T 850 09h



error vs time, T 850 05h



error vs time, T 850 11h



Two sources of errors affecting the forecasts and the data assimilation process:

- initial condition errors
- model errors

See *C. Nicolis, Rui A. P. Perdigao, and S. Vannitsem* Dynamics of Prediction Errors under the Combined Effect of Initial Condition and Model Errors, *Journal of the Atmospheric Sciences* , 66, 766–778, 2009.

How can we improve the forecast quality (Besides the improvement of the model and the observational system)?

**Use of techniques to post-process the forecasts in order to improve their quality.**

**What is corrected by the post-processing method?**

# The Model Output Statistics technique

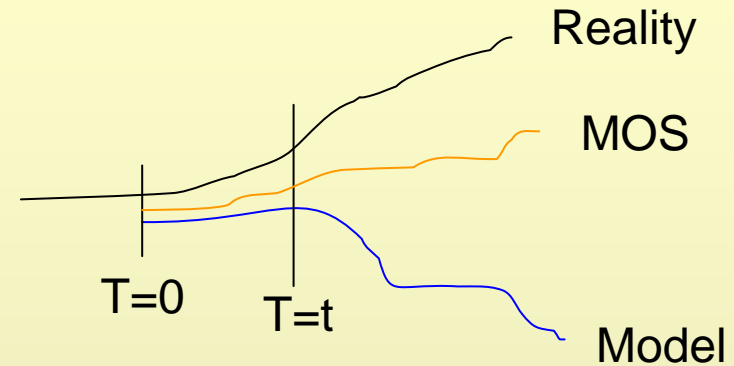
Post-processing of forecasts in order to improve their quality based on information gathered from past forecasts

- **'Deterministic' approach:** Linear techniques (Classical MOS, perfect prog), adaptative Kalman filtering. Ref: Glahn and Lowry, 1972, JAM, 11, 1203; Wilks, 2006, Academic Press;....
- **'Deterministic' approach:** Nonlinear techniques (Neural networks, nonlinear fits). Ref: Marzban, 2003, MWR, 131, 1103; Casaioli et al, 2003, NPG, 10, 373;....
- **Probabilistic approaches:** correction of the (deterministic) precipitation forecasts. Ref: Lemcke and Kruizinga, 1988, MWR, 116, 1077;....
- **(True) Probabilistic approaches:** Calibration of the ensemble forecasts. Ref: Roulston and Smith, 2003, Tellus, 55A, 16; Gneiting et al, 2005, MWR, 133, 1098; Wilks, 2006, MA, 13, 243; Hamill and Wilks, 2007, MWR, 135, 2379;.....

# The Linear MOS technique

Linear regression between a set of observables of forecasts and observations

$$X_c(t) = \alpha(t) + \sum_{i=1}^n \beta_i(t) V_i(t)$$



$$J(t) = \sum_{k=1}^M (X_{c,k}(t) - X_k(t))^2$$

M past forecasts

## Minimization of J(t)

n=1

$$\alpha(t) = \langle X(t) \rangle - \beta(t) \langle V(t) \rangle$$

$$\beta(t) = \frac{\langle X(t)V(t) \rangle - \langle X(t) \rangle \langle V(t) \rangle}{\langle V(t)^2 \rangle - \langle V(t) \rangle^2}$$

# Example of fit: 2-m Temperature, Uccle

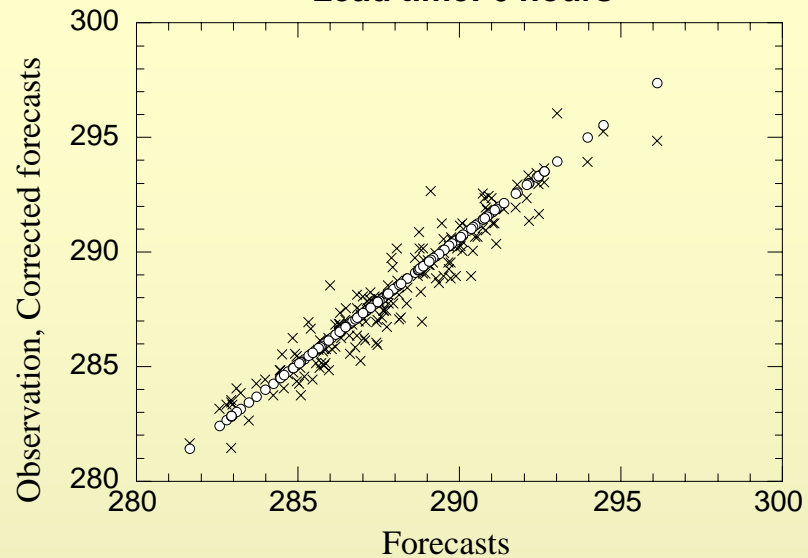
Training:

2 Summer seasons  
2002-2003

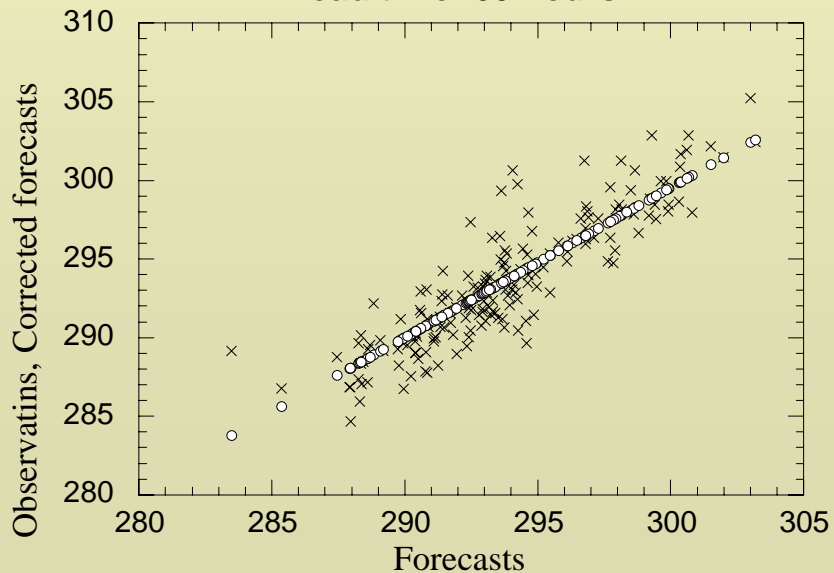
Verification:

2004-2005

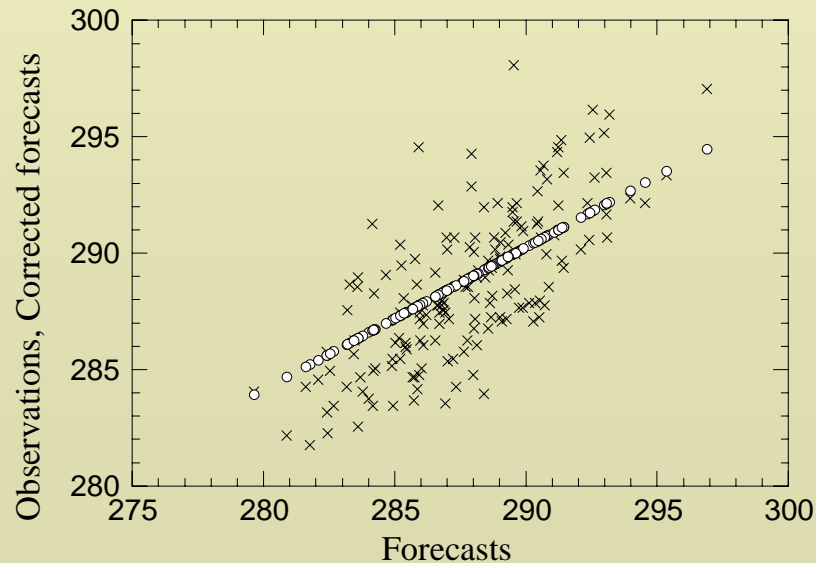
**Lead time: 6 hours**



**lead time: 60 hours**



**Lead time: 120 hours**





# Properties of the MOS forecast

$$\langle X_c(t) \rangle = \langle X(t) \rangle$$

$$\sigma_c^2(t) = \langle (X_c(t) - \langle X(t) \rangle)^2 \rangle = \frac{C(X(t), V(t))^2}{\sigma_{V(t)}^2 \sigma_X^2} \sigma_X^2 = \beta(t)^2 \sigma_{V(t)}^2$$

↘ Progressive decrease

## The mean square error

$$\langle (X_c(t) - X(t))^2 \rangle = \langle (V(t) - X(t))^2 \rangle - (\sigma_c(t) - \sigma_v(t))^2 - \underbrace{(\langle X(t) \rangle - \langle V(t) \rangle)^2}_{\text{Drift correction}}$$

Drift correction

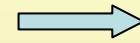
↙ Variability correction

# Short term dynamics of MOS forecasts (Formalism based on Nicolis, 2004, JAS)

The real system

$$\frac{d\vec{X}}{dt} = \vec{F}(\vec{X}, \vec{Y}, \{\mu\})$$

$$\frac{d\vec{Y}}{dt} = \vec{S}(\vec{X}, \vec{Y}, \{\lambda\})$$



Variables  
not described  
by the model

$\vec{X}$  and  $\vec{V}$  span the same phase space

The model

$$\frac{d\vec{V}}{dt} = \vec{G}(\vec{V}, \{\mu'\})$$

Formal sols

$$\vec{V}(t) = \vec{V}(0) + \int_0^t d\tau \vec{G}(\vec{V}(\tau), \{\mu'\})$$

$$\vec{X}(t) = \vec{X}(0) + \int_0^t d\tau \vec{F}(\vec{X}(\tau), \vec{Y}(\tau), \{\mu\})$$

# Typical initial errors

$$\vec{V}(0) = \vec{X}(0) + \vec{\varepsilon}(0) \longrightarrow \begin{array}{l} \text{Gaus random noise} \\ + \\ \text{Systematic error} \end{array} \quad \vec{\varepsilon} = \langle \vec{\varepsilon} \rangle + \sigma_{\varepsilon} N(0,1)$$

$$(\sigma_C(t) - \sigma_V(t))^2 = S_0 + S_1 t + \frac{1}{2} S_2 t^2 + O(t^3)$$

$$S_0 = \frac{\sigma_{\varepsilon(0)}^4}{\sigma_{\varepsilon(0)}^2 + \sigma_{X(0)}^2}$$

$$S_1 \propto 2\sigma_{\varepsilon(0)}^2 \underline{C(V(0), G(\vec{X}(0)) - F(\vec{X}(0)))}$$

$$S_2 \propto \underline{2C(V(0), G(\vec{X}(0)) - F(\vec{X}(0)))^2}$$

$$C(V(0), G(\vec{X}(0)) - F(\vec{X}(0))) = \langle V(0)(G - F) \rangle - \langle V(0) \rangle \langle G - F \rangle$$

FOR 2 PREDICTORS: C(V1,G-F), C(V2,G-F)

$$(\langle X(t) \rangle - \langle V(t) \rangle)^2 = S'_0 + S'_1 t + \frac{1}{2} S'_2 t^2 + O(t^3)$$

$$S'_0 = (\langle X(0) \rangle - \langle V(0) \rangle)^2 = \langle \varepsilon \rangle^2$$

Systematic initial error

$$S'_1 = 2(\langle X(0) \rangle - \langle V(0) \rangle)(\langle F(\vec{X}(0)) - G(\vec{V}(0)) \rangle)$$


Systematic model error

$$S'_2 = 2(\langle F(\vec{X}(0)) - G(\vec{V}(0)) \rangle)^2 + 2(\langle X(0) \rangle - \langle V(0) \rangle) \left( \left\langle \frac{d}{dt} (F(\vec{X}(0)) - G(\vec{V}(0))) \right\rangle_0 \right)$$

# Summary

## A. Correction of initial condition errors

- Systematic initial error is corrected.
- Random initial errors are only partially corrected.


$$S_0 = \frac{\sigma_{\varepsilon(0)}^4}{\sigma_{\varepsilon(0)}^2 + \sigma_{X(0)}^2}$$

## B. Correction of model errors

- The systematic part is corrected.
- The time-dependent part can be corrected provided that the covariance of the model error with the predictors is high.

# The Lorenz system

$$\frac{dx}{dt} = -y^2 - z^2 - ax + aF$$

$$\frac{dy}{dt} = xy - bxz - y + G$$

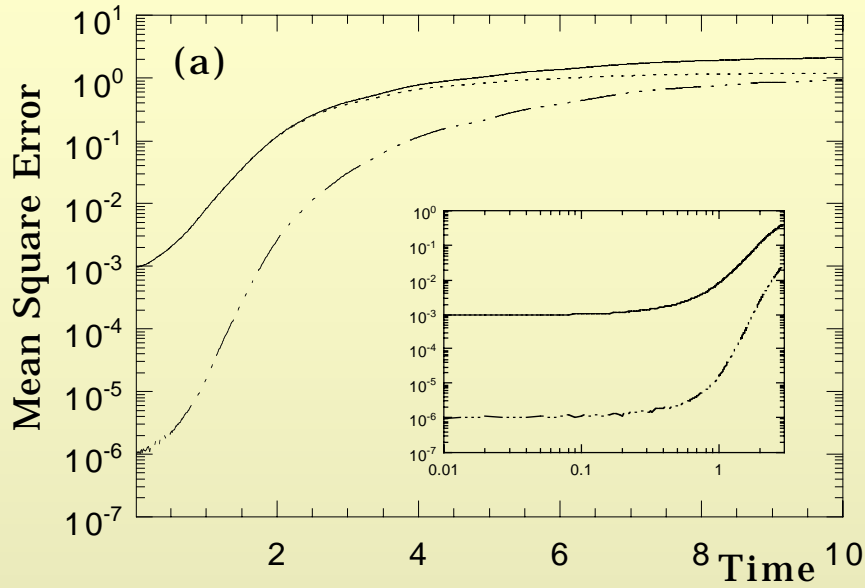
$$\frac{dz}{dt} = -bxy + xz - z$$

Chaotic for  $a=0.25$ ,  $b=6$ ,  $F=16$ ,  $G=3$

- Initial condition errors:  $N(0,s)$ ,  $s$  small
- Model errors (parametric error on  $a$ ,  $b$ ,  $F$  or  $G$ )

Results obtained with 100,000 realizations starting from different initial conditions

# The Lorenz system (continued)

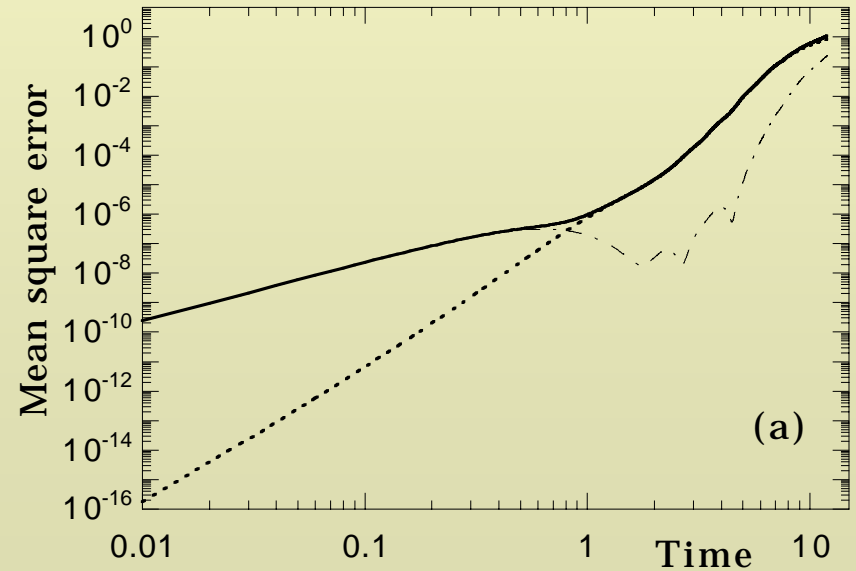


Correction of variable  $x$

Initial condition error only

Error in parameter  $a$

Model error only



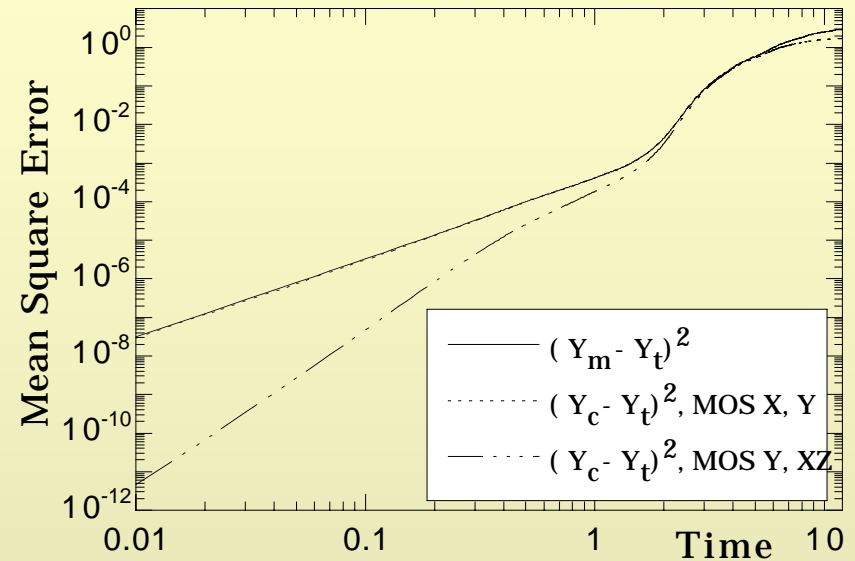
# The Lorenz system (continued)

Model errors only in the parameter  $b$

Correction of variable  $y$

Different MOS schemes

- Predictors:  $X, Y$
- Predictors:  $Y, XZ$



Partial conclusions

- IC not well corrected by MOS
- MOS can correct model errors when model predictors well chosen



## Application to the ECMWF forecasting system: temperature

**Data:** temperature for the period December 1 2001 to November 30 2005

- **Training period:** December 1 2001 to November 30 2003
- **Independent evaluation period:** December 1 2003 to November 30 2005

-Temperature 500 hPa, 850 hPa

-Temperature at 2 meters

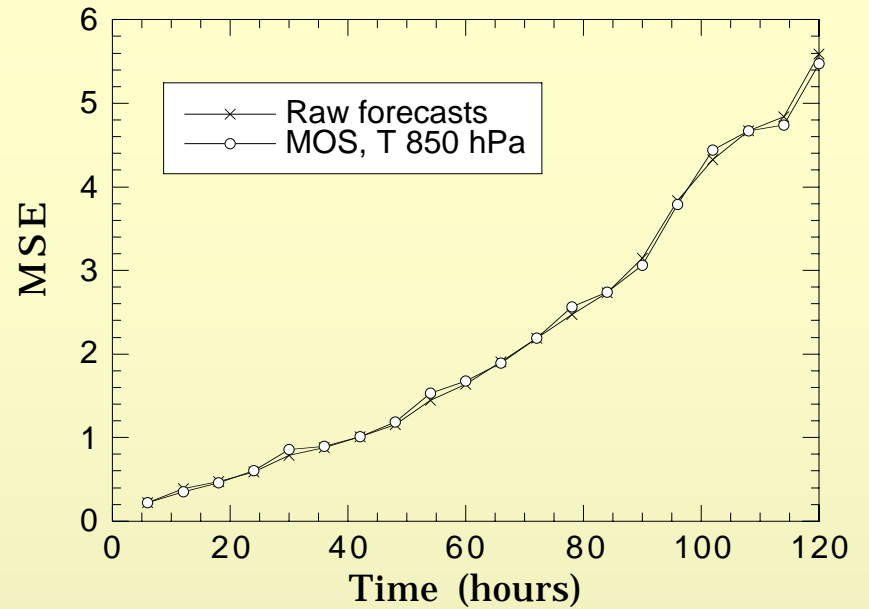
→Evaluation on a grid covering Belgium (verification using the set of analyses)

(52 N, 2 E) to (49 N, 7 E)

→Evaluation at some synoptic stations: model forecast given by the closest gridpoint

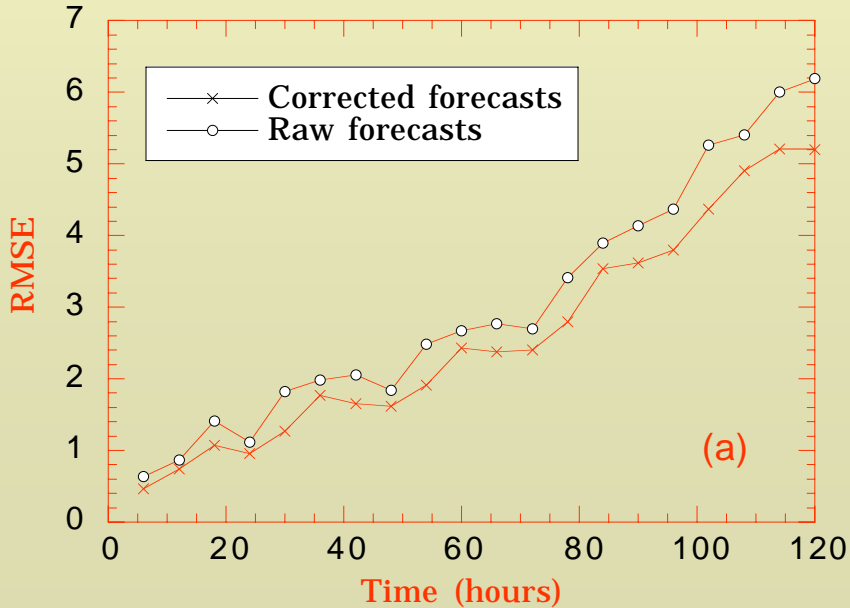
# Temperature at 850 hPa

Results averaged over the whole grid

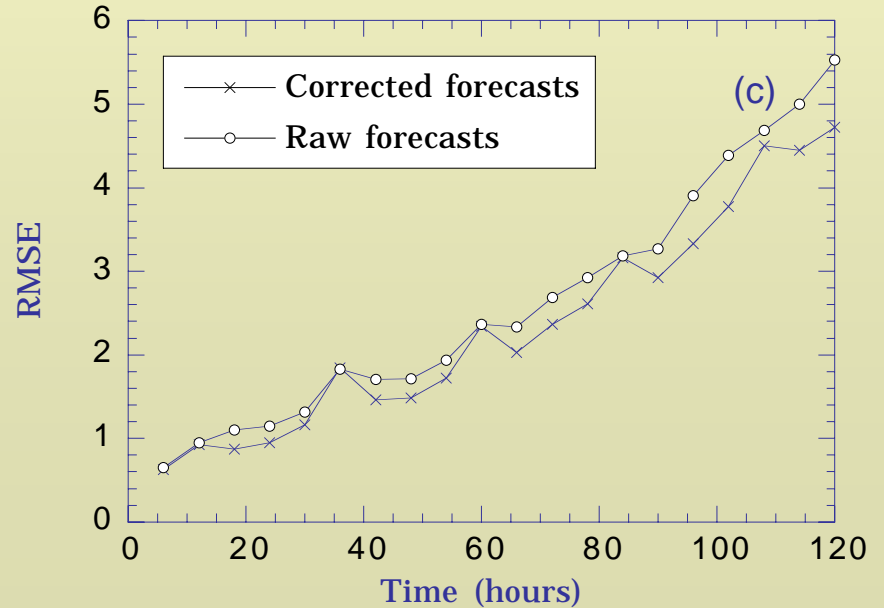


# Temperature at 2 meters

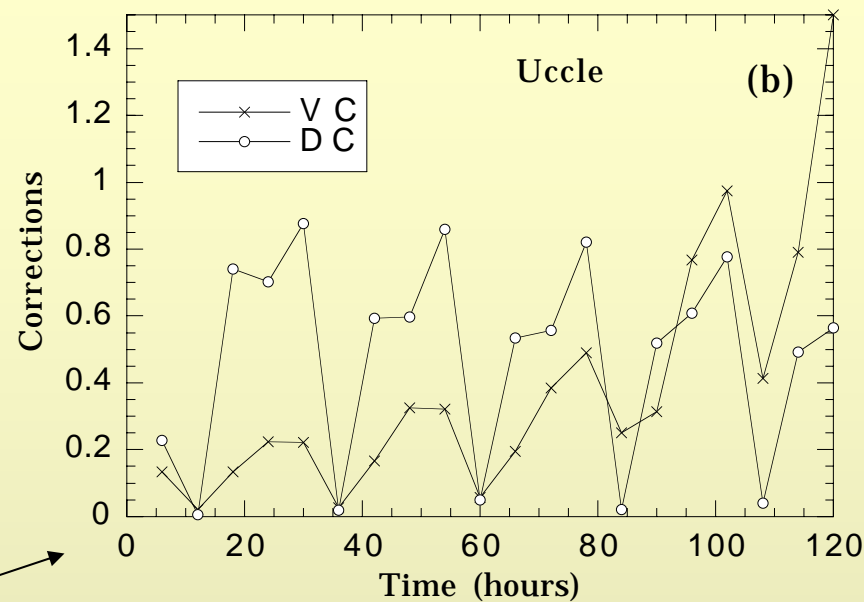
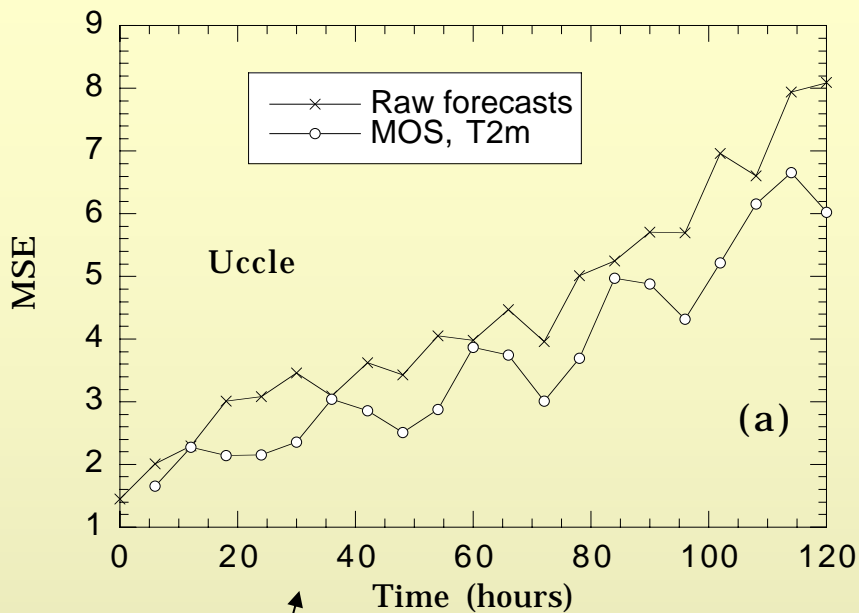
2002-2003



2004-2005

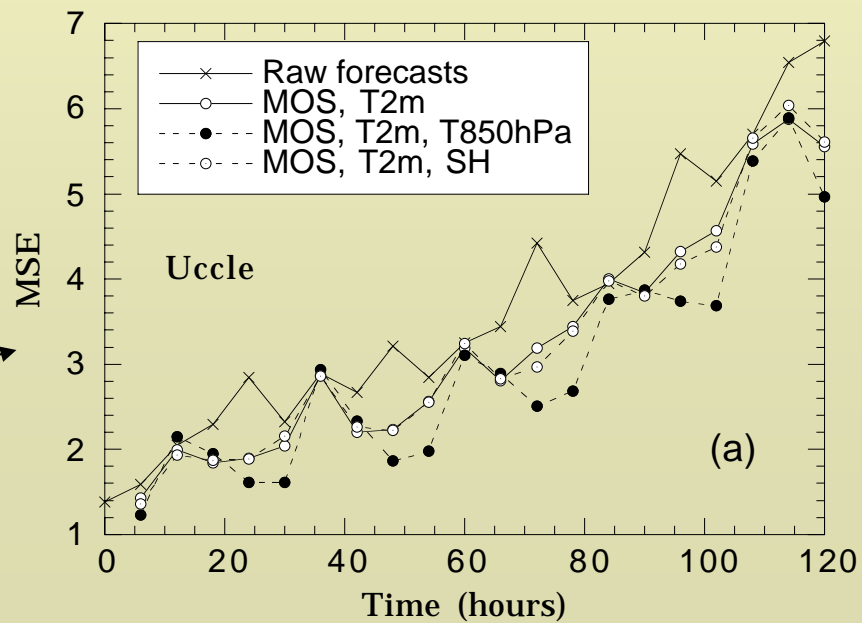


# Uccle-Ukkel

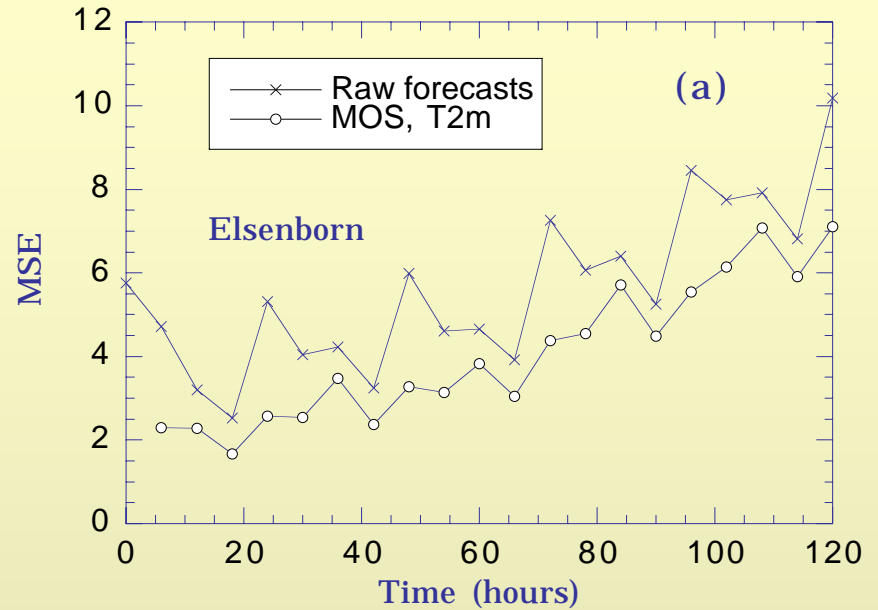
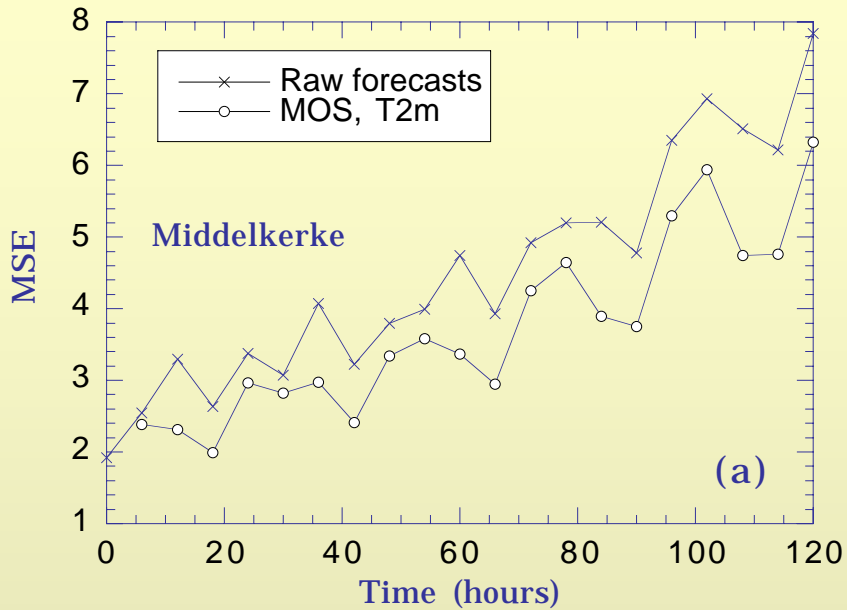


Training + verif on 2002-2003

Training: 2002-2003  
Analysis on 2004-2005



# Other stations



Training + verif on 2002-2003

A second predictor?

Sensible heat flux

A second predictor?

TMP 850 hPa

# New Linear MOS

$$X = \alpha + \beta\zeta + \kappa$$

$$V = \zeta + \delta$$

$$J(t) = \sum_{k=1}^M \frac{(V_k(t) - \zeta_k(t))^2}{\sigma_\delta^2(t)} + \sum_{k=1}^M \frac{(X_k(t) - (\alpha + \beta\zeta_k(t)))^2}{\sigma_\kappa^2(t)}$$

Intermediate cost function:

$$J(t) = \sum_{k=1}^M \frac{\overbrace{((\alpha(t) + \beta(t)V_k(t)) - X_{c,k}(t))^2}^{X_{c,k}(t)}}{\sigma_\kappa^2(t) + \beta^2(t)\sigma_\delta^2(t)}$$

One 'free' parameter:

$$\lambda = \frac{\sigma_\delta^2}{\sigma_\kappa^2} \text{ fixed to } \frac{\sigma_V^2}{\sigma_X^2}$$

Needs some knowledge about the sources of errors

Minimization

$$\alpha(t) = \langle X(t) \rangle - \beta(t) \langle V(t) \rangle$$

$$\beta(t) = \sqrt{\frac{\langle X(t)^2 \rangle - \langle X(t) \rangle^2}{\langle V(t)^2 \rangle - \langle V(t) \rangle^2}} = \sqrt{\frac{\sigma_X^2(t)}{\sigma_V^2(t)}}$$

## Statistical Properties

$$\langle X_C(t) \rangle = \langle X(t) \rangle$$

$$\sigma_C^2(t) = \langle (X_C(t) - \langle X(t) \rangle)^2 \rangle = \beta(t)^2 \sigma_{V(t)}^2 = \sigma_{X(t)}^2$$

And the third moment,

$$\langle (X_C(t) - \langle X_C(t) \rangle)^3 \rangle = \beta^3(t) \langle (V(t) - \langle V(t) \rangle)^3 \rangle$$

This development can be extended to two predictors.



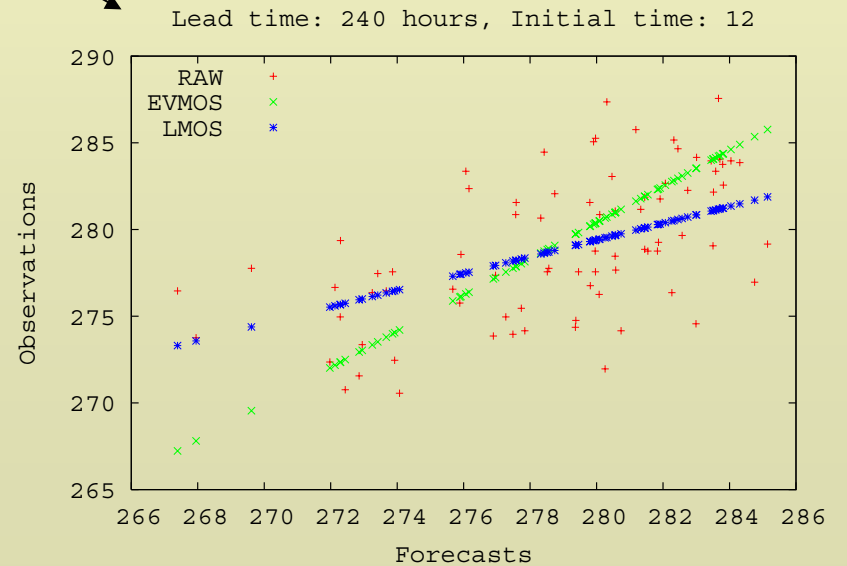
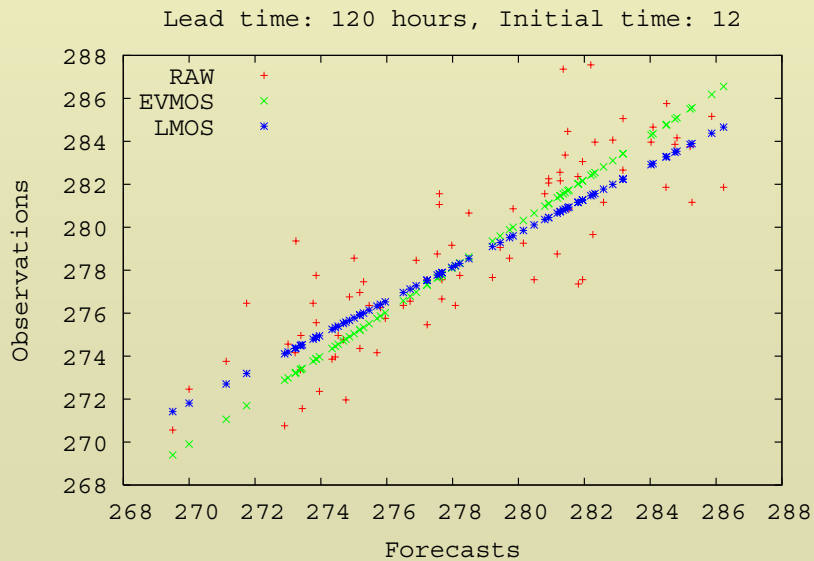
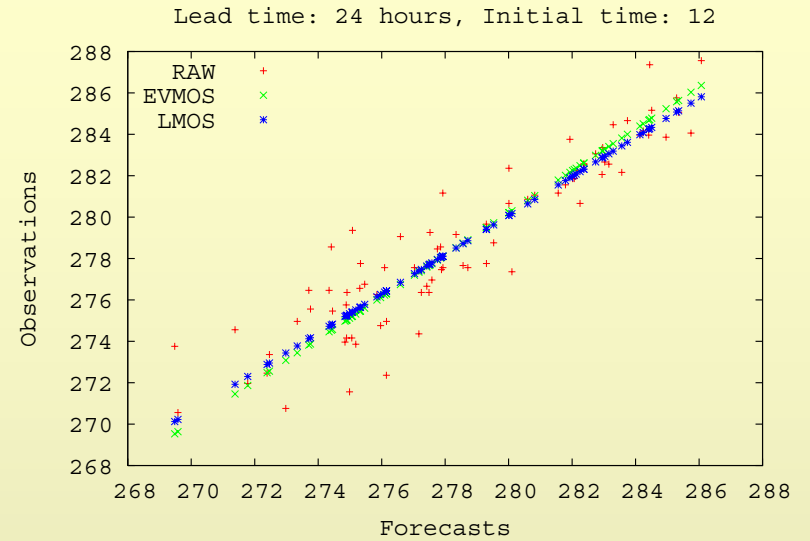
Vannitsem, S., A unified Linear Model Output Statistics scheme for both deterministic and ensemble forecasts. Accepted in *Quart. J. Roy. Met. Soc.*, 2009

# MOS at Station 'Uccle'.

Control Forecast is used.

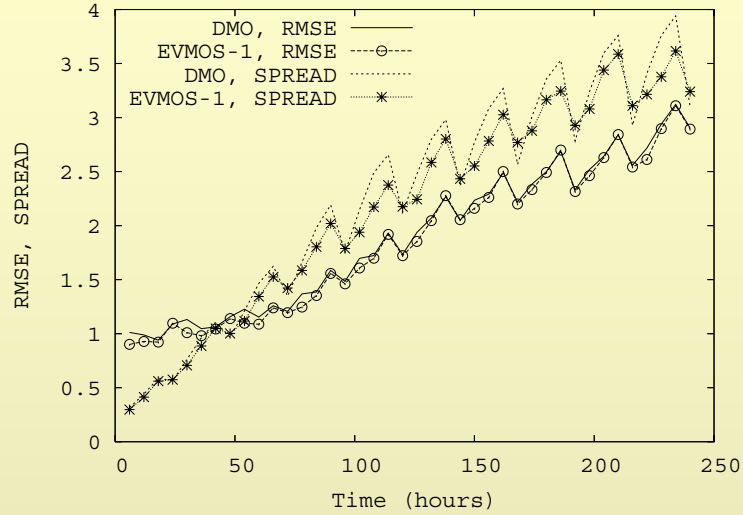
Training: DJF 2002-2004

Verif: DJF 2002

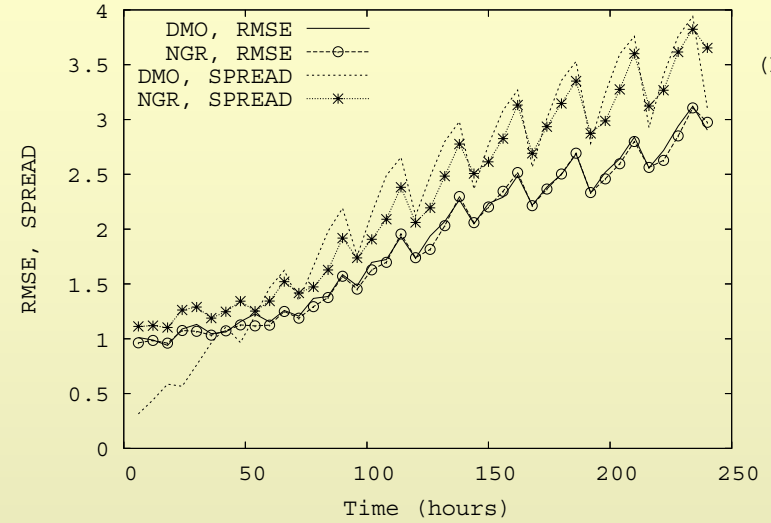


# Application to an operational ensemble: ECMWF, Winter, Uccle

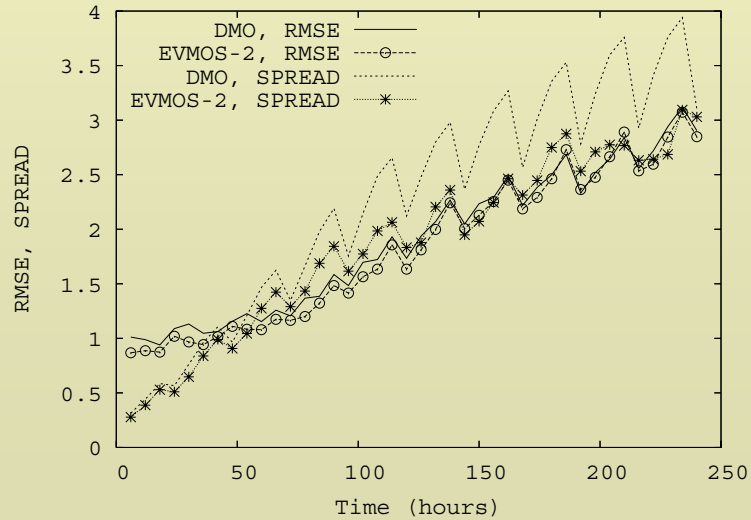
## 2 meter Temperature



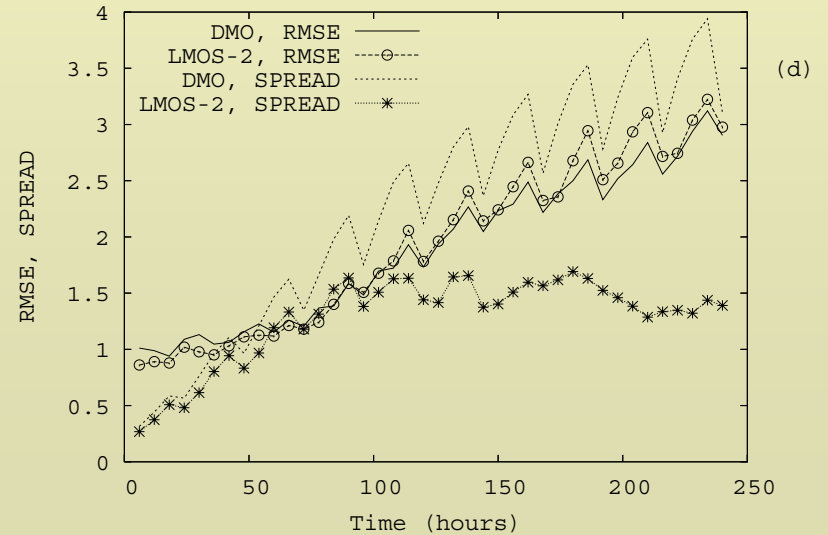
(a)



(b)



(c)



(d)



# Conclusions on MOS

A dynamical analysis of the MOS correction has been undertaken with emphasis on the correction of

- initial condition errors
- Model errors

Both partly corrected but correction more sensitive to model errors.

One central quantity

$$C(V(0), G(\vec{X}(0)) - F(\vec{X}(0)))$$

An additional model observable can improve considerably the correction if proportional to G-F

## For the ECMWF deterministic forecast:

### TMP 2m

- Substantial corrections with MOS with 1 observable
- MOS with 2 observables:
  - In the central part of Belgium: Tmp 850 hPa
  - On the coast: Sensible Heat Flux

### TMP 850 hPa

- No correction. → Mainly initial condition errors

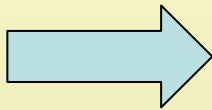
#### More infos:

Vannitsem S. and C. Nicolis, Dynamical properties of Model Output Statistics forecasts. *Mon. Wea. Rev.*, 136, 405-419, 2008.

Vannitsem S., Dynamical properties of MOS forecasts. Analysis of the ECMWF operational forecasting system. *Weather and Forecasting*, 23, 1032-1043, 2008.

## Correction of Ensemble forecasts based on EVMOS

A unified Scheme for both deterministic and ensemble forecasts is proposed, based on the assumption that the forecast displays errors.



- Do not damp the ensemble spread anymore.
- The correction provides a mean and a variability in agreement with the observations
- The scheme can be trained on the control forecast only.

### References:

Vannitsem, S., A unified Linear Model Output Statistics scheme for both deterministic and ensemble forecasts. Accepted in *Quart. J. Roy. Met. Soc.*, 2009