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Hidden Order in a Spatiotemporal Chaotic System

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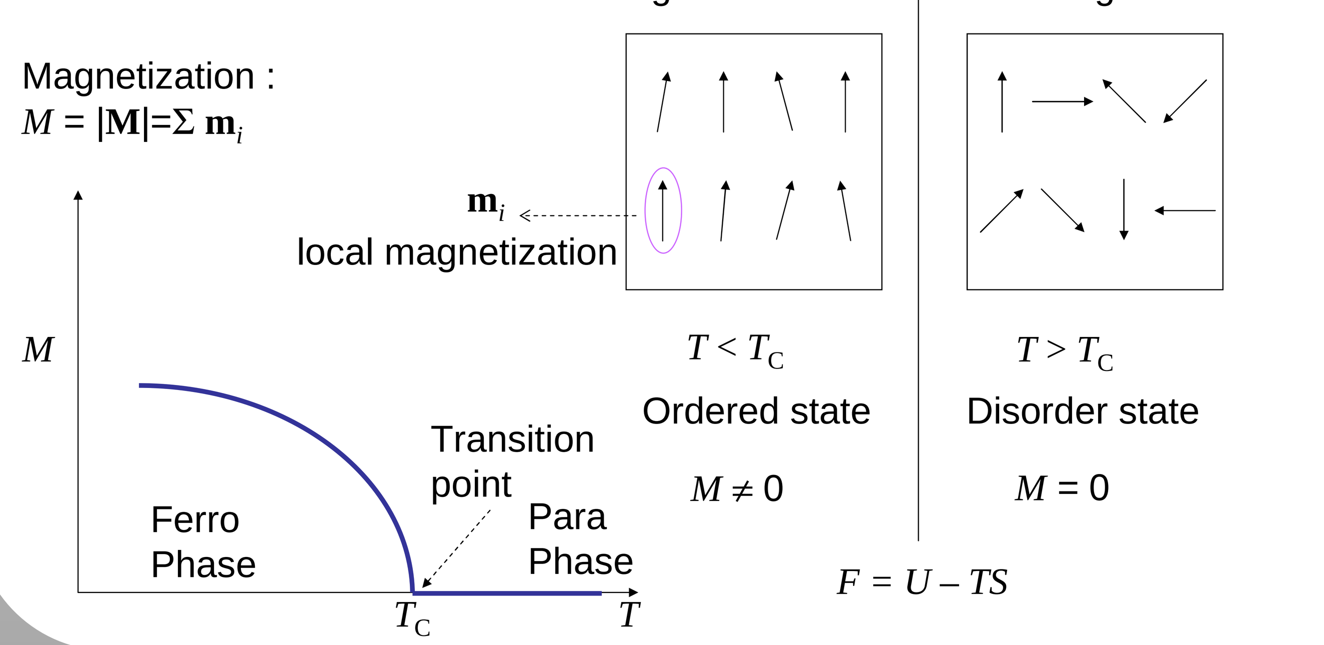
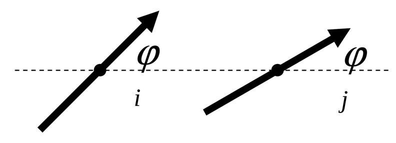
Introduction

Ferromagnetic-Paramagnetic Transition

Magnetization : $M = |\mathbf{M}| = \Sigma\, \mathbf{m}_i$



2D XY Model



Spin $\mathbf{s}_i = (\cos\varphi_i, \sin\varphi_i)$

$H = -J\sum_{i,j}\mathbf{s}_i\cdot\mathbf{s}_j = -J\sum_{i,j}\cos(\phi_i - \phi_j)$

Mermin-Wagner Theorem

$\lim_{r\to\infty}\langle\varphi(\mathbf{r})\varphi(\mathbf{0})\rangle = 0$

Mermin and Wagner, *Phys. Rev. Lett.* **17**, 1133 (1966)

Magnetization in 2D XY

$M_p = \langle\cos(\varphi - \langle\varphi\rangle)\rangle$

Sellitto and Holdsworth, *Fractals* **11**, 73 (2003).

Kosterlitz-Thouless transition (T_{KT})

■ $T < T_{KT}$: $C(r) \propto r^{-\zeta}$

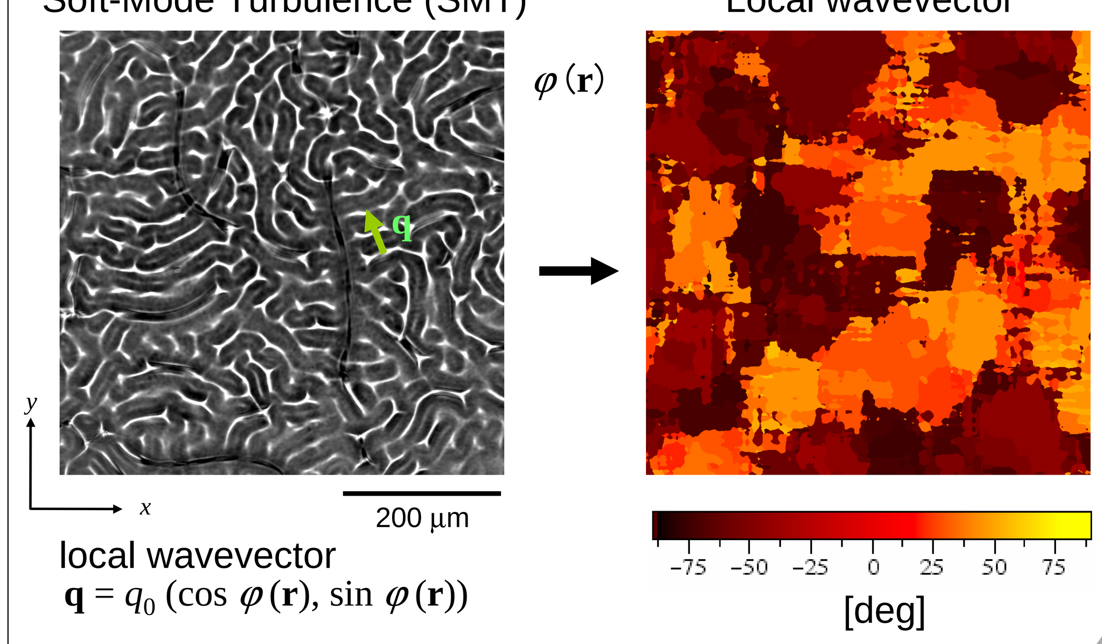
■ $T > T_{KT}$: $C(r) \propto \exp(-r/\xi_{XY})$

$C(r\to\infty) = 0$

$\xi_{XY} \propto \exp(b(T/T_{KT}-1)^{1/2})$

Kosterlitz and Thouless, *J. Phys. C* **6**, 1181 (1973)

SMT as a 2D-XY system

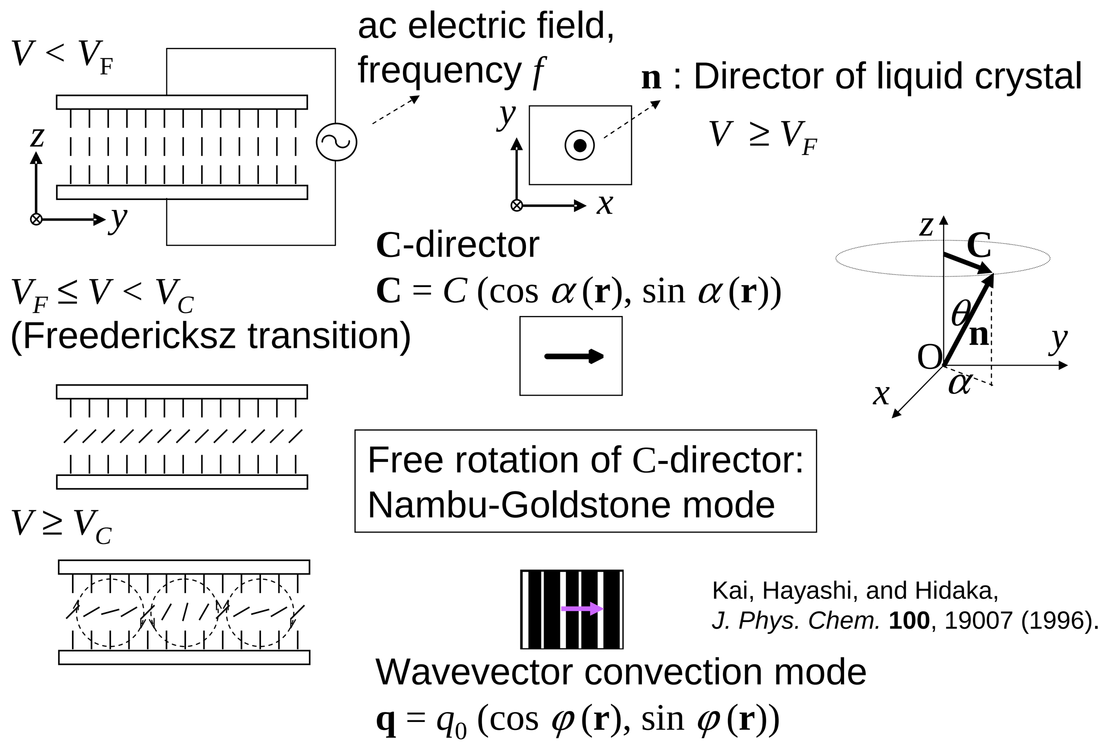


local wavevector
$\mathbf{q} = q_0(\cos\varphi(\mathbf{r}), \sin\varphi(\mathbf{r}))$

Tamura, Anugraha, Matsuo, Hidaka, Kai, *J. Phys. Soc. Jpn.* **75**, 063801 (2006)

Experimental

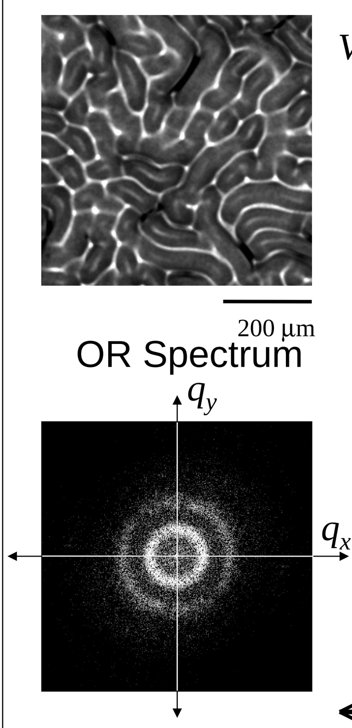
Electroconvection in Homeotropic Nematic



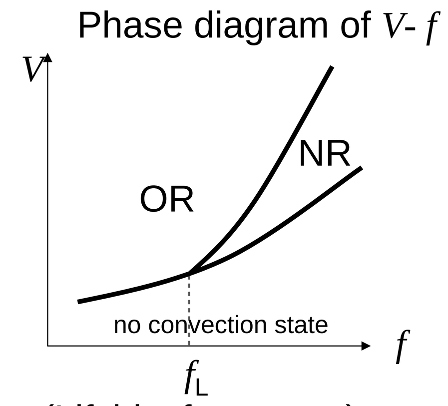
SMT : Spatiotemporal Chaos induced by nonlinear interaction between **C** and **q**.

Oblique and Normal Rolls

Oblique Rolls (ORs) $\varepsilon = 0.1$, $f = 200$ Hz



Phase diagram of V- f

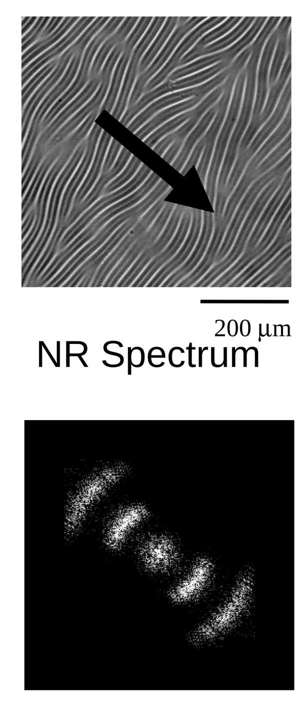


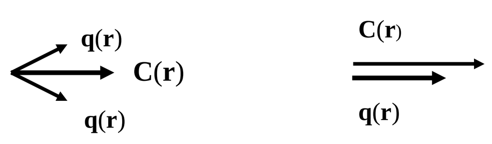
(Lifshitz frequency)

Hidaka, Huh, Hayashi, Kai, Tribelsky *Phys. Rev. E* **56**, R6256 (1997)

Control parameter
$\varepsilon = (V/V_c)^2 - 1$

Normal Rolls (NRs) $\varepsilon = 0.1$, $f = 1000$ Hz





Problems and Purpose

Nonlinear interactions for NR and OR

NR

$\tau_0\partial_t A = \left[\varepsilon + \xi_{xx}^2\partial_x^2 + \xi_{yy}^2(\partial_y - iq_c\alpha)^2 - g|A|^2 + i\beta_y(\partial\alpha/\partial y)\right]A$

$\gamma_1\partial_t\alpha = K_1\partial_y^2\alpha + K_3\partial_x^2\alpha + (\Gamma/4)(-iq_c A^*(\partial_y - iq_c\alpha)A + \text{c.c.})$

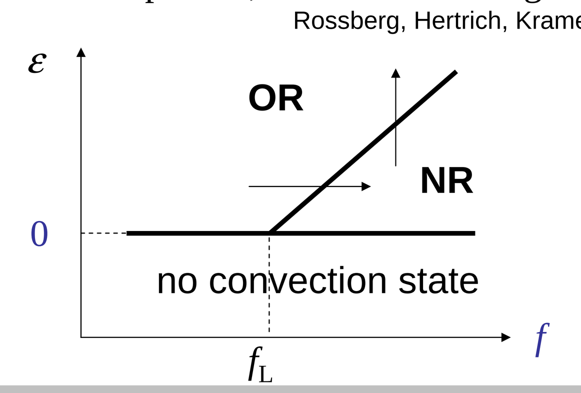
OR

$\tau_0\partial_t A_{1/2} = [\varepsilon + \xi_{xx}^2(\partial_x \pm ip_c\alpha)^2 + \xi_{yy}^2(\partial_y - iq_c\alpha)^2 \pm 2\xi_{xy}^2(\partial_x \pm ip_c\alpha)(\partial_y - iq_c\alpha)$
$- g|A_{1/2}|^2 - h|A_{2/1}|^2 \pm i\beta_x(\partial\alpha/\partial x) + i\beta_y(\partial\alpha/\partial y)]A_{1/2}$

$\gamma_1\partial_t\alpha = K_1\partial_y^2\alpha + K_3\partial_x^2\alpha + (\Gamma q_c p_c/2)(|A_1|^2 - |A_2|^2)$

A = Amplitude, α = azimuthal angle of **C**-director

Rossberg, Hertrich, Kramer, and Pesch, *Phys. Rev. Lett.* **76**, 4729 (1996).



Transition between the SMT patterns:
◆Increase f → OR to NR
◆Increase ε → NR to OR

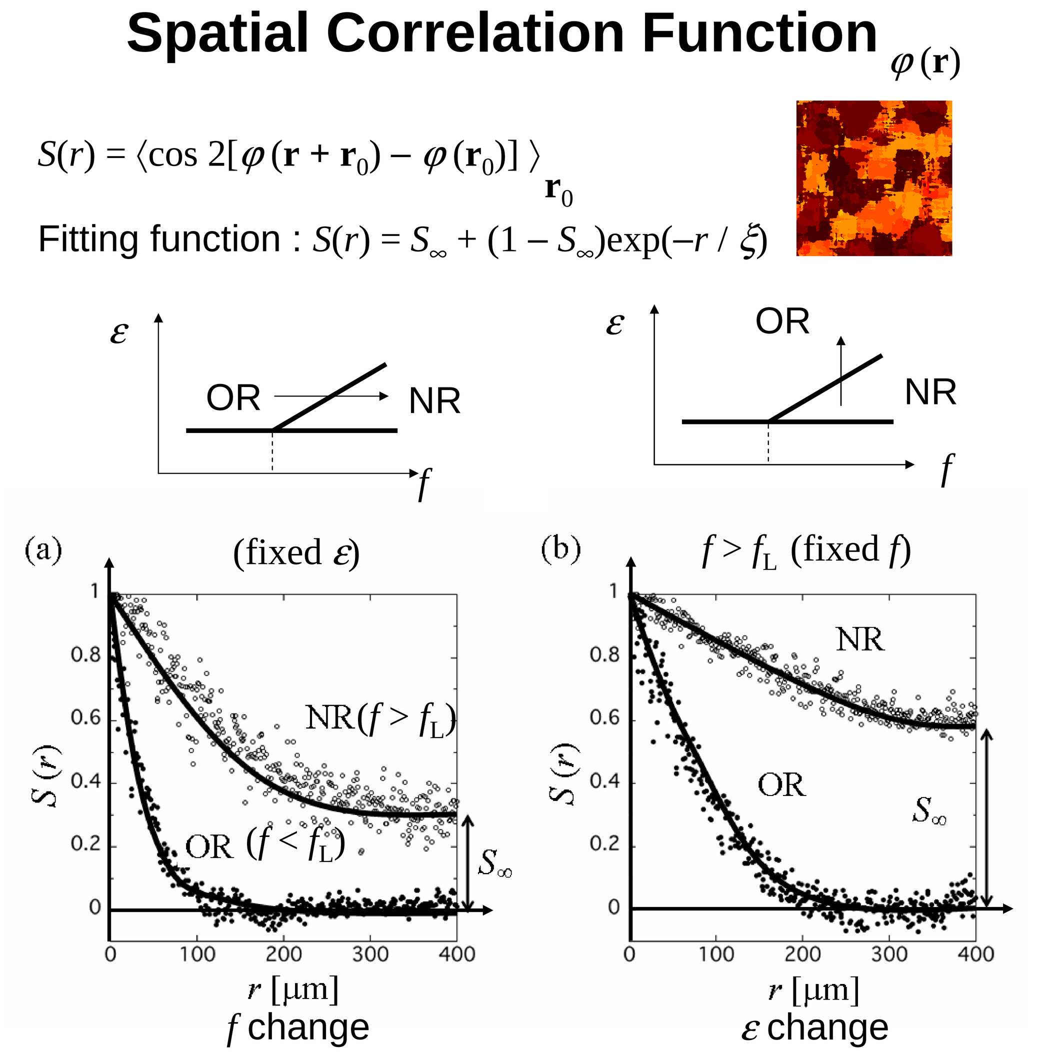
PURPOSE:
To investigate transition from a chaotic to another chaotic state

Results and Discussion

Spatial Correlation Function

$S(r) = \langle\cos 2[\varphi(\mathbf{r}+\mathbf{r}_0) - \varphi(\mathbf{r}_0)]\rangle_{\mathbf{r}_0}$

Fitting function : $S(r) = S_\infty + (1 - S_\infty)\exp(-r/\xi)$



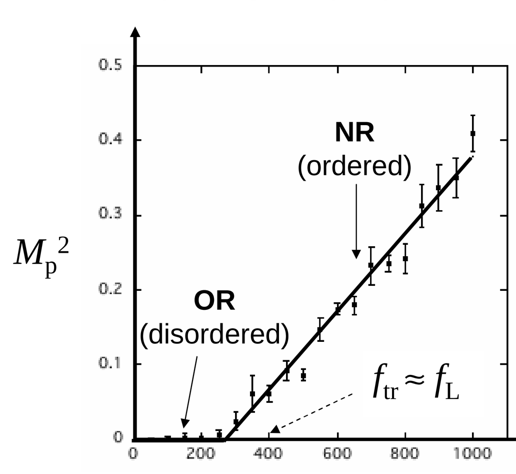
Pattern Magnetization

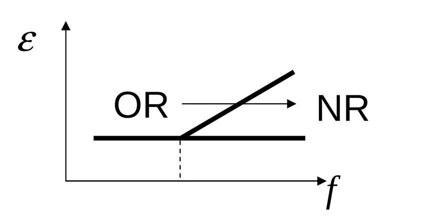
$M_p = \langle\cos 2[\varphi - \langle\varphi\rangle]\rangle \quad (0 \leq M_p \leq 1)$

$= \left[\int_{-\pi/2}^{\pi/2} B(\varphi)\cos 2[\varphi - \langle\varphi\rangle]d\varphi\right] / \left[\int_{-\pi/2}^{\pi/2} B(\varphi)d\varphi\right]$

averaged φ — distribution function of φ

2 particular conditions:
1. Completely isotropic pattern $B(\varphi) = \text{constant} \to M_p = 0.$
2. Completely stripe pattern $B(\varphi) = \delta(\varphi) \to M_p = 1.$





Order-Disoder Transition

For OR, $S_\infty = 0 \to$ No long-range order.
For NR, $S_\infty \neq 0 \to$ long range-order exists.

For OR, $M_p = 0 \to$ Disordered state.
For NR, $M_p \neq 0 \to$ Ordered state.

The transition point for the order-disorder is the Lifshitz frequency.

OR and NR belong to spatiotemporal chaotic pattern. Spatiotemporal chaos is associated with randomness and disorder. However, by defining new order parameters such as spatial correlation function and pattern magnetization, hidden order is revealed in NR regime.

The SMT and the conventional 2D XY model have the same dimensions and the same degree of freedom of vector fields. However, the SMT is induced by non-thermal fluctuations, whereas in 2D XY model, no long-range order for any finite temperature is due to thermal fluctuations.

OR and NR have different symmetry of nonlinear interaction.
Spatially uniform case :
OR : $\frac{\partial\alpha}{\partial t} = -K|A|^2$ NR : $\frac{\partial\alpha}{\partial t} = K|A|^2\alpha \quad K < 0$

SMT in OR regime:
☐ No stationary solution.
☐ C-director always rotates.
☐ Wavevector **q** also rotates.
☐ Global fluctuations break initial anisotropy of **q**.
☐ Neither macroscopic order nor long-range correlation exists.

SMT in NR regime:
☐ A stationary solution exists, but is unstable.
☐ C-director fluctuates around an initial direction.
☐ Wavevector **q** follows C-director.
☐ Local fluctuations cannot break initial anisotropy of **q**.
☐ Macroscopic order and long-range correlation exist.

Conclusions

1. In a spatiotemporal chaotic state (i.e. NR), hidden order exists.
2. Order–disorder phase transition occurs in the SMT. The transition point is the Lifshitz frequency.
3. In OR pattern, the global nonthermal fluctuations break long-range correlation. Meanwhile in NR pattern, the local ones cannot break the long-range correlation.
4. Phase Transition in the SMT occurs by the change of symmetry of nonlinear interaction.

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Journal reference

R. Anugraha, K. Tamura, Y. Hidaka, N. Oikawa, and S. Kai, *Phys. Rev. Lett.* **100**, 164503 (2008)