Energy Redistribution in Higher Dimensions and Chaotic Motion

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Abstract

A system plus environment conservative model is used to characterize the nonlinear dynamics when the time averaged energy for the system particle starts to decay. The system particle dynamics is regular for low values of the N environment oscillators and becomes chaotic in the interval $13 \le N \le 15$, where the system time averaged energy starts to decay. To characterize the nonlinear motion we estimate the Lyapunov exponent (LE) and determine the power spectrum. For much larger values of N the energy of the system particle is completely transferred to the environment and the corresponding LEs decrease.

1 Introduction

Small dissipation is inevitable in real systems. A possibility to describe dissipation is to consider an *open system* interacting with its *environment* by collision processes. The whole problem (System + Environment + Interaction) is conservative but, due to energy exchange between system and environment, the *system* can be interpreted as an open system with dissipation. In this work the environment is composed by a *finite* number N of uncoupled harmonic oscillators. Changing N we are able to study the interesting transition from low- to high-dimensional dynamical systems which simultaneously experiment dissipation. We focus on the nonlinear dynamics of the time series obtained for the system particle under the N harmonic oscillators. The tools used in this analysis are the Lyapunov spectrum [1], phase space dynamics and the power spectrum. Our system is deterministic and we use the TISEAN [2] package to analyse the time series.

2 The model

We considered the problem composed by a particle under the influence of an asymmetric potential (the system) interacting with N independent harmonic oscillators (the environment). The dimensionless equations of motion [3] are

$$\ddot{X} + \frac{dV(X)}{dX} - \sum_j^N \gamma_j x_j + X \sum_j^N \frac{\gamma_j^2}{\mu_j w_j^2} = 0, \qquad (1)$$

$$\ddot{x}_j + w_j^2 x_j - \frac{\gamma_j}{\mu_j} X = 0, \qquad (2)$$

where X and $\vec{x}$ are, respectively, system and oscillators $(x_j, j = 1, 2, ..., N)$ coordinates. The coupling parameter γ_j is a measure of the strength of the bilinear coupling between the system and the j-oscillator. The dimensionless anharmonic potential [see Fig. 1] is defined in the interval $X = (-0.38, 0.62)$ by

$V(X) = C - \frac{1}{4\pi^2\delta}\left[\sin 2\pi(X - X_0) + \frac{1}{4}\sin 4\pi(X - X_0)\right]$, where the constant C is such that $V(0) = 0$ and $\delta = \sin 2\pi|X_0'| + \sin 4\pi|X_0'|$. The time and the oscillators frequencies w_j are written in units of $w_0 = 1.0$, which is the frequency of the linear motion around the minimum of the potential and is determined from $w_0^2 = 4\pi^2 V_0 \delta / M$. In the numerical experiments we used fourth-order Runge-Kutta integrator with fixed step $\Delta t = 10^{-3}$. For all cases studied, dimensionless mass $\mu_j = 0.1$ and $\gamma_j = 0.01$ are used (the energy is admensional too). The interaction energy is assumed to be zero, the energy of the system is very close to the total energy $E_S \sim E_T = 0.02$, and the oscillators energy is close to zero ($E_O \sim 0.0$).

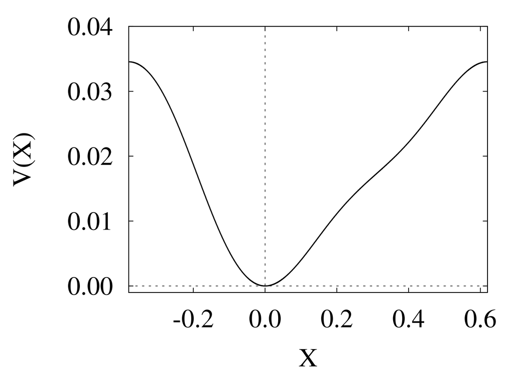

Figure 1: Dimensionless anharmonic potential $V(X)$.

3 Emergence of the time averaged energy decay

For lower values of $N \lesssim 15$, the Poincaré recurrence times (PRT) are not very large and the energy return can be observed in simulations. However, for higher values of N the PRT increase very much to be observable with finite integration times. In such cases we can say that the energy transferred to the environment will not return, within the integration times, to the system particle [see Fig. 2].

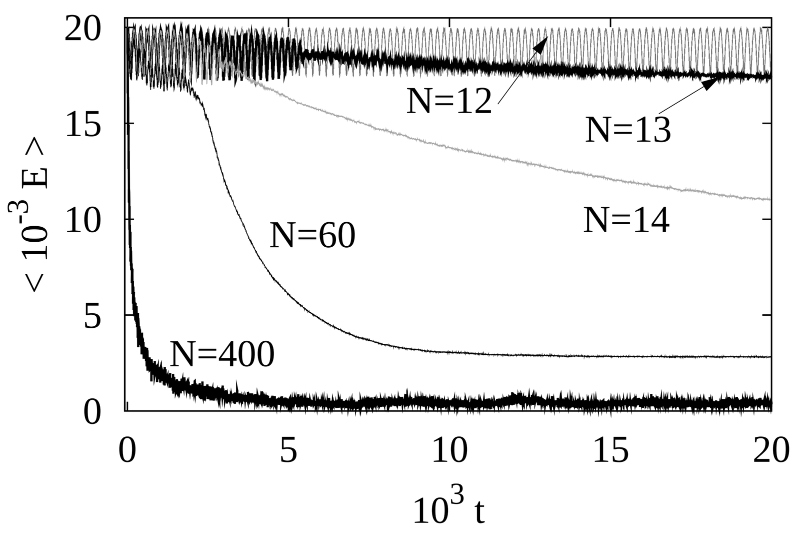

Figure 2: Mean energy (over 800 environmental initial conditions) as a function of time for $N = 12, 13, 14, 60, 400$.

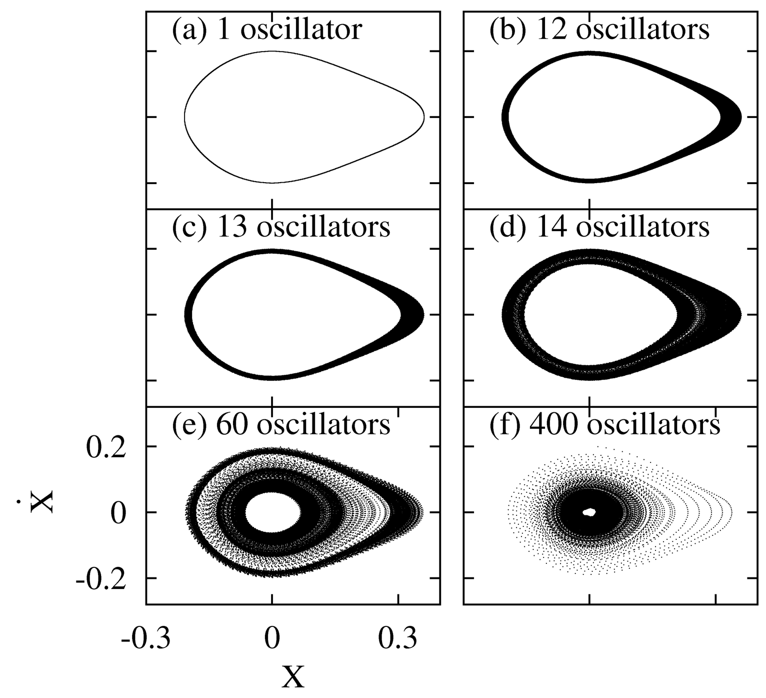

Figure 3: Phase space dynamics of the system particle.

Figure 3 shows the phase space dynamics of the system particle for $N = 1, 12, 13, 14, 60, 400$. As N increases the system particle exchanges energy with the oscillators and moves within a layer inside the deformed ellipsis. For higher values of N (60 and mainly 400) the system particle rapidly loses its energy to the environment, and ends up moving close to the minimum of the anharmonic potential. When $N = 4000$ (not shown) the system particle energy is close to zero and the energy per oscillator is also close to zero.

4 Nonlinear analysis

By integrating Eqs. (1) and (2) for many values of N, we generated the time series (TS) for the variable $X(t)$ for the system particle only. In order to perform the nonlinear analysis it is necessary to determine the dimension m of the reconstructed attractor [4]. The adequate values of m were determined using the false-nearest neighbors method [5]. For values of $N = 1 \to 4$, the appropriate embedding dimension is $m = 2$, for $N = 5 \to 12$ it is $m = 3$, for $N = 13$ and $N = 14$ it is $m = 4$ and for $N \ge 15$ it is $m = 5$.

In the interval $1 \le N \le 12$, all LEs are time independent and are estimated to be $\lesssim 10^{-3}$. The power spectra for $N = 2 \to 12$ (not shown) shows some small frequencies around the peaks from Fig. 4(a).

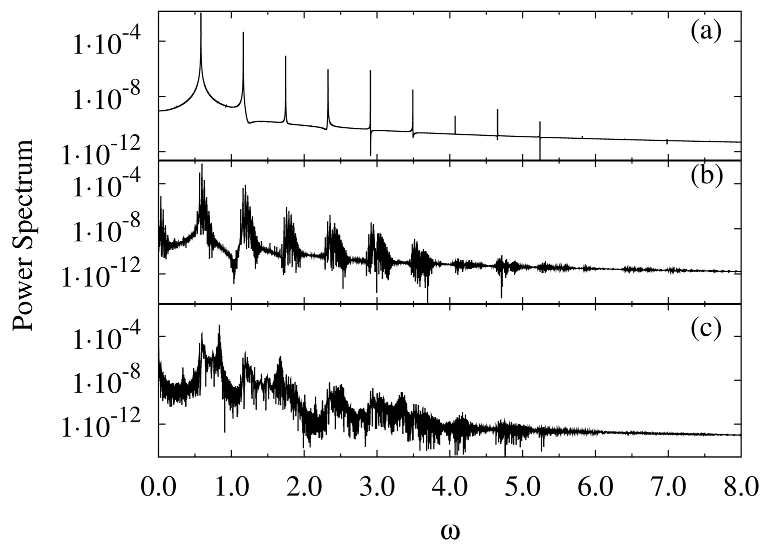

Figure 4: Power spectrum for (a) $N = 1$ (main frequency is $\omega = 0.56$ Hz and the other peaks are its high-harmonics), (b) $N = 13$ and (c) $N = 20$.

Figure 5 shows the (a) energy from the system particle and (b) the four LEs as a function of time for $N = 13$. Corresponding power spectrum Fig. 4(b) new smaller peaks appear close to the main frequencies. We can see [Fig. 4(c)] that almost all main frequencies dissapear and a complicated spectrum is obtained for $N = 20$ (a chaotic dynamics is therefore expected).

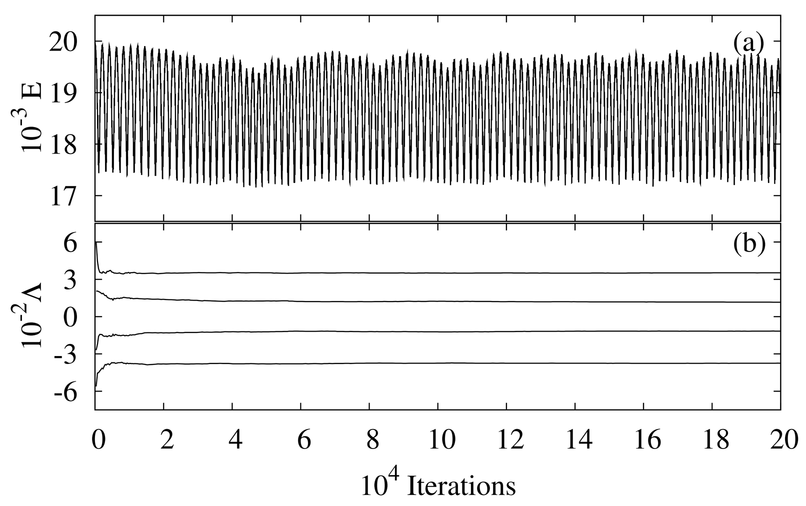

Figure 5: (a) Energy from the system particle and (b) LE spectrum for $N = 13$ as a function of the iterations of the time series.

Figure 6 shows the (a) energy of the system particle and (b) the LEs as a function of time for $N = 20$. Each time the amplitude of the oscillation of energy decreases, energy starts to be transferred to the environment and the positive LEs decrease in time. Such local decreasing of the LEs in conservative systems is due to "sticky" (trapped) trajectories [6]. As N increases, the regular islands are broken and the chaotic trajectory may find another regular island which will decrease its LEs.

Figure 7 shows the power spectra for $N = 60, 150, 4000$. Clearly the Fig. 7 (power spectra) shows is possible to observe the complexity induced by the environment oscillators. The main frequencies observed at low values of N [see Fig. 4(a)] are now mixed to other (new) frequencies which arise due to the particle collisions with the oscillators.

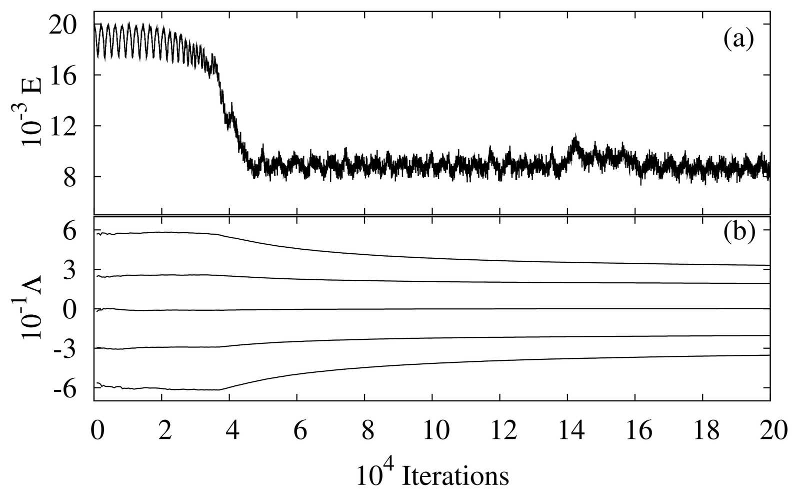

Figure 6: (a) Energy for the system particle and (b) LE spectrum as a function of the iterations of the time series for $N = 20$.

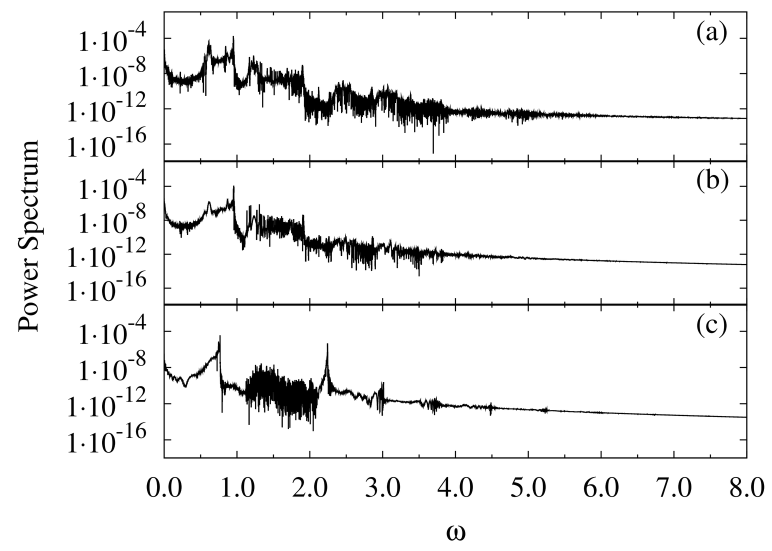

Figure 7: Power spectra for (a) $N = 60$, (b) $N = 150$ and (c) $N = 4000$.

Figure 8 shows the (a) final time average energy from the system particle and (b) positive estimated LEs as a function of N. We observe that for values of $N \lesssim 12$, the initial energy (~ 0.02) equals the final energy. Close to $N \sim 13, 14$ the final mean energy starts to decrease, meaning that a portion of the system energy is transferred to the environment. For higher values of N the LEs decrease slowly [Fig. 8(b)] following the qualitative behavior of the mean final energy [Fig. 8(a)].

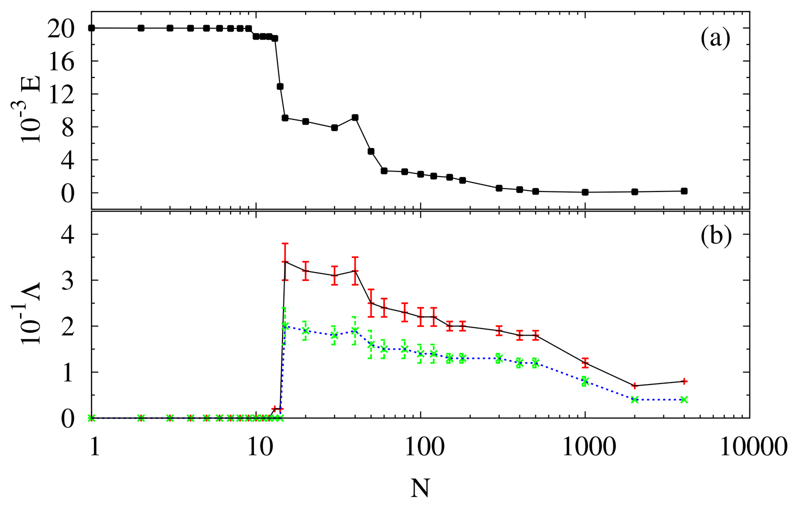

Figure 8: (a) Final time average energy from the system particle and (b) positive LEs as a function of N.

5 Conclusions

We observed that the time averaged energy decay for the system particle starts to occur in the interval $N = (10, 20)$. For lower values of N the system continuously exchange energy with the environment but the time average is constant. For much higher values of N the system particle energy is transferred to the environment for times very close to zero. Numerical evidences show a connection between the variation (in time) of the *amplitude* of the particle energy with the energy decay and the decrease of the Lyapunov exponents. This is explained in terms of chaotic trajectories from the system particles trapped close to regular island. Since this trajectory is restricted to a smaller portion of the phase space, part of the total energy is transferred to other degrees of freedom thus explaining the origin of the time averaged energy decay.

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References

[1] M. Sano and Y. Sawada. Phys. Rev. Lett. **55**, 1082 (1985).

[2] R. Hegger, H. Kantz and T. Schreiber. Chaos **9**, 413 (1999).

[3] J. Rosa and M. W. Beims. Phys. Rev. E **78**, 031126 (2008).

[4] F. Takens. Dynamical Systems and Turbulence. Lecture Notes in Mathematics; Springer-Verlag: New York; 1981.

[5] M. B. Kennel, R. Brown and H. D. I. Abarbanel. Phys. Rev. A 45, 3403 (1992).

[6] M. W. Beims, C. Manchein and J. M. Rost. Phys. Rev. E **76**, 056203 (2007).