Accounting for model error in data assimilation

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Exploring complex dynamics in High dimensional Chaotic Systems: From Weather Forecasting to Oceanic Flows

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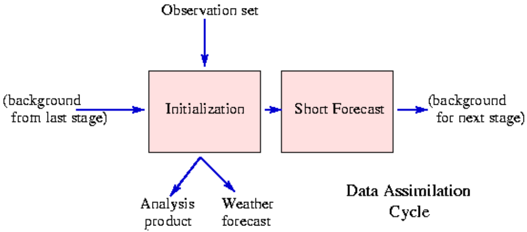
The main fields of applications in geophysics are:

- initialize weather prediction
- produce reanalysis
- parameter estimation (especially for seasonal and climate prediction)



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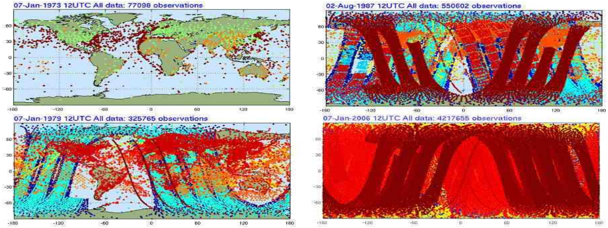
- evolution dynamics (set of differential equations, numerical model ...)



from M.G.Bosilovich (2008)

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Fundamental problems making difficult an adequate treatment of model error in data assimilation:

1. large variety of possible error sources
2. the amount of available data insufficient to realistically describe the model error statistics
3. lack of a general framework for model error dynamics and statistics

Formulation

Let assume to have the model:

$$\frac{d\mathbf{x}(t)}{dt} = f(\mathbf{x}, \lambda)$$

used to describe the true process:

$$\frac{d\mathbf{y}(t)}{dt} = f(\mathbf{y}, \lambda^{'}) + \epsilon g(\mathbf{y}, \lambda^{'})$$

$g(\mathbf{y}(t), \lambda^{'})$ represents the dynamics associated to processes not accounted for by the model.

Estimation error evolution in the linear approximation

$$\delta\mathbf{x}(t) \approx \mathbf{M}_{t,t_0}\delta\mathbf{x}_0 + \int_{t_0}^{t} d\tau \mathbf{M}_{t,\tau}\delta\mu(\tau) = \delta\mathbf{x}^{ic}(t) + \delta\mathbf{x}^{m}(t)$$

where

$$\delta\mu = \frac{\partial\mathbf{f}}{\partial\lambda}|_{\lambda}\delta\lambda + \gamma g(\mathbf{y}(t), \lambda')$$

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- In view of the presence of the propagator M, the flow instabilities are expected to influence the model error dynamics

Model error covariance and correlation

Model error covariance

$$\mathbf{P}^m(t) = \int_{t_0}^{t} d\tau \int_{t_0}^{t} d\tau^{'} \mathbf{M}_{t,\tau} < (\delta\mu(\tau))(\delta\mu(\tau^{'}))^T) > \mathbf{M}^T_{t,\tau^{'}}$$

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These equations are NOT suitable for realistic geophysical applications - Some approximation is necessary

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Can these short-time approximations be incorporated in data assimilation procedures to account for model error ?

Extended Kalman Filter in the presence of model error - Deterministic formulation

Input $\mathbf{x}^a$, $\mathbf{P}^a$

Forecast: $\mathbf{x}^f = \mathcal{M}(\mathbf{x}_i)$ — $\mathbf{P}^f = \mathbf{M}\mathbf{P}^a\mathbf{M}^T + \mathbf{P}^m$

Analysis: $\mathbf{x}^a = [\mathbf{I} - \mathbf{K}\mathbf{H}]\,\mathbf{x}^f + \mathbf{K}\mathbf{y}^o$ — $\mathbf{P}^a = [\mathbf{I} - \mathbf{K}\mathbf{H}]\mathbf{P}^f$

$\mathbf{P}^m$ - Model Error Covariance Matrix

Estimate $\mathbf{P}^m$ using the short time approximation: $\mathbf{P}^m = \mathbf{Q}\tau^2$

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In standard EKF, model error is assumed to be a white-noise and $\mathbf{P}^m$ represents the covariance of the process

Results EKF - Case (1) **Q** statistical estimate

Carrassi,Vannitsem&Nicolis (2008) *QJRMS*

Observation System Simulation Experiments

- Prototype of nonlinear chaotic dynamics (Lorenz, 1996): $\frac{dx_i}{dt} = \alpha(x_{i+1} - x_{i-2})x_{i-1} - \beta x_i + F \qquad 1 \leq i \leq 36$

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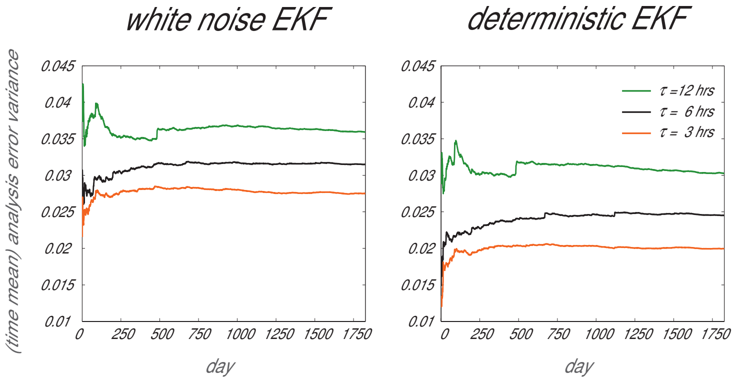

Results EKF - Case (2) $\mathbf{Q}$ online estimate

Q estimated online by assimilating observations - State Augmented formulation

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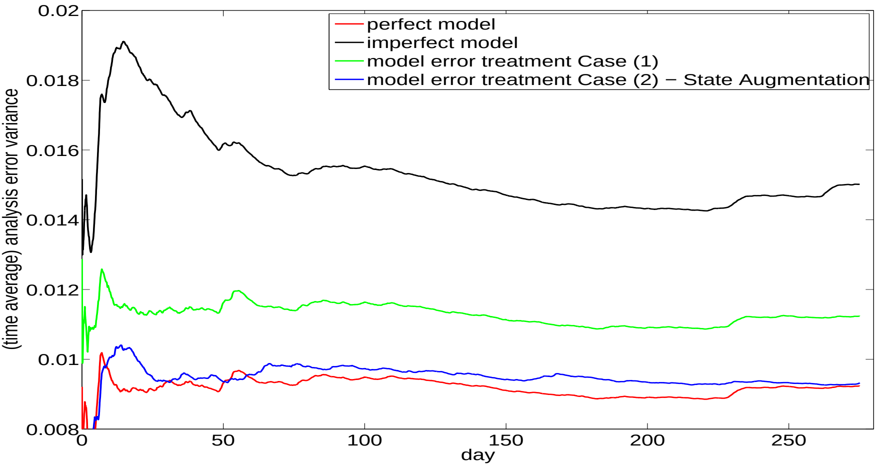
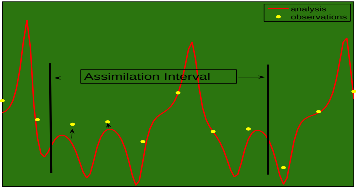


4DVar in the presence of model error - Weak Constraint deterministic formulation



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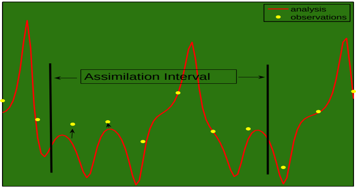

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$$2J = \int_0^{\tau}\int_0^{\tau} (\delta \mathbf{x}^m_{t_1})^T (\mathbf{P}^m)^{-1}_{t_1 t_2} (\delta \mathbf{x}^m_{t_2}) dt_1 dt_2 + \sum_{k=1}^{M} \epsilon_{\mathbf{k}}{}^T \mathbf{R}_k^{-1} \epsilon_{\mathbf{k}} + \epsilon_b^T \mathbf{B}^{-1} \epsilon_b$$

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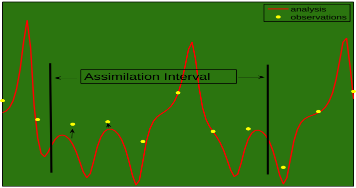

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Estimate model error correlation using $\mathbf{P}(t_1, t_2) \approx \mathbf{Q}(t_1 - t_0)(t_2 - t_0)$

- In the standard weak-constraint 4DVar, model error is assumed to be white in time $\Rightarrow$ Only the model error covariances $\mathbf{P}^m_t$ need to be estimated

Results weak-constraint 4DVar

Carrassi&Vannitsem (2010) *MWR*

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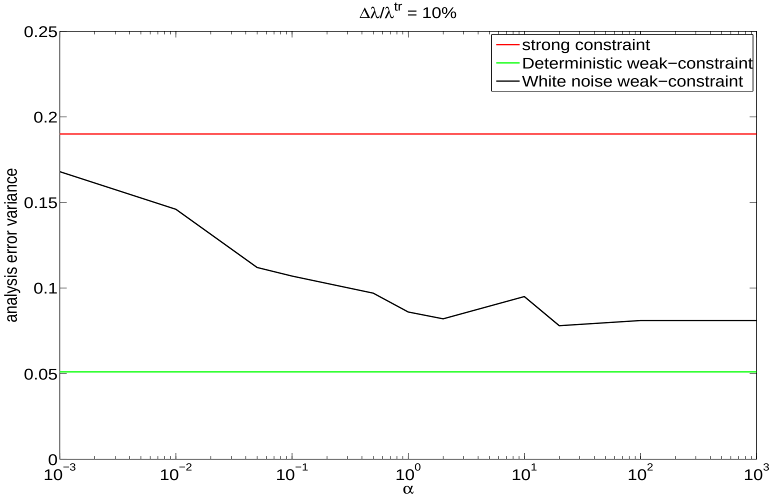

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