Targeting Synchronization by Design of Coupling

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Abstract: *Design of coupling function are proposed for targeting a desired synchronized state in two chaotic oscillators. We proposed two methods, one using Hurwitz stability and another using Lyapunov function stability. We implemented the methods in electronic circuits.*

Design of Coupling:

1. Open-Plus-Closed Loop Method

Driver: $\dot{y} = F(y) + \Delta F(y), y \in R^n$

Coupling: $D(x, \alpha y) = \alpha \dot{y} - F(\alpha y) + [H - JF(\alpha y)](x - \alpha y)$

Response: $\dot{x} = F(x) + D(x, \alpha y)$

Error Dynamics: $\dot{e} = He$

$e \rightarrow 0 \quad if \quad H$ has all eigenvalues with negative real parts

Example: *Sprott Model*

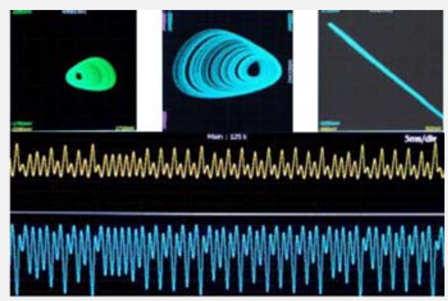
Driver: $\dot{y}_1 = -ay_2; \dot{y}_2 = y_1 + y_3; \dot{y}_3 = y_1 + y_2^2 - y_3$

Response: $\dot{x}_1 = -ax_2; \dot{x}_2 = x_1 + x_3$

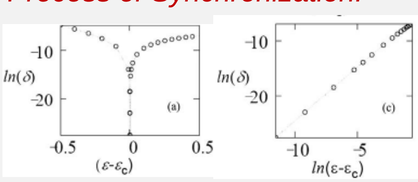
$\dot{x}_3 = x_1 + x_2^2 - x_3 + \alpha(1-\alpha)y_2^2 + (p - 2\alpha y_2)(x_2 - \alpha y_2)$

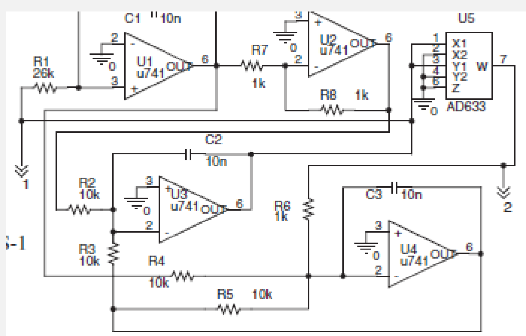
$$H = \begin{bmatrix} 0 & -a & 0 \\ 1 & 0 & 1 \\ 1 & p & 1 \end{bmatrix}$$

Experiment: Sprott Circuit

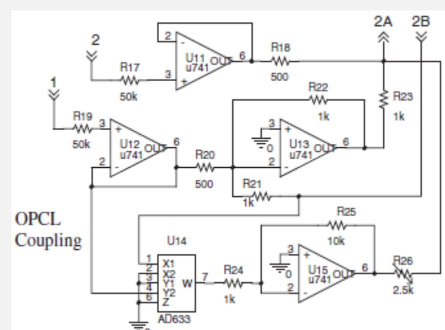


Process of Synchronization:





Dana et al, PRL 100, 234102 (2008);
PRE 80, 016212 (2009)

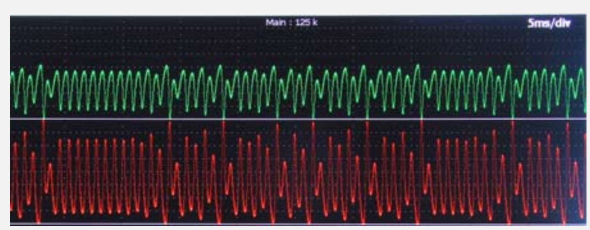


2. Lyapunov Function Stability Method

$\dot{x} = f(x); \dot{y} = f(y) + U; \quad e = y - \alpha y$

$\dot{V}(e) = -e_1^2 - e_2^2 - e_3^3$

$e \rightarrow 0 \; ; \; \alpha = -2$



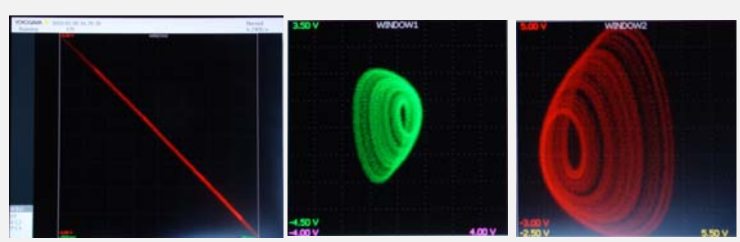
Example: *Sprott Model*

Response:

$\dot{y}_1 = -ay_2 + ae_2 - e_1$

$\dot{y}_2 = y_1 + y_3 - e_1 - e_2 - e_3$

$\dot{y}_3 = y_1 + y_2^2 - y_3 - e_1 + \alpha(1-\alpha)x_2^2$



Mutual Amplification is also possible.