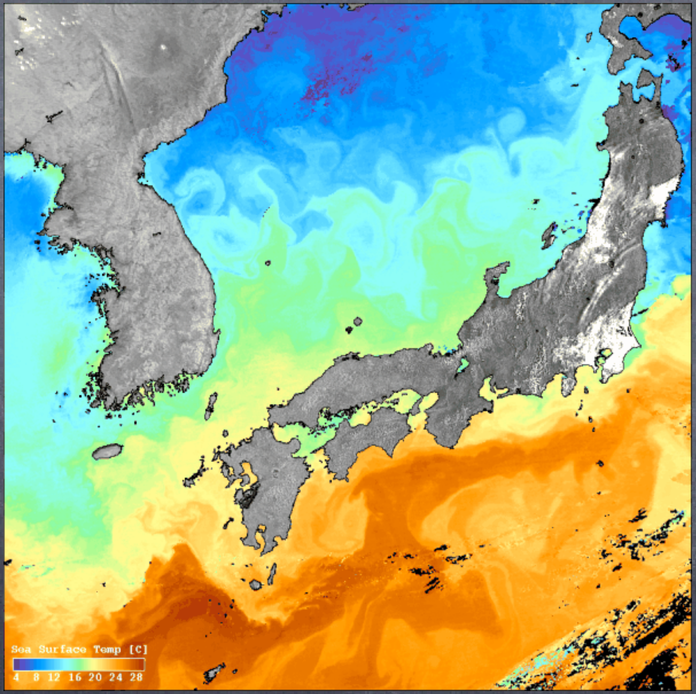
A nonlinear theory of the bimodality of the Kuroshio Extension

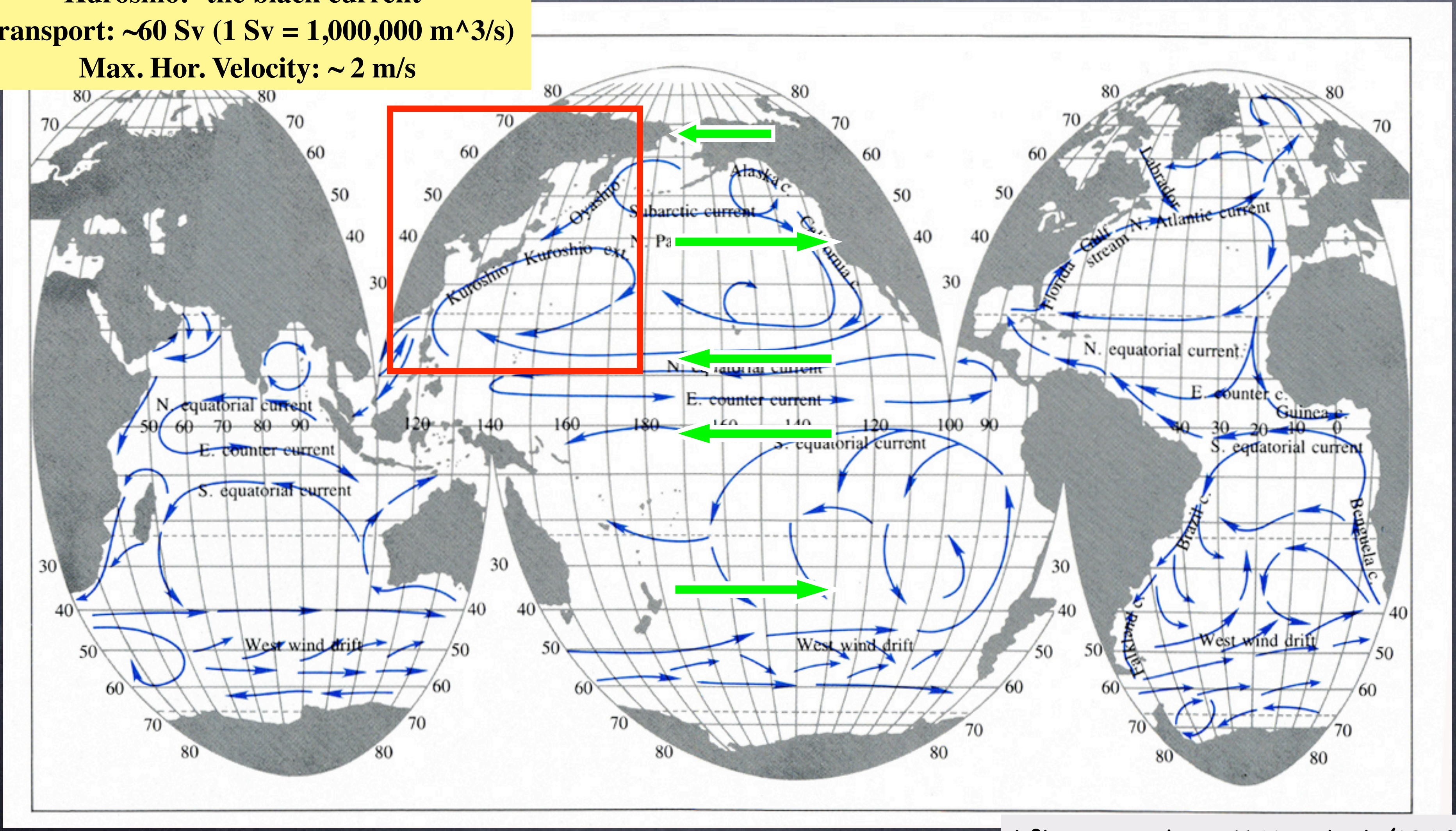
Exploring COmplex DYnamics in high-dimensional Chaotic systems: from weather forecasting to oceanic flows (ECODYC10, Dresden)



Stefano Pierini (Naples/Italy) and Henk Dijkstra (Utrecht/Netherlands)

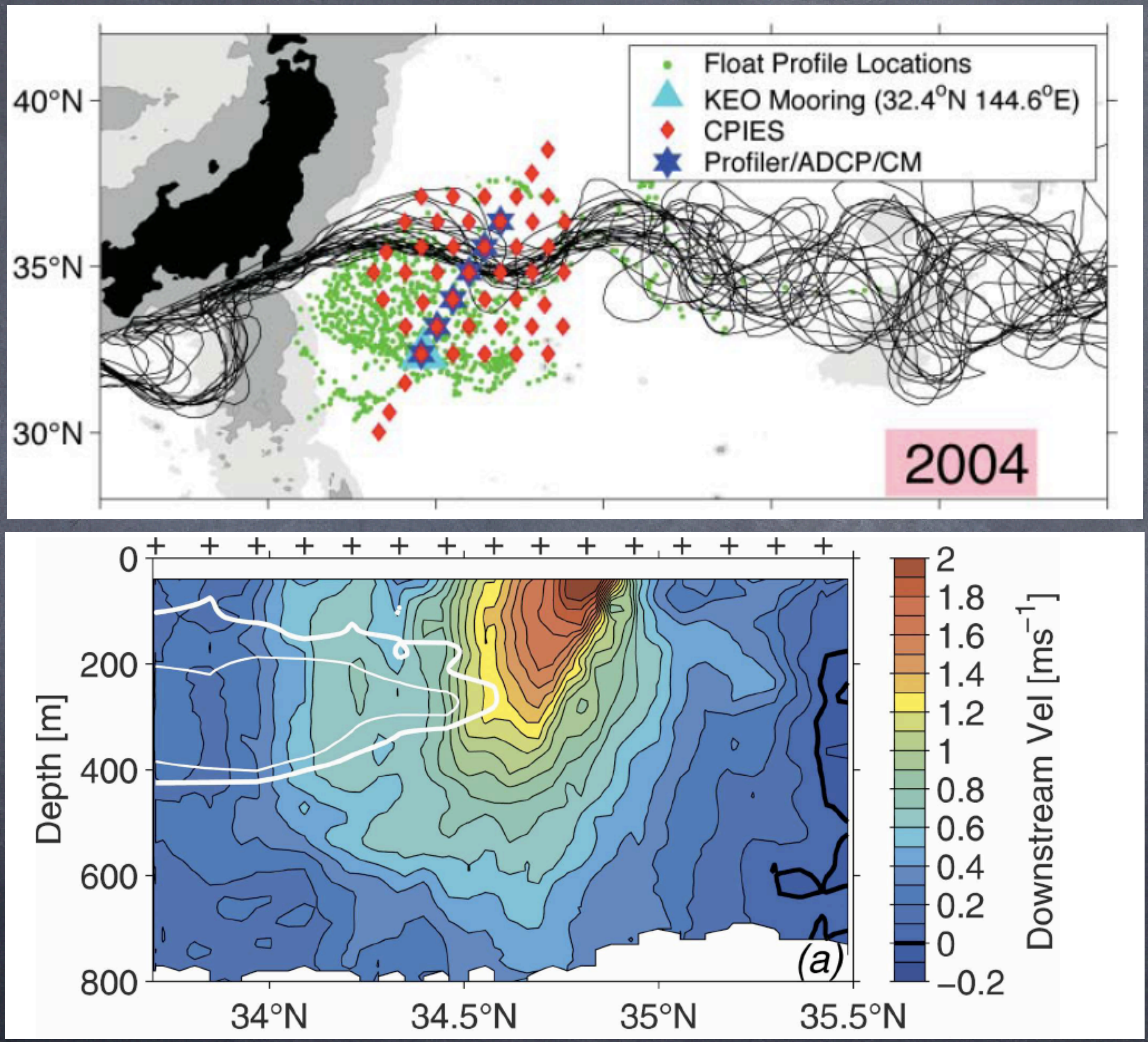
Surface Ocean Circulation

Kuroshio: 'the black current'
Transport: ~60 Sv (1 Sv = 1,000,000 m^3/s)
Max. Hor. Velocity: ~ 2 m/s



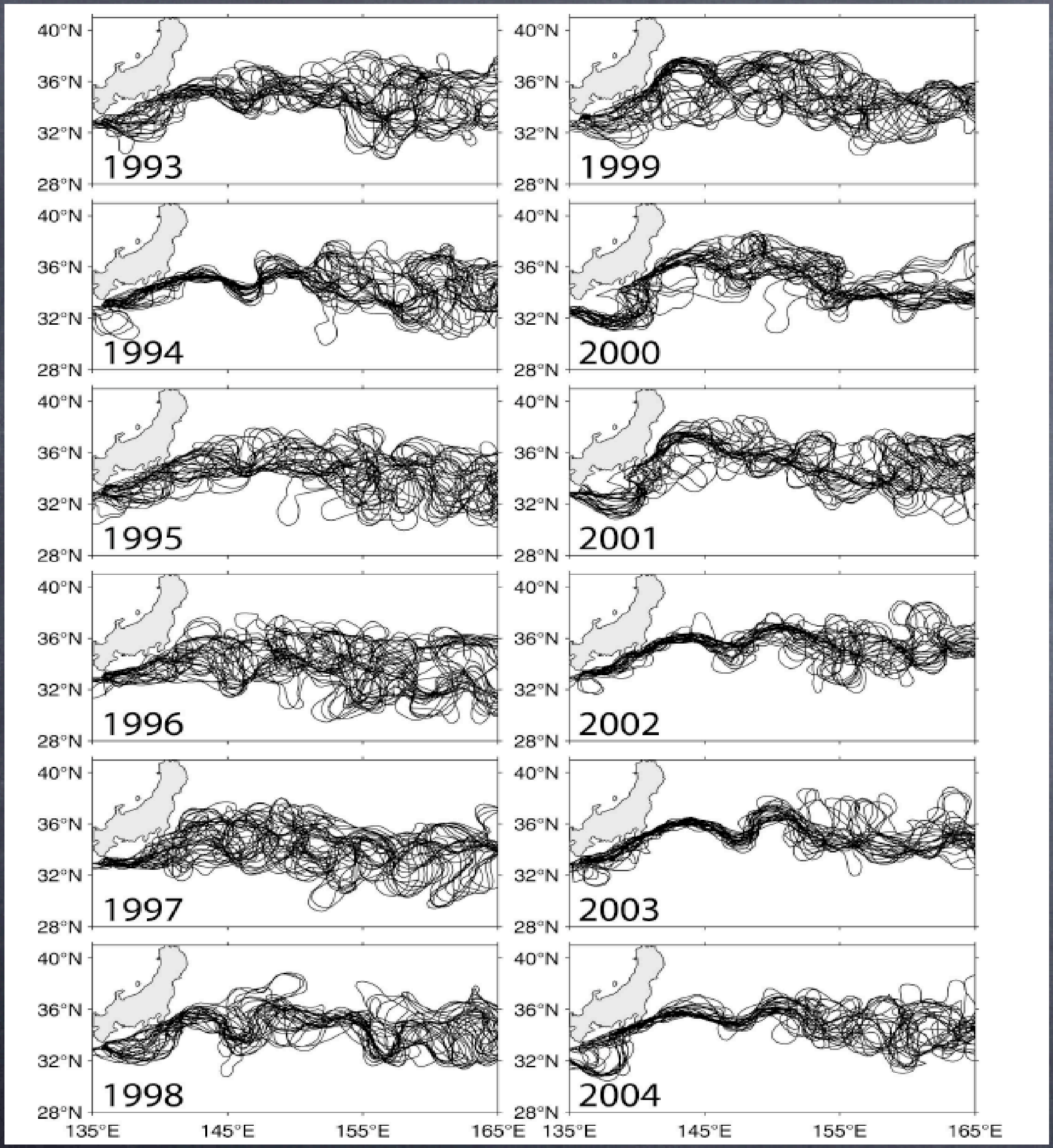
After: Sverdrup, H.U. et al. (1942)

The Kuroshio Extension System Study (KESS)



Qiu et al., JPO, 2006

Kuroshio path: 1993-2004

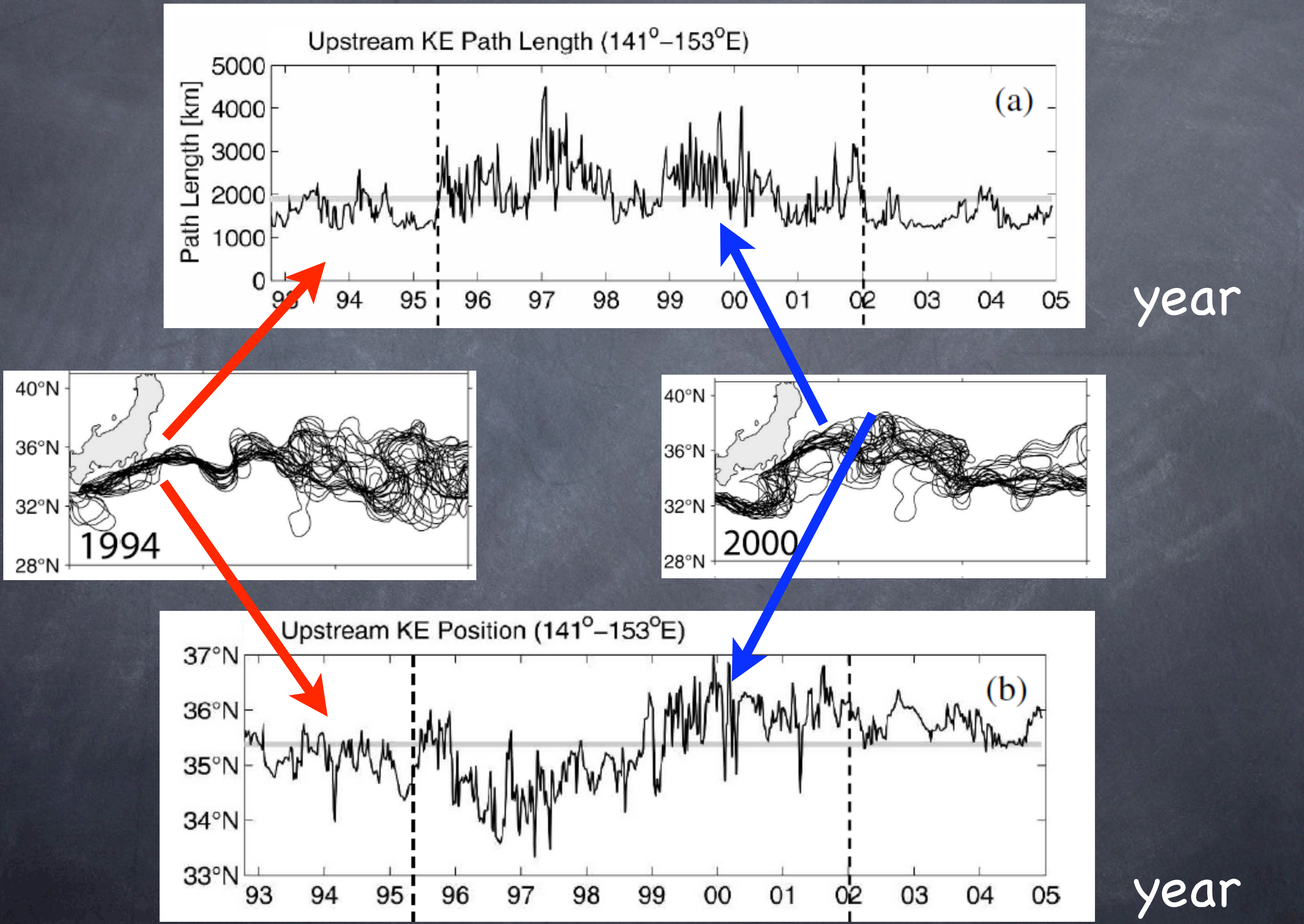


Bi-weekly mean Kuroshio path from altimetry (170 cm sea level contour)

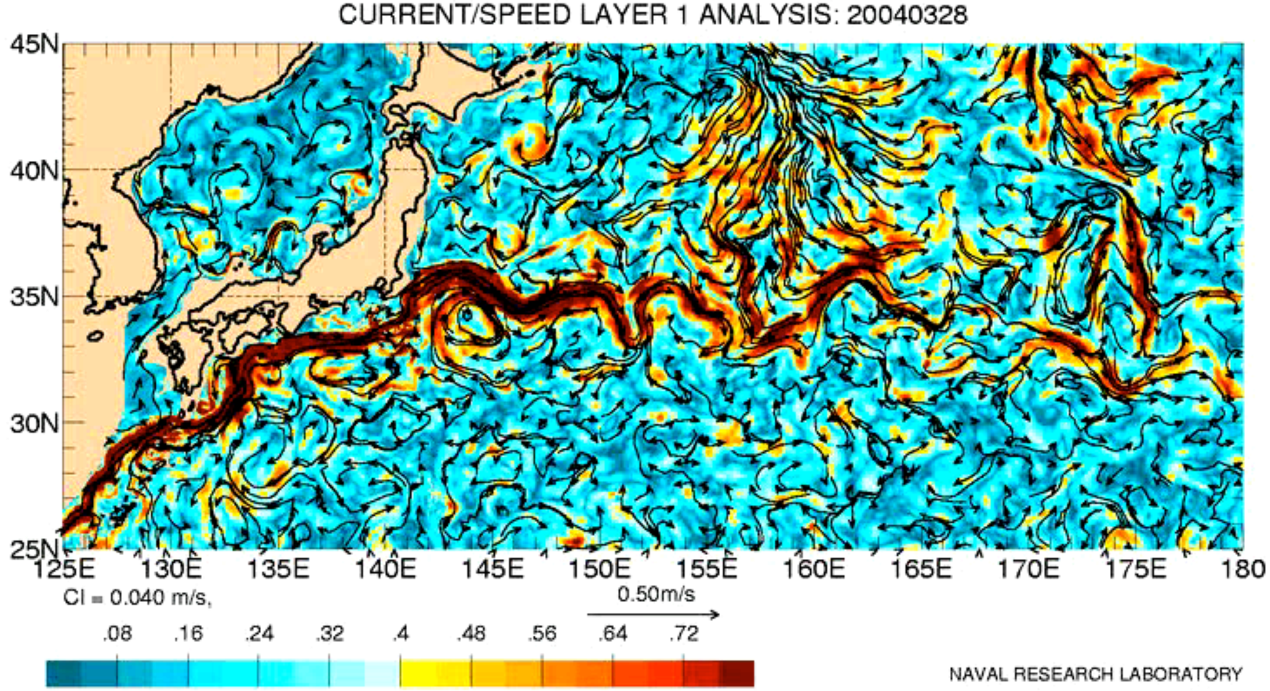
Low frequency (decadal time scale) transitions between different Kuroshio paths.

Qiu and Chen, JPO, 2005

Important characteristics



High resolution modeling results (april 2004 - feb 2005)



Surface Current Speed

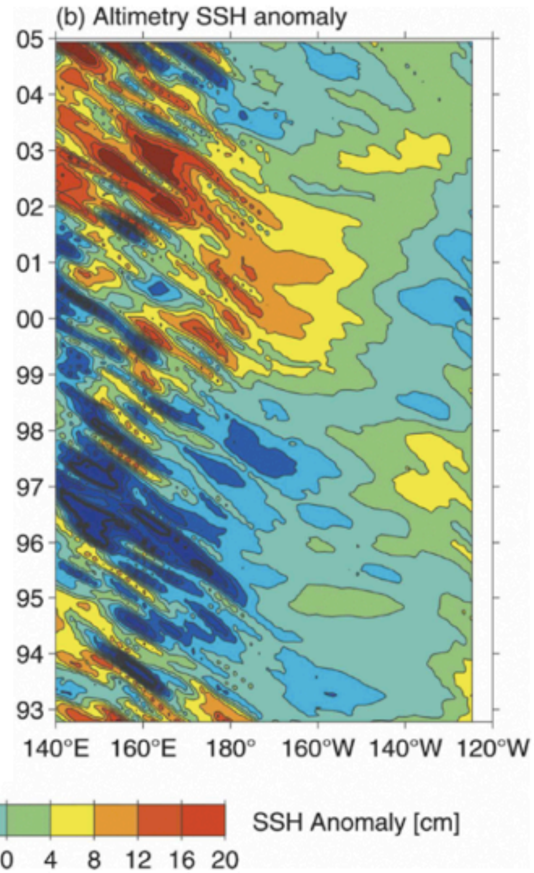
http://www7320.nrlssc.navy.mil/global_nlom32/skill.html

Which physical processes control the path transitions (time scale/patterns) of the Kuroshio?

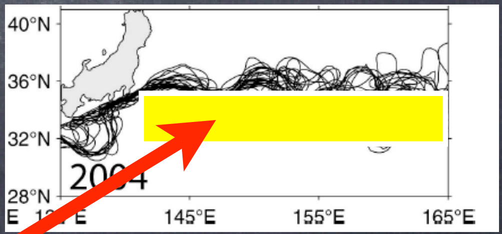
Theory I: Adjustment to time varying wind forcing

Observations

Time



+ mesoscale eddy field!!

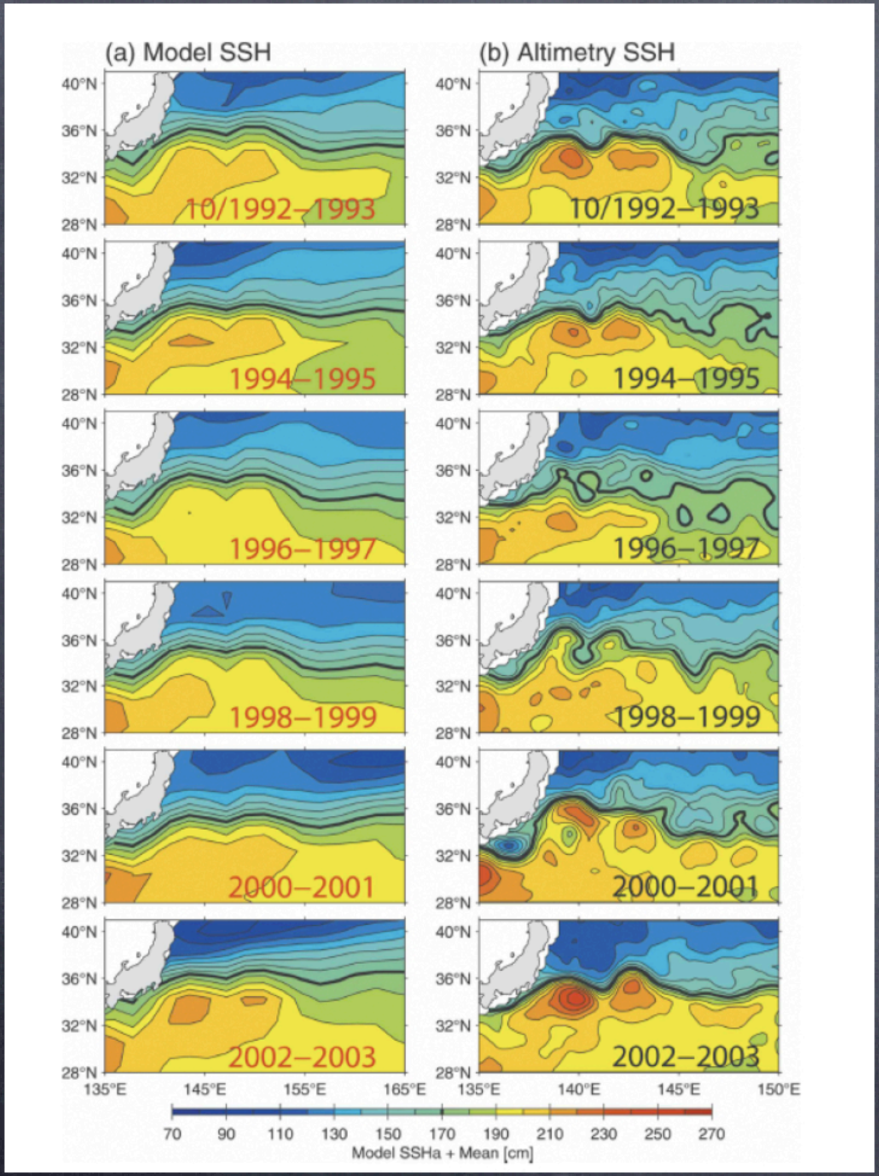


Sea surface height (SSH) anomalies along the zonal band of 32N - 34N

Qiu & Chen, JPO, 2005

Theory I: Problems

Model Observations



1. Effect of (Rossby) waves on jet is very small

2. Processes involved with the meso-scale eddy field are highly complicated

3. Need to explain wind variations

Qiu & Chen, JPO, 2005

Theory II: Intrinsic Ocean Variability

Dynamical systems approach

- Use a PDE model hierarchy of the surface (wind-driven) ocean circulation (e.g., Quasi Geostrophic (QG), Shallow Water (SW), Primitive Equation (PE)) and determine a `minimal' model which is able to describe central observational characteristics

- Within each member of the model hierarchy, look for the behavior of the model flows when the 'viscosity' is decreased (from `syrup' to `water')

- Use small dimensional systems (ODEs) for detailed analysis of dynamics

Quasi-geostrophic (QG) barotropic model

$$\frac{\partial \zeta}{\partial t} + u\frac{\partial \zeta}{\partial x} + v\frac{\partial \zeta}{\partial y} + \beta\frac{\partial \psi}{\partial x} = \frac{1}{Re}\nabla^2\zeta + \alpha\left(\frac{\partial \tau^y}{\partial x} - \frac{\partial \tau^x}{\partial y}\right)$$

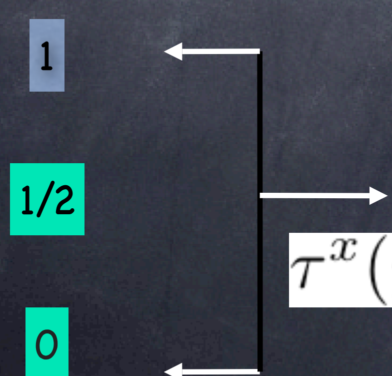
advection `friction' wind forcing

$$u = \frac{\partial \psi}{\partial y} \; ; \; v = -\frac{\partial \psi}{\partial x}$$

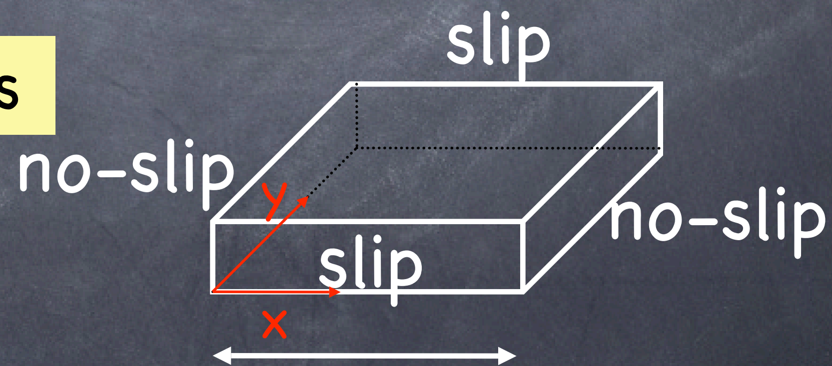
$$\mathrm{Re} = \frac{UL}{A_H}$$ → `Viscosity'

$$\zeta = \nabla^2\psi$$ Vorticity - Streamfunction relation

Forcing: double-gyre wind stress



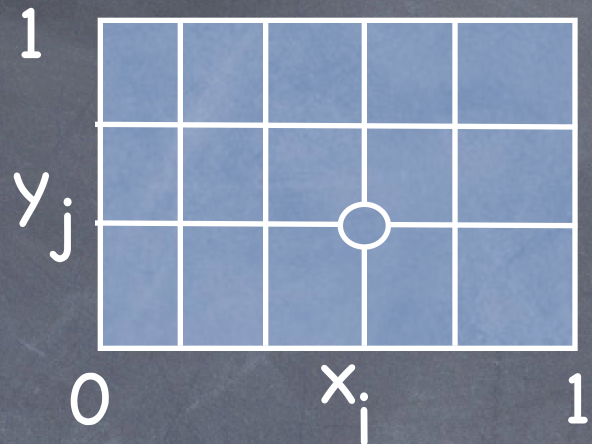
$$\tau^x(y) = -\cos\ 2\pi y$$



L = 1000 km

Domain: square basin

The computational problem



Ex: $$\nabla^2 \zeta_{i,j} \approx \frac{\zeta_{i+1,j} + \zeta_{i-1,j} - 2\zeta_{i,j}}{\Delta x^2} + \frac{\zeta_{i,j+1} + \zeta_{i,j-1} - 2\zeta_{i,j}}{\Delta y^2}$$

State vector: $$\mathbf{x} = (\zeta_{1,1}, \psi_{1,1}, \cdots, \zeta_{N,M}, \psi_{N,M})$$

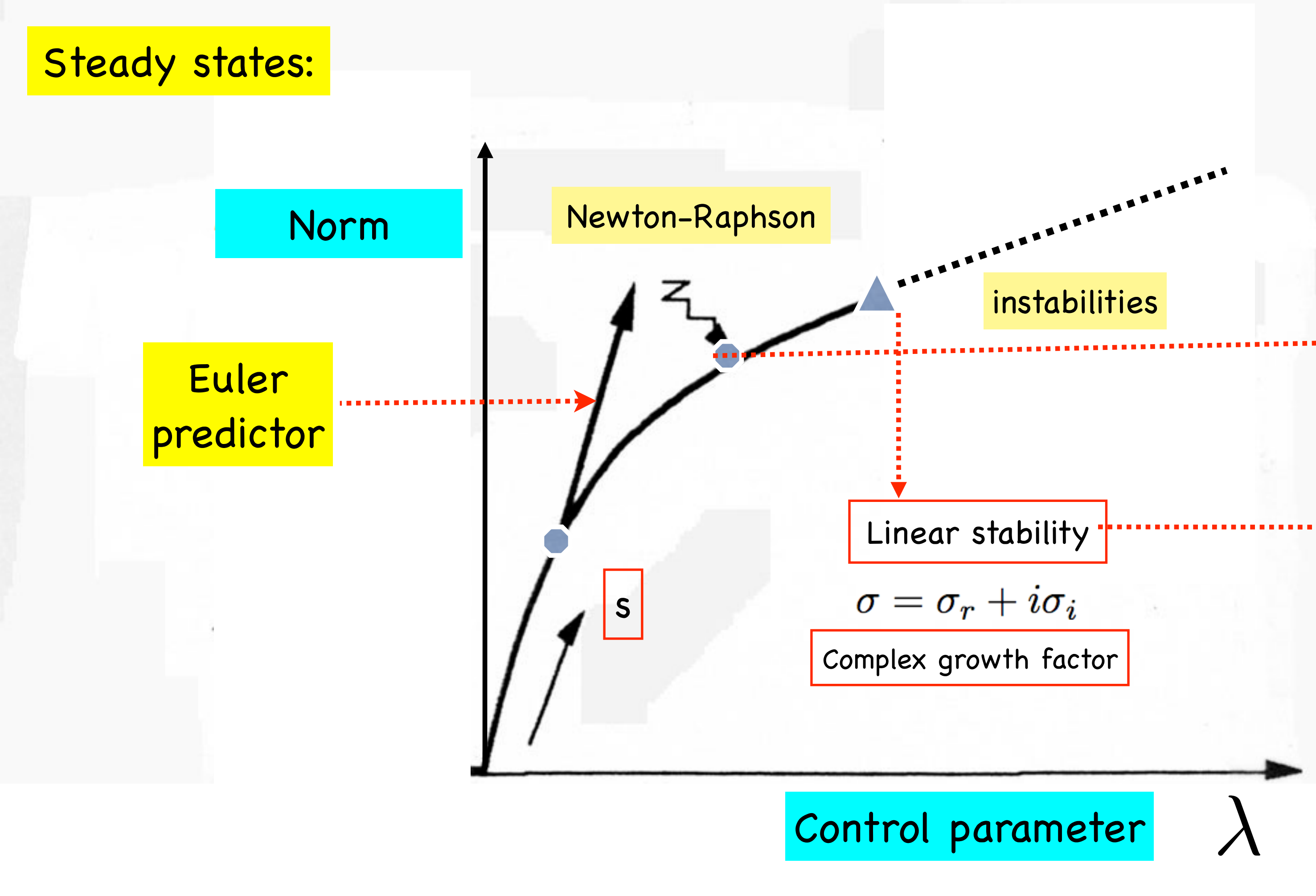
Dynamical system: $$M\frac{d\mathbf{x}}{dt} + \mathbf{G}(\mathbf{x}, \lambda) = 0$$

Continuation Methods

Pseudo-arclength method

$$\mathbf{G}(\mathbf{x}(s), \lambda(s)) = 0$$
$$N(\dot{\mathbf{x}}(s), \dot{\lambda}(s)) = 0$$

d: # degrees of freedom



$Ax = b$

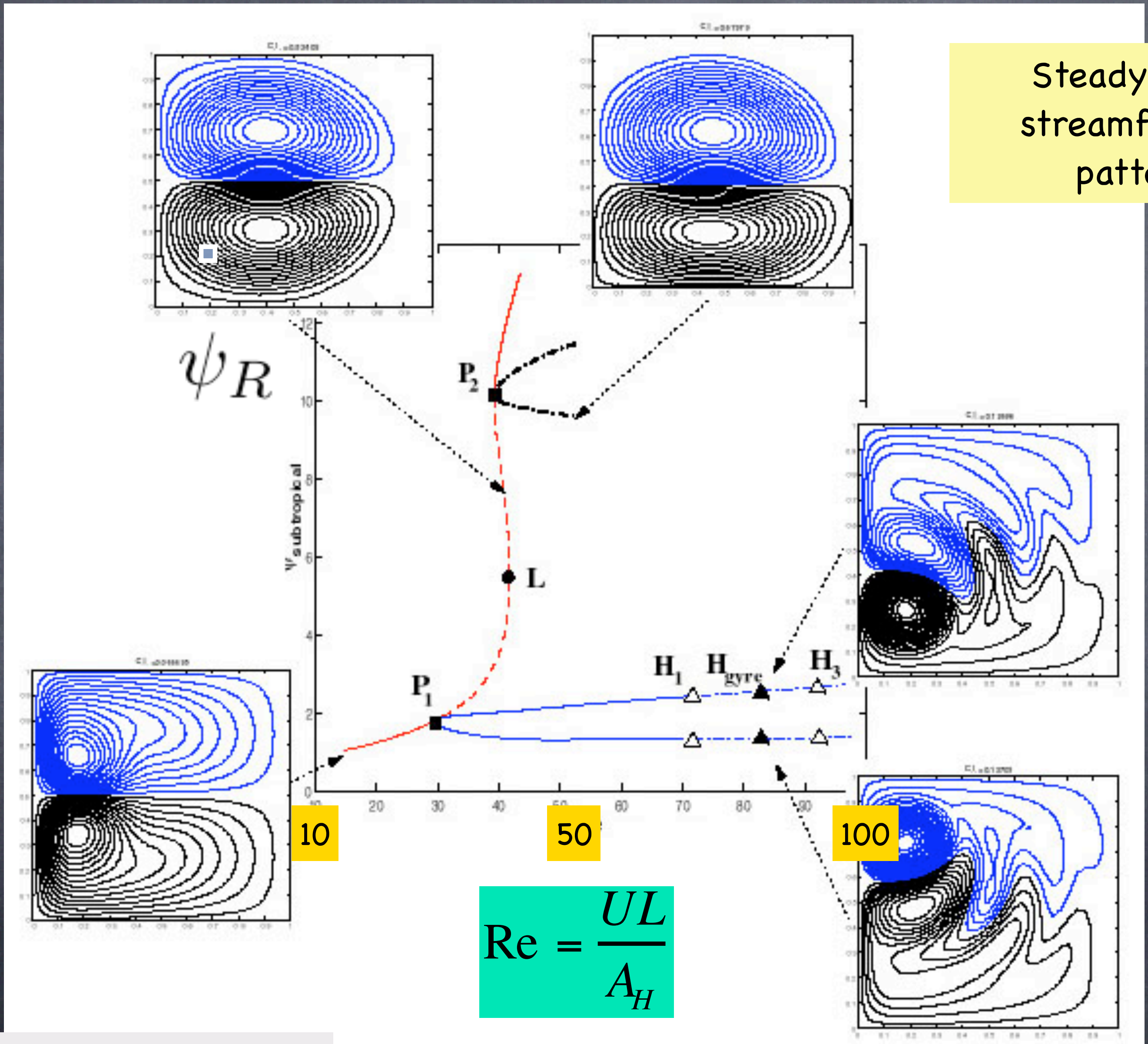
$A : d \times d$

Numerics

$Ax = \sigma Bx$

$A, B : d \times d$

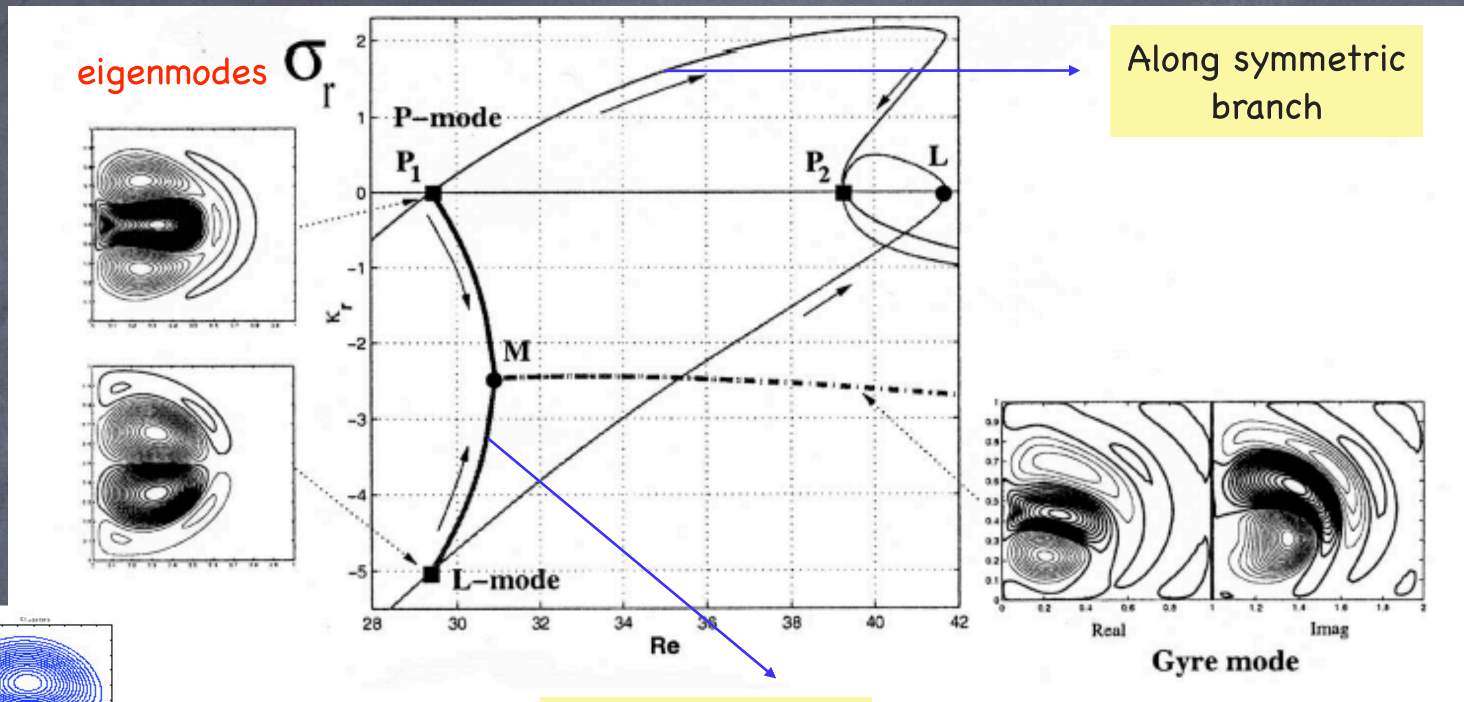
QG - barotropic bifurcation diagram, I



Cessi & Ierley, JPO, (1995)
Dijkstra & Katsman, GAFD, (1997)

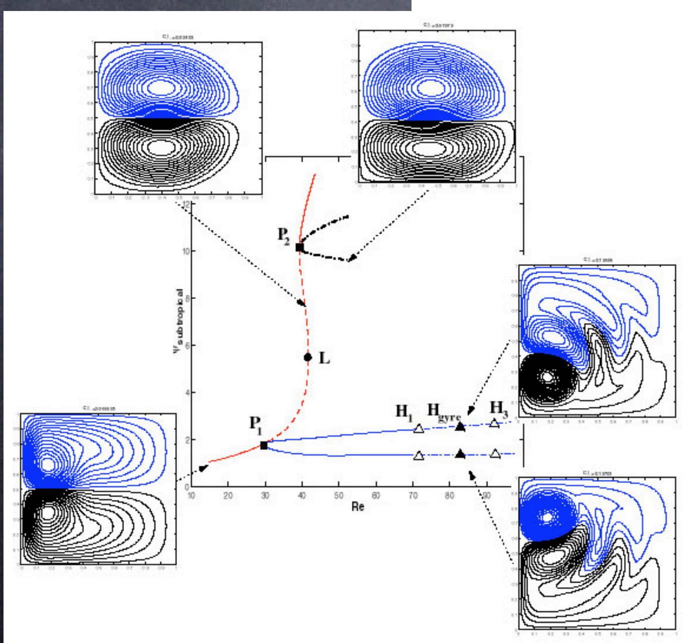
d = 128 x 128 x 2 = 32,768

Mode merging: low-frequency oscillatory modes



Along symmetric branch

Along asymmetric branch



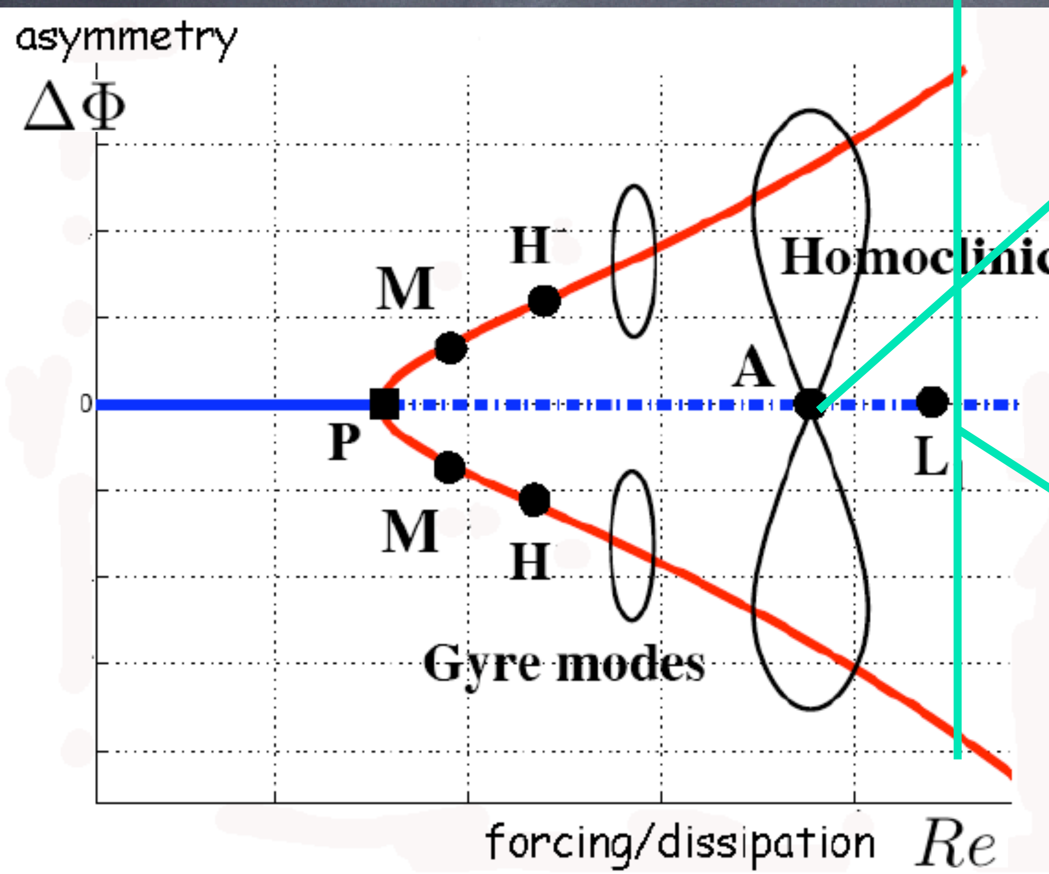
Period of gyre mode at Hopf bifurcation (Re~80): ~ 1.5 year

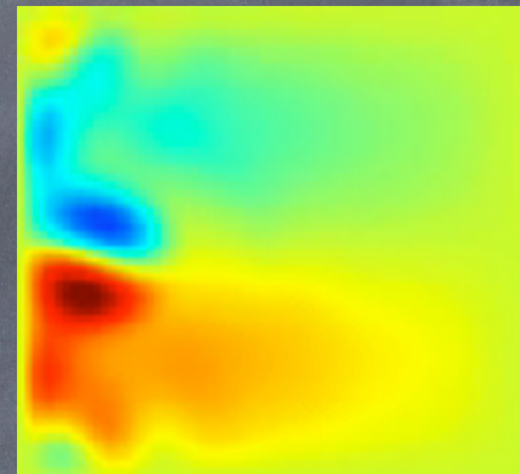
Period of other (Rossby-basin) modes: ~ 2-4 months

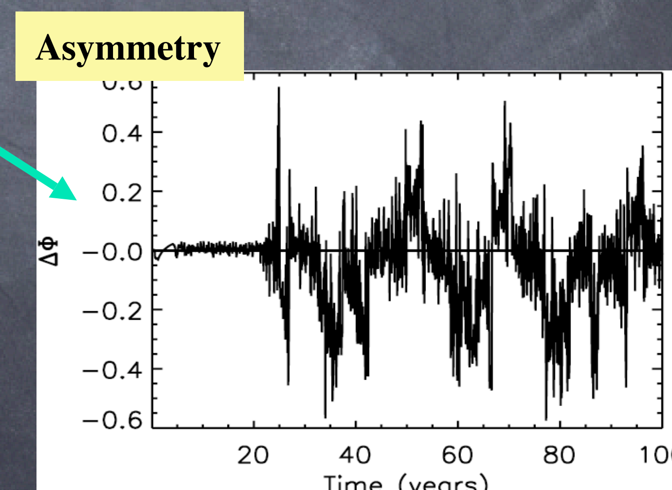
Simonnet & Dijkstra, JPO, (2002)

Bifurcation diagram QG-model, II

$$\Delta\Phi = \frac{\psi_{min} + \psi_{max}}{|\psi_{max}|}$$







Simonnet et al., JMR, (2005)

Summary: QG model

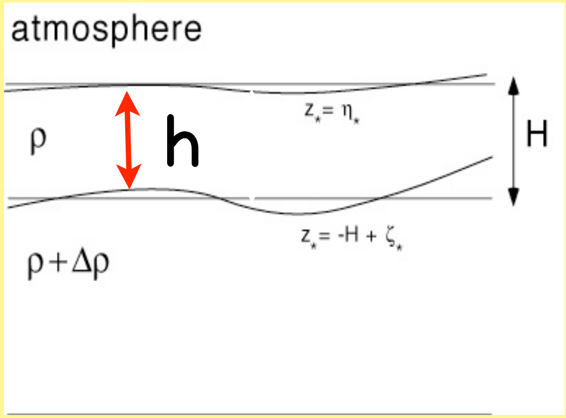
- The barotropic QG model provides a low-dimensional DS view of the low-frequency variability of the double-gyre circulation
  - Pitchfork - > symmetry breaking + multiple equilibria
  - Gyre mode -> transition to oscillatory flow
  - Homoclinic bifurcation -> low-frequency variability

Different Kuroshio patterns ⟷ Multiple equilibria

?

Decadal Time scale ⟷ 2 x period gyre mode

Search for a 'minimal' model: the spherical reduced gravity shallow-water model



Idealization of stratification

$$g' = g\frac{\Delta\rho}{\rho}$$

Control parameter:

$$E = \frac{A_H}{2\Omega r_0^2}$$ ⟶ 'Viscosity'

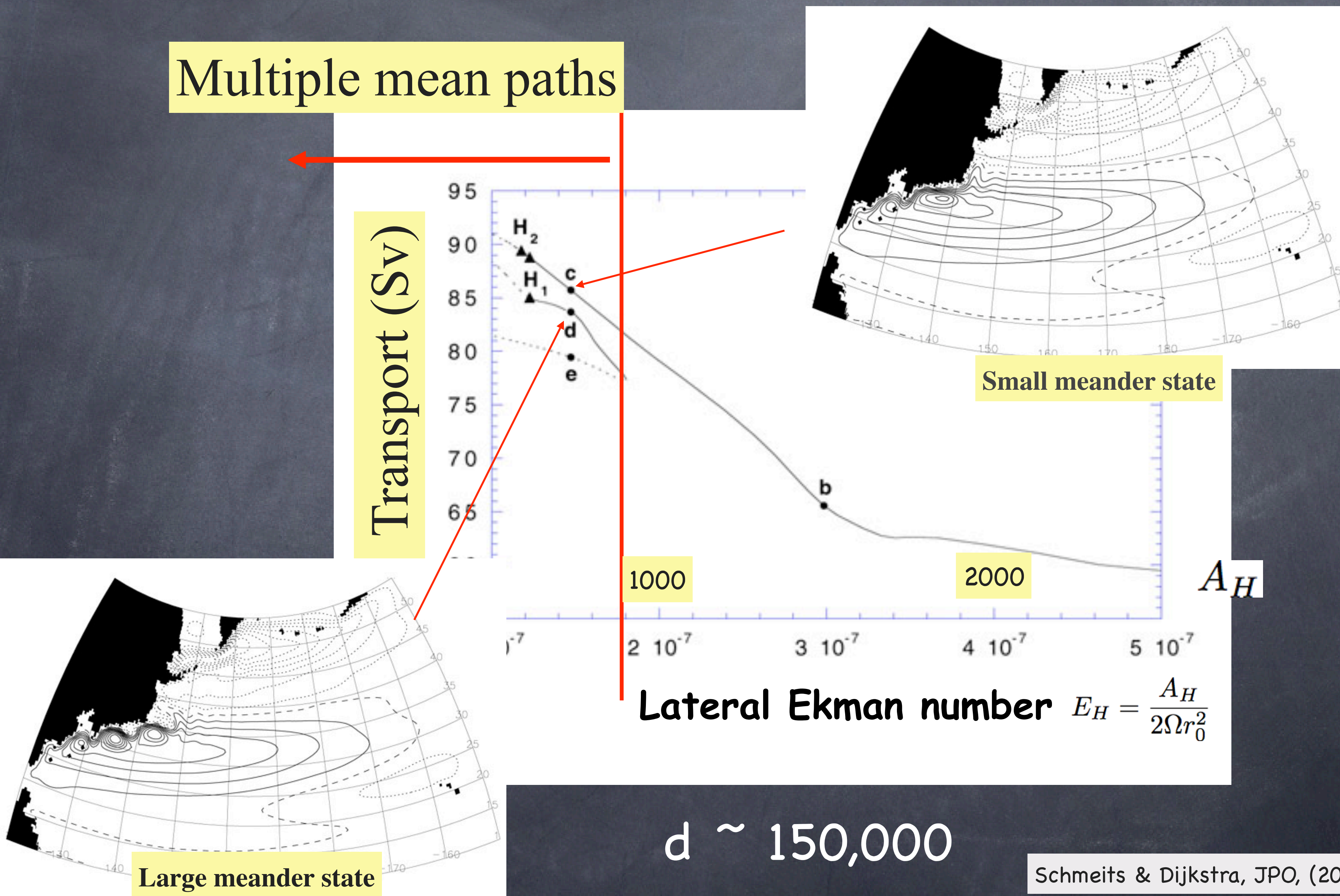
$$\epsilon\left(\frac{\partial u}{\partial t} + \frac{u}{\cos\theta}\frac{\partial u}{\partial\phi} + v\frac{\partial u}{\partial\theta} - uv\tan\theta\right) - v\sin\theta = -\frac{\epsilon F}{\cos\theta}\frac{\partial h}{\partial\phi} + E\left(\nabla^2 u - \frac{u}{\cos^2\theta} - \frac{2\sin\theta}{\cos^2\theta}\frac{\partial v}{\partial\phi}\right) + \alpha\frac{\tau^\phi}{h}$$

$$\epsilon\left(\frac{\partial v}{\partial t} + \frac{u}{\cos\theta}\frac{\partial v}{\partial\phi} + v\frac{\partial v}{\partial\theta} + u^2\tan\theta\right) + u\sin\theta = -\epsilon F\frac{\partial h}{\partial\theta} + E\left(\nabla^2 v - \frac{v}{\cos^2\theta} + \frac{2\sin\theta}{\cos^2\theta}\frac{\partial u}{\partial\phi}\right) + \alpha\frac{\tau^\theta}{h}$$

$$\frac{\partial h}{\partial t} + \frac{1}{\cos\theta}\left(\frac{\partial(hu)}{\partial\phi} + \frac{\partial(hv\cos\theta)}{\partial\theta}\right) = 0.$$

Pacific basin

Multiple mean paths

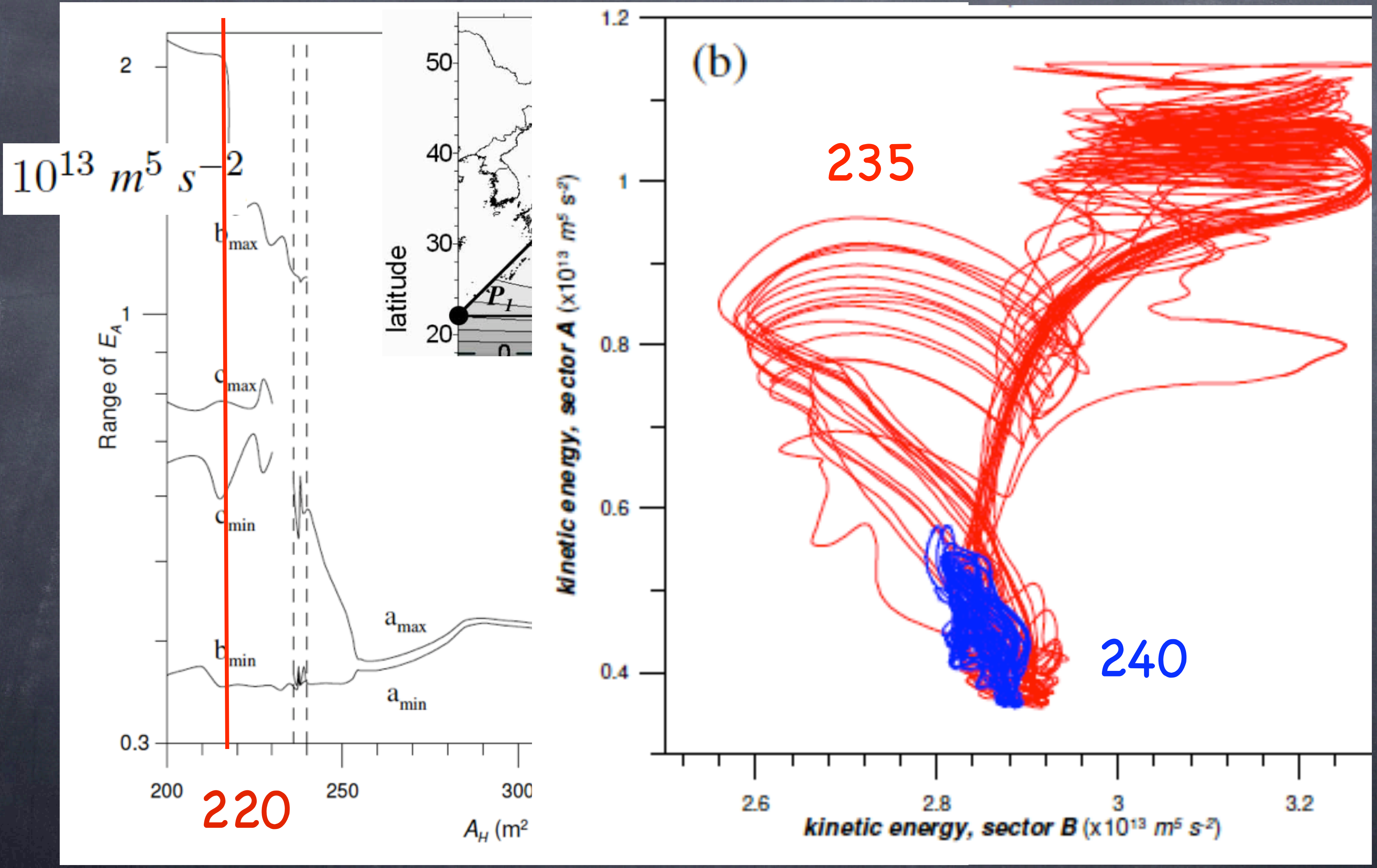


Lateral Ekman number $E_H = \frac{A_H}{2\Omega r_0^2}$

d ~ 150,000

Schmeits & Dijkstra, JPO, (2001)

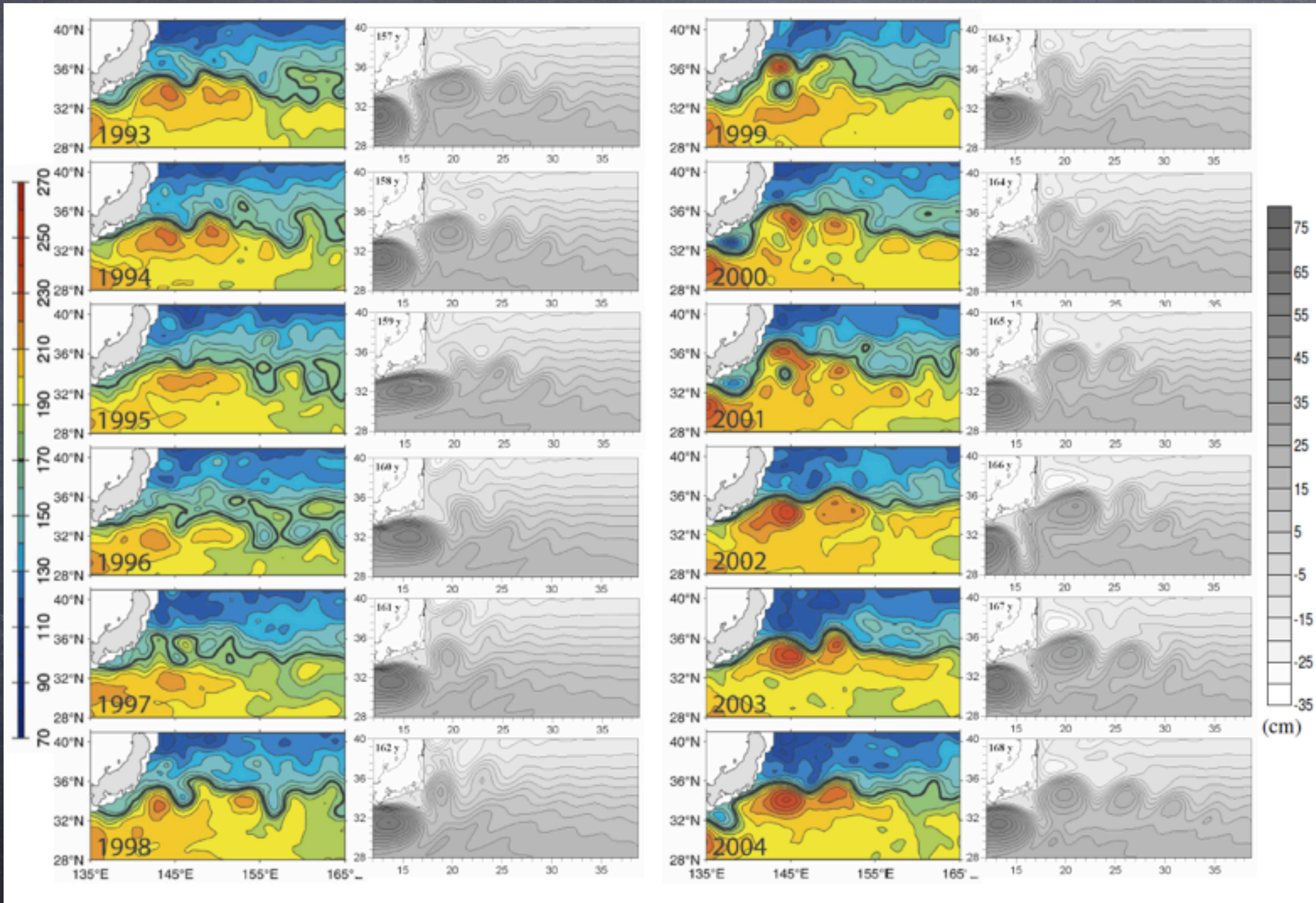
Variations kinetic energy in two regions



d ~ 500,000

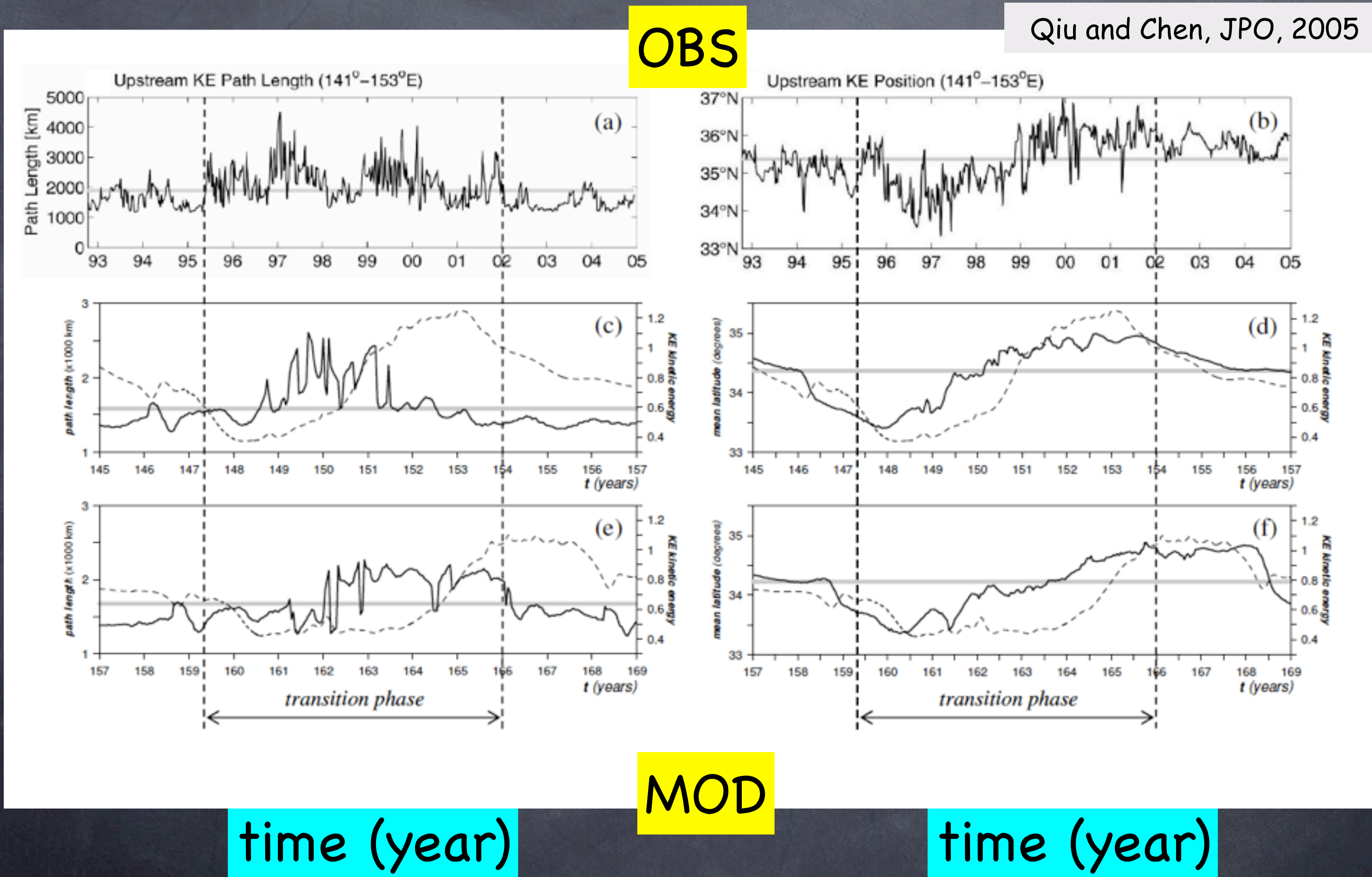
Pierini, Dijkstra & Riccio, JPO, (2009)

Comparison to SSH observations: patterns



Comparison to SSH characteristics

Qiu and Chen, JPO, 2005



Pierini, Dijkstra & Riccio, JPO, (2009)

Summary: SW model

- The reduced-gravity shows qualitatively similar transition behavior in 'viscosity' as the QG model
  - (Imperfect) Pitchfork - > multiple equilibria
  - Gyre mode -> transition to oscillatory flow
  - Homoclinic bifurcation -> low-frequency variability
- Qualitatively good agreement with observations on important observational characteristics
  - Path length & northward KE position
  - Transition time scale

SW model: a `minimal' model ?

What about the meso-scale eddies?

Stochastic representation of `meso-scale' eddies.
I: reduced model

$$\frac{\partial \zeta}{\partial t} + u\frac{\partial \zeta}{\partial x} + v\frac{\partial \zeta}{\partial y} + \beta\frac{\partial \psi}{\partial x} = \frac{1}{Re}\nabla^2\zeta + \alpha\left(\frac{\partial \tau^y}{\partial x} - \frac{\partial \tau^x}{\partial y}\right) \quad -\mu\zeta$$

$$\zeta = \nabla^2\psi$$

QG - model

Choose wind-stress strength as control parameter

$$\begin{aligned} \psi(x,y,t) &= A_1(t)G(x)\sin y + A_2(t)G(x)\sin 2y + \\ &+ A_3(t)G(x)\sin 3y + A_4(t)G(x)\sin 4y \\ G(x) &= e^{-sx}\sin x \qquad x \in [0,\pi], y \in [0,\pi] \end{aligned}$$

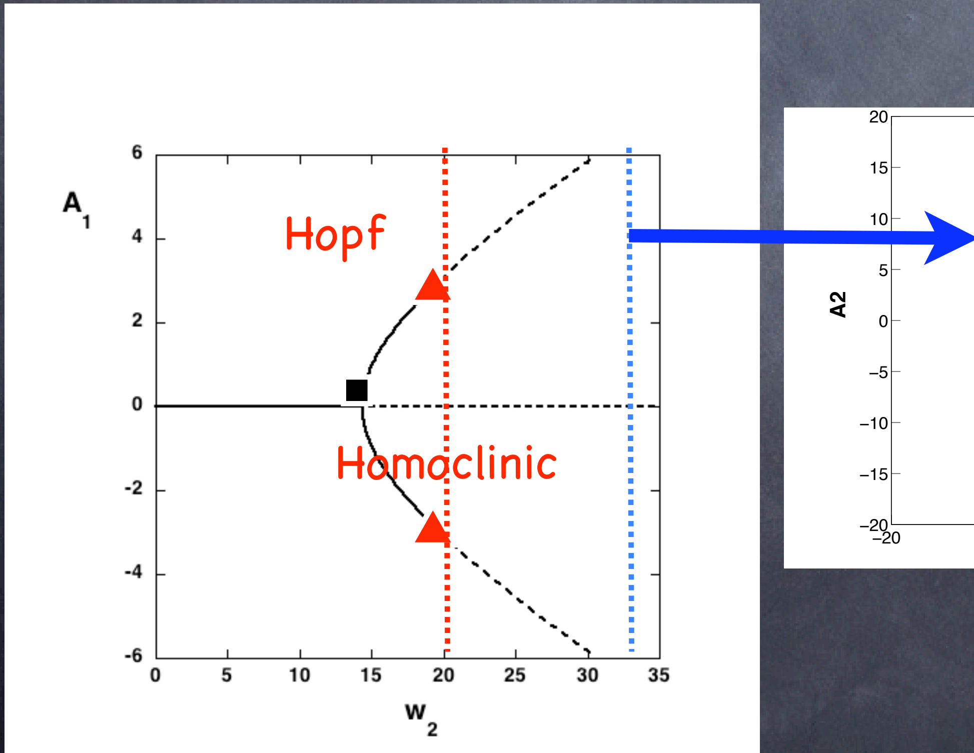
+ Galerkin projection - >

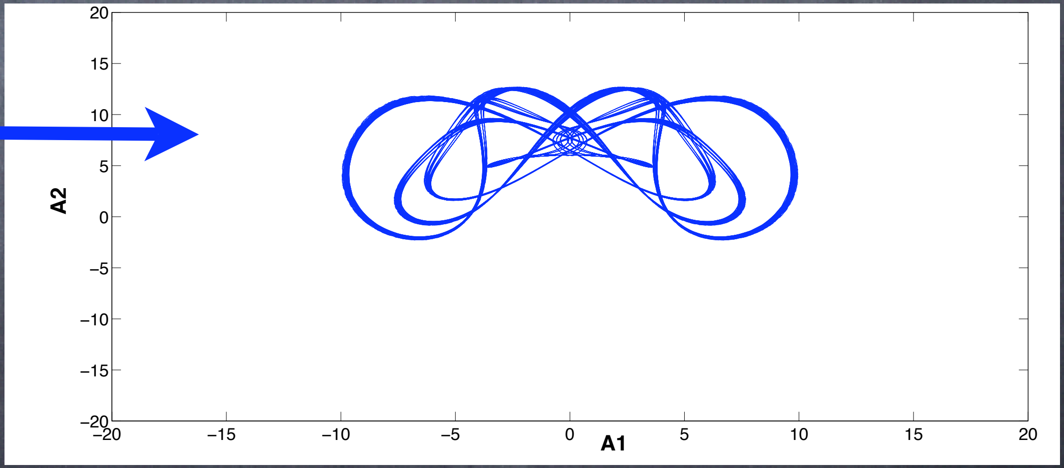
Reduced model

$$\frac{dA_1}{dt} = -A_1 + c_1(A_1A_2 + A_2A_3 + A_3A_4)$$
$$\frac{dA_2}{dt} = c_5w_2 - c_2A_1^2 - A_2 + 2c_2(A_1A_3 + A_2A_4)$$
$$\frac{dA_3}{dt} = -A_3 + A_1(-c_3A_2 + c_3A_4)$$
$$\frac{dA_4}{dt} = -c_4A_2^2 - A_4 - 2c_4A_1A_3$$

w_2

Wind-stress amplitude



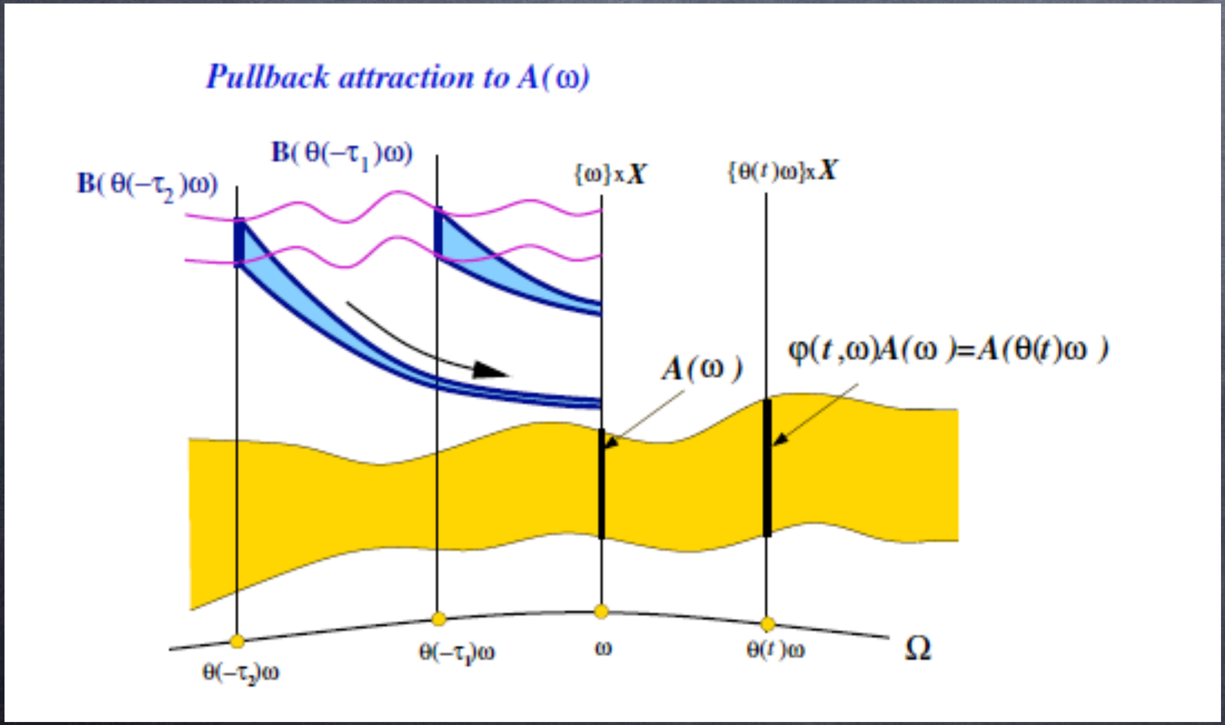


$$\lambda_1 > 0$$

Random dynamical system

choose: $$-A_i dt \to -(A_i dt + \epsilon A_i dW_t)$$

as representation of the effect of the `meso-scale' eddies



Computation of invariant measures

on

random attractor

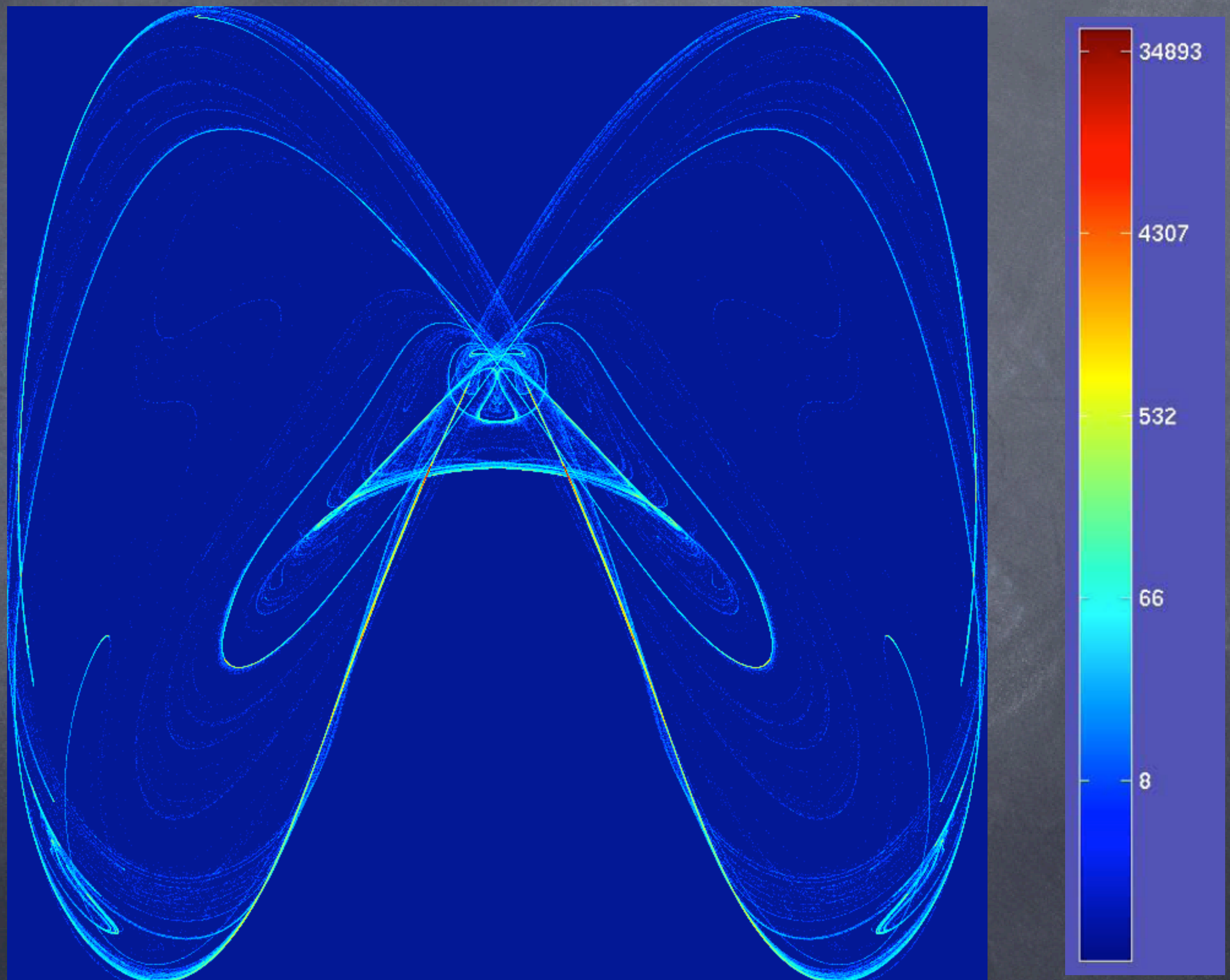
Density of trajectories of 4-mode model at different times

10

A_2

10^6 initial conditions

single noise realization



-10

A_1

10

w_2 = 20

Summary: reduced model

- The reduced model displays the same transition behavior as in the full PDE (QG) model
- Interesting invariant measures on the random (pull back) attractor
- The reduced model seems capable of investigating the detailed physics of the transition time scale

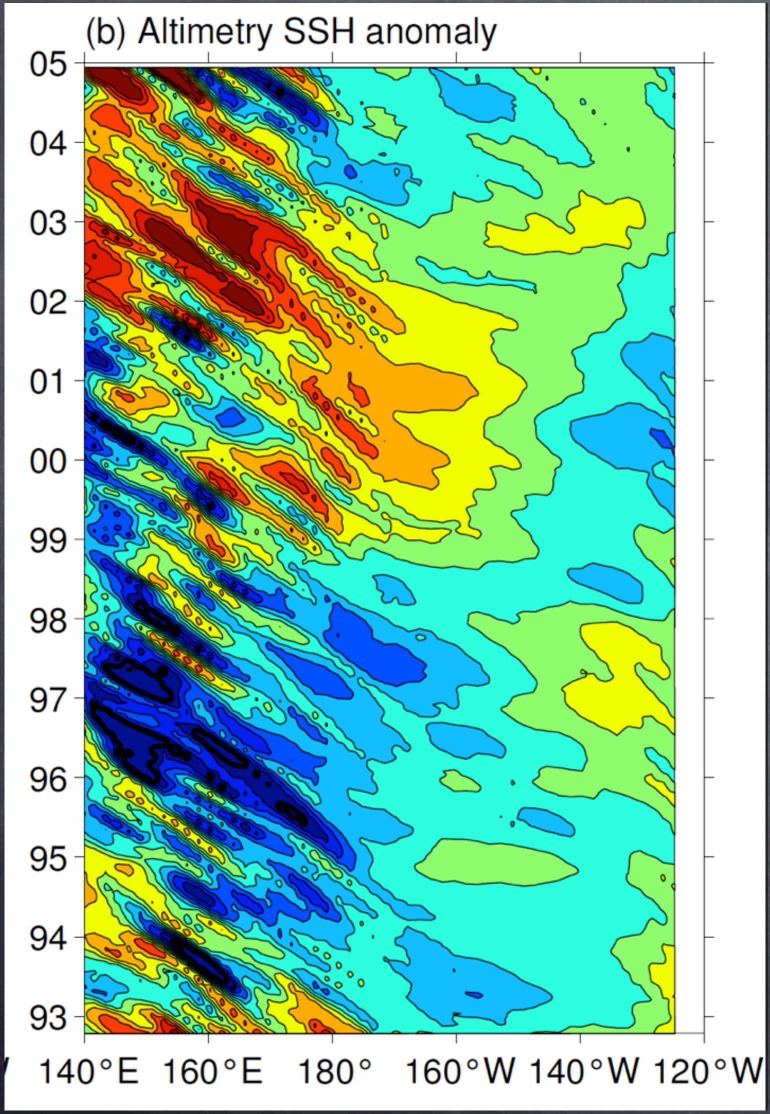
Summary theory II: nonlinear intrinsic low-frequency variability

- In a hierarchy of models, there appear to be multiple equilibria related to different separation states of the Kuroshio; the origin is a symmetry-breaking shear instability

- Transitions between the different states are caused by internal (gyre) modes of variability and/or global bifurcations.

Problem: no clear physical mechanism for the transition time scale yet

Finally: Possible unification of Theory I + II



- The intrinsic variability may cause wind - stress variations downstream

- The resulting (Rossby) waves may be strongly correlated
with the jet variations.