Stretching fields and lines from finite-size Lyapunov exponents: Ocean transport and biological impact

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IFISC

OUTLINE

- The dynamical systems approach to fluid transport: hyperbolic points, manifolds, Lyapunov …
- Finite-size Lyapunov exponents. Impact of flow structures on
  - Abiotic tracers
  - Phytoplankton
  - Frigatebirds

The dynamical systems approach to fluid transport

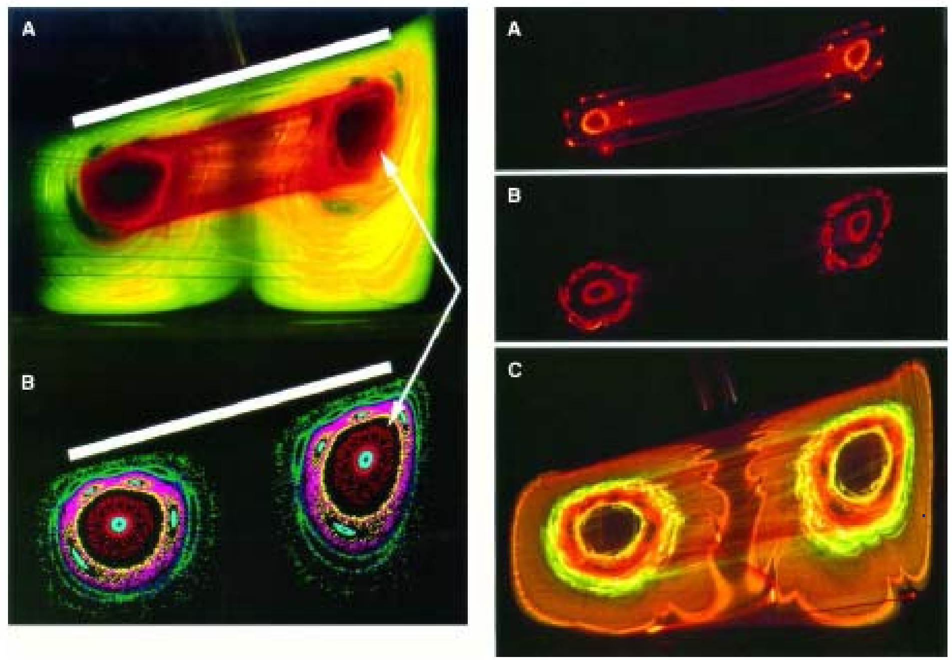
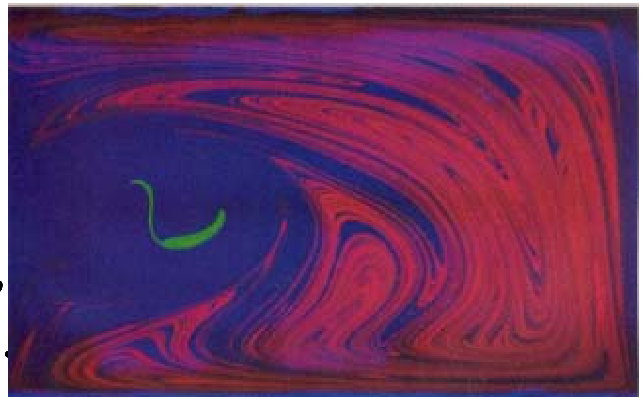
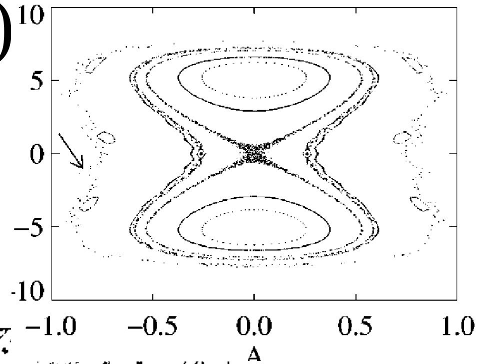


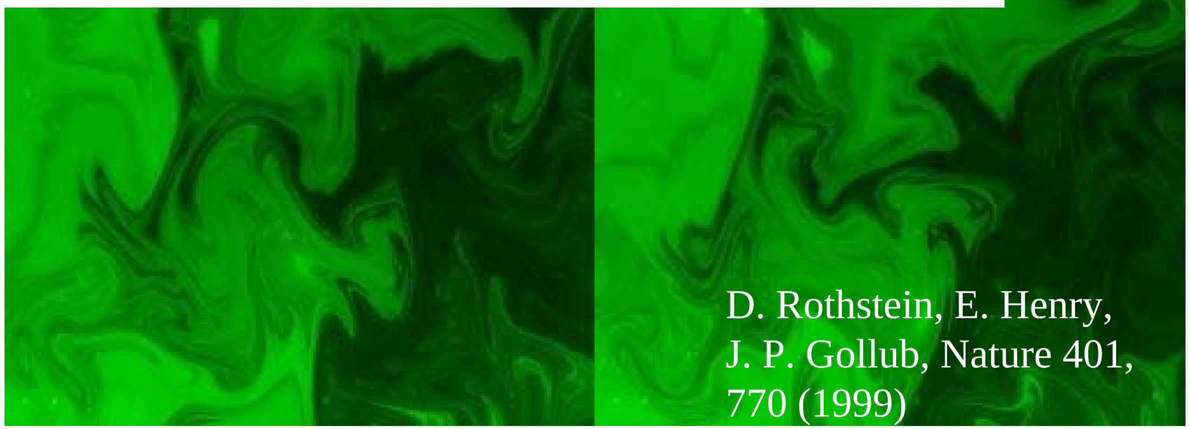
Figure 2.7: KAM tori and elliptic islands visualized by fluorescent dye in an experiment with a steady three-dimensional flow in a viscous fluid (from Fountain et al. (1998)).

Chaotic seas, KAM tori, …

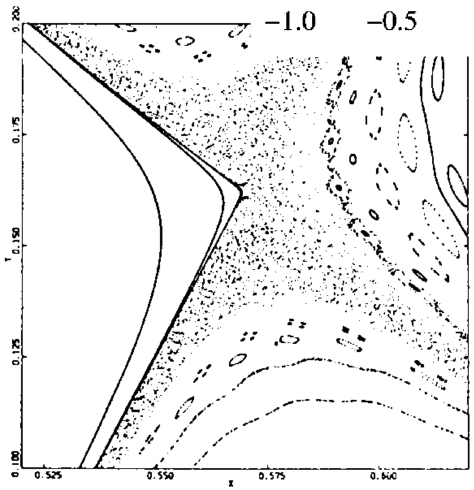


$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{v}(\mathbf{x}(t), t)$$





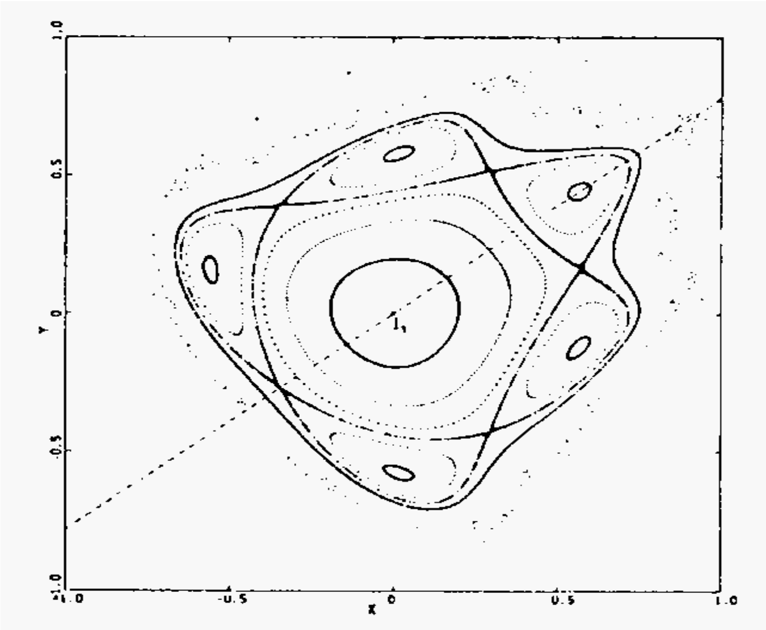
D. Rothstein, E. Henry, J. P. Gollub, Nature 401, 770 (1999)



- Dynamical system: **particle trajectories** in a given velocity field. **Twodimensional** (+ time), threedimensional. NOT the dynamics of the velocity field itself.

- Phase space = physical space

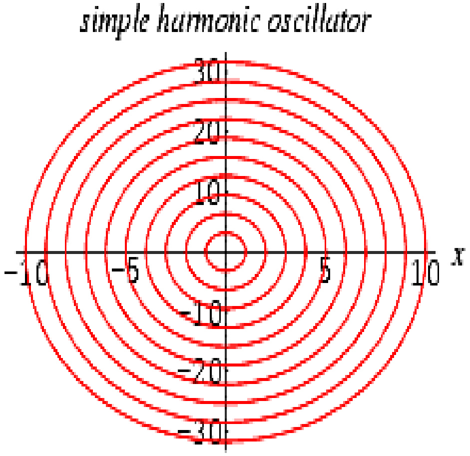
- Global behavior in phase space is organized by some relevant lines

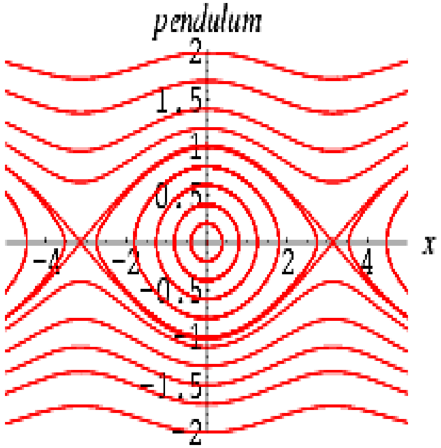


$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{v}(\mathbf{x}(t), t)$$

WHICH ARE THE RELEVANT LINES?

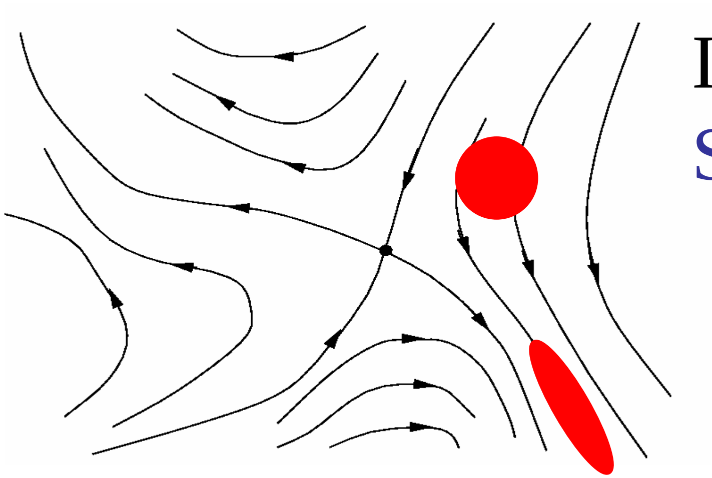
Trajectories of two-dimensional steady or periodic flows are





organized by the fixed points, or periodic orbits of the dynamical system

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{v}(\mathbf{x}(t))$$



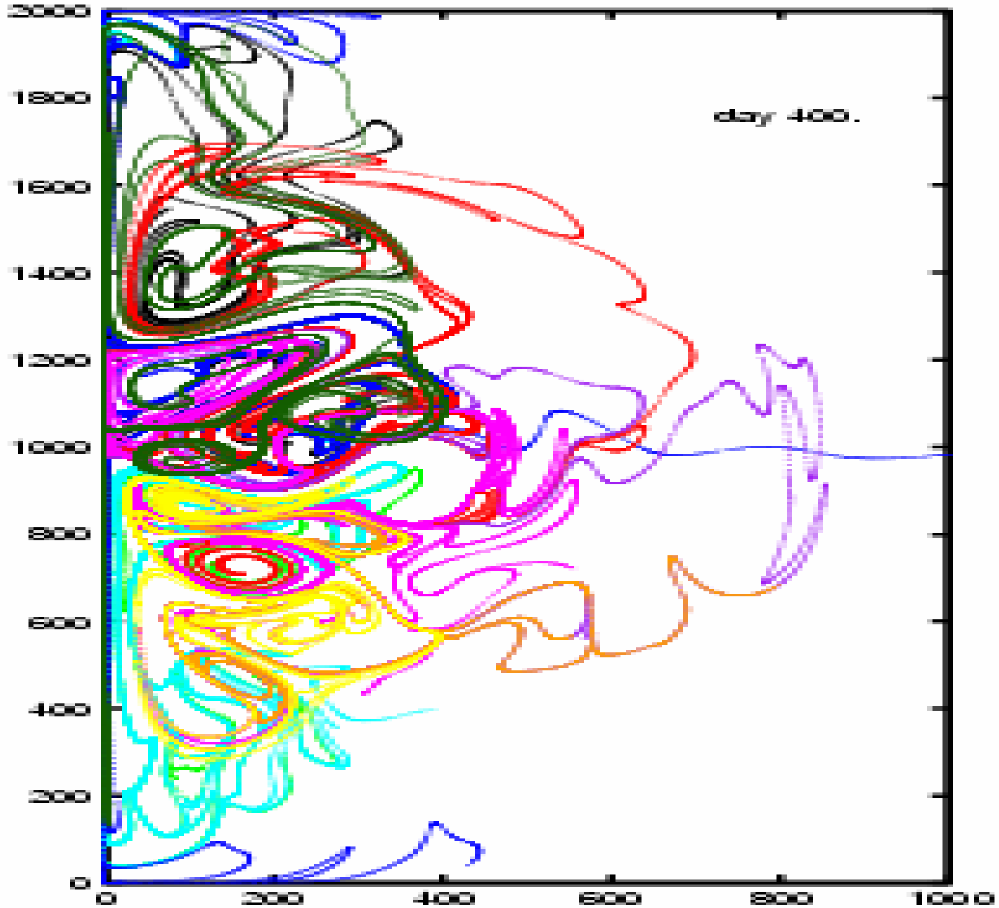
If hyperbolic:

Stable and

unstable manifolds → separatrices

Tracers tend to approach unstable manifolds

But
unsteady flows …



From Mancho, Small and Wiggins, 2005

Is there any particular subset of hyperbolic points and manifolds organizing the dynamics (the equivalent to the fixed points in autonomous systems) ?
How to select them among this mess ?

Identifying the relevant trajectories and manifolds in time-aperiodic dynamical systems

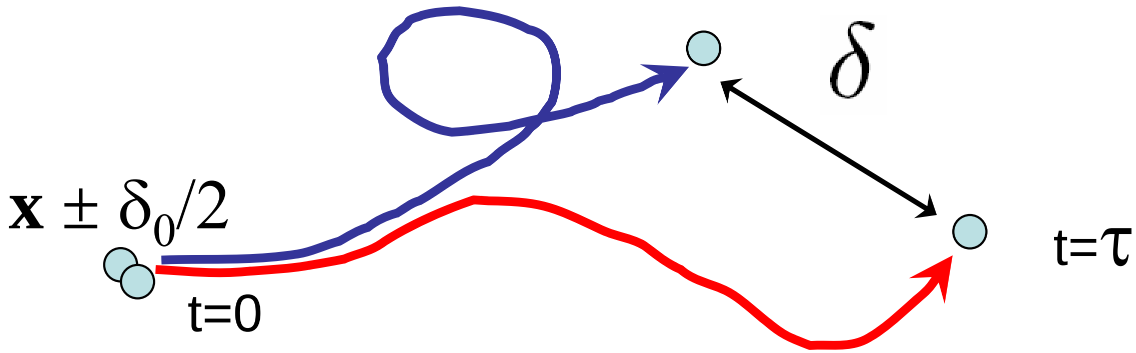
- Attracting or repelling material lines (Haller, Poje, …)
- Leaking, escape,or residence time methods (Tél, Schneider, …)
- Distinguished hyperbolic trajectories (Wiggins, Ide, Mancho, …)
- **Stretching-field methods**: Finite-time Lyapunov exponents, Finite-size Lyapunov exponents, …(Vulpiani, Cencini, Legras, Artale, Haller, Lekien, …). M function (Mancho, Jiménez-Madrid, Mendoza)
- …

$$\lambda(t) = \lim_{\|\delta(0)\|\to 0} \frac{1}{t} \ln \frac{\|\delta(t)\|}{\|\delta(0)\|}$$

Finite-**time** Lyapunov exponent

$$\lambda = \lim_{t\to\infty} \lambda(t)$$

Lyapunov exponent



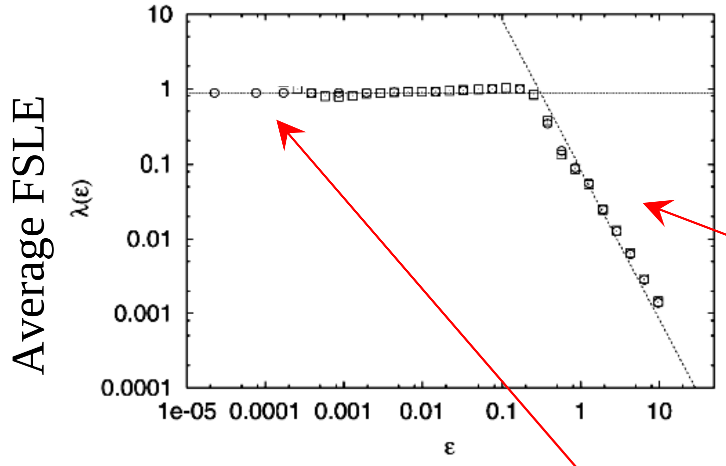
$$\lambda(\delta_0, \delta_f) \equiv \frac{1}{\tau} \log \frac{\delta_f}{\delta_0}$$

Finite-size Lyapunov exponent
FSLE

All the quantities are also functions of the initial position and time:

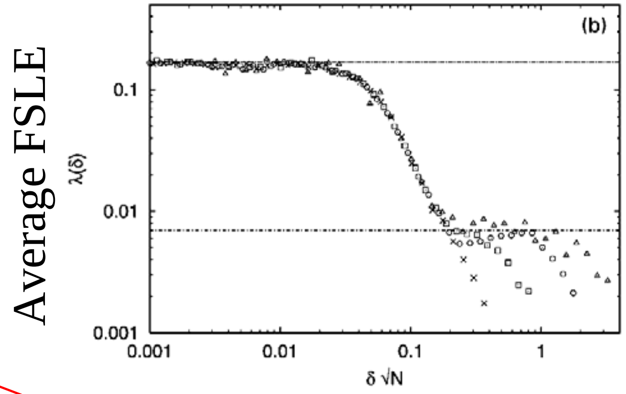
$$\lambda(\mathbf{x}, t, \delta_0, \delta_f)$$

A chaotic map



δ_0

System with several time scales



(coupled maps)

δ_0

Subexponential growth (diffusion regime)

Exponential growth of separations (chaotic regime)

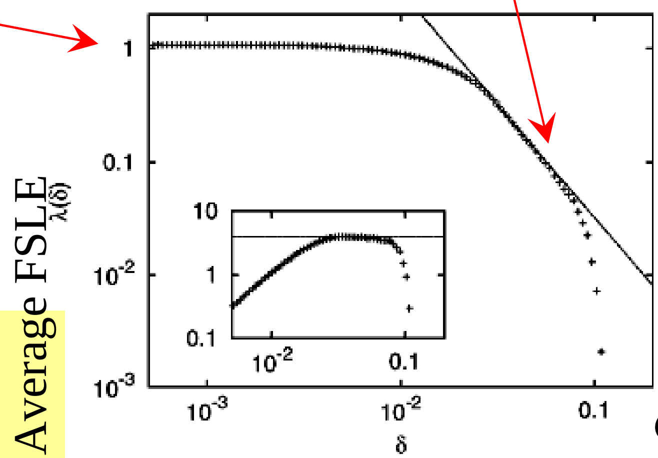
When $\delta_0 \to 0$,

FSLE→Lyapunov

and when $t \to \infty$,

FTLE→Lyapunov

G. Boffetta et al. / Physics Reports 356 (2002) 367–474

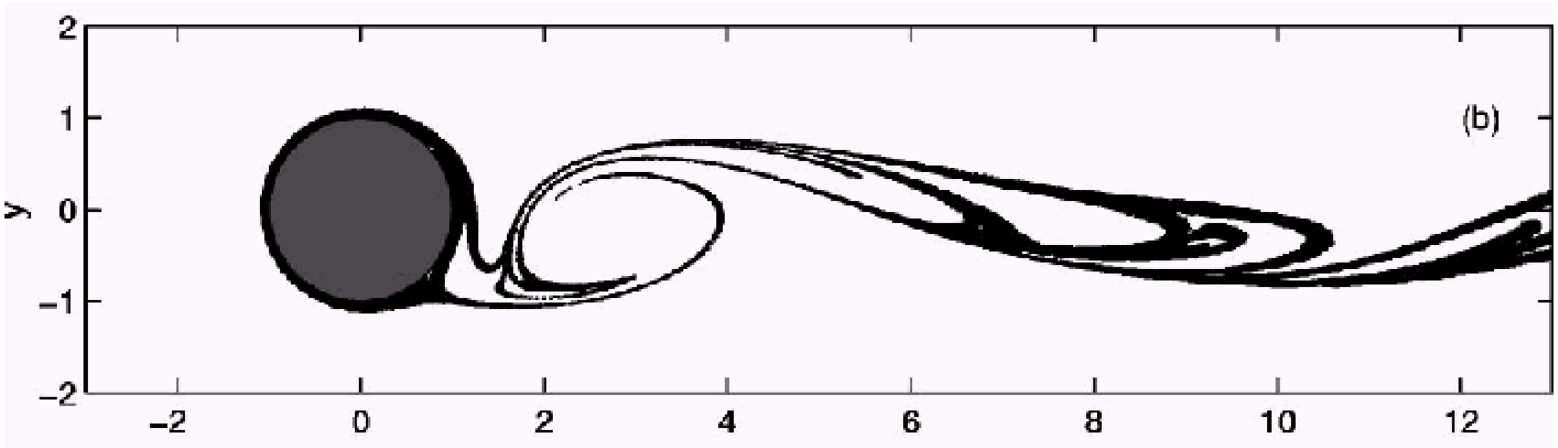


δ_0

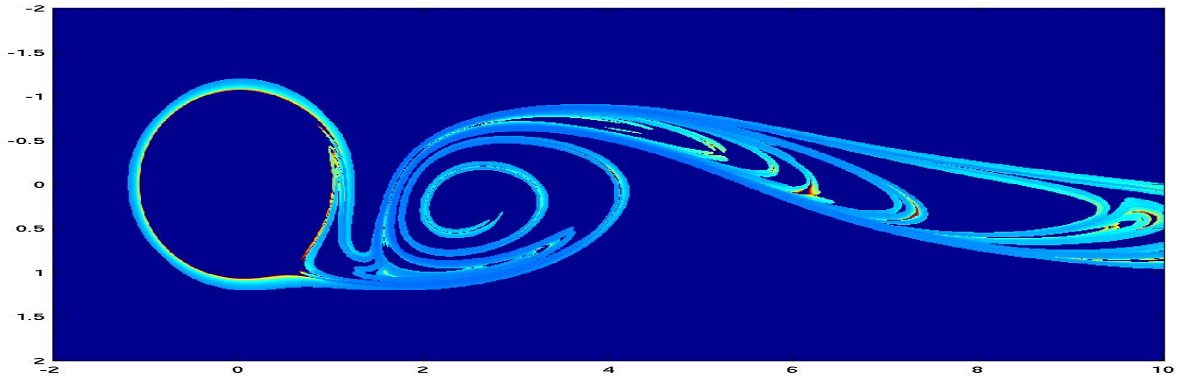
2D turbulence

The FSLE was originally introduced to quantify dispersion from non-infinitesimal initial separations

The spatial dependence of the FSLE allows the detection of stable and unstable manifolds of hyperbolic objects

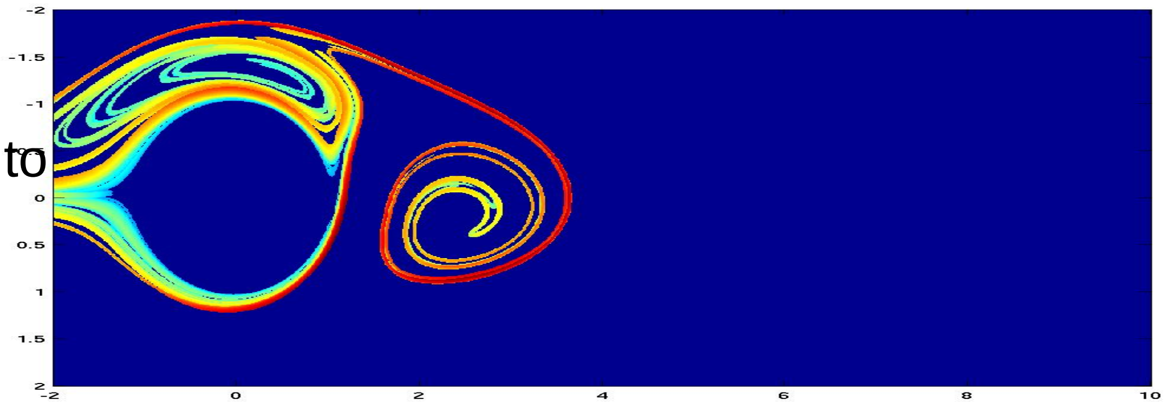


MAXIMA of FSLE: Lagrangian Coherent Structures (LCSs)



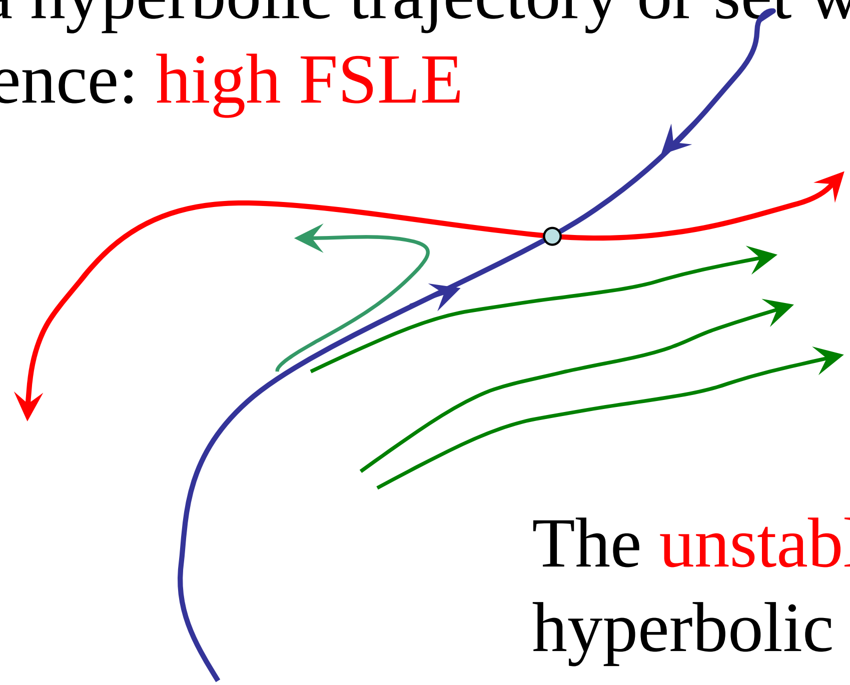
FSLE values from **time-backwards** trajectories

The lines organizing the flow seem to be the manifolds associated to strongest local Lyapunov exponents (backards and forward)



FSLE values from **time-forward** trajectories

The idea is that initial conditions close to the stable manifold of a hyperbolic trajectory or set will show strong divergence: high FSLE

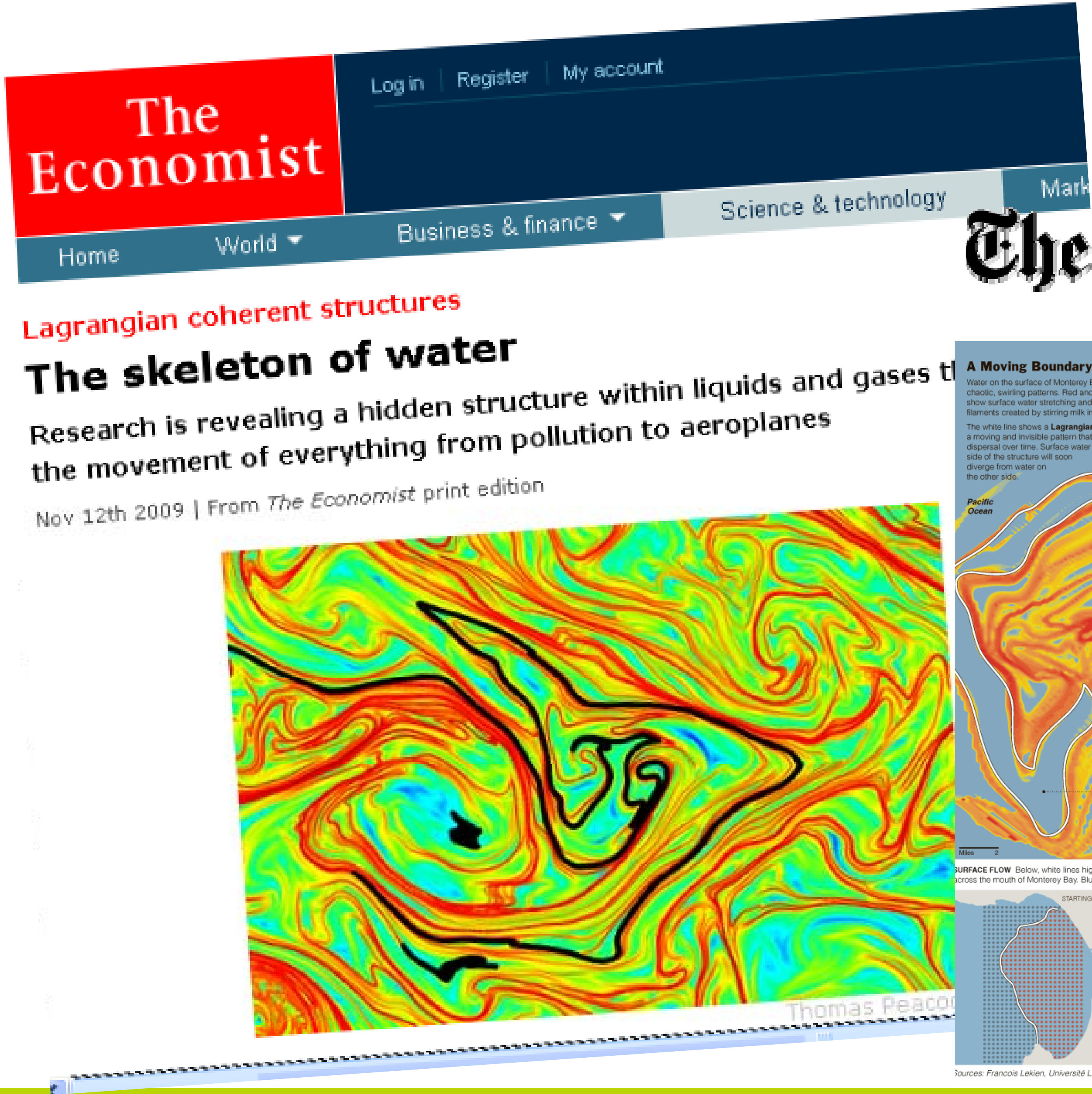


The unstable manifold of hyperbolic sets would be marked by high FSLE in the time backwards direction

Other types of Lyapunov exponents would display similar information, but FSLE is less afected by saturation

REMARK: these are heuristic consideration. Theorems needed (some available for FTLE)

Lagrangian Coherent Structures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Lagrangian coherent structures

The skeleton of water

Research is revealing a hidden structure within liquids and gases th

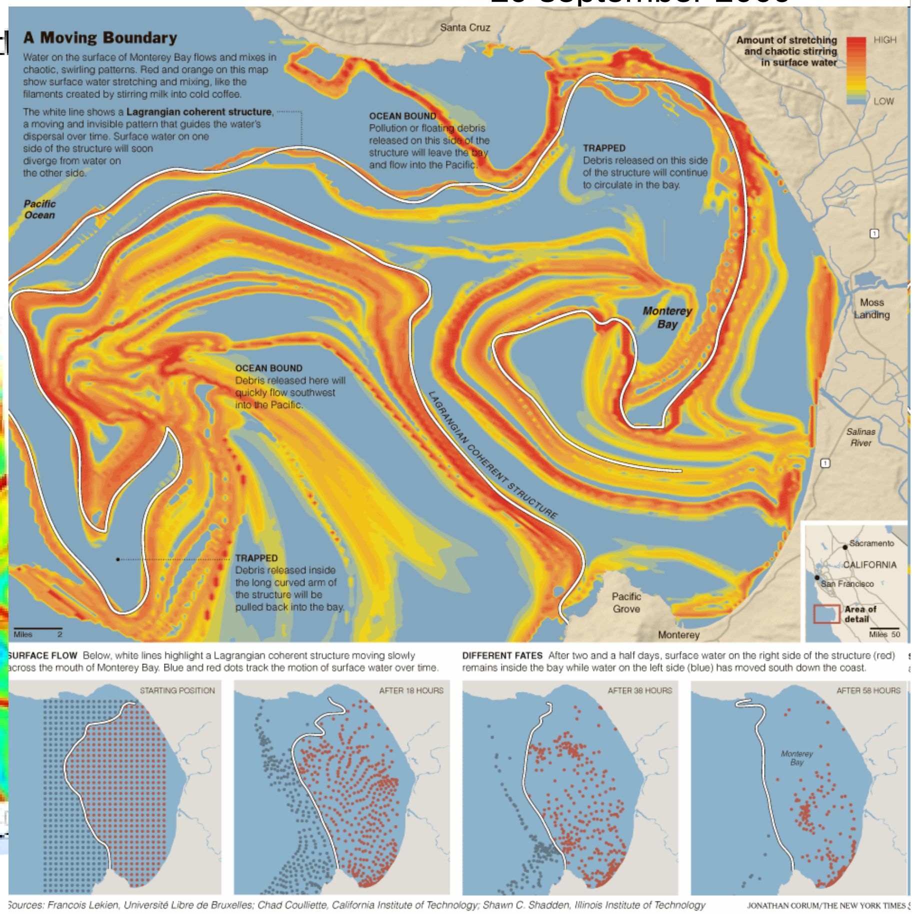
the movement of everything from pollution to aeroplanes

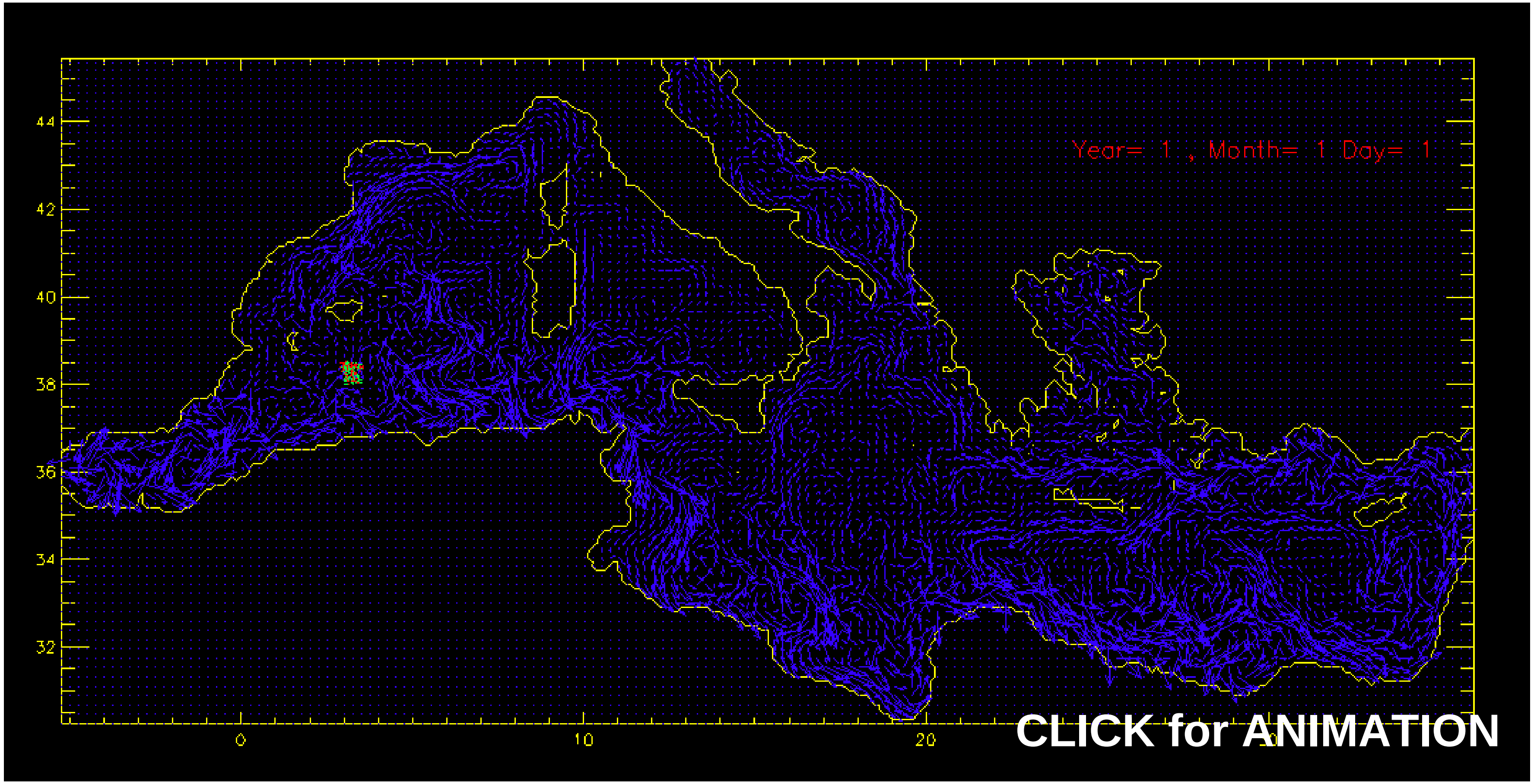
Nov 12th 2009 | From *The Economist* print edition

Thomas Peaco

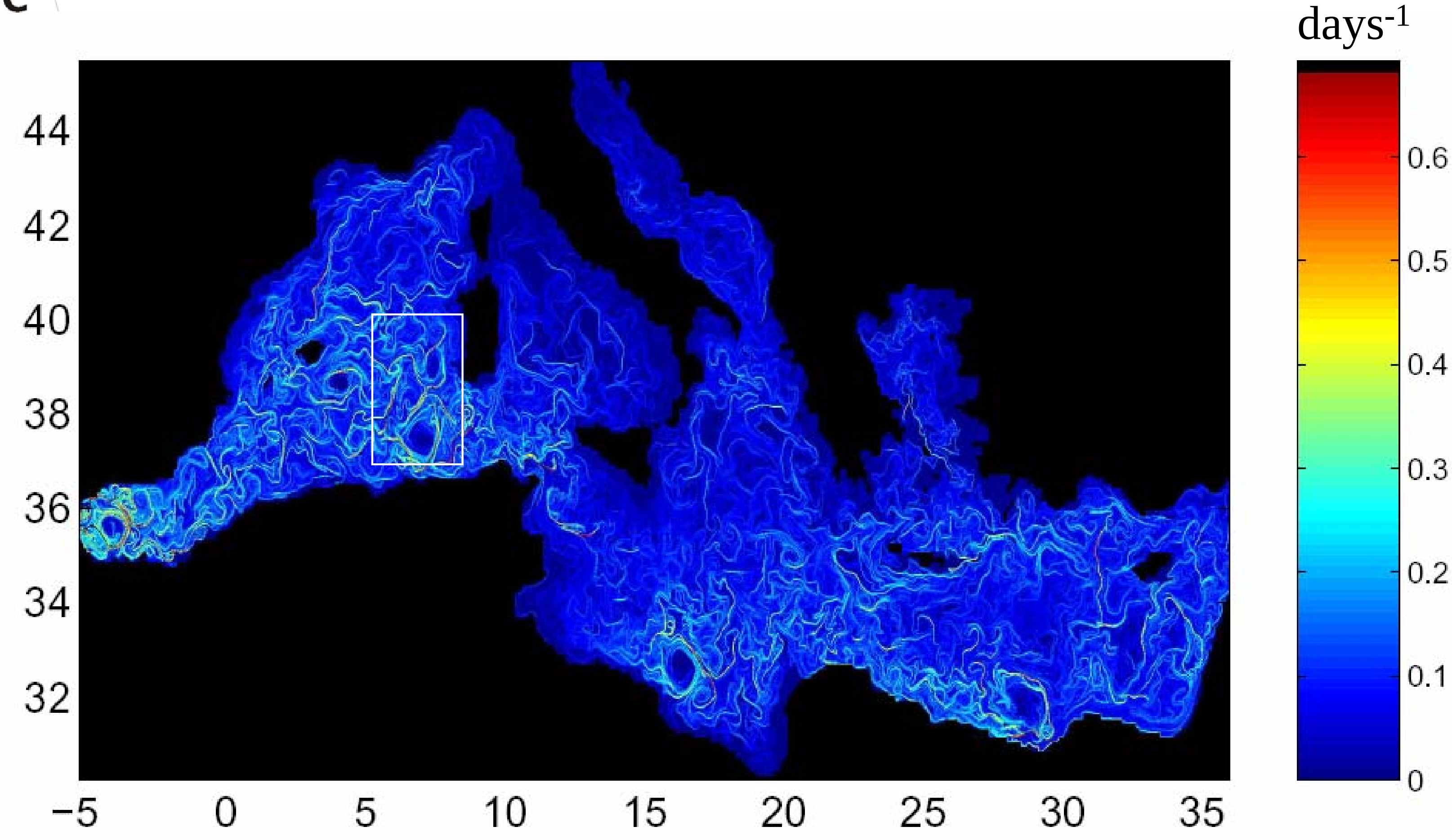
The New York Times

29 september 2009





DieCAST model for the full Mediterranean
Primitive equations,
48 vertical levels, 1/8 º horizontal resolution,
climatological forcings … → 5 years of dayly velocity fields

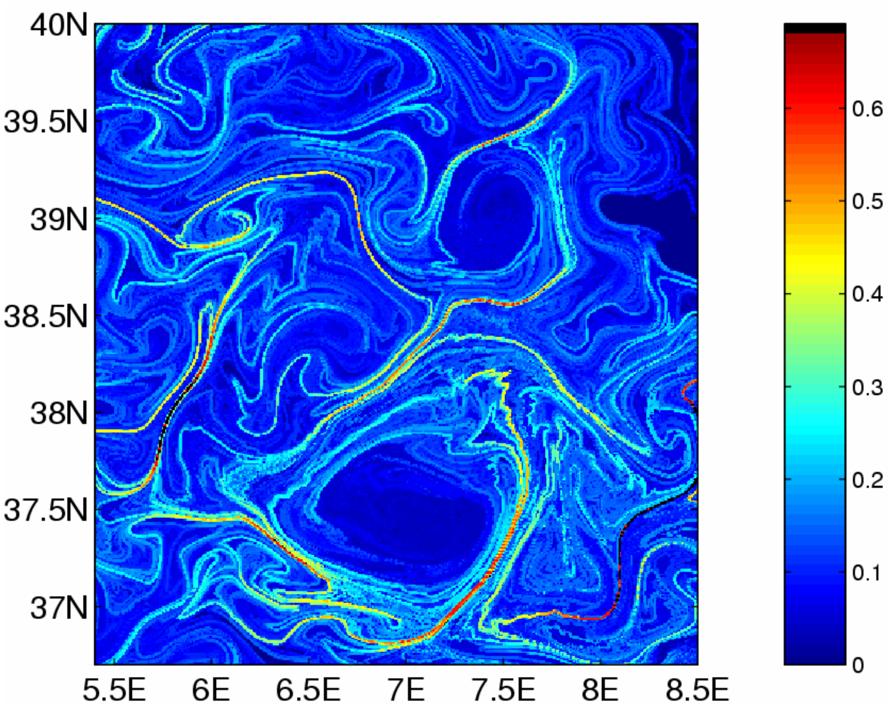


$\delta_0 = 0.02^{\circ} \rightarrow \delta_f = 1^{\circ}$ (mesoscale transport)

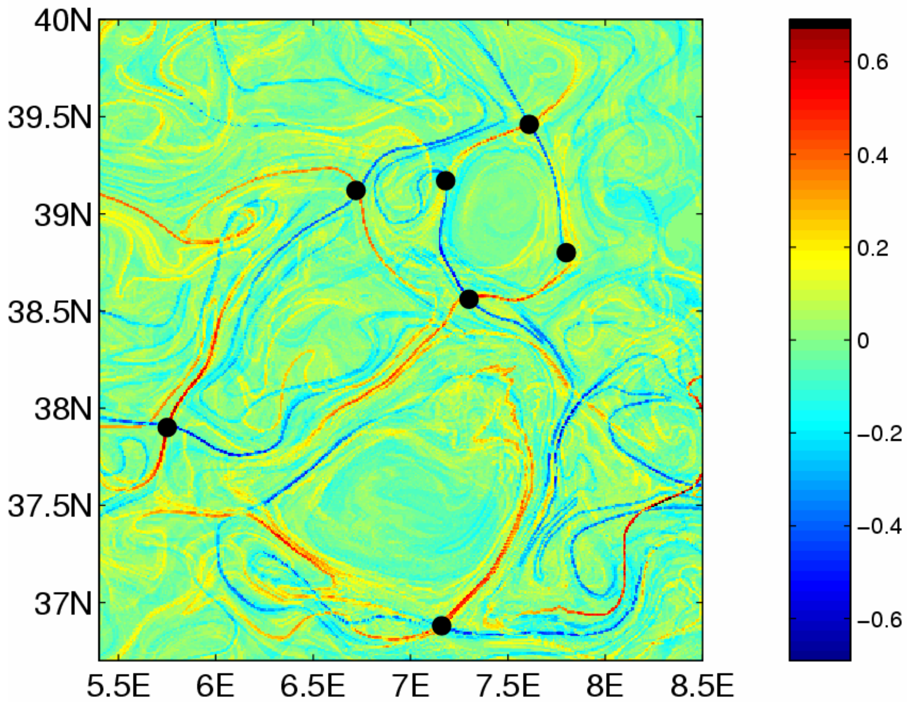
$\delta_0 \approx 2$ km $\rightarrow \delta_f \approx 110$ km twodimensional

d'Ovidio, Fernández, Hernández-García, López, Geophys. Res. Lett. 31, L17203 (2004)

FSLE from time-backwards Integrations.
Are they really unstable manifolds of hyperbolic trajectories?

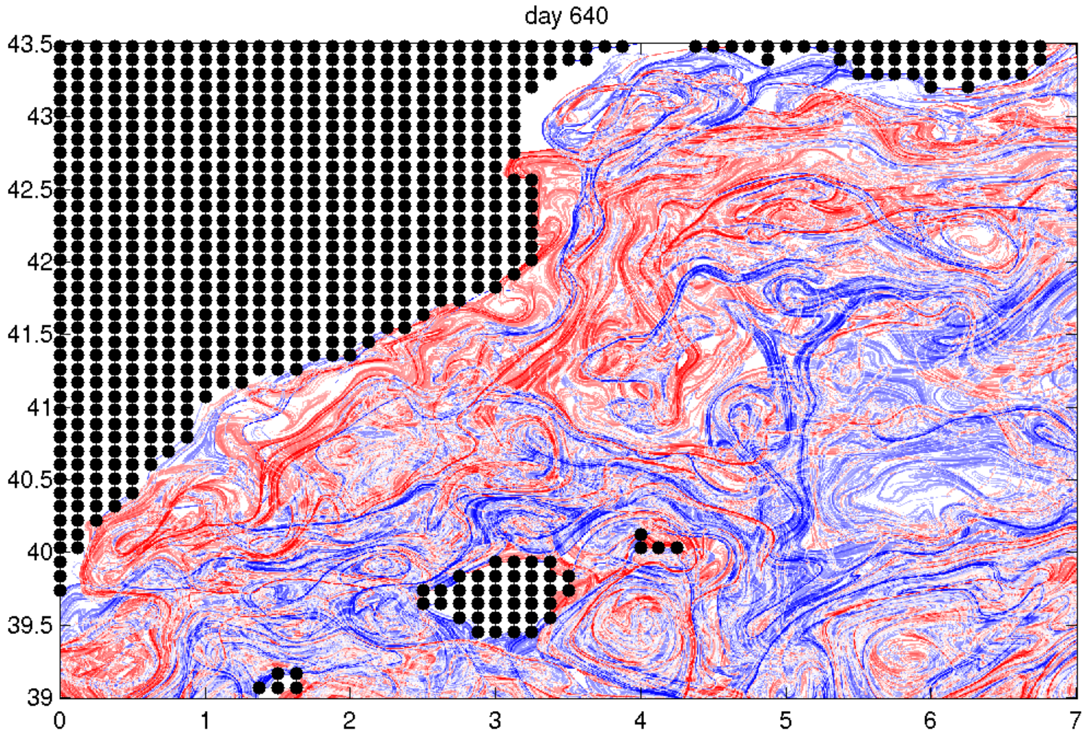


FSLE from forward and backwards integrations

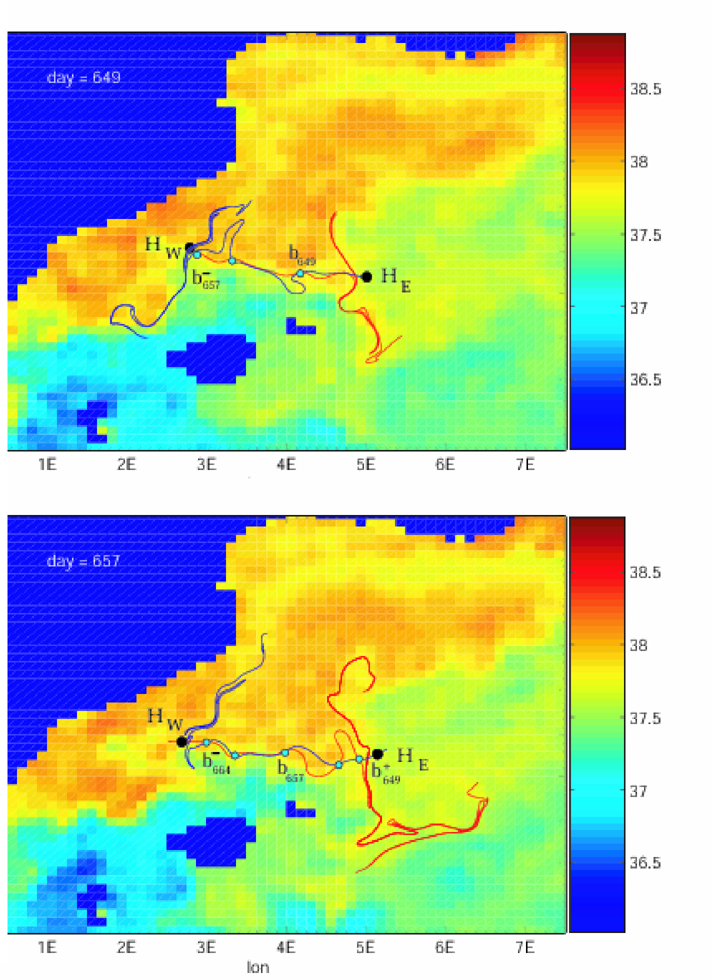


Click figures for movies

Mancho, Hernández-García, Small, Wiggins, Fernández, J. Phys. Ocean. 38, 1222 (2008)

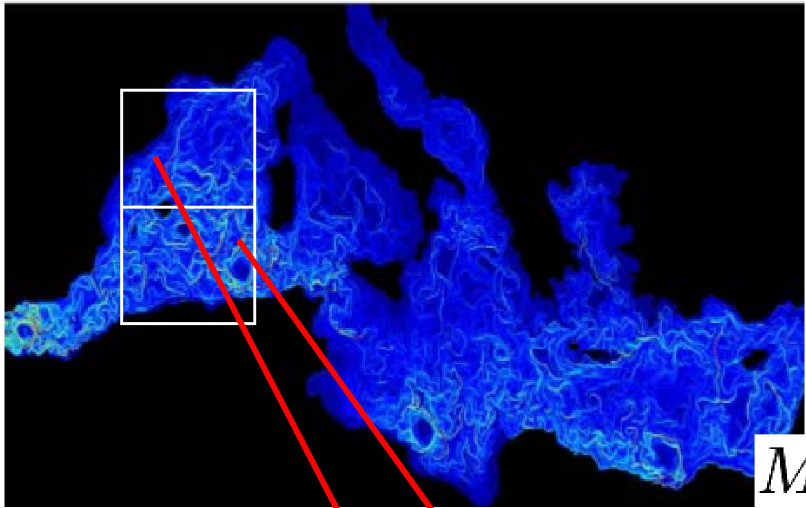


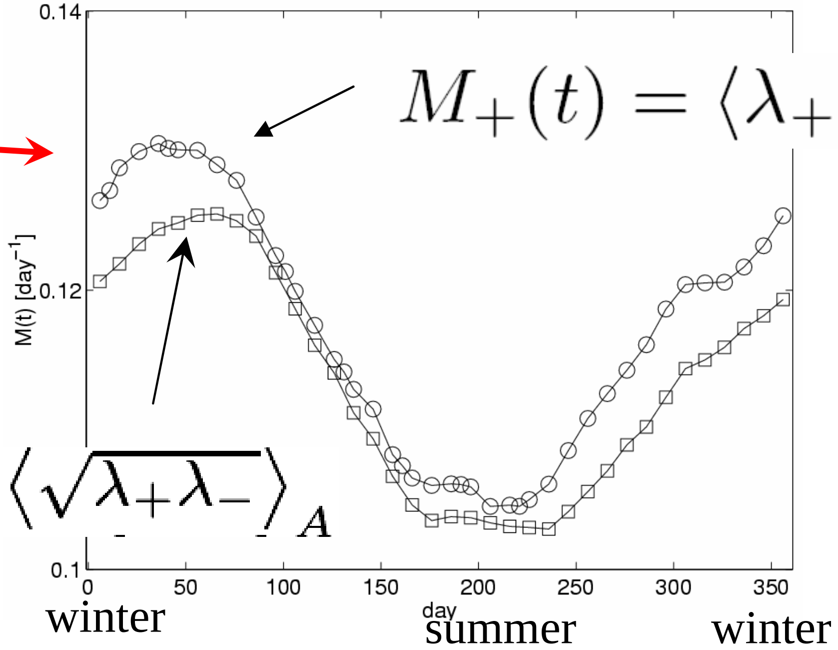
CLICK FOR MOVIE



High backwards Lyapunov values (unstable manifolds)

High forward Lyapunov values (stable manifolds)

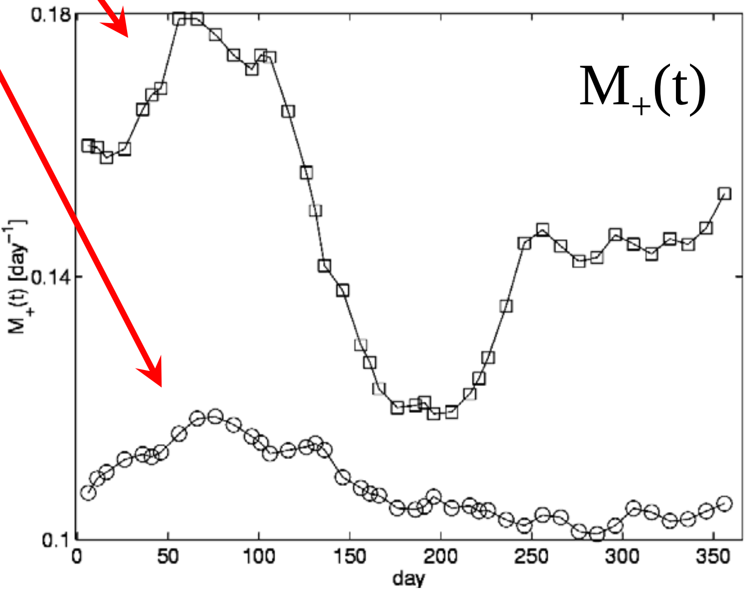


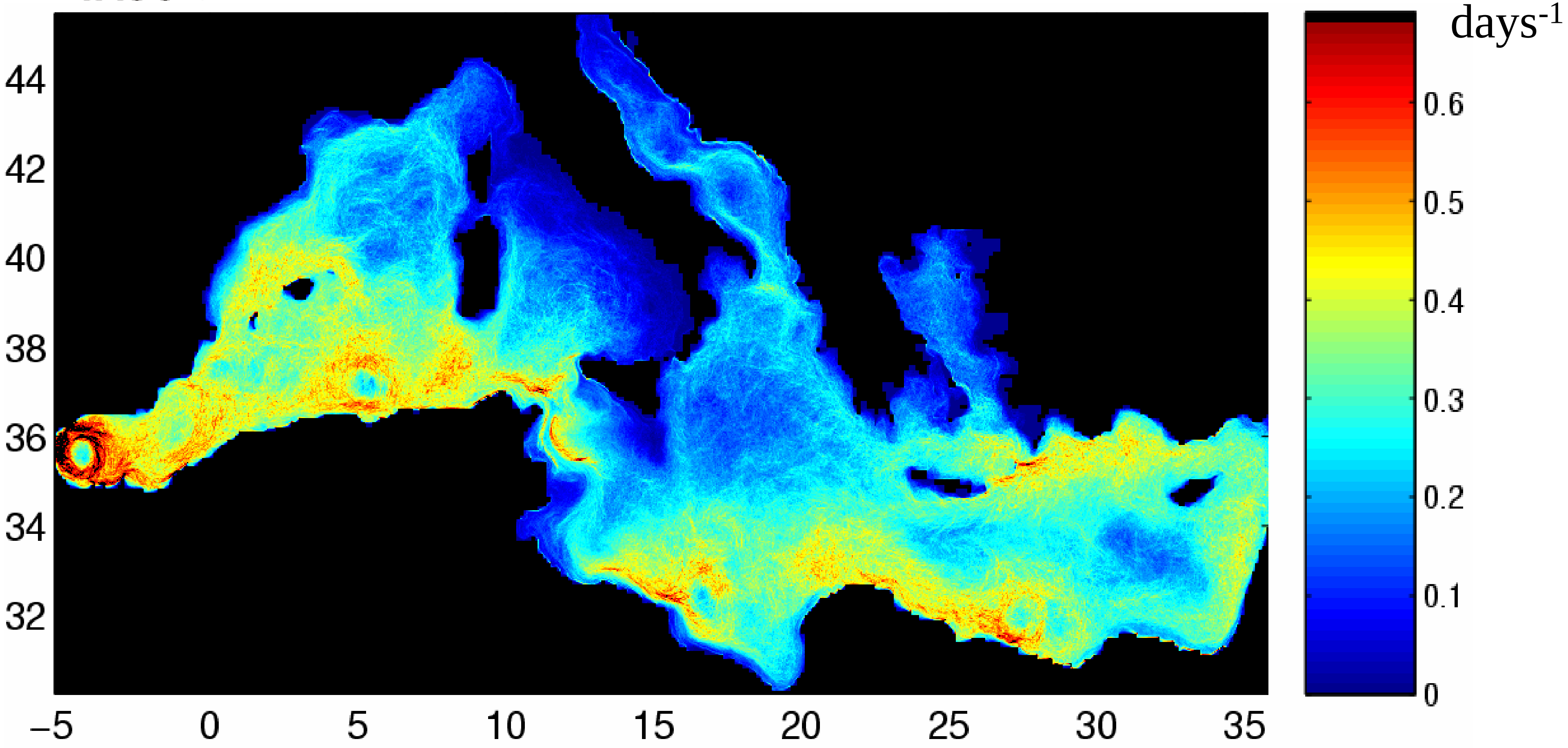


$$M_+(t) = \langle \lambda_+ \rangle_A$$

$$M_\pm(t) \equiv \langle \sqrt{\lambda_+ \lambda_-} \rangle_A$$

Mixing strenght
in different areas



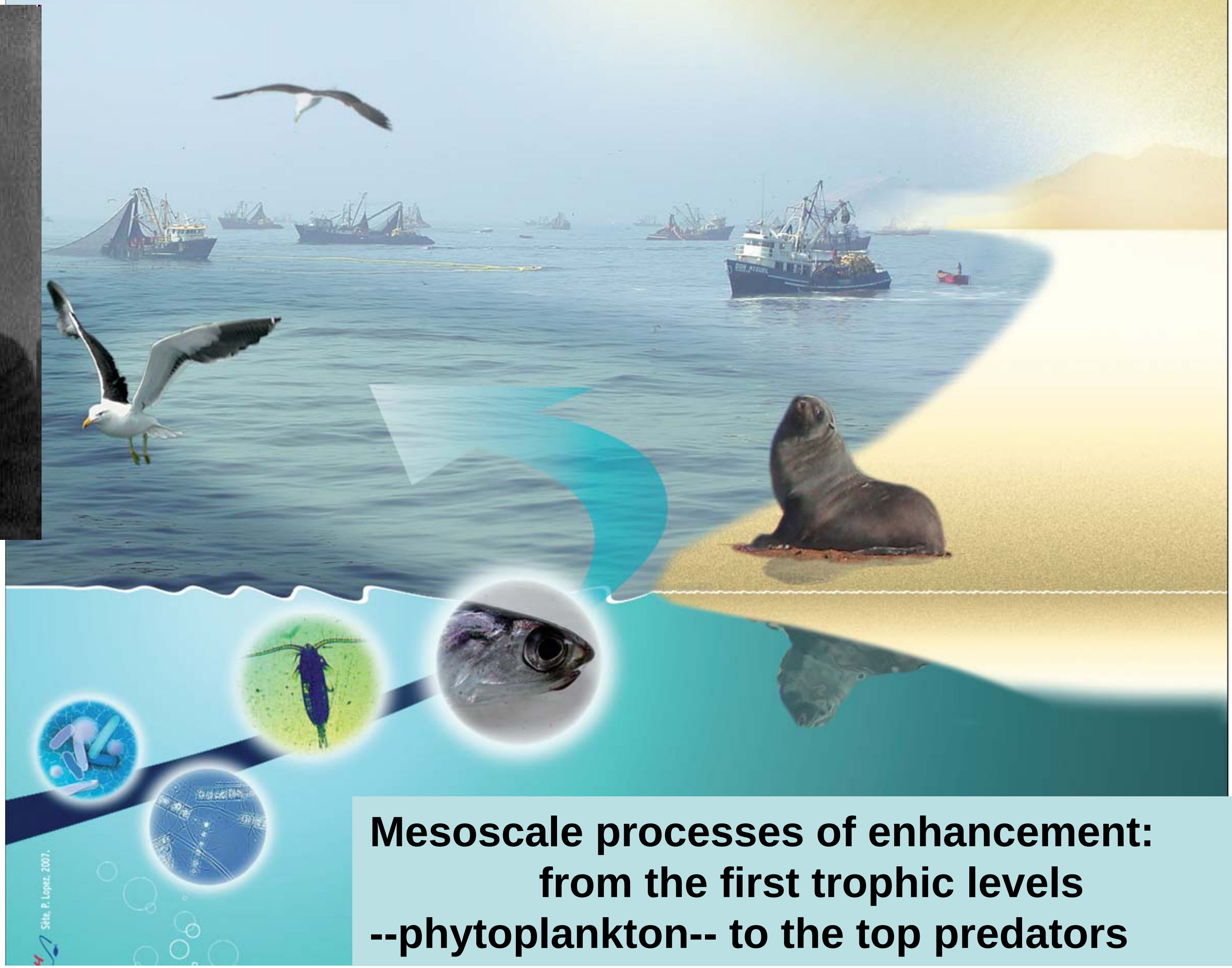


$$M_+(\mathbf{x}) = \langle \lambda_+ \rangle_t$$

Local mixing strength averaged over 5 years

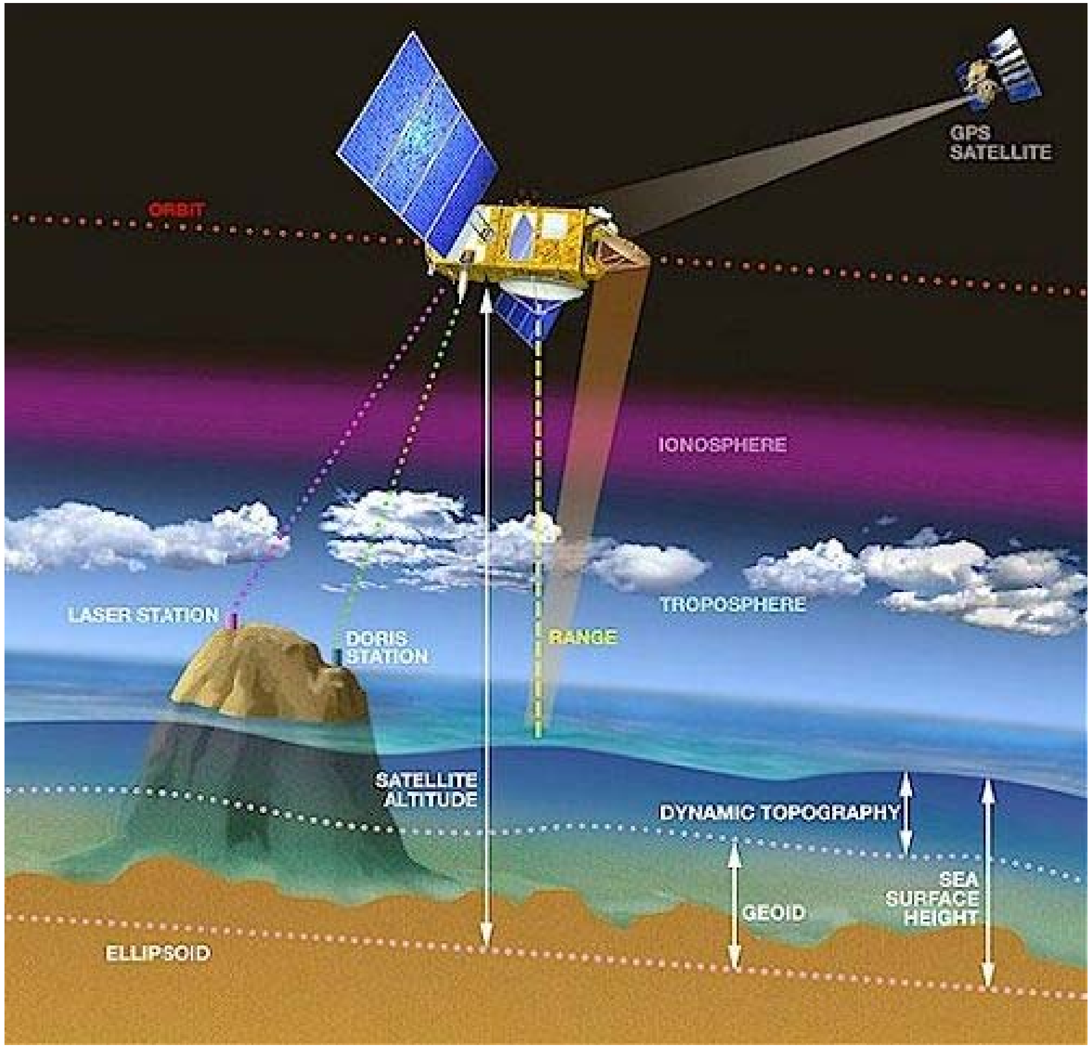
d'Ovidio, Fernández, Hernández-García, López, Geophysical Research Letters **31**, L17203 (1-4) (2004).

Describing biological process with Lyapunov eyes …



Mesoscale processes of enhancement: from the first trophic levels --phytoplankton-- to the top predators

SATELLITE ALTIMETRY FROM TOPEX/POSEIDON, ERS-2, JASON, ENVISAT, …



Dynamic Topography (DT)=
Sea Surface Heigh (SSH) – Geoid (G)

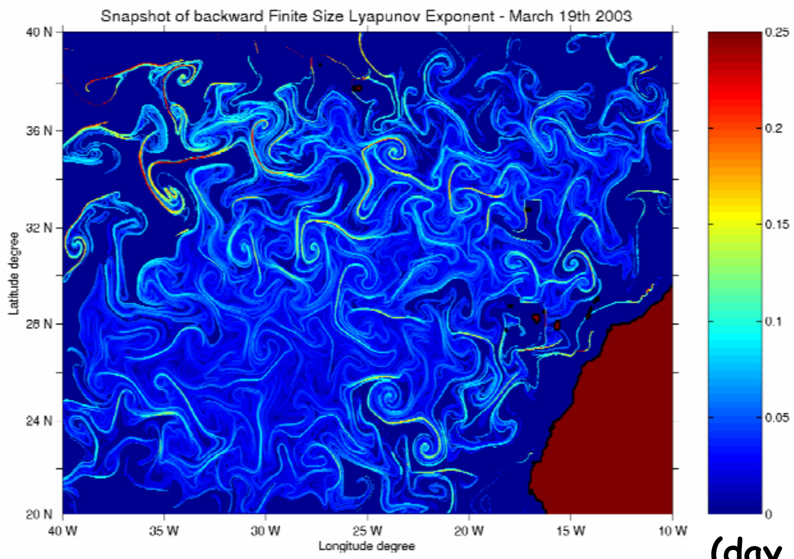
SSH ≈ 3 cm
G ≈ meters …

Sea Level Anomalies (SLA) =
SSH - $\langle SSH \rangle_t$ = DT - $\langle DT \rangle_t$

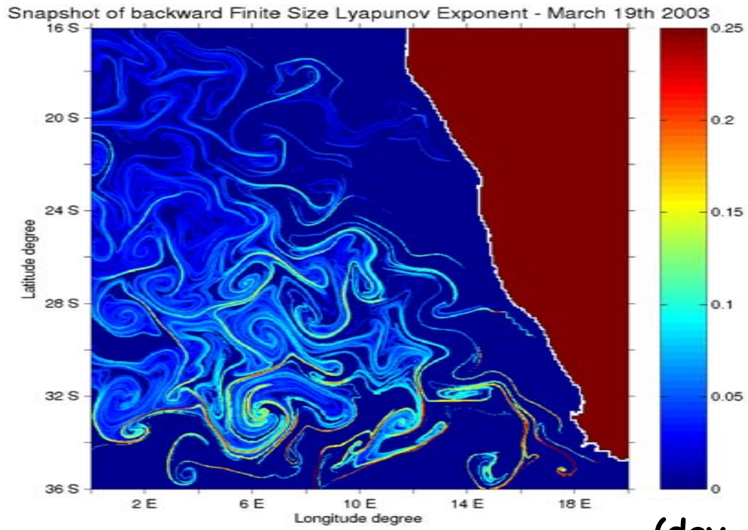
Dynamic topography determines, via the Colioris force, the velocity field (at large scales, geostrophic approximation)

FROM ALTIMETRY DATA

BENGUELA

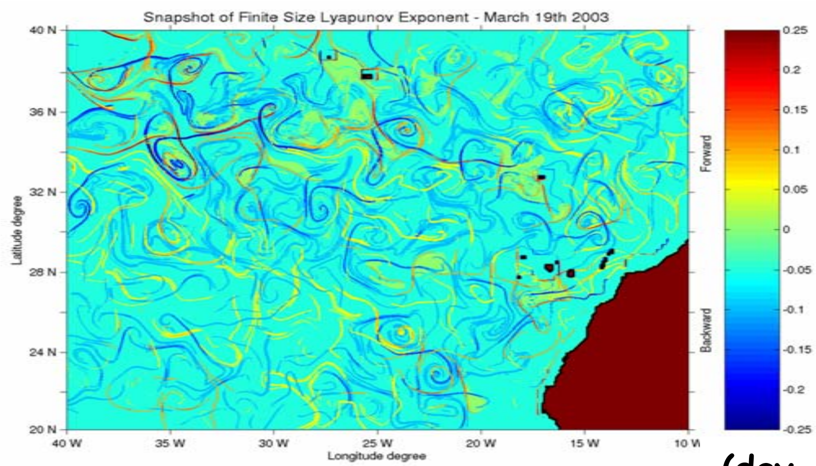


(day s^{-1})

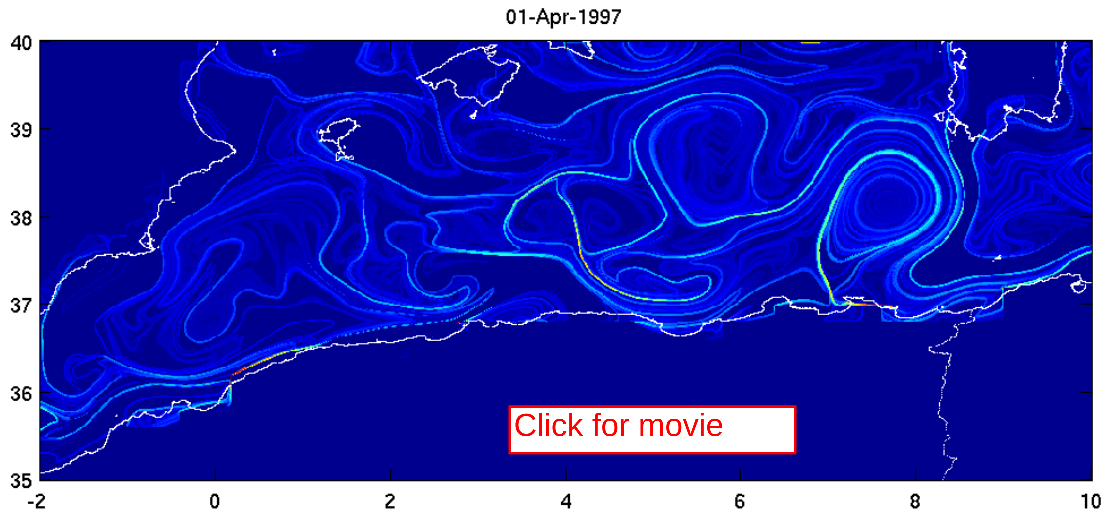


(day s^{-1})

March 19 2003 snapshots



(day s^{-1})



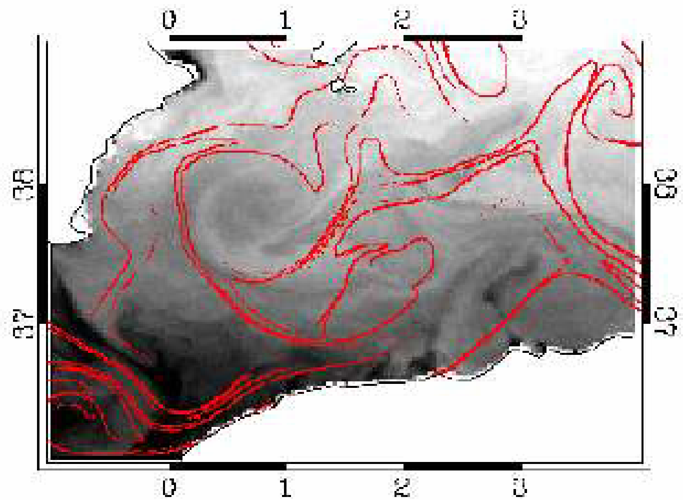
CANARY

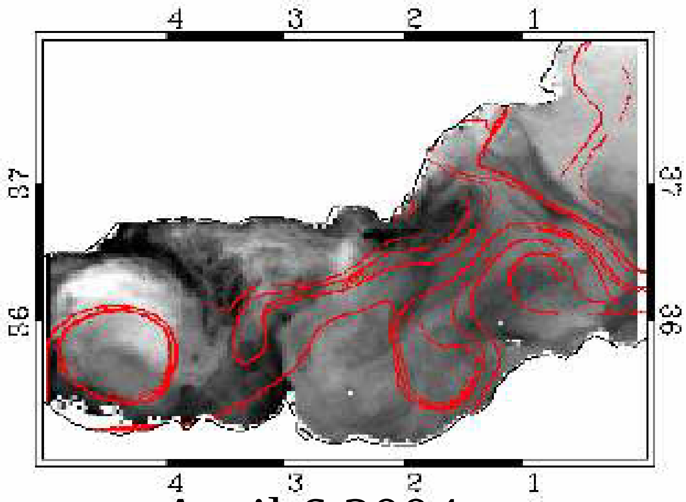
Note the presence of SUB-MESOSCALE detail

d'Ovidio et al. Deep-Sea Res. I 56, 15 (2009)
V. Rossi et al. Nonlin. Proc. Geophys. 16, 557 (2009)

Sea Surface Temperature vs lines of FSLE > 0.1 day^{-1} (LCSs)

July 9 2003

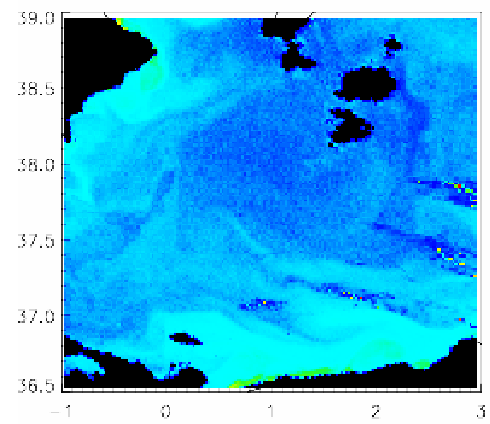




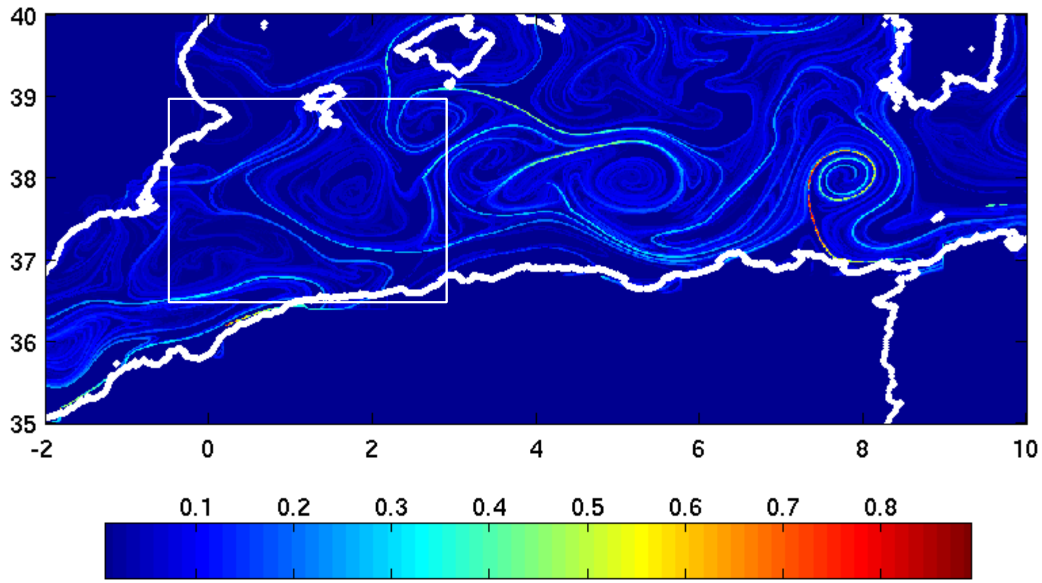
April 6 2004

d'Ovidio et al.
Deep-Sea Res. I (2009)

Chlorophyill

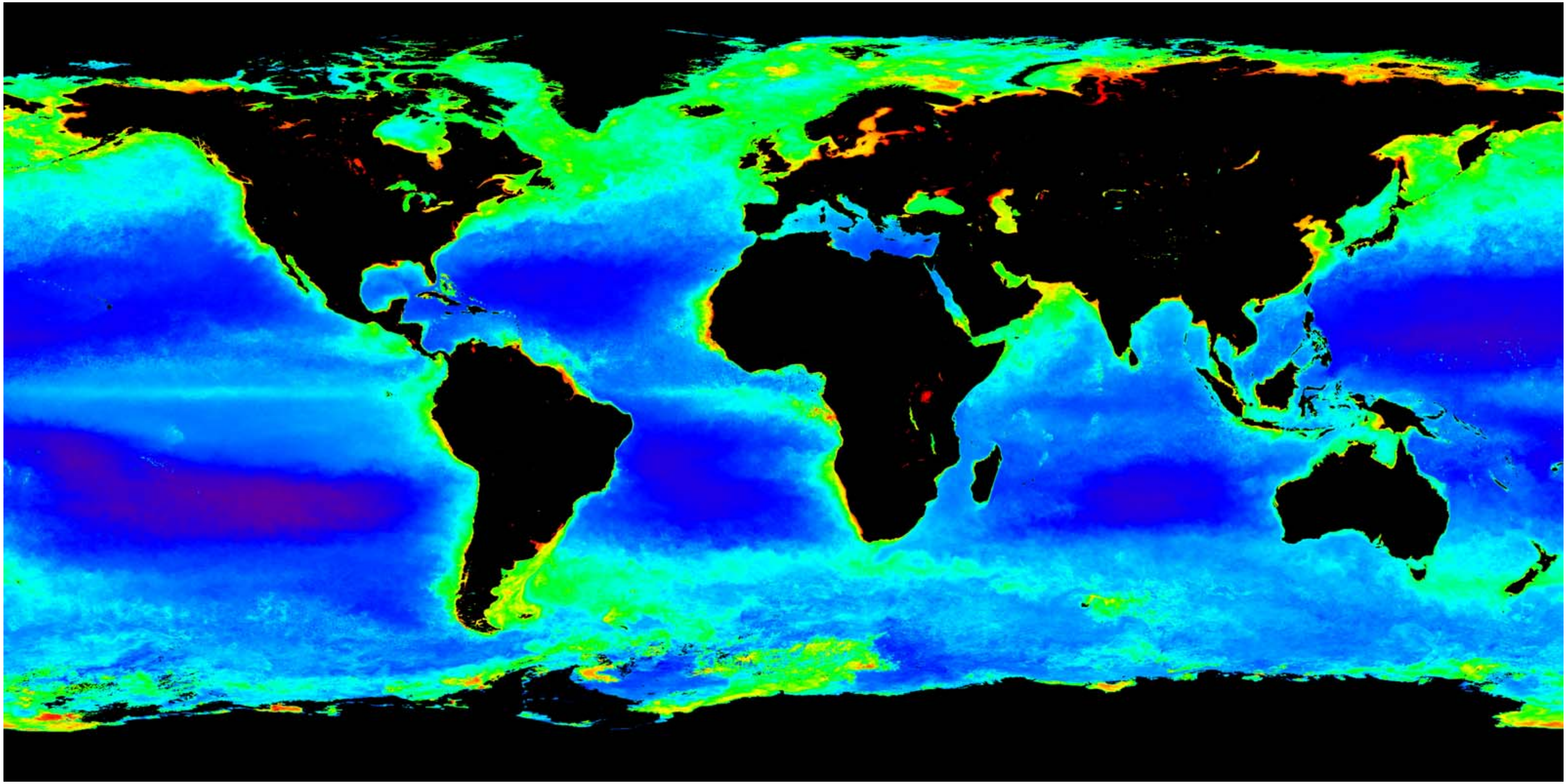


18 May 1998

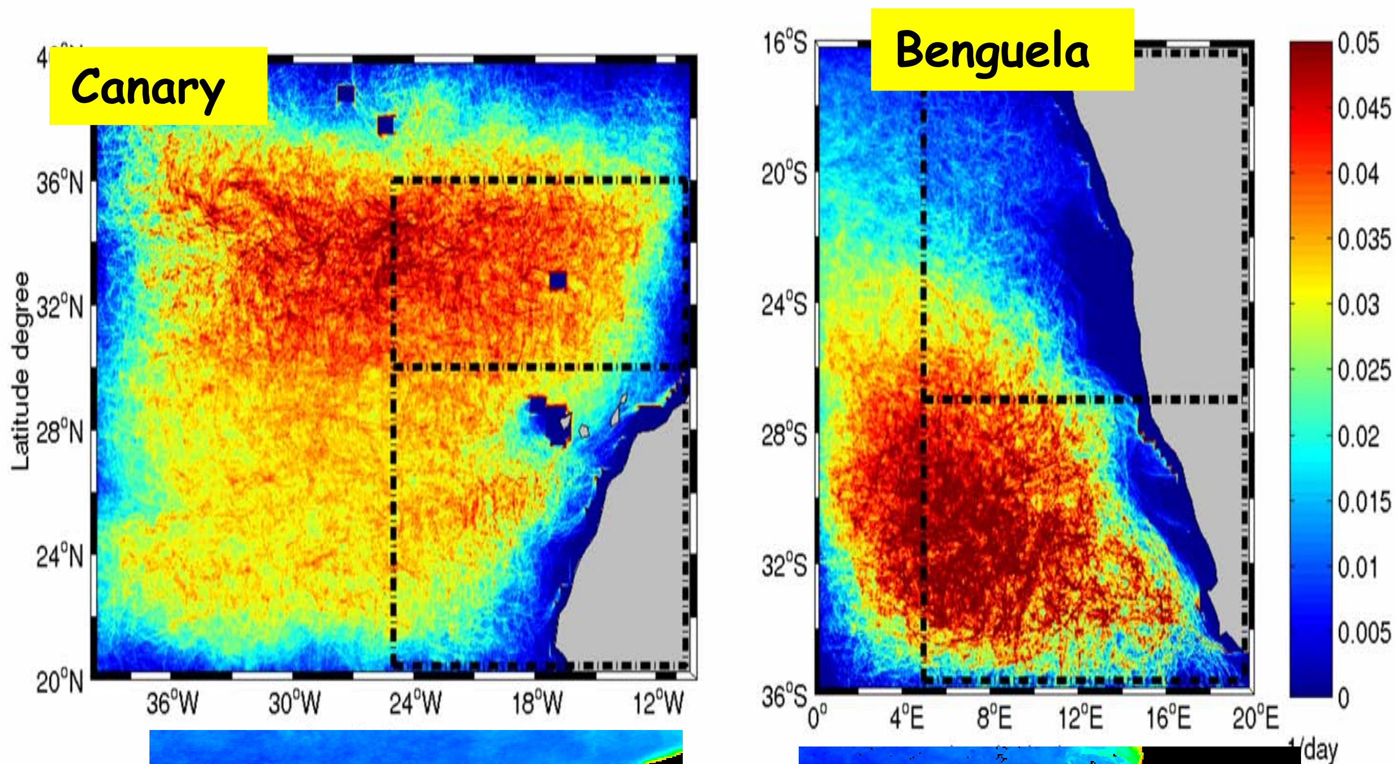


days^{-1}

Chlorophyll-a (≈ phytoplankton) from space

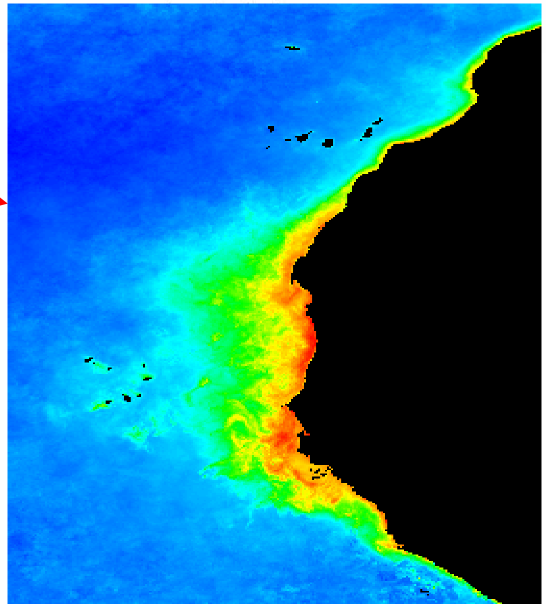


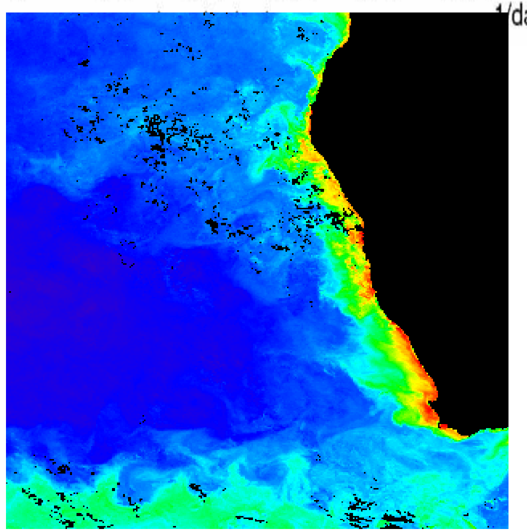
MODIS Image



Backward FSLE (λ-): Temporal average **(a measure of horizontal MIXING)** from June 2000 till June 2005

Phytoplankton and in the world major upwelling areas

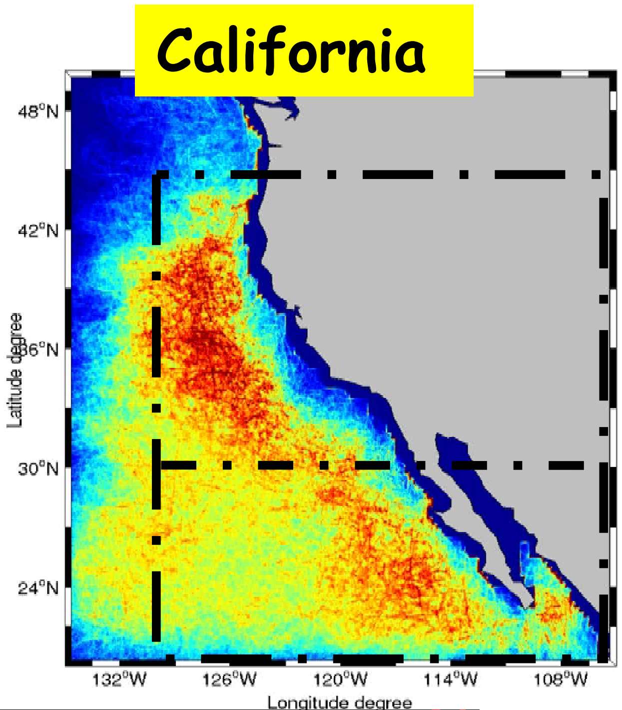


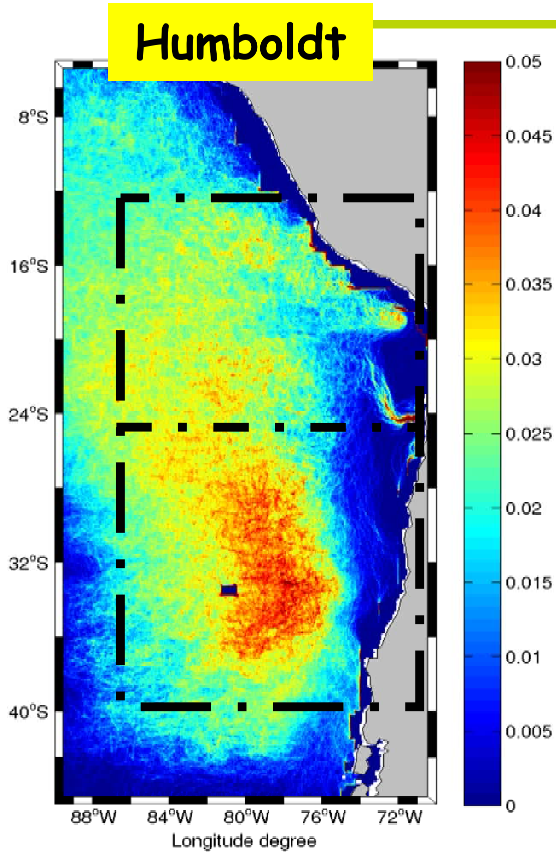


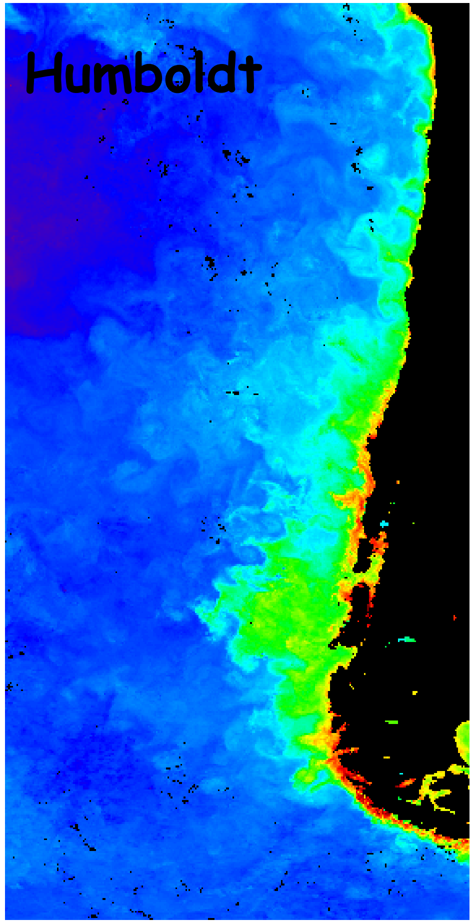
Rossi et al.,

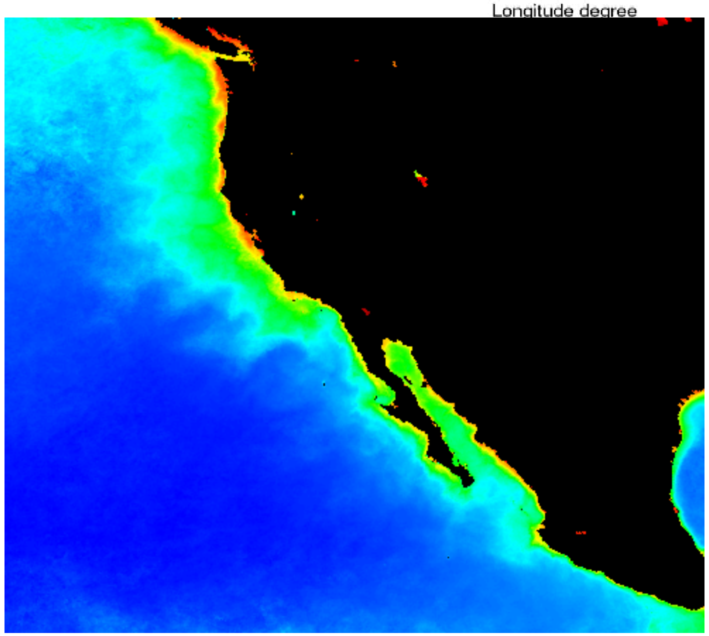
Geophys, Res. Lett. 2008

Nonlin. Proc. Geophys. 2009



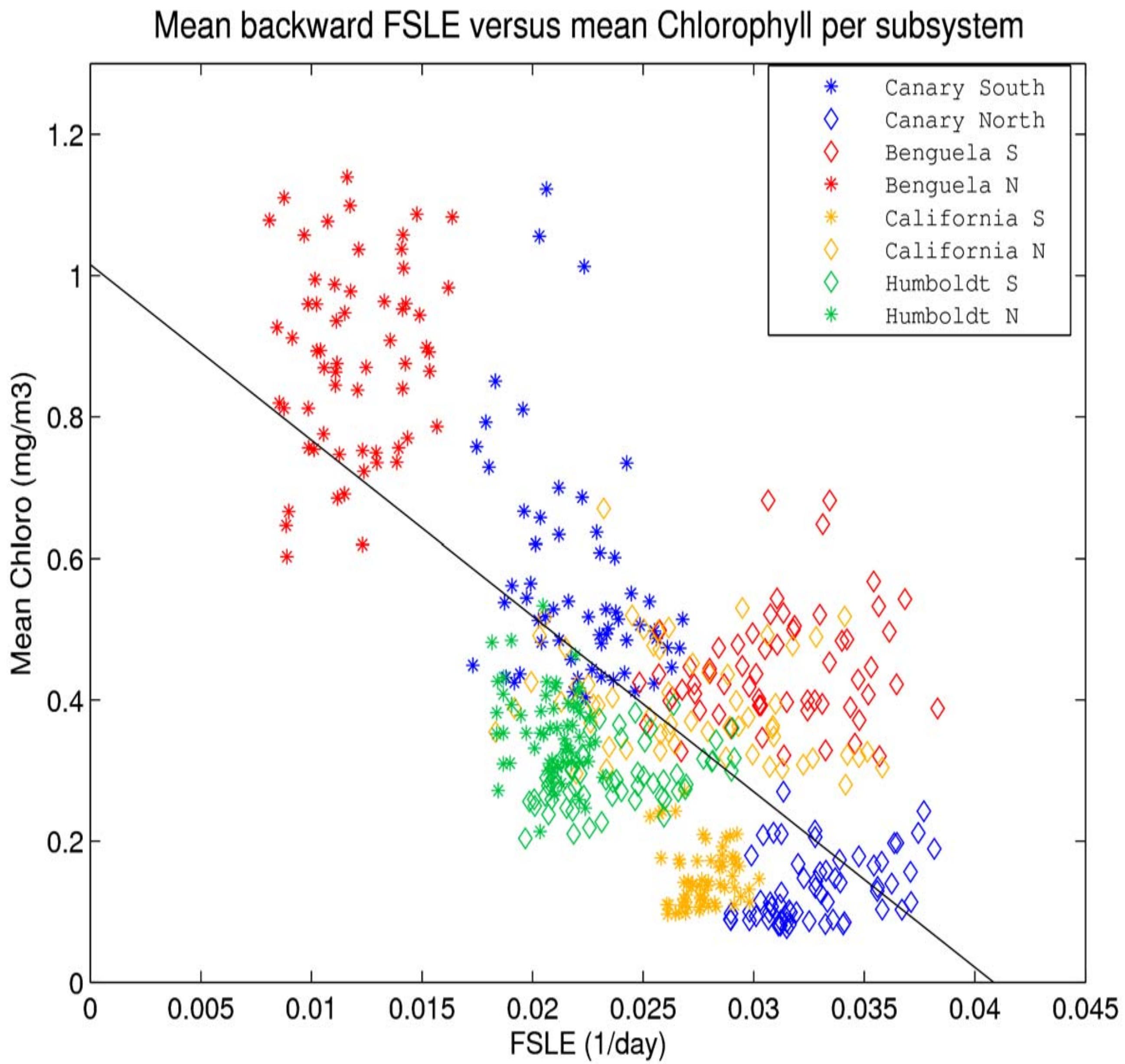






Backward FSLE (λ-):
Temporal average
(a measure of horizontal MIXING)
from June 2000 till June 2005

Rossi et al.,
Geophys, Res. Lett. 2008
Nonlin. Proc. Geophys. 2009

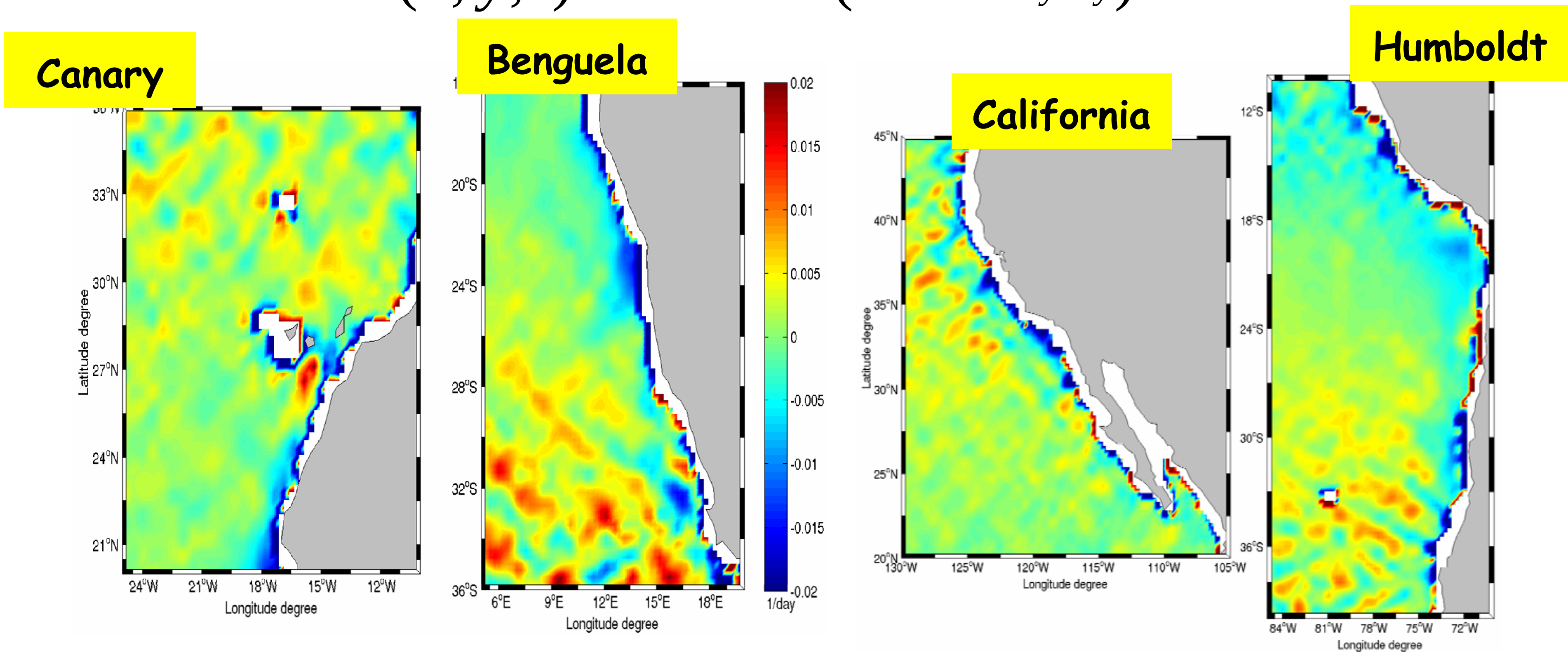


- Negative correlation
- Clustering
- Less turbulent systems are characterized by: LOW FSLE / HIGH CHLOROPHYLL.
- Most turbulent systems: HIGH FSLE / LOW CHLOROPHYLL.

Opposite to behavior seen in less enriched systems

Temporal averages of vertical velocities from incompressibility condition

$$\Delta(x, y, t) \equiv \partial_z V_z = -(\partial_x V_x + \partial_y V_y)$$



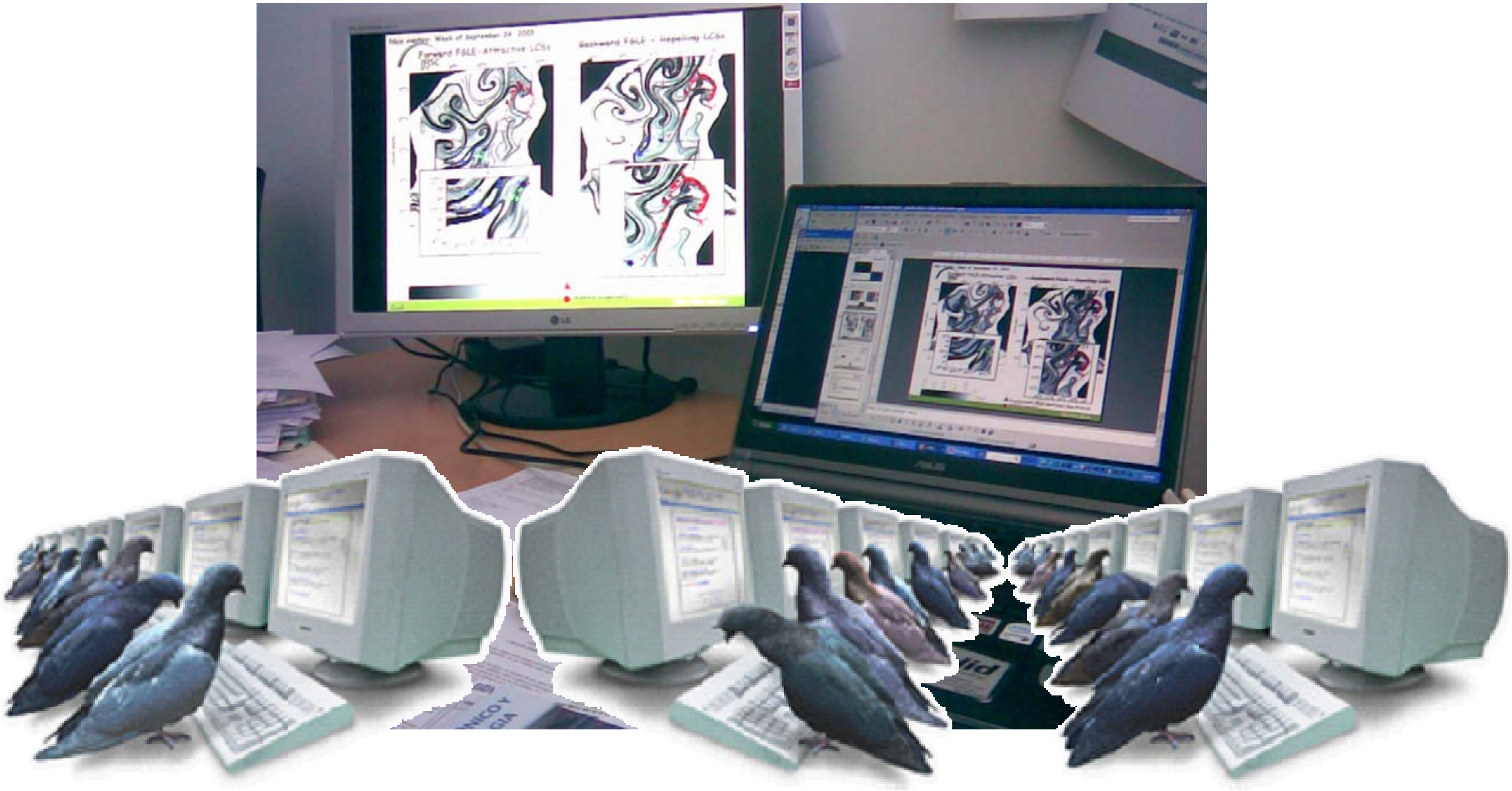
• Dominance of (small) upwelling vertical velocities in the less turbulent subsystem.

• Thus, probably the influence of horizontal stirring on plankton is only indirect: need to understand the 3d flow structure: high FSLE associated to low Eckman transport.

Rossi et al., Geophys, Res. Lett. 2008, Nonlin. Proc. Geophys. 2009

- Lagrangian Coherent Structures give the skeleton of horizontal transport
- This certainly influences abiotic quantities: temperature, nutrients, …
- This certainly influences plankton distribution
- From there, impact is expected in plankton consumers, their predators, … cascades up along the food chain …

Do birds know about FSLE calculations?

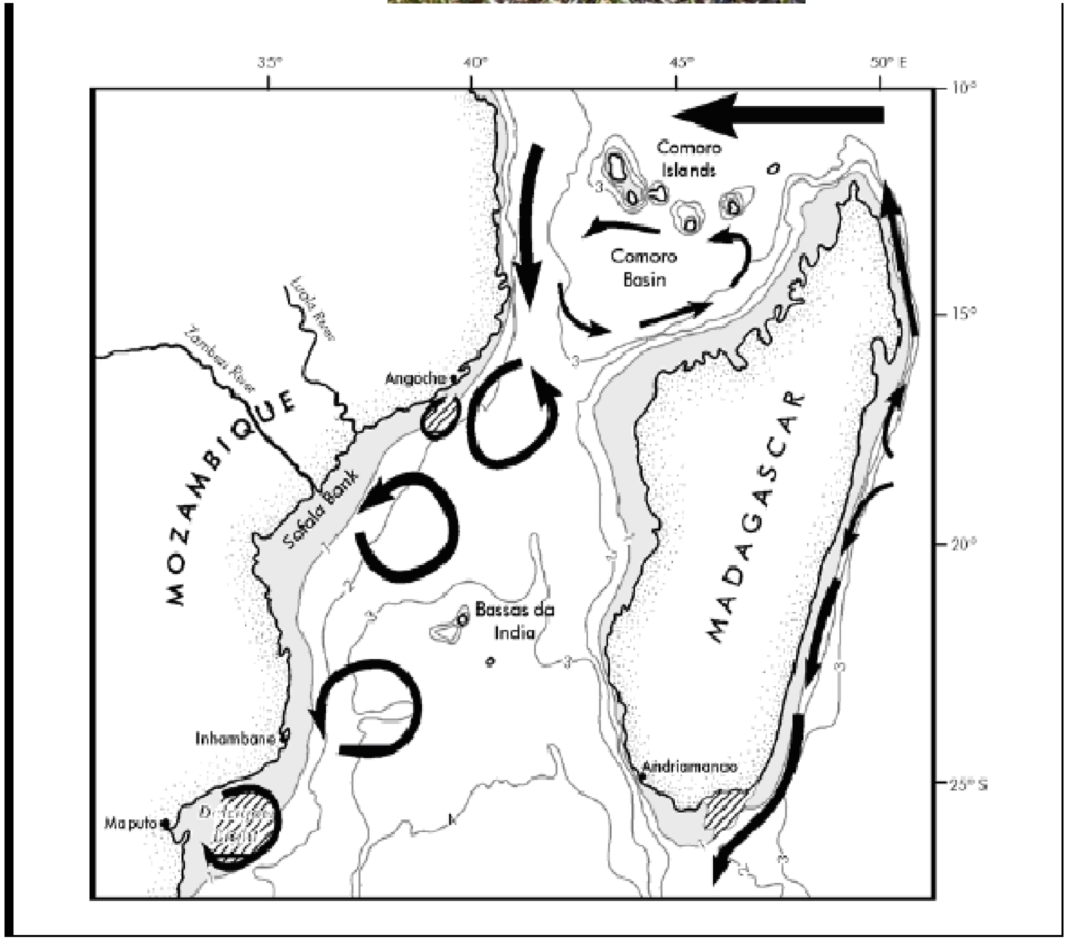


Tew Kai, Rossi, Sudre, Weimerskirch, Lopez, Hernandez-Garcia, Marsac, Garçon, PNAS 106, 8245 (2009)

FRIGATEBIRDS in the MOZAMBIQUE CHANNEL

Particular topography (channel/islands) linked with strong mesoscale activity:

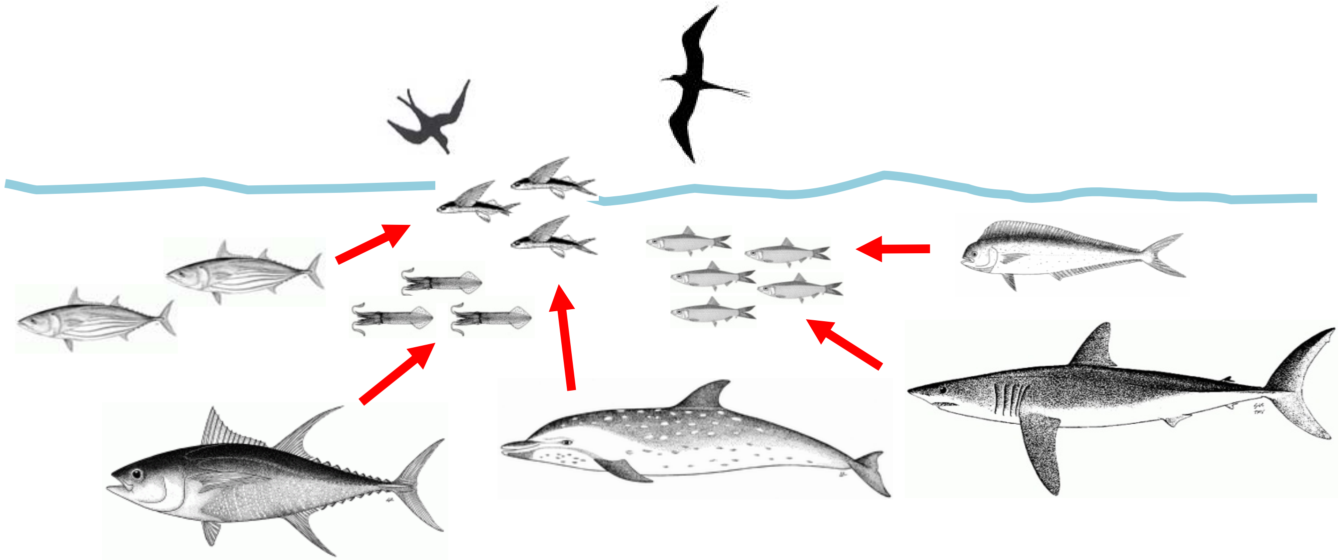
- Large anticyclonic cell at the north
- Local upwellings
- Anticyclonic and cyclonic mesoscale eddies moving southward permanently.



(De *Ruijter et al., 2004*)

Great frigatebird (*fregata minor*):

- Large seabirds (light weight < 5 kg and large wings > 2m). Use thermals to soar before gliding over long distances and time (days/nights over weeks).
- Traveling at high altitudes to locate patches of prey and come close to surface to feed (reduced flight speed indicates foraging).
- Feeding occurs only during daytime (peaks in the morning and evening).
- Unable to dive or rest on the water surface (permeable plumage) → in association with subsurface predators (tuna, …): **fisheries indicators**



Weimerskirch et al. 2004; Jaquemet et al. 2005

8 birds (from Europa Island community) fitted with satellite transmitter and altimeter.

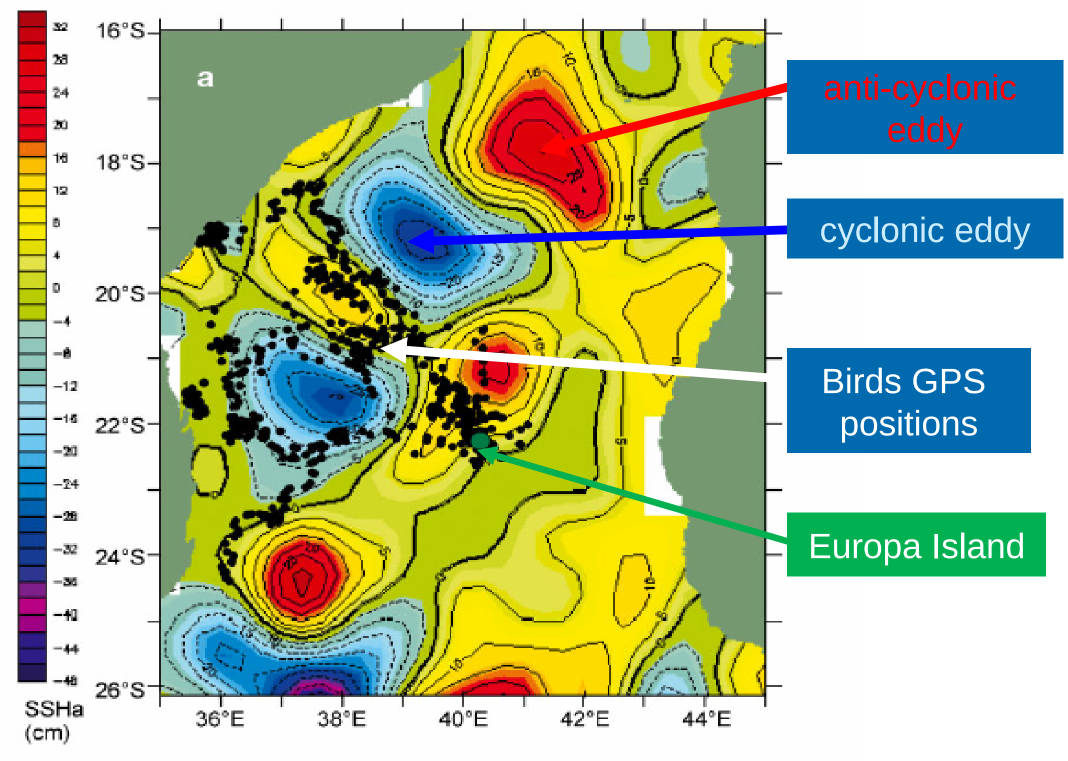
Followed for their foraging trips from August 18 to September 30, 2003.

1600 Argos from 50 trips positions, distributed into 17 long trips (> 614 km) and 33 short trips.

(Weimerskirch et al., 2004)

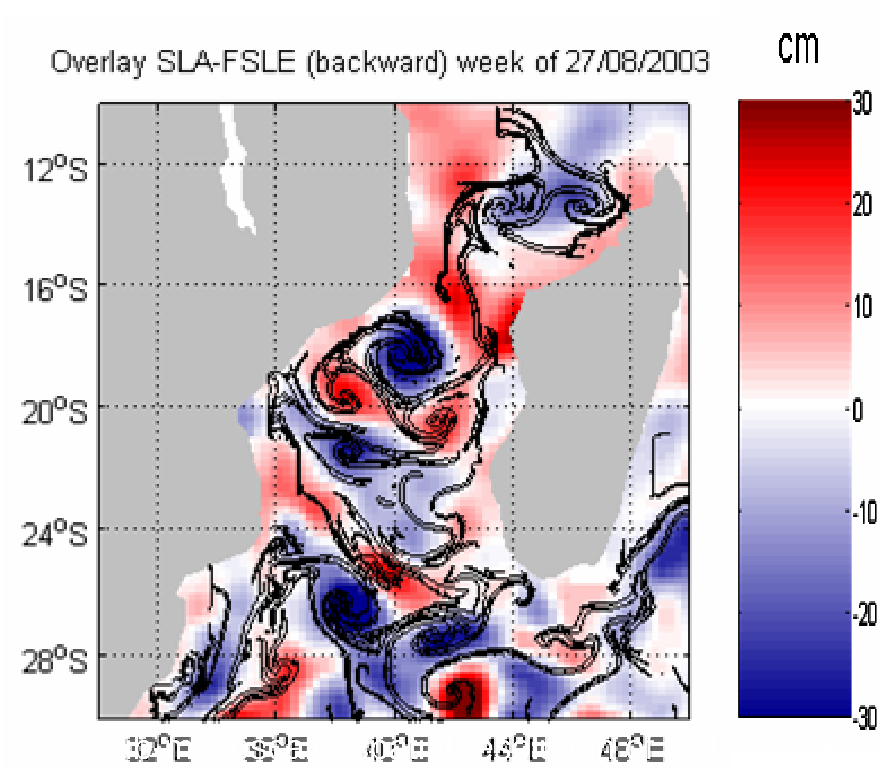
Satellite transmitter and altimeter (total weight : 1 to 3% mass of adults, max 45g)

SSH (cm): Eulerian view



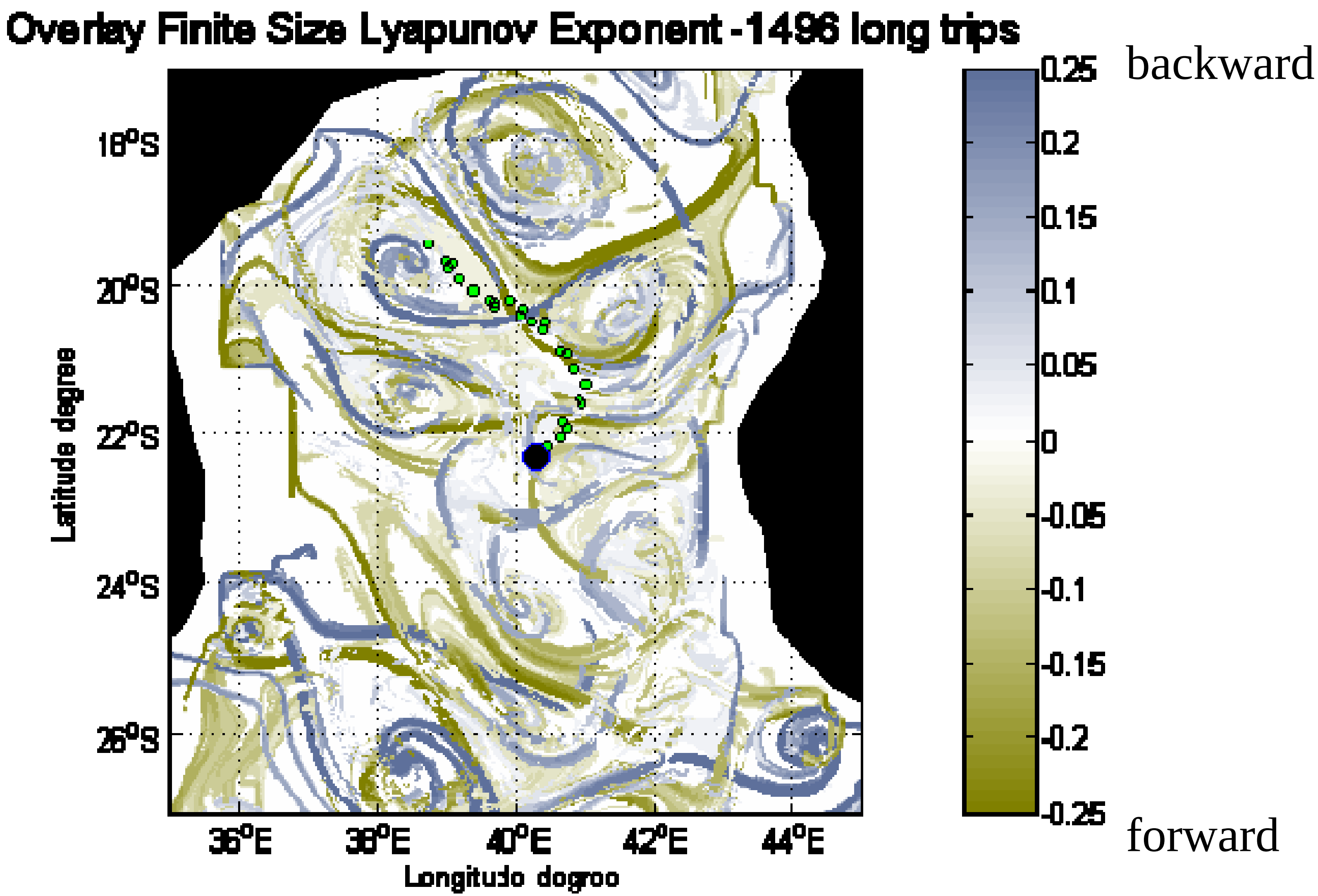
Weimerskirch et al,2004

Lagrangian FSLEs versus SSH

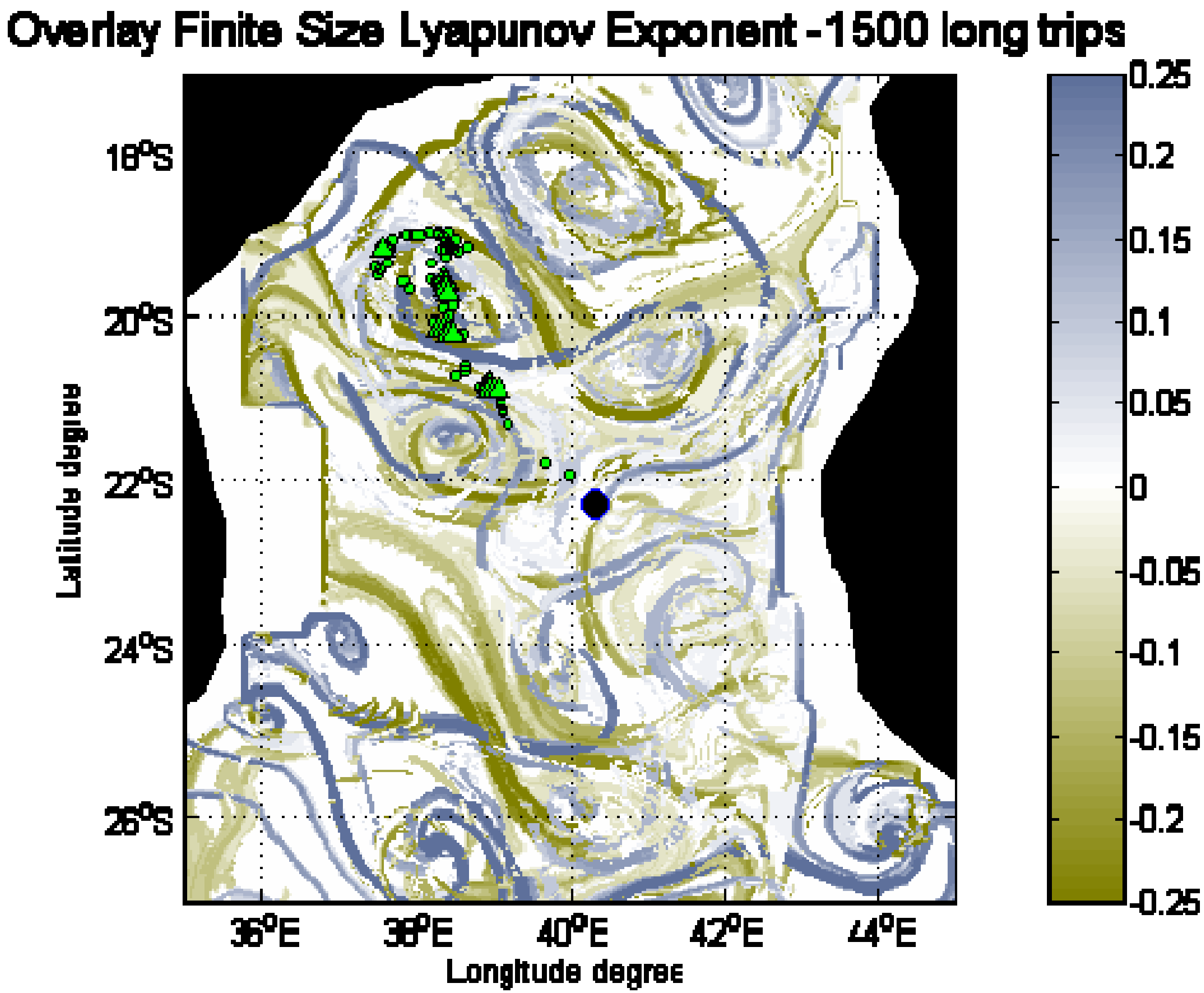


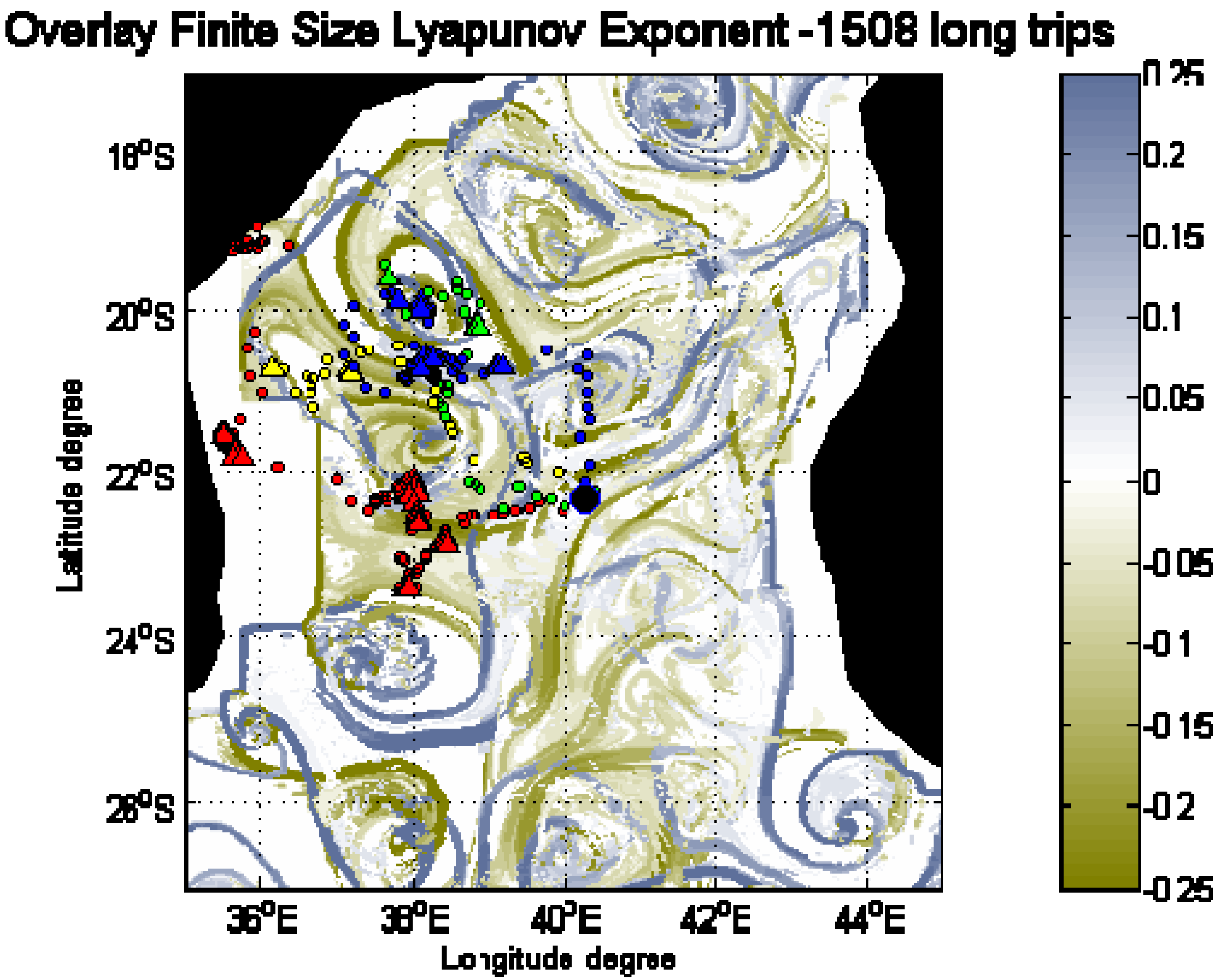
The Lagrangian FSLE gives access to submesoscale structures

Lagrangian Coherent Structures: **|FSLE| > 0.1 day^{-1}**

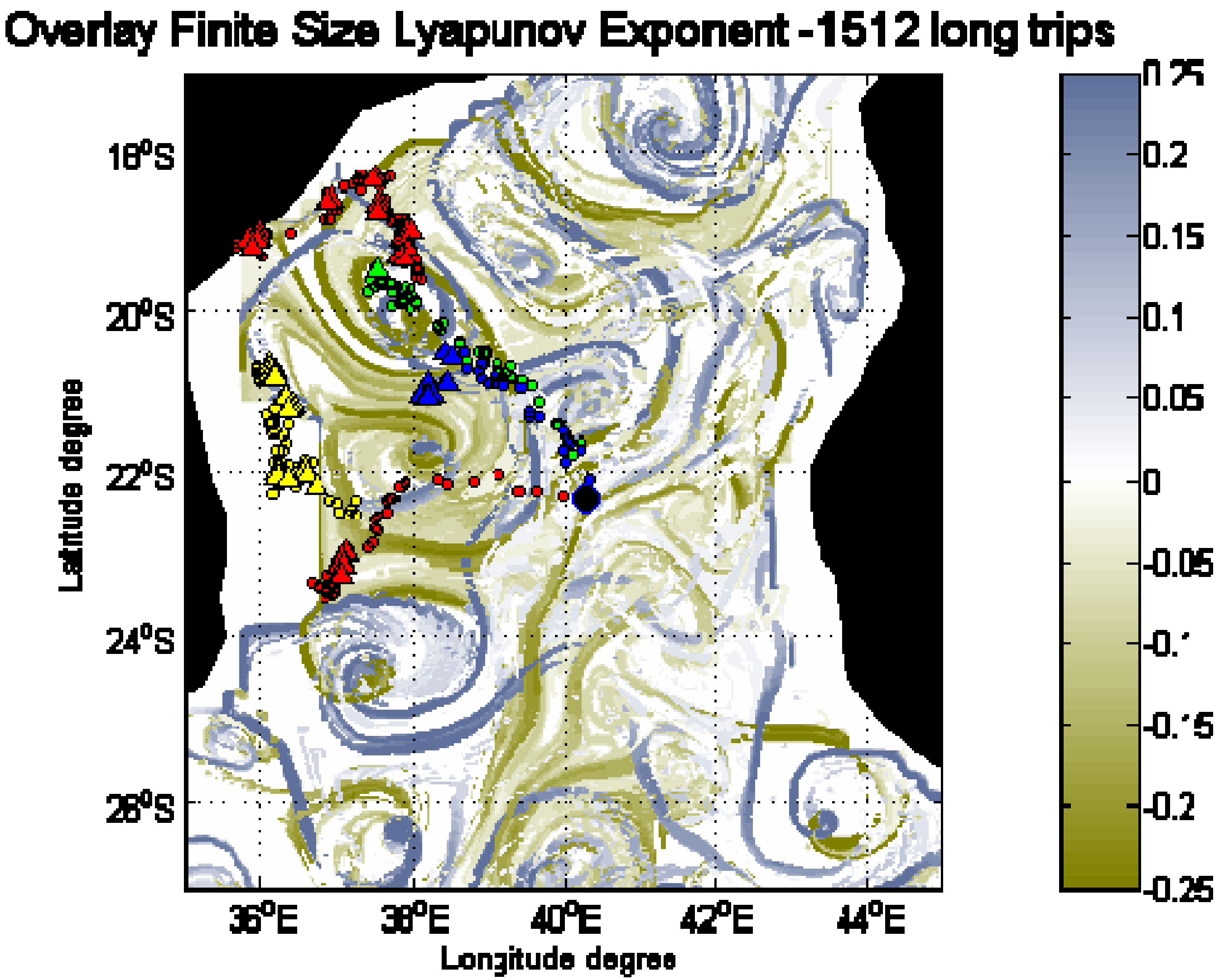


Overlay Finite Size Lyapunov Exponent -1496 long trips
18°S
20°S
22°S
24°S
26°S
36°E
38°E
40°E
42°E
44°E
Longitude degree
Latitude degree
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
backward
forward


Overlay Finite Size Lyapunov Exponent -1500 long trips
18°S
20°S
22°S
24°S
26°S
36°E
38°E
40°E
42°E
44°E
Longitude degree
Latitude degree
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25



Overlay Finite Size Lyapunov Exponent -1508 long trips
18°S
20°S
22°S
24°S
26°S
36°E
38°E
40°E
42°E
44°E
Longitude degree
Latitude degree
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25


Overlay Finite Size Lyapunov Exponent -1512 long trips
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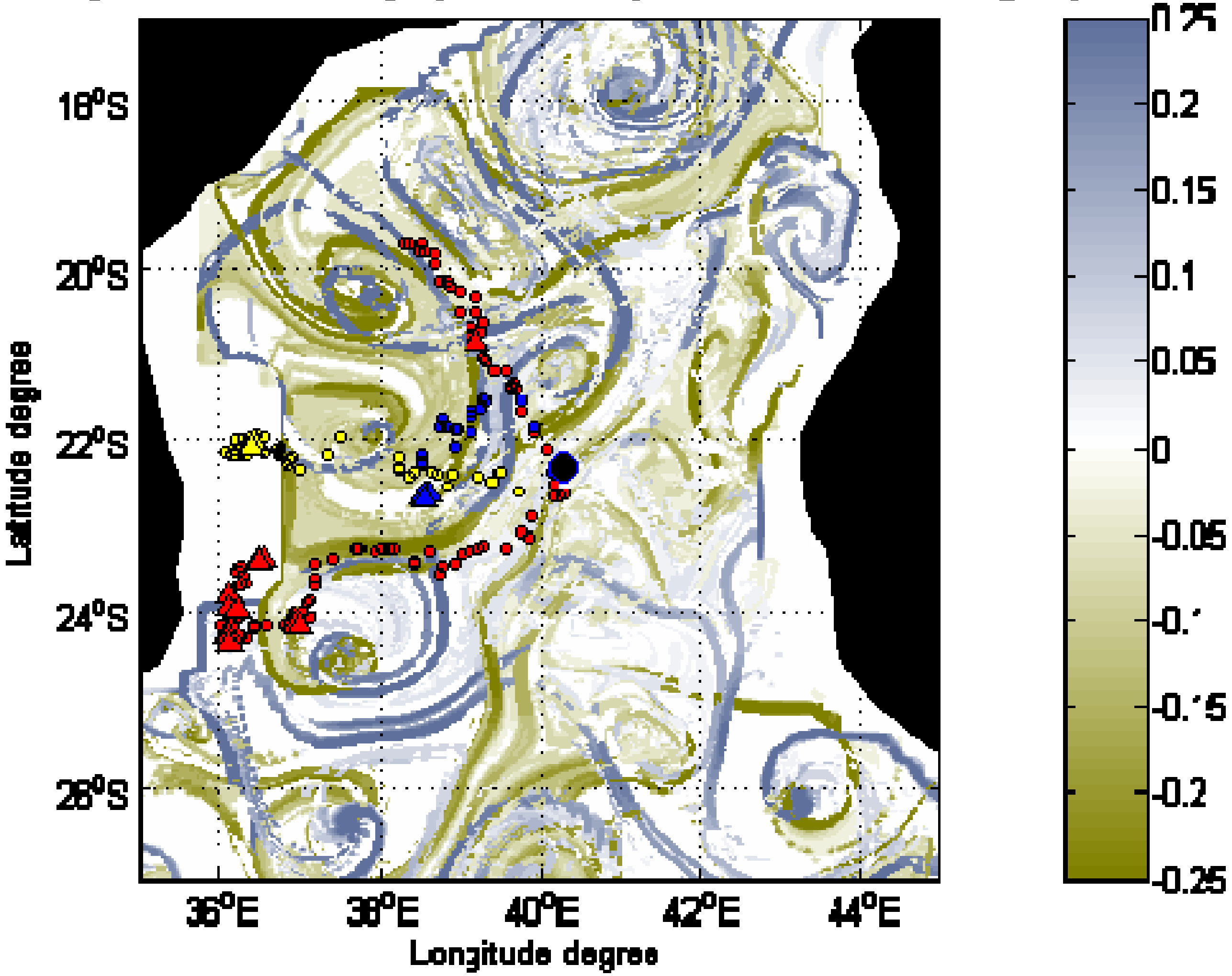
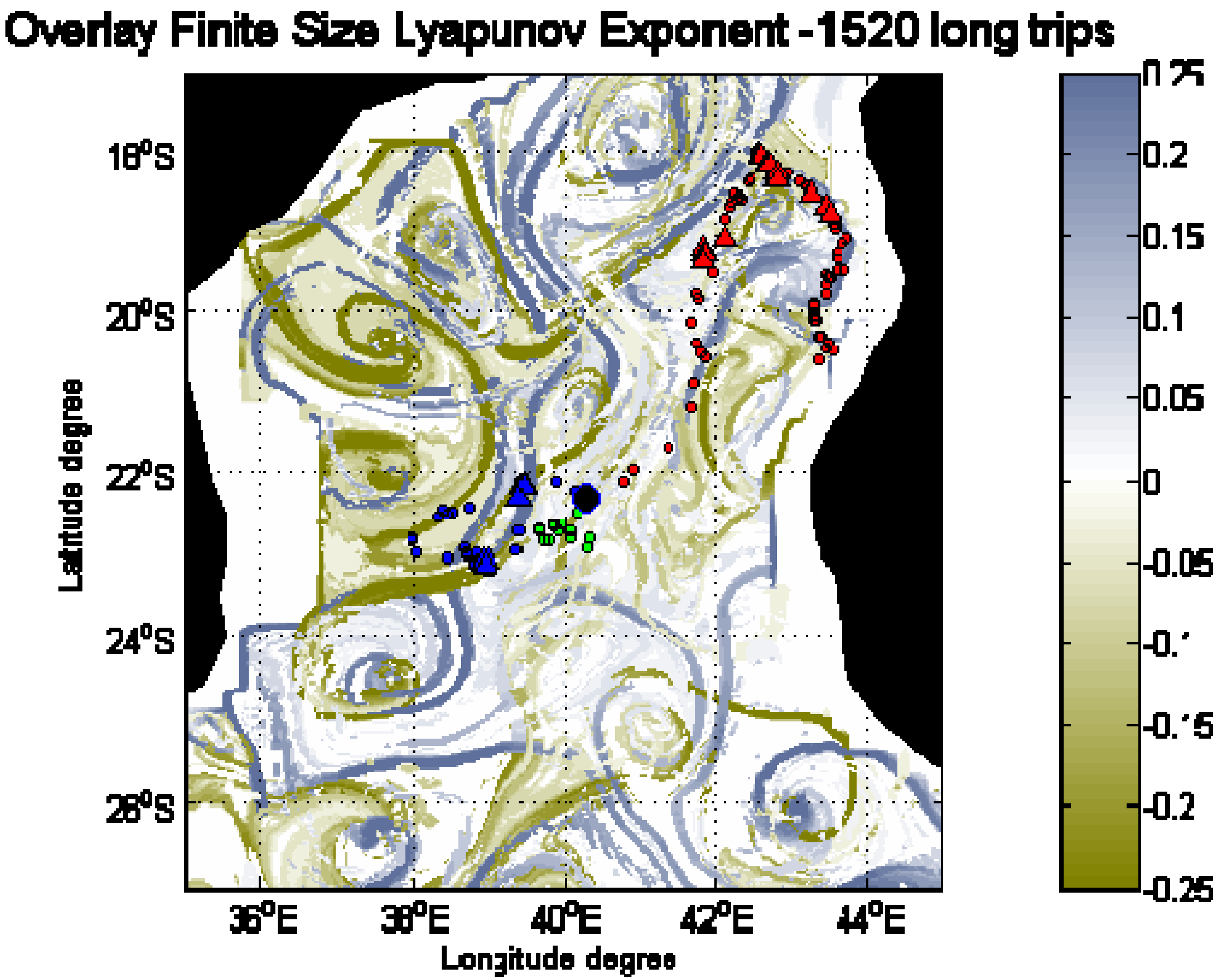
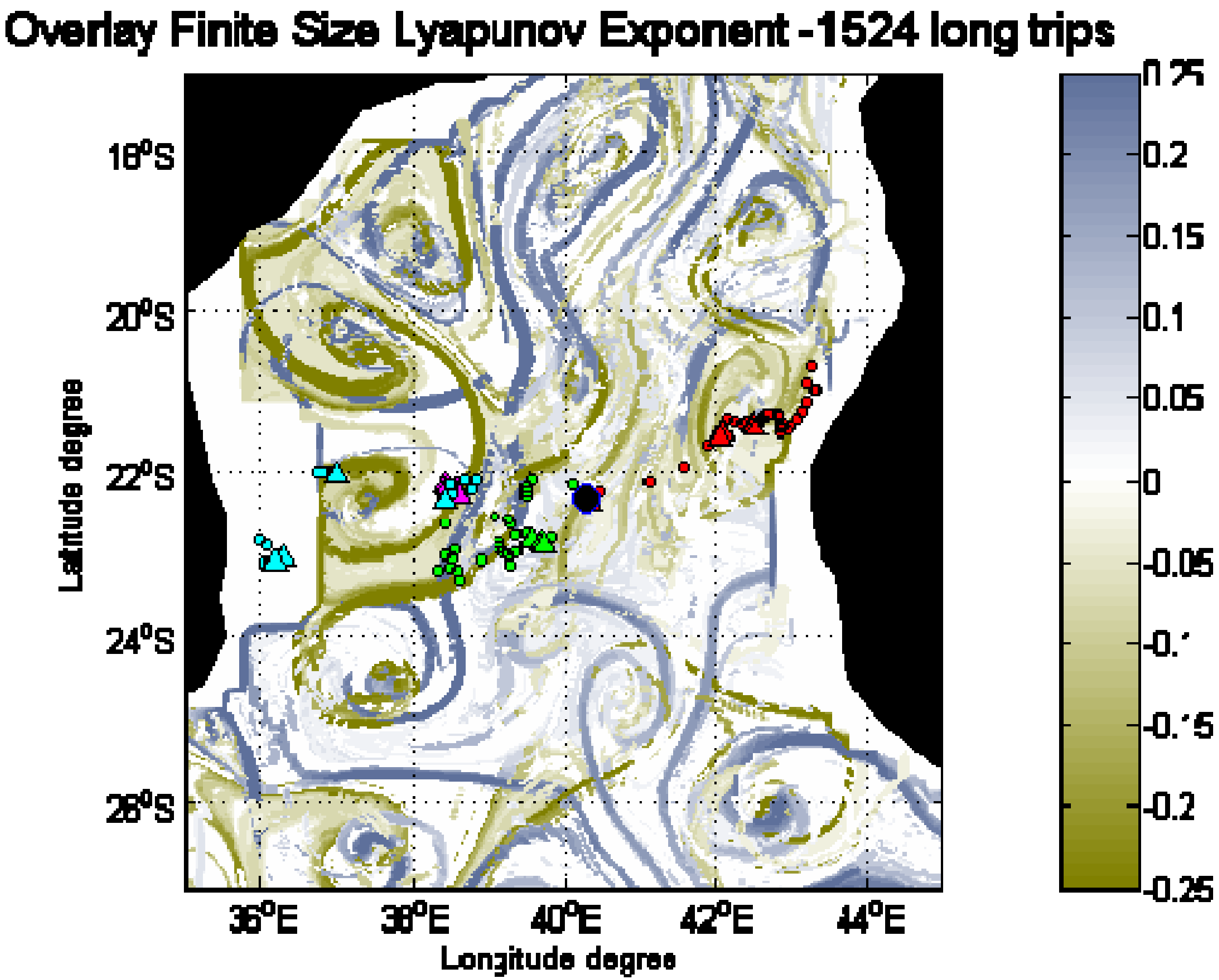
20°S
22°S
24°S
26°S
36°E
38°E
40°E
42°E
44°E
Longitude degree
Latitude degree
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25

Overlay Finite Size Lyapunov Exponent -1516 long trips




Overlay Finite Size Lyapunov Exponent -1520 long trips
18°S
20°S
22°S
24°S
26°S
36°E
38°E
40°E
42°E
44°E
Longitude degree
Latitude degree
0.2
0.15
0.1
0.05
0
-0.05
-0.2
-0.25


Overlay Finite Size Lyapunov Exponent -1524 long trips
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20°S
22°S
24°S
26°S
Latitude degree
36°E
38°E
40°E
42°E
44°E
Longitude degree
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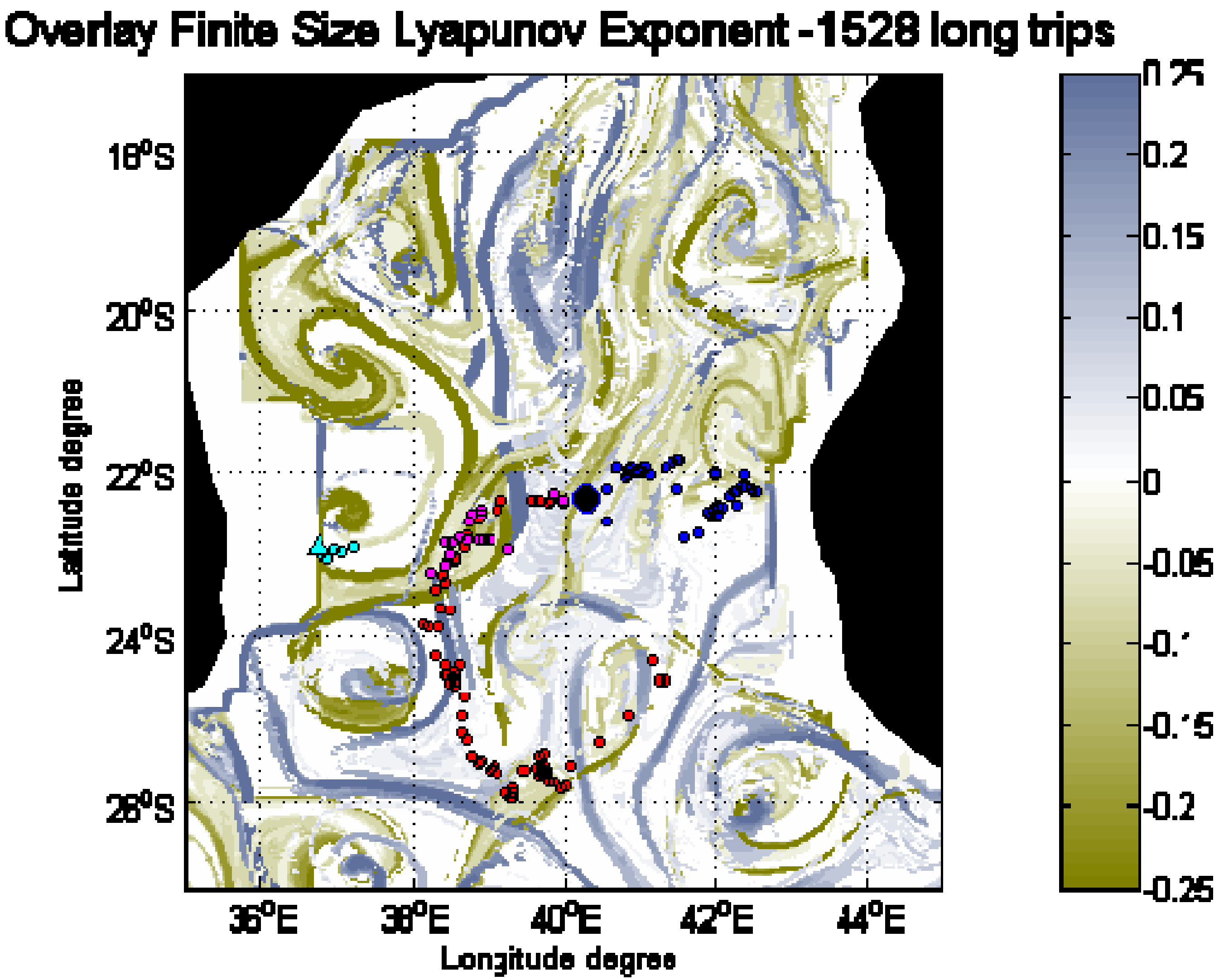
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25


Overlay Finite Size Lyapunov Exponent -1528 long trips
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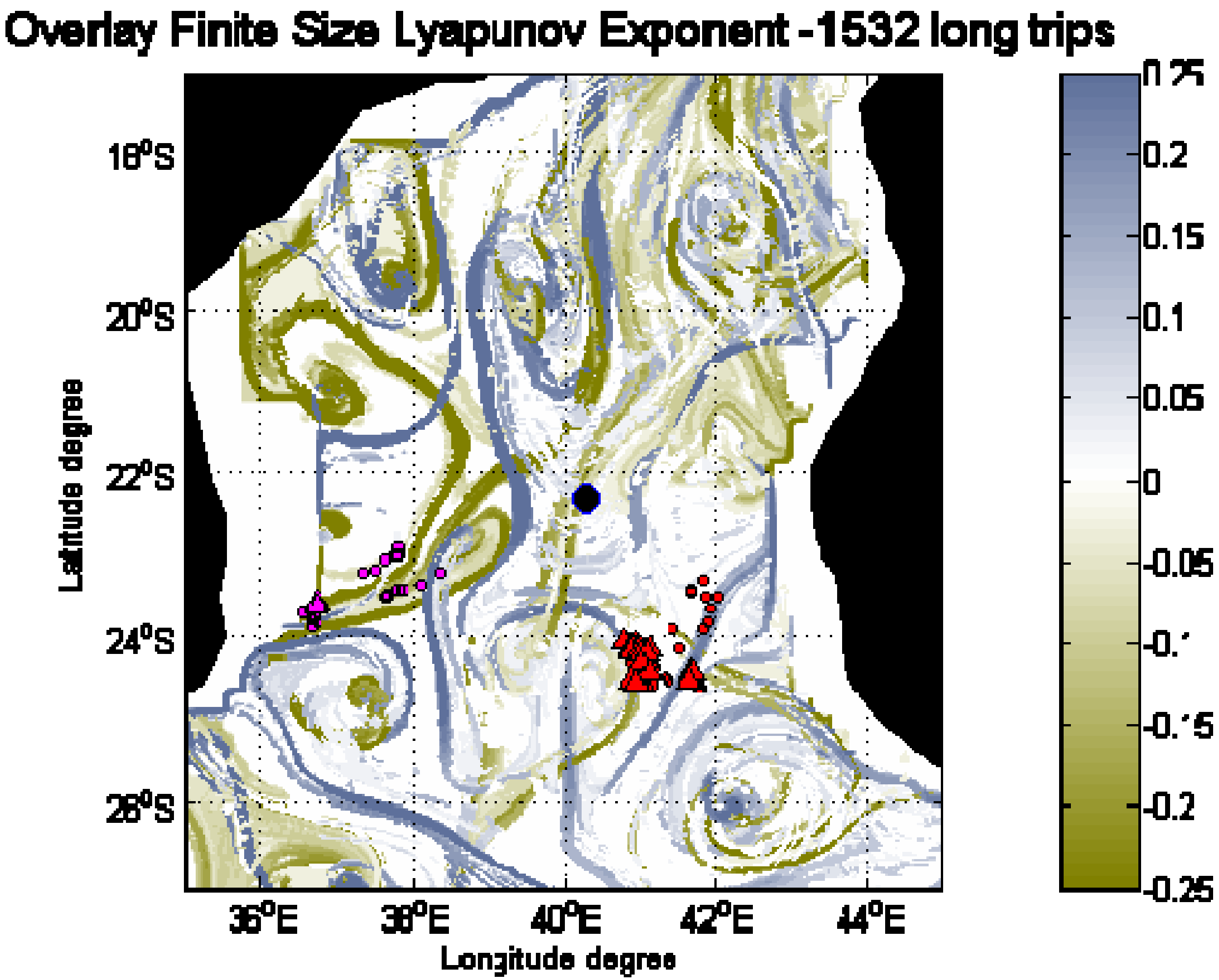
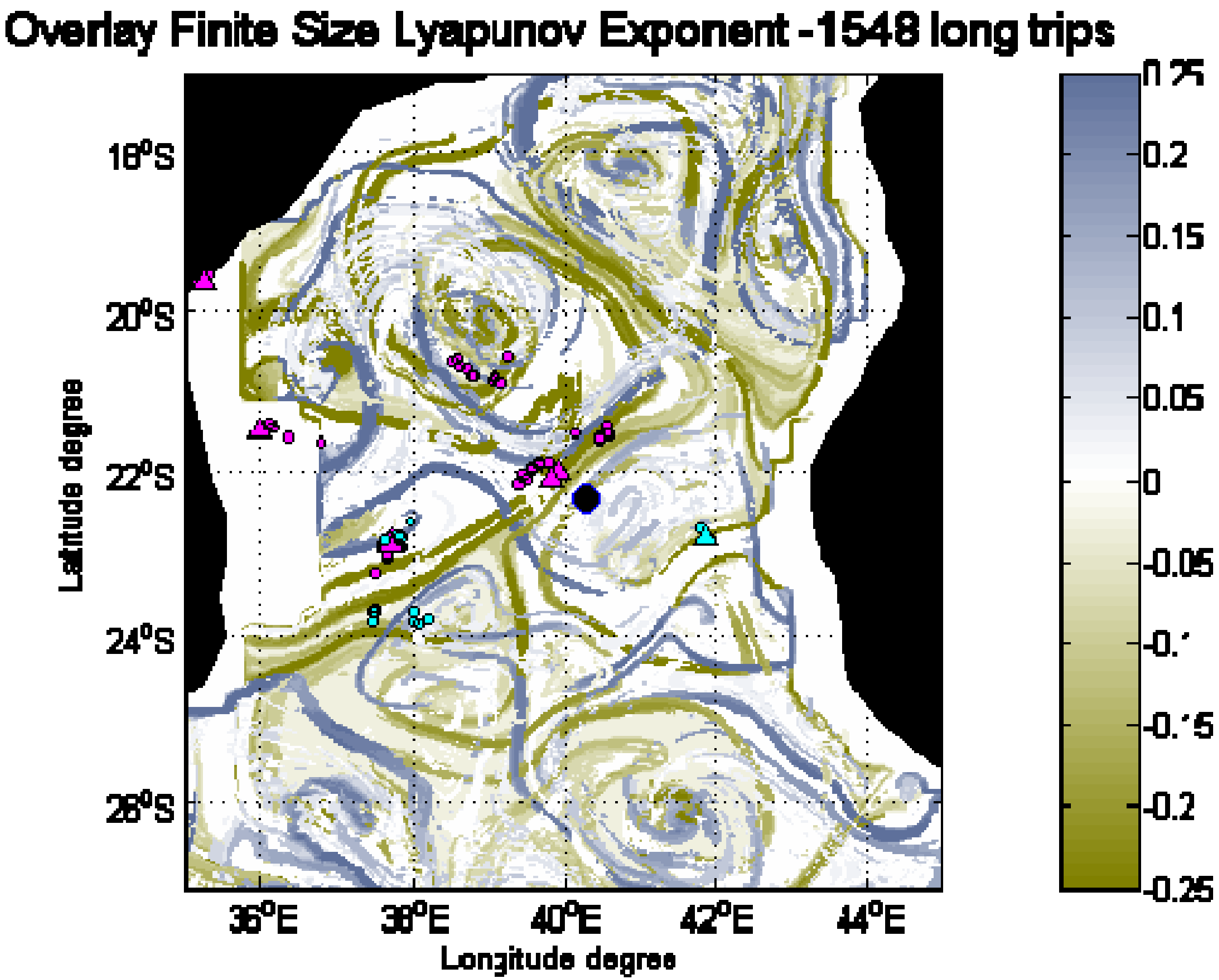
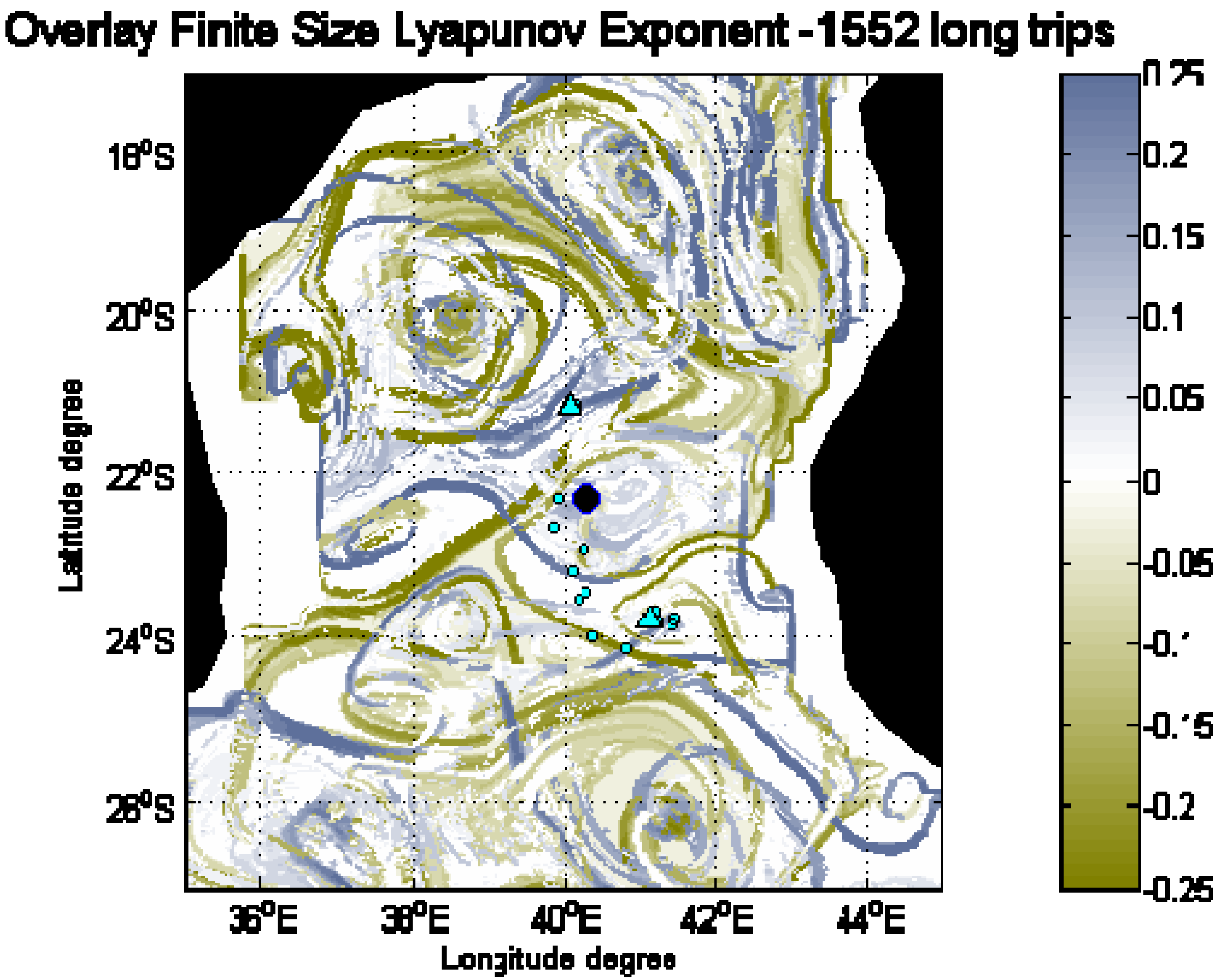
20°S
22°S
24°S
26°S
Latitude degree
36°E
38°E
40°E
42°E
44°E
Longitude degree
0.2
0.15
0.1
0.05
0
-0.05
-0.2
-0.25

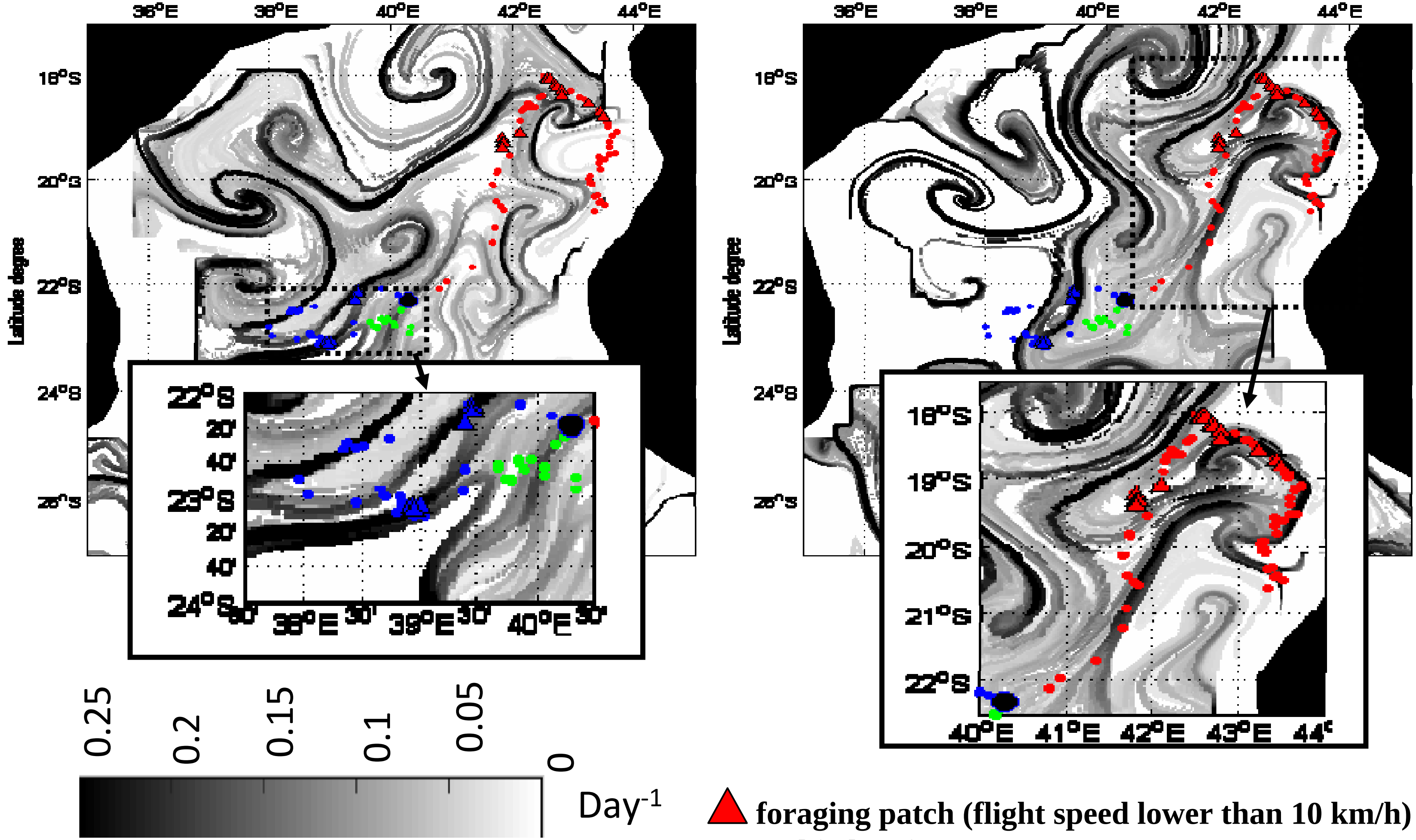

Overlay Finite Size Lyapunov Exponent -1532 long trips
18°S
20°S
22°S
24°S
26°S
Latitude degree
36°E
38°E
40°E
42°E
44°E
Longitude degree
0.2
0.15
0.1
0.05
0
-0.05
-0.2
-0.25


Overlay Finite Size Lyapunov Exponent -1548 long trips
18°S
20°S
22°S
24°S
26°S
36°E
38°E
40°E
42°E
44°E
Longitude degree
Latitude degree
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25


Overlay Finite Size Lyapunov Exponent -1552 long trips
18°S
20°S
22°S
24°S
26°S
36°E
38°E
40°E
42°E
44°E
Longitude degree
Latitude degree
0.2
0.15
0.1
0.05
0
-0.05
-0.2
-0.25

Week of September 24, 2003

Backward FSLE=Attractive LCSs | **Forward FSLE = Repelling LCSs**

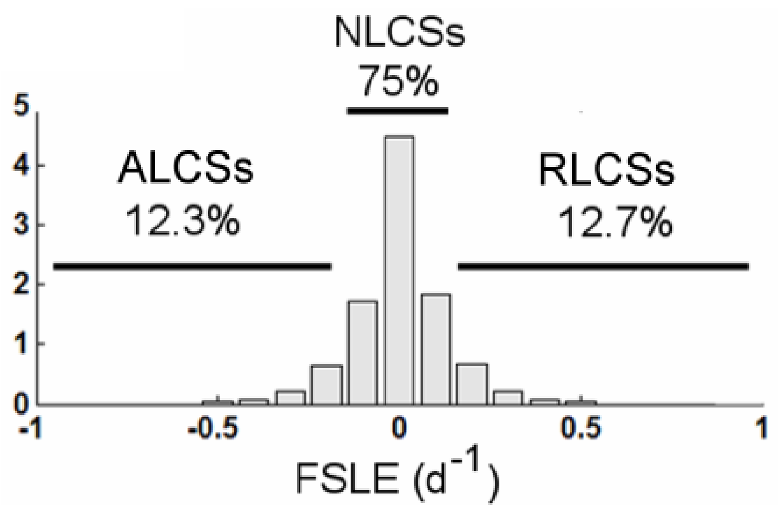


▲ **foraging patch (flight speed lower than 10 km/h)**

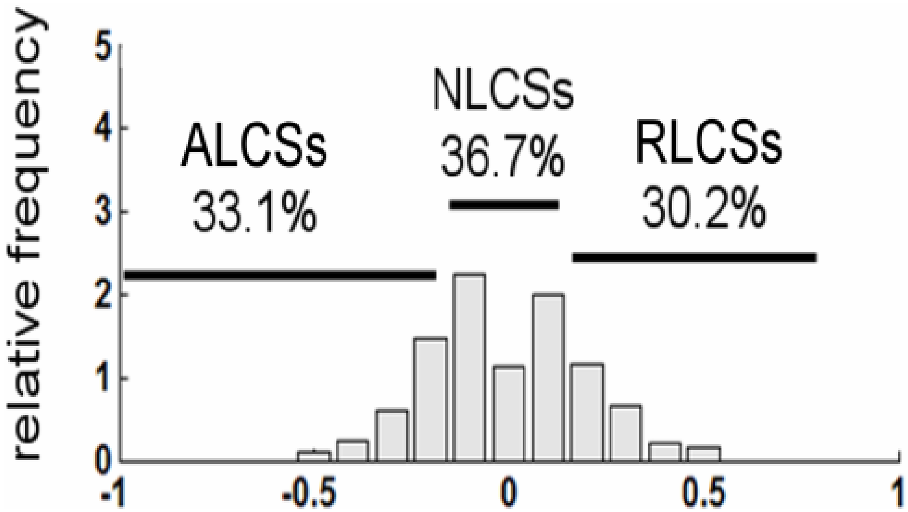
● **seabird trajectory**

Histograms of FSLE values

On the whole area



On the birds positions



ALCS: attracting LCS, i.e. FSLE (backwards) < - 0.1 day^{-1}
RLCS: repelling LCS, i.e. FSLE (forwards) > 0.1 day^{-1}
NLCS: not LCS (small FSLE)

Despite LCS occupy only 25% of space, 63% of bird's positions are on them

Table 1. Absolute frequency of seabird positions on LCSs and on no Lagrangian structures for long and short trips per week and result of the G-test for goodness of fit

Week	All trips: LCSs: $\|FSLE\| > 0.1$ day^{-1}	All trips: $\|FSLE\| < 0.1$ day^{-1}	Long trips: LCSs: $\|FSLE\| > 0.1$ day^{-1}	Long trips: $\|FSLE\| < 0.1$ day^{-1}	Short trips: LCSs: $\|FSLE\| > 0.1$ day^{-1}	Short trips: $\|FSLE\| < 0.1$ day^{-1}
1	38	9	19	7	19	2
2	78	40	55	12	23	28
4	208	85	147	54	61	31
5	167	109	137	84	30	25
6	120	77	89	51	31	26
7	79	55	72	32	7	23
8	53	34	53	34	—	—
9	61	59	61	59	—	—
10	55	31	45	24	10	7
14	35	12	35	12	—	—
15	10	5	10	5	—	—
%	63.7	36.3	65.9	34.1	56.0	44.0
G-test (log-likelihood ratio)						
n	1420		1097		323	
k	11		11		7	
df	10		10		6	
G	28.119		30.613		32.057	
P	0.00173		0.001		0.000	

One-tailed tests. Null hypothesis Ho: Seabird positions share equally LCSs ($|FSLE| > 0.1$ day^{-1} and on no LCSs. $\alpha = 5\%$.

STATISTICAL TESTS

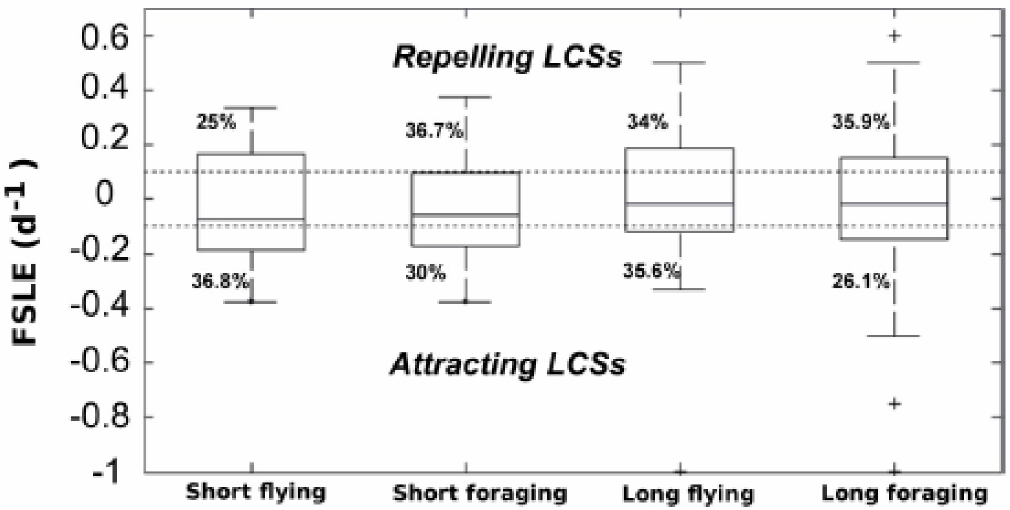


Table S2. Result of G-test statistics for comparison between frequency of bird positions on repelling or attracting LCS during flying and foraging and short and long trips

Variable	Flying		Foraging
Long trips			
Repelling LCS (FSLE > 0.1 day^{-1})	318		50
Attracting LCS (FSLE<-0.1 day^{-1})	333		37
n		738	
G		2.29	
P		0.13021	
Short trips			
Repelling LCS (FSLE > 0.1 day^{-1})	76		9
Attracting LCS (FSLE<-0.1 day^{-1})	112		10
n		207	
G		0.34	
P		0.55993	

Two-tailed tests. Null hypothesis Ho: seabirds share out equally on repelling and attracting structures when they fly or forage. $\alpha = 5\%$.

Results of statistical tests:

- Frigate birds fly on top of LCSs both for travelling as for foraging
- No significant difference between day and night positions
- No significant difference between come and return trip

Frigatebirds 'follow' LCSs not only to find there prey, but as biological corridors which bring them to foraging places

Aggregation of prey on LCSs? or aggregation of subsurface predators?

Olfatory clues (DMS produced by zooplankton) ? thermal air currents?

Puzzling issue: no significant difference between attracting and repelling LCSs

- Tangencies between manifolds?
- Interleaving between them?
- 3d dynamics associated both to ALCS and RLCS?
- Do they simply avoid low FSLE regions?

Tew Kai et al. PNAS (2009)

FINITE-SIZE LYAPUNOV EXPONENT FIELDS

- Able to reveal globally the dynamical structures in the flow: main hyperbolic trajectories, their manifolds, …
- Simple enough to be applied in a practical way to real and complex ocean velocity fields.
- On the negative side: relationship with material lines is not rigorous, and lobe areas not easy to compute.
- Reveals impact of fluid flow on biological dynamics at all scales: from plankton to top predators
- Relationship with 3d dynamics desirable

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