# Amidst the Chaos, good predictions: How Meteorology manages to beat the odds

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# Chaos in Numerical Weather Prediction and how we can fight it

- Lorenz (1963) introduced the concept of “chaos” in meteorology. (Yorke, 1975, coined the name chaos)
  - Even with a perfect model and perfect initial conditions we cannot forecast beyond two weeks: butterfly effect
  - In 1963 this was only of academic interest: forecasts were useless beyond a day or two anyway!
  - Now we exploit “chaos” with ensemble forecasts and routinely produce skillful forecasts beyond a week
  - The El Niño coupled ocean-atmosphere instabilities are allowing one-year forecasts of climate anomalies
- “Breeding” is a simple method to find instabilities in a complex flow
  - With the exact energy equations for bred vectors we can determine the physical origin of the instabilities

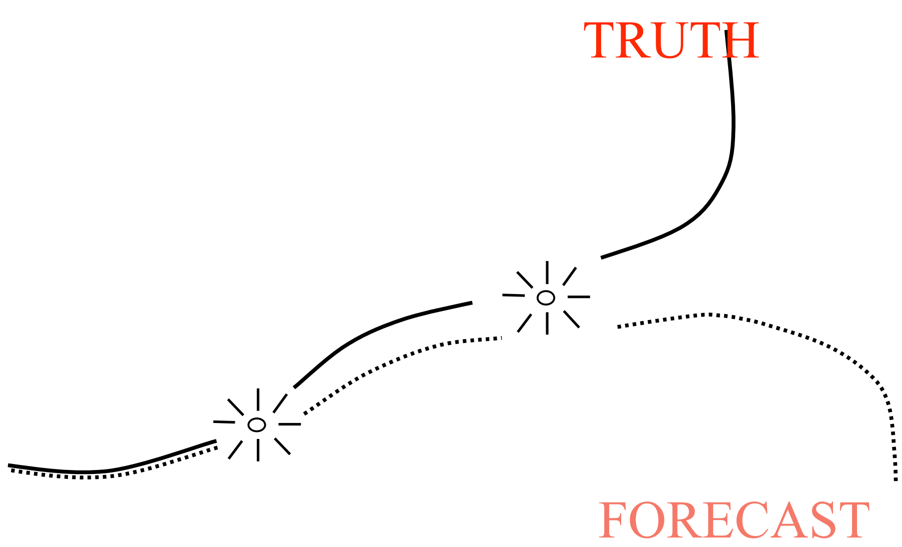
- Chaos-Weather research led to the UMD Local Ensemble Transform Kalman Filter (LETKF, Hunt et al., 2007)

Central theorem of chaos (Lorenz, 1960s):

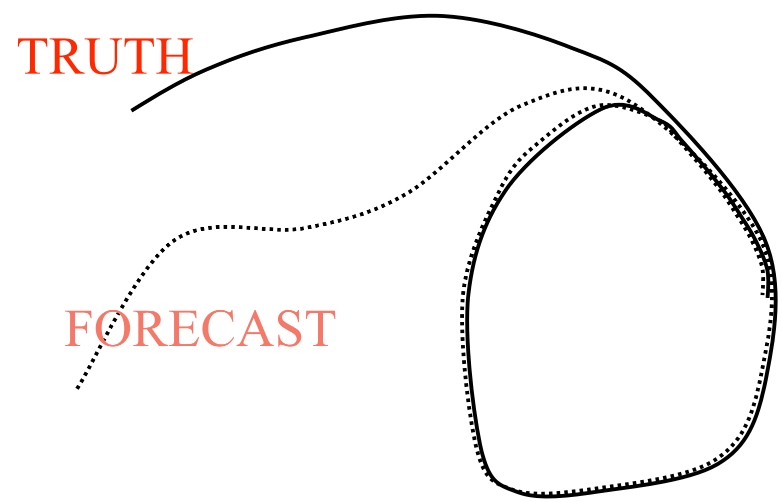
**a) Unstable systems have finite predictability (chaos)**

**b) Stable systems are infinitely predictable**

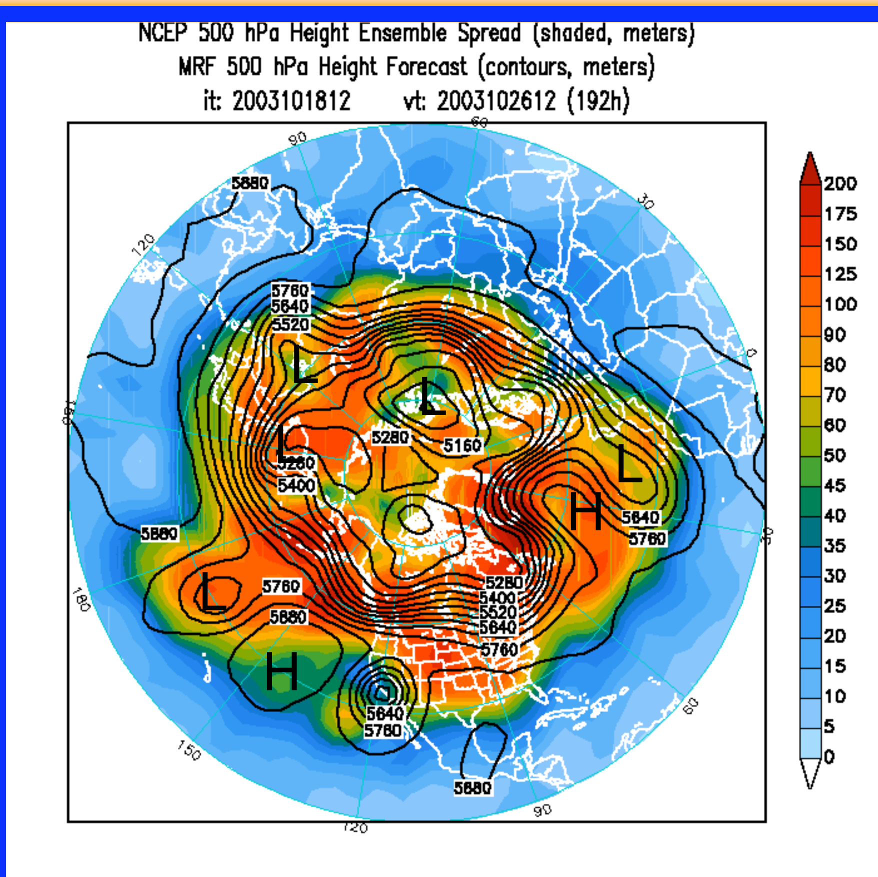
a) Unstable dynamical system

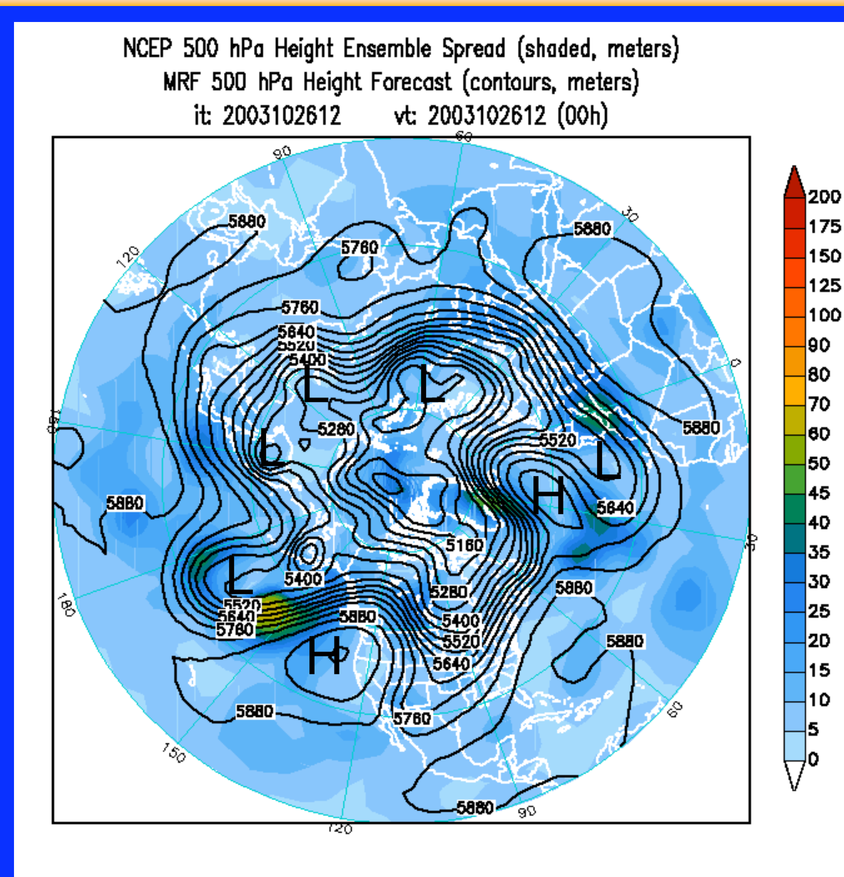



b) Stable dynamical system



# 8-day forecast and verification

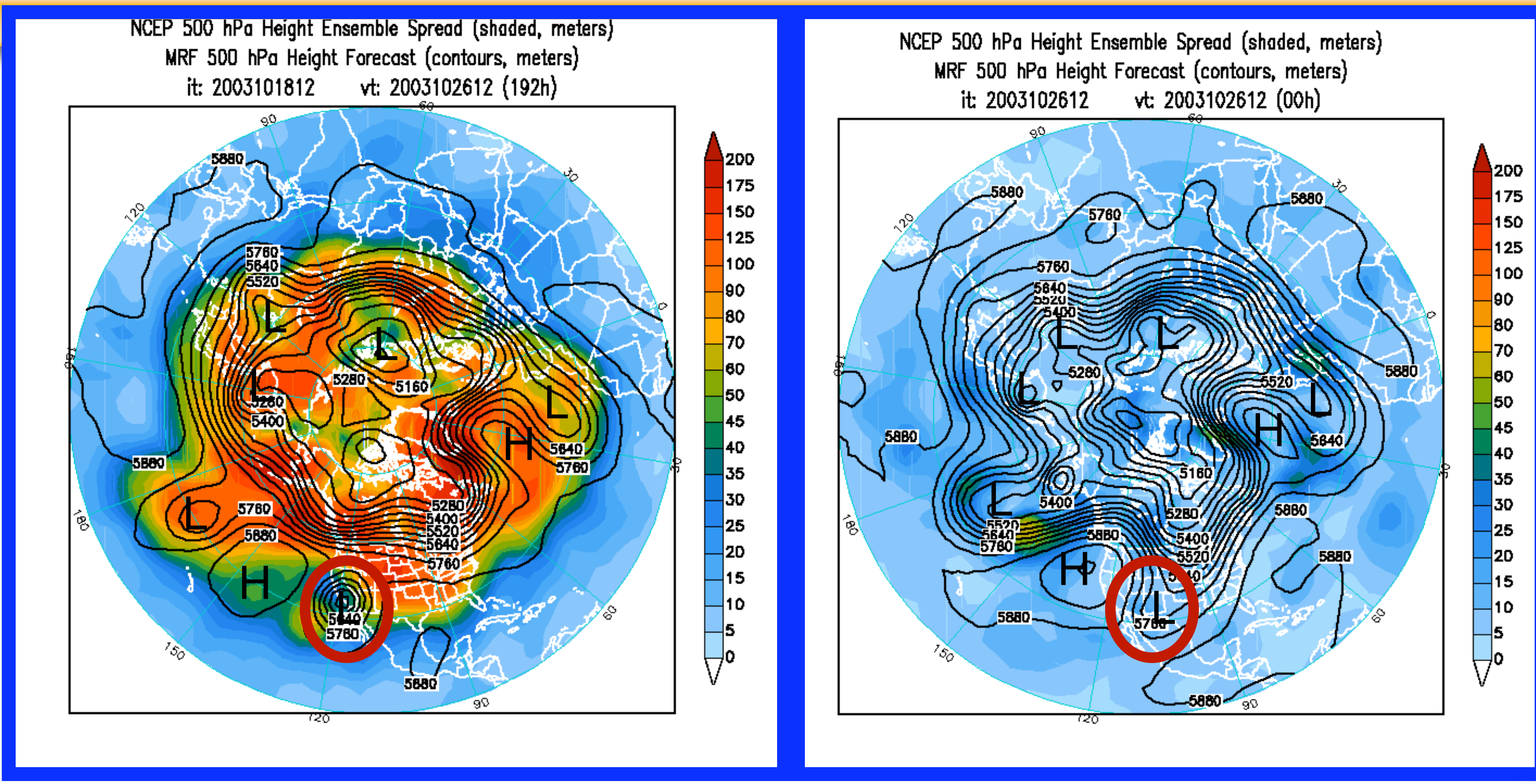



Almost all the centers of low and high pressure are very well predicted after 8 days!

Need good models, good observations, good data assimilation

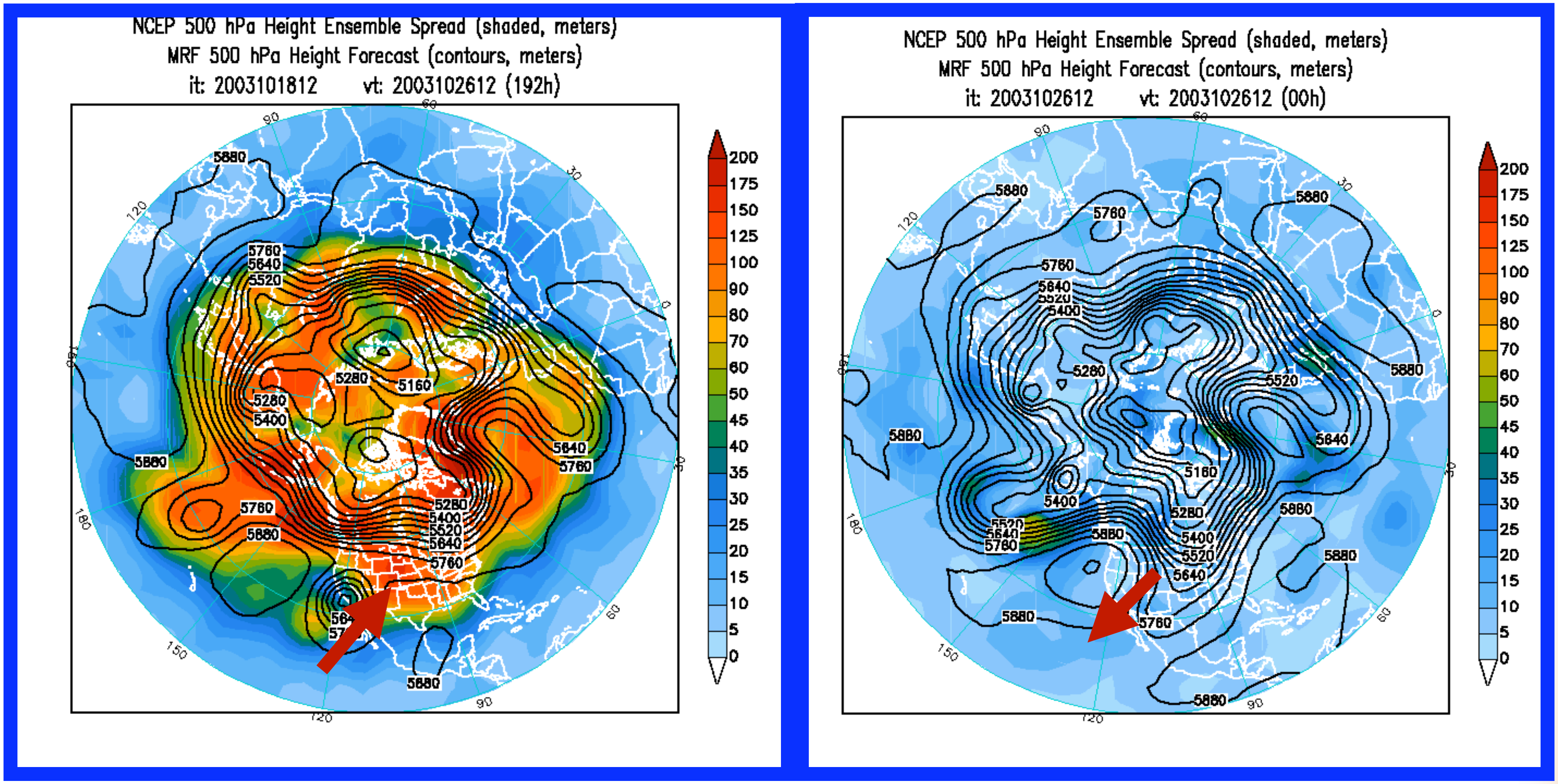
# 8-day forecast and verification




Almost all the centers of low and high pressure are very well predicted after 8 days!

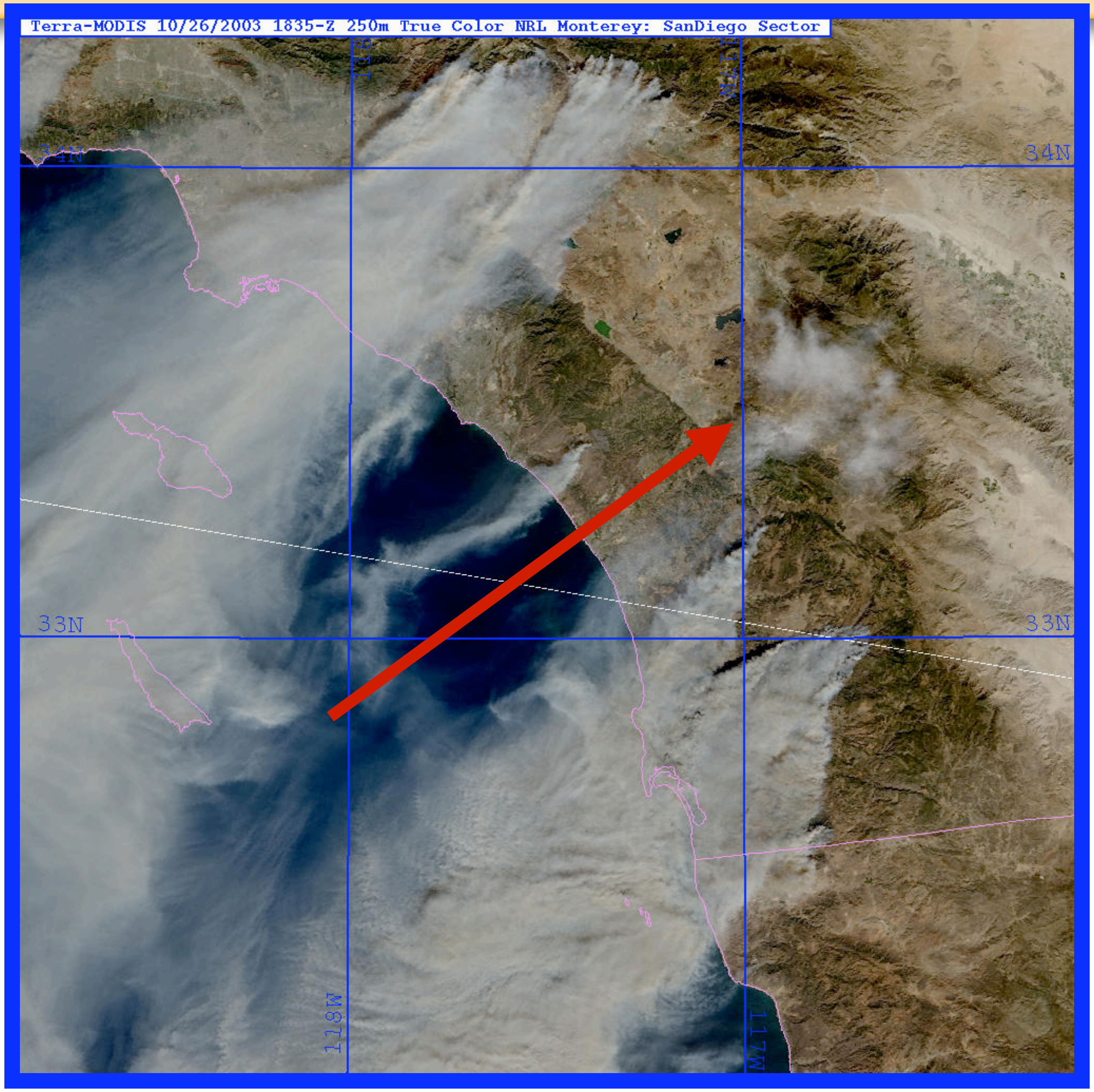
Over Southern California forecast has a cut-off low, not a trough

# 8-day forecast and verification



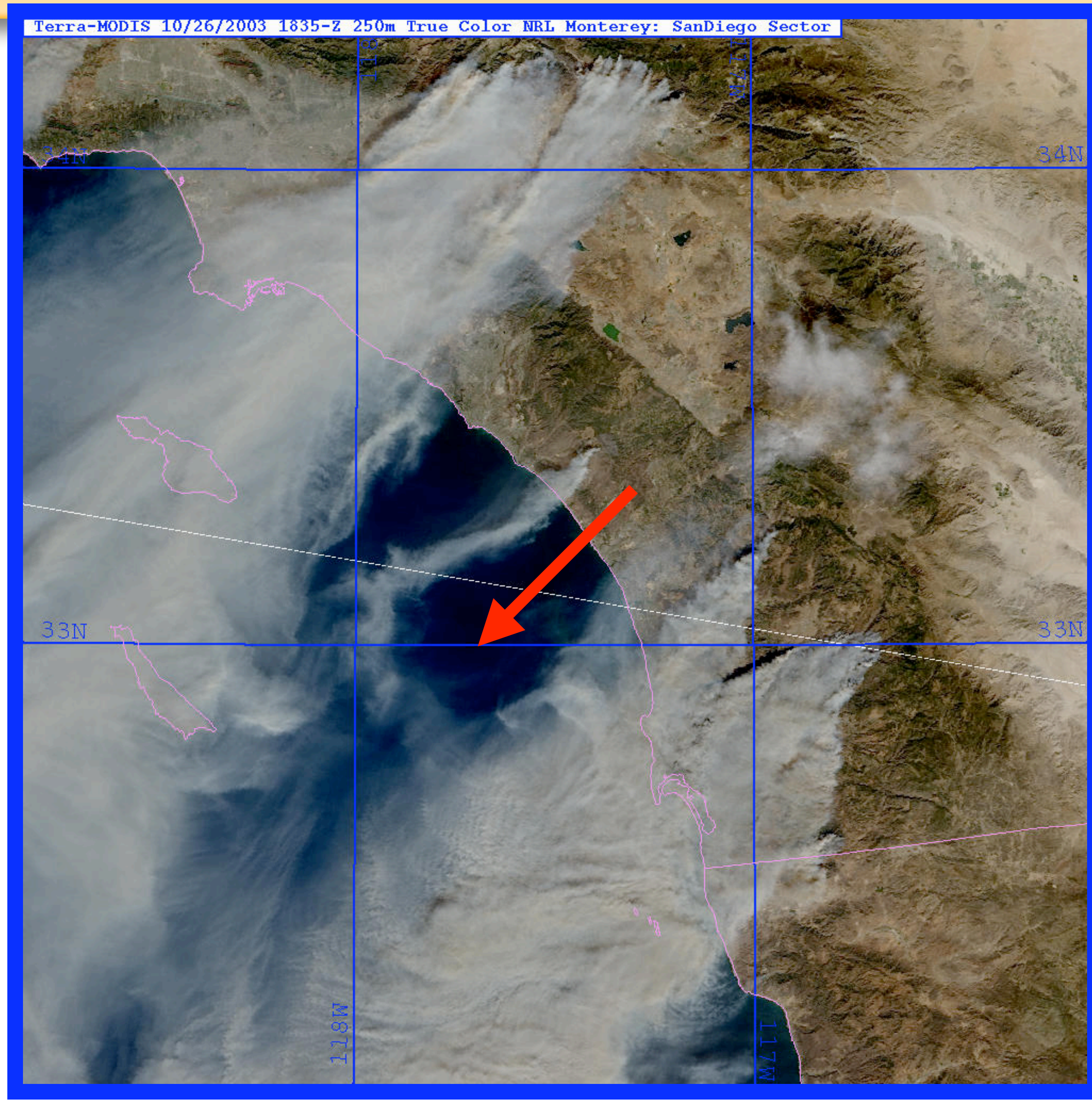

Southern California: winds are from the wrong direction!

# Fires in California (2003)




8-day cool, moist wind forecast: It would have stopped the fires

# Fires in California (2003)




Warm, dry, Santa Ana winds: locally wrong prediction (8 days in advance!)

# A simple chaotic model: Lorenz (1963) 3-variable model

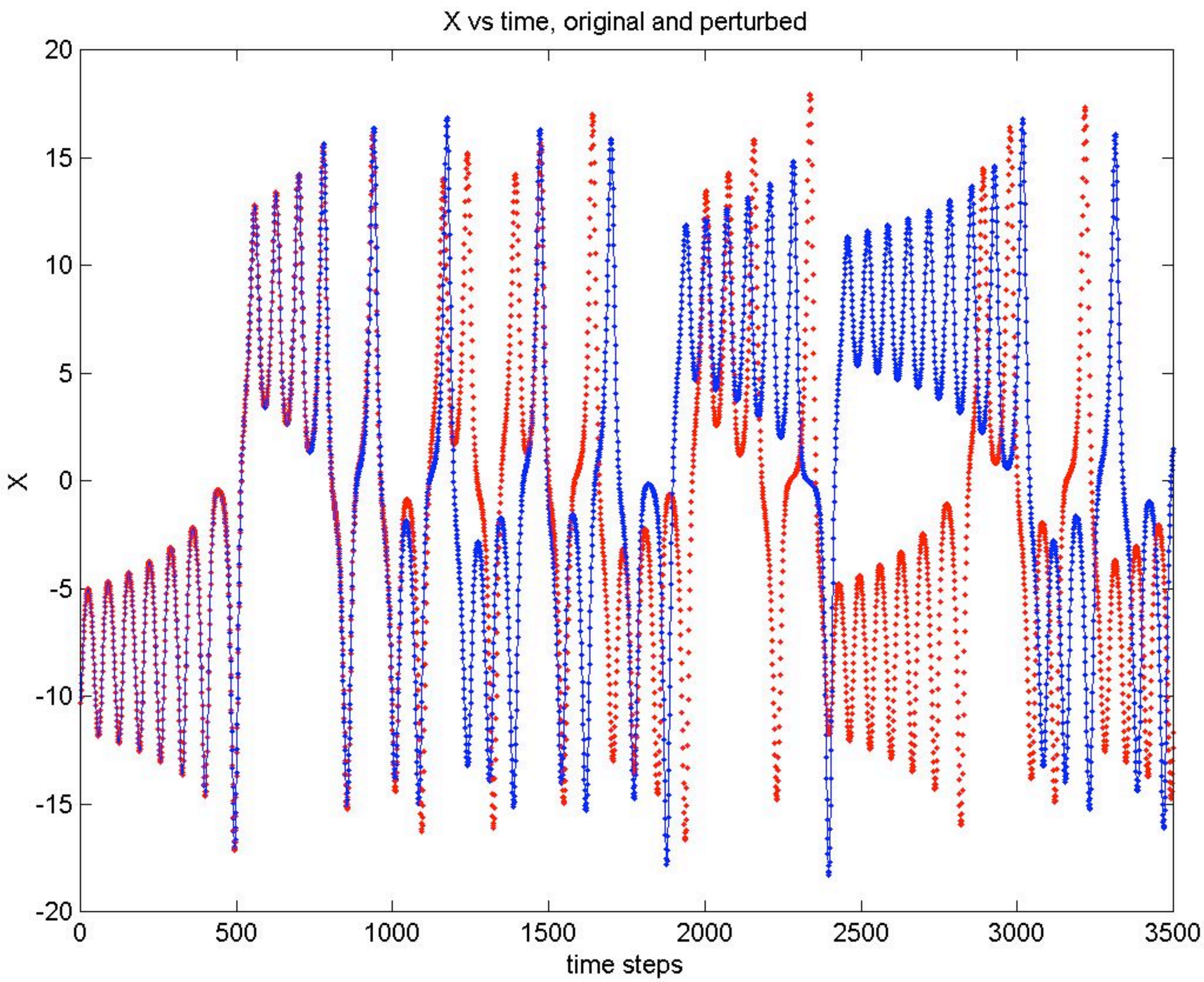
$$\frac{dx}{dt} = \sigma(y - x)$$

$$\frac{dy}{dt} = rx - y - xz$$

$$\frac{dz}{dt} = xy - bz$$

**Has two regimes and the transition between them is chaotic**

# If we introduce an infinitesimal perturbation in the initial conditions, the forecast soon loses all skill



# Definition of Chaos
## (Lorenz, March 2006, 89 years old)

**WHEN THE PRESENT DETERMINES**

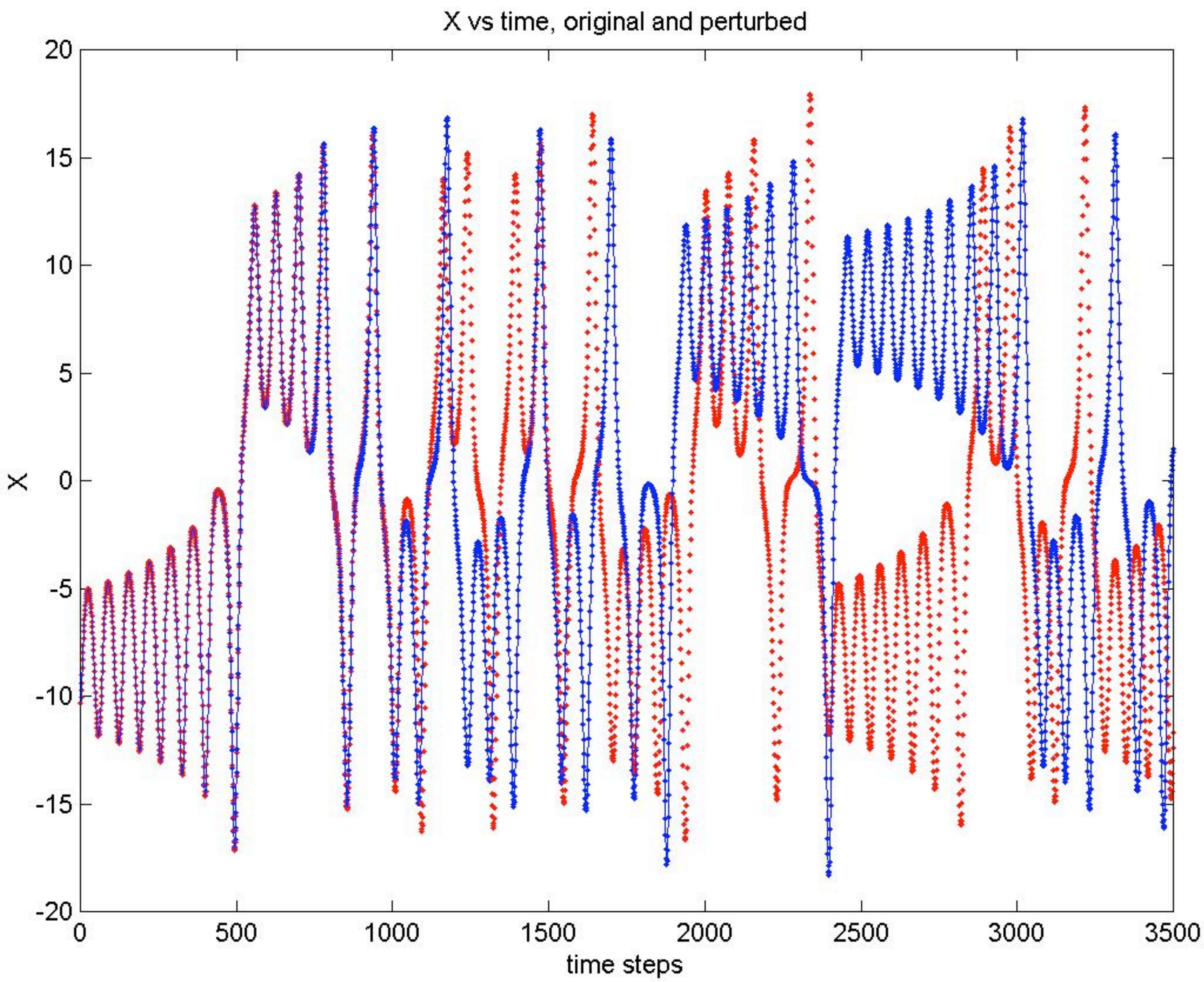
**THE FUTURE**

**BUT**

**THE APPROXIMATE PRESENT DOES NOT**

**APPROXIMATELY DETERMINE THE FUTURE**

# The approximate present does not approximately determine the future!

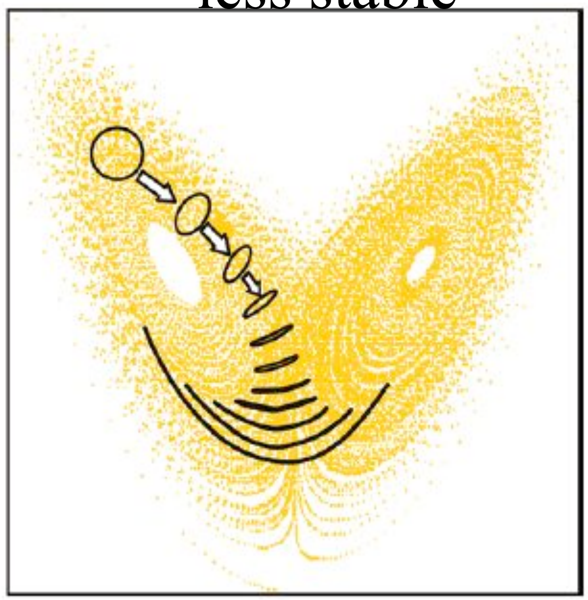


Predictability depends on the initial conditions (Palmer, 2002):

stable

less stable

unstable



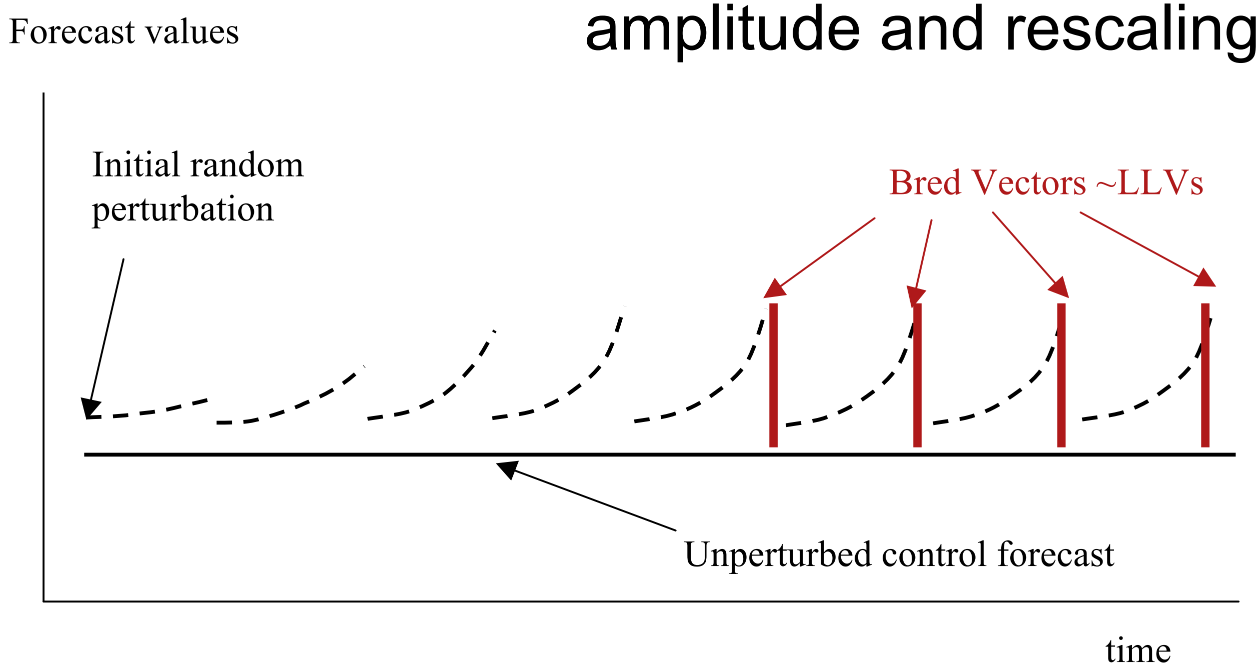
**Initial conditions that are unstable (with growing “errors of the day”) grow much faster**

# An 8 week RISE project for undergraduate women (2002)

- We gave a team of 4 RISE intern undergraduates a problem: Play with the famous Lorenz (1963) model, and explore its predictability using "breeding" (Toth and Kalnay 1993), a very simple method to study the growth of errors.
- We told them: "Imagine that you are forecasters that live in the Lorenz 'attractor'. Everybody living in the attractor knows that there are two weather regimes, the 'Warm' and 'Cold' regimes. But what the public needs to know is when will the change of regimes take place, and how long are they going to last!!".
- "Can you find a forecasting rule to alert the public that there is an imminent change of regime?"

# Breeding: simply running the nonlinear model a second time, from perturbed initial conditions.

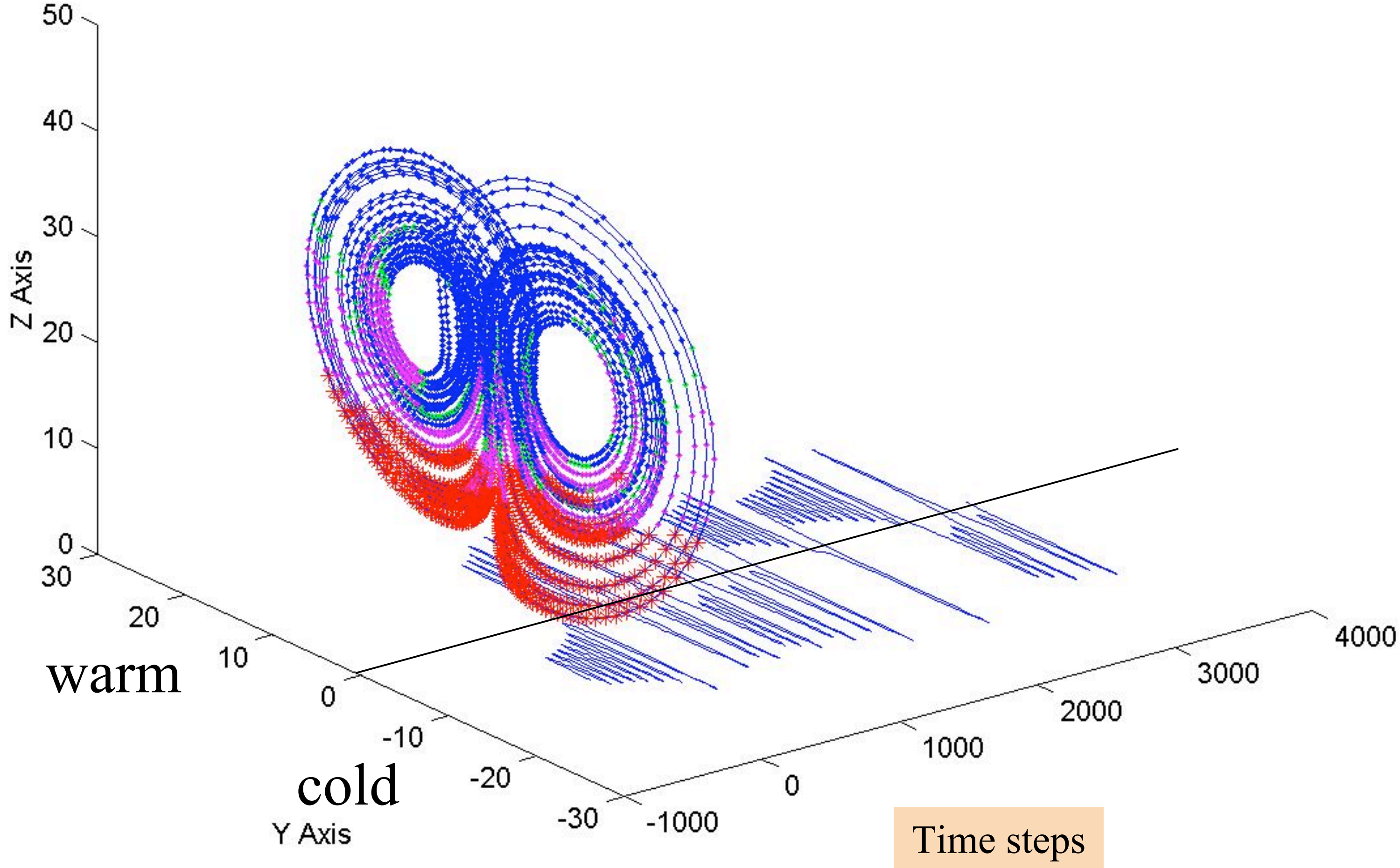
Only two tuning parameters: rescaling amplitude and rescaling interval



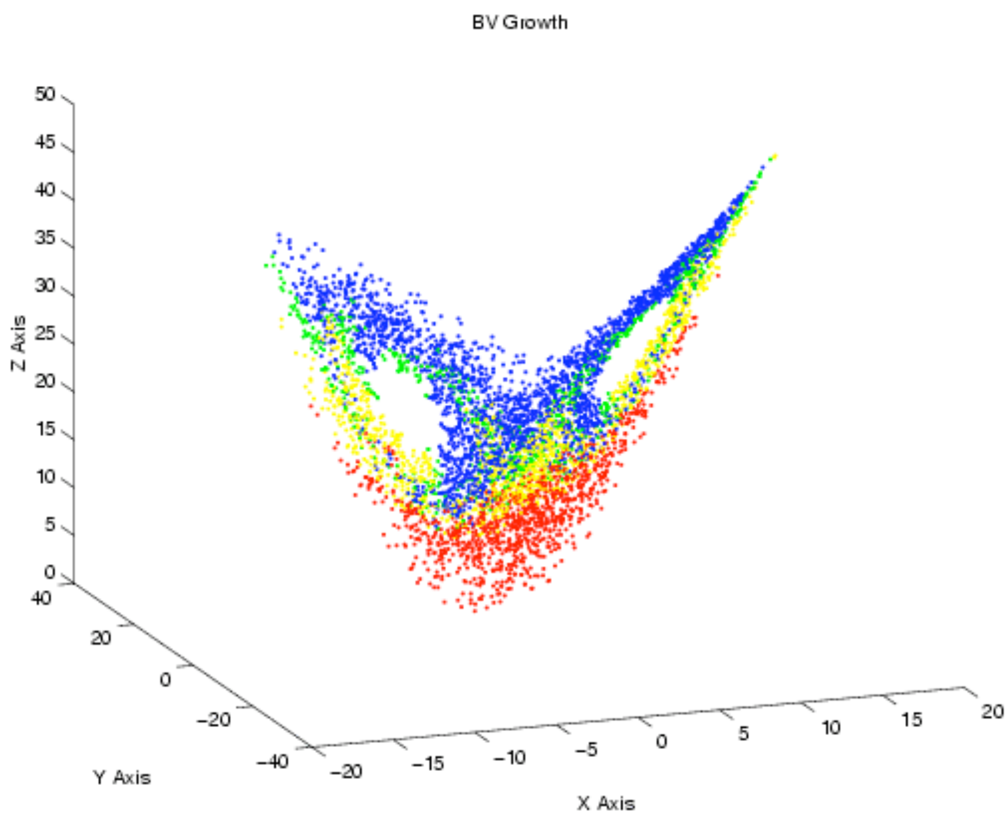

Local breeding growth rate:

$$g(t) = \frac{1}{n\Delta t} \ln\left( \left|\delta \mathbf{x}\right| / \left|\delta \mathbf{x}_0\right| \right)$$

# 4 summer interns computed the Lorenz Bred Vector growth rate: red means large BV growth, blue means perturbations decay



# In the 3-variable Lorenz (1963) model we used breeding to estimate the local growth of perturbations:

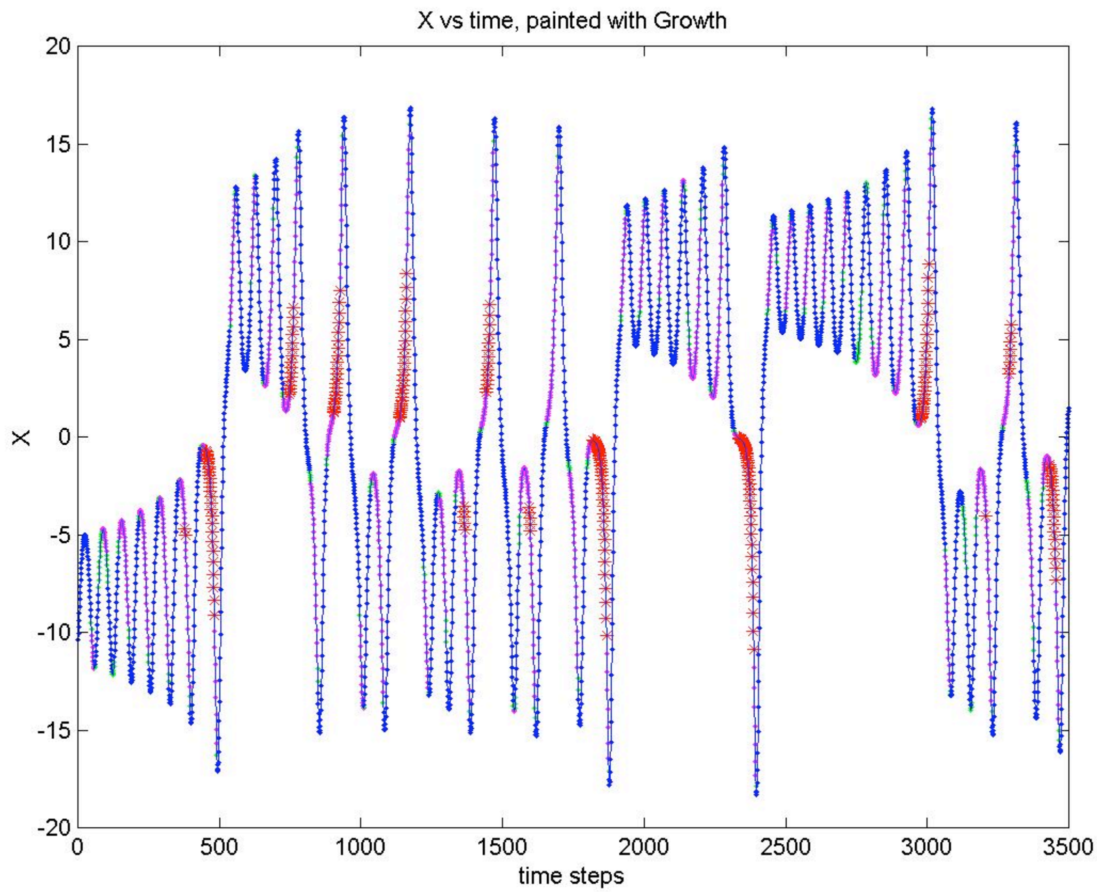



Bred Vector Growth:
red, high growth;
yellow, medium;
green, low growth;
blue, decay

With just a single breeding cycle, we can estimate the stability of the whole attractor (Evans et al, 2004)

## This looked promising, so we asked the interns to "paint" x(t) with the bred vector growth, and the result almost made me faint:

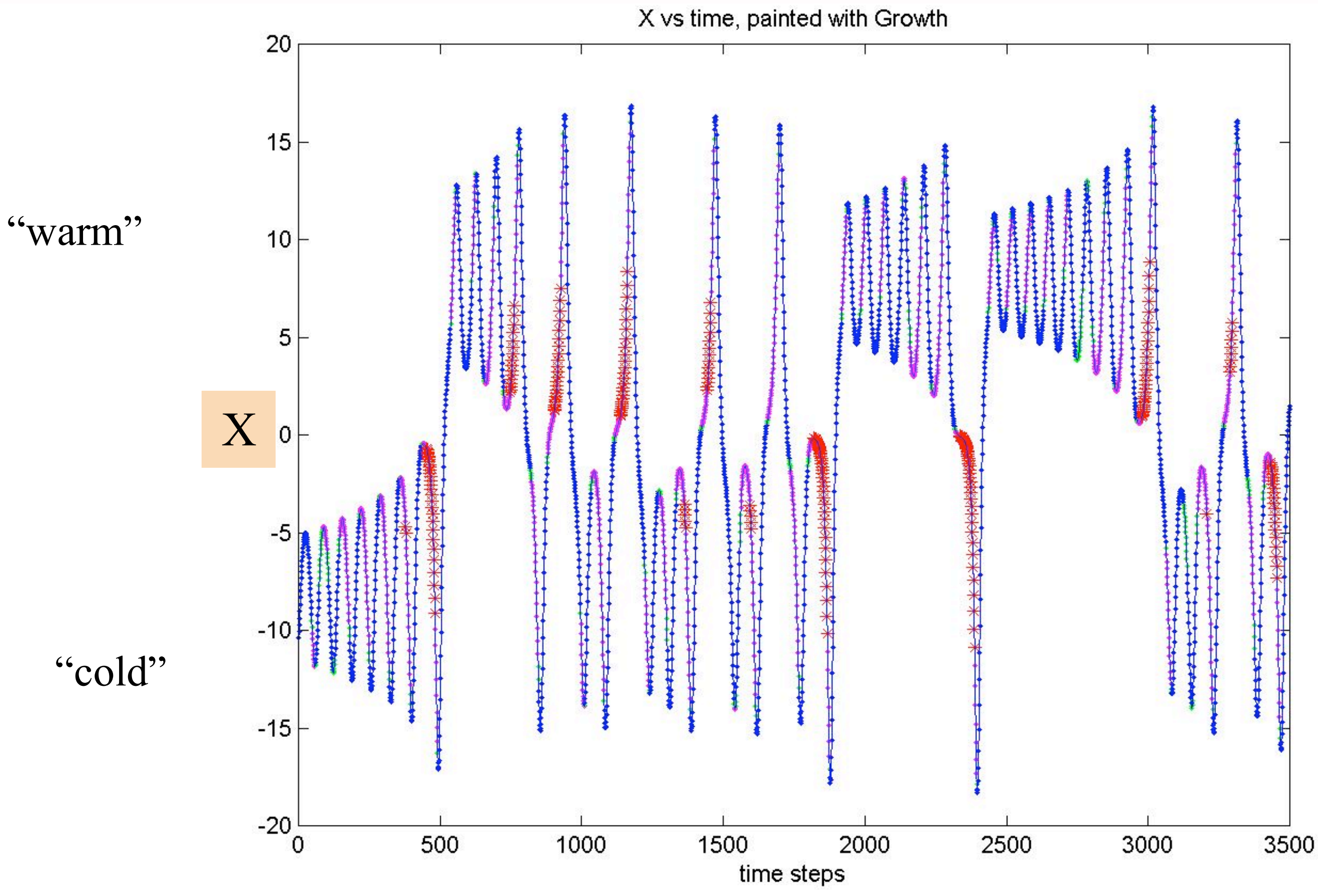
# This looked promising, so we asked the interns to “paint” x(t) with the bred vector growth, and the result almost made me faint:




Growth rate of bred vectors:

A * indicates fast growth (>1.8 in 8 steps)

# Forecasting rules for the Lorenz model:




Growth rate of bred vectors:

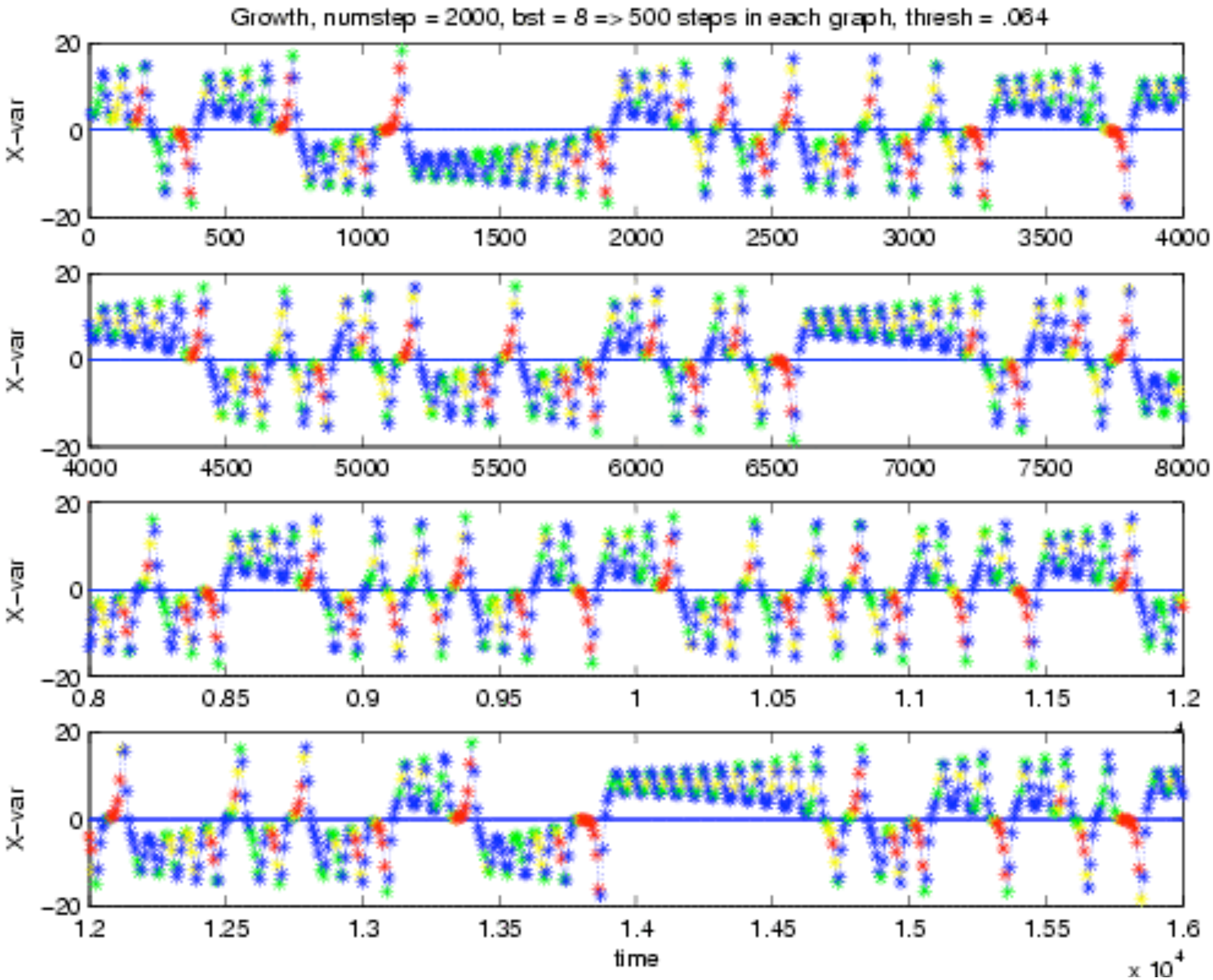
A * indicates fast growth (>1.8 in 8 steps)

1. Regime change: The presence of red stars (fast BV growth) indicates that the next orbit will be the last one in the present regime.

2. Regime duration: One or two red stars, next regime will be short. Several red stars: the next regime will be long lasting.

These rules surprised Lorenz himself!

# These are very robust rules, with skill scores > 95%



# Summary for this part and rest of the talk

- Breeding is a simple generalization of Lyapunov vectors, for finite time, finite amplitude: simply run the model twice, take the difference and rescale…
- Breeding in the Lorenz (1963) model gives accurate forecasting rules for the "chaotic" change of regime and duration of the next regime that surprised Lorenz!

**Rest of the talk:**

- The same ideas can be applied to fight chaos in the full forecast models that have dimension 10-100 million rather than just 3!
- In the atmosphere, in the ocean, and in coupled systems
- We can also use breeding to understand the **physical mechanisms** of the instabilities that create chaos
- Apply it to Mars!

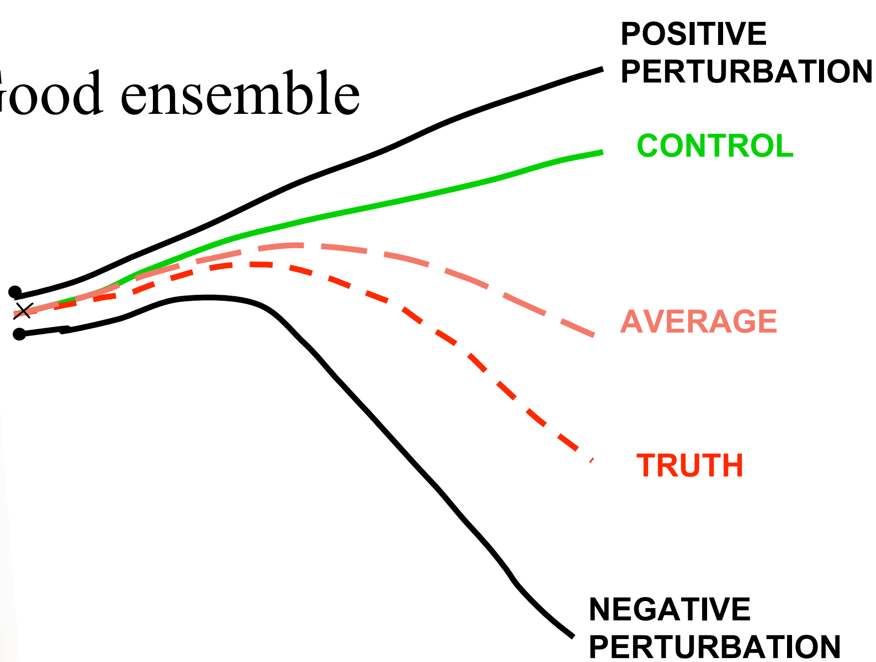
# A major tool to "fight chaos" is ensemble forecasting

An ensemble forecast starts from initial perturbations to the analysis…

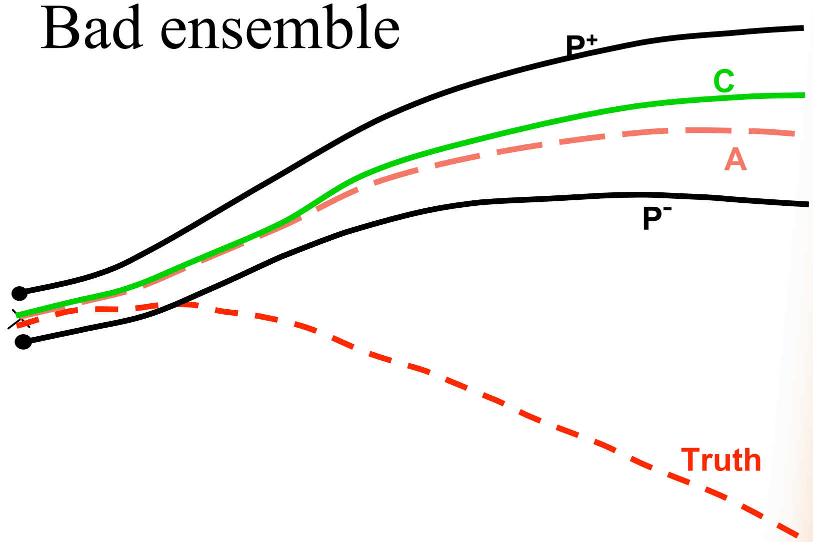
In a good ensemble "truth" looks like a member of the ensemble

The initial perturbations should reflect the analysis "errors of the day"

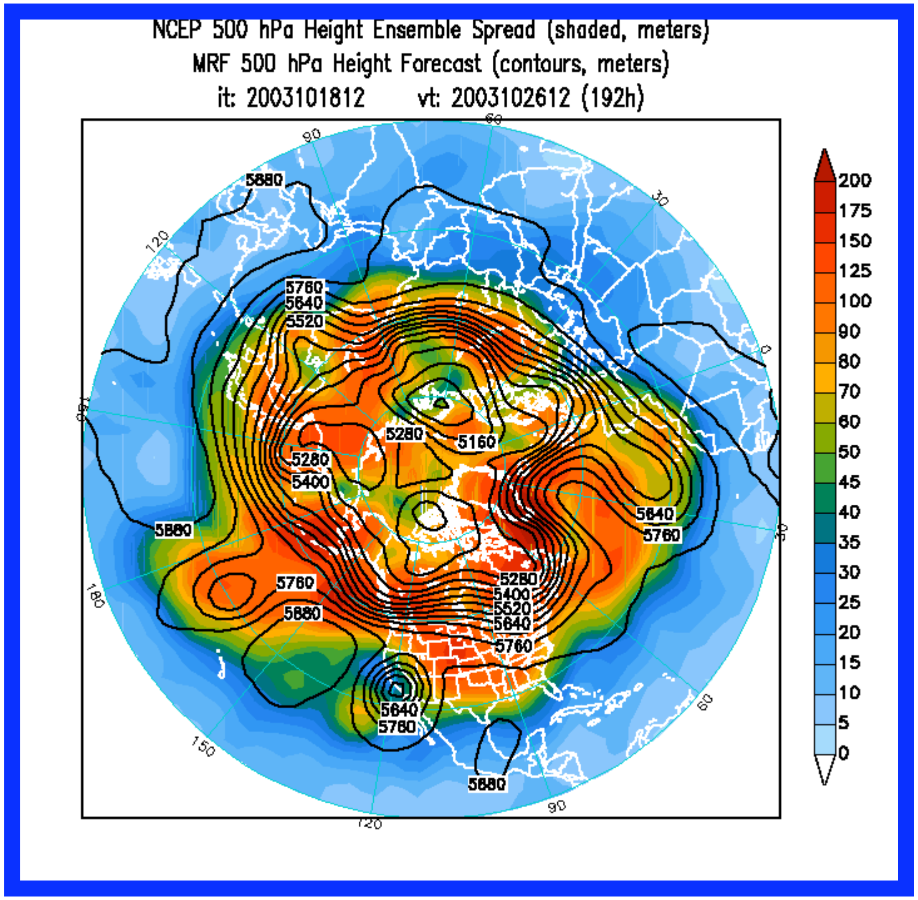
Good ensemble

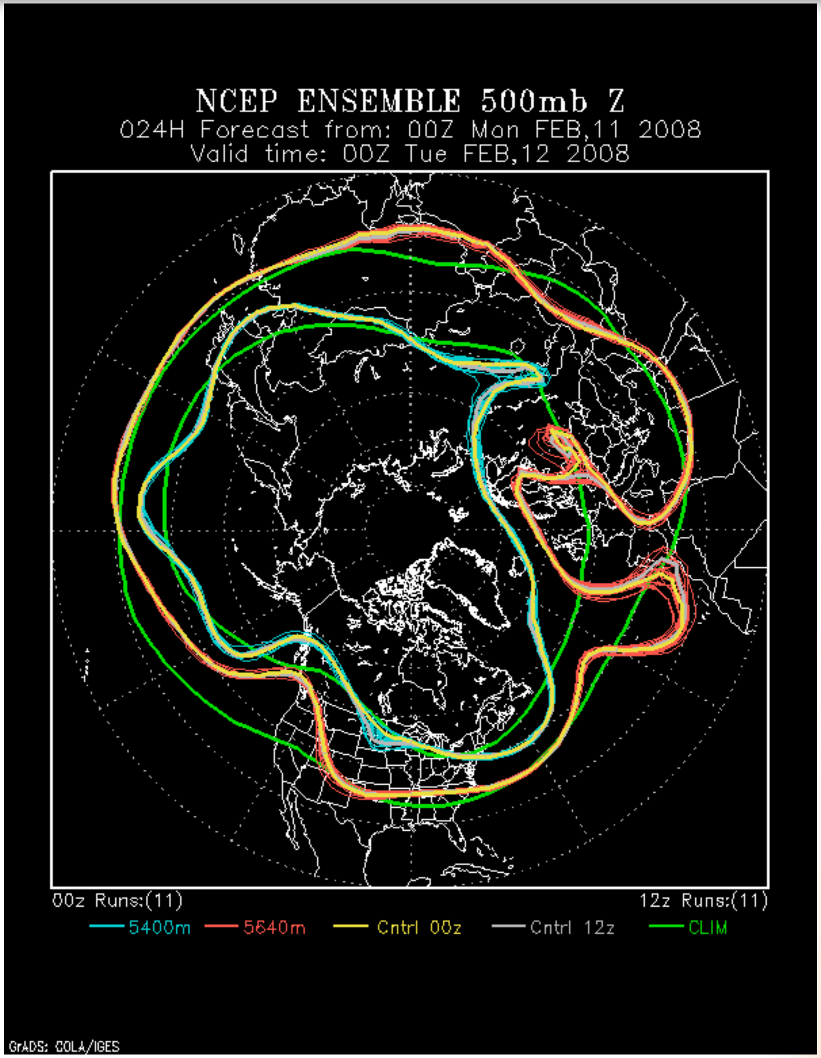



Bad ensemble

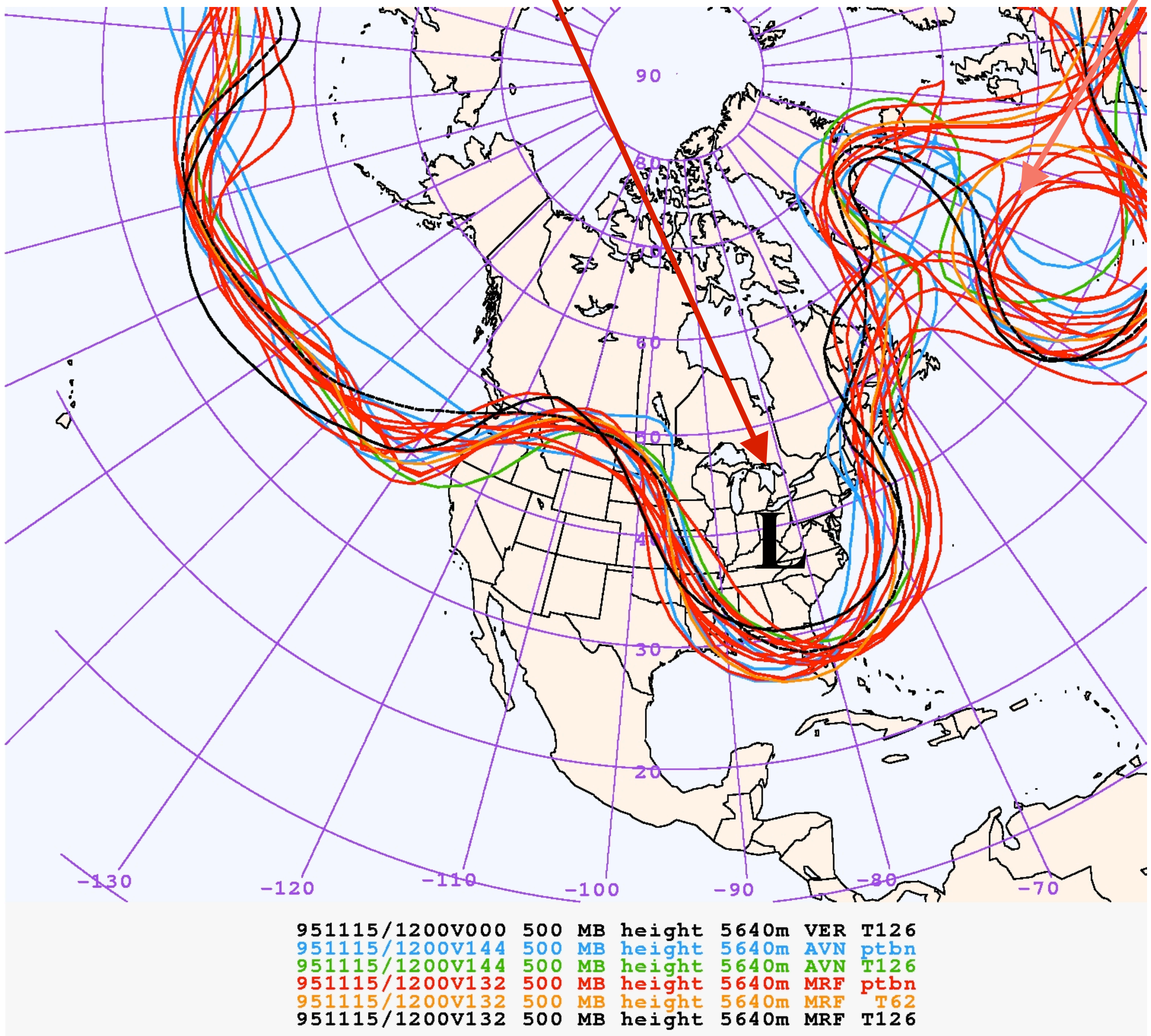


# In ensemble forecasting we need to represent the uncertainty: spread or "spaghetti plots"



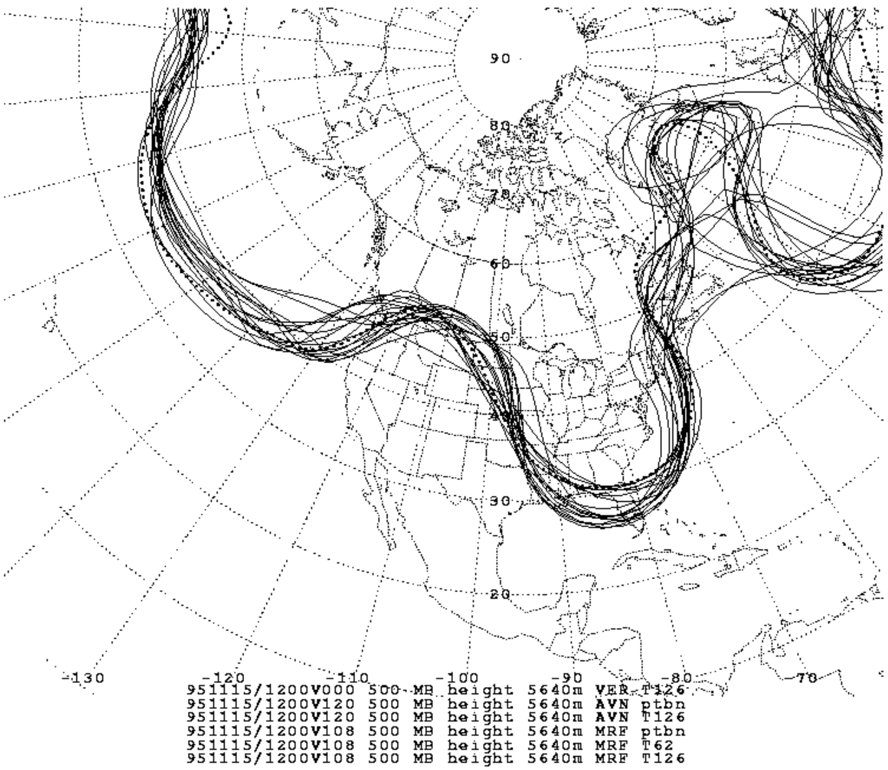
Example of a very predictable 6-day forecast, with "errors of the day"

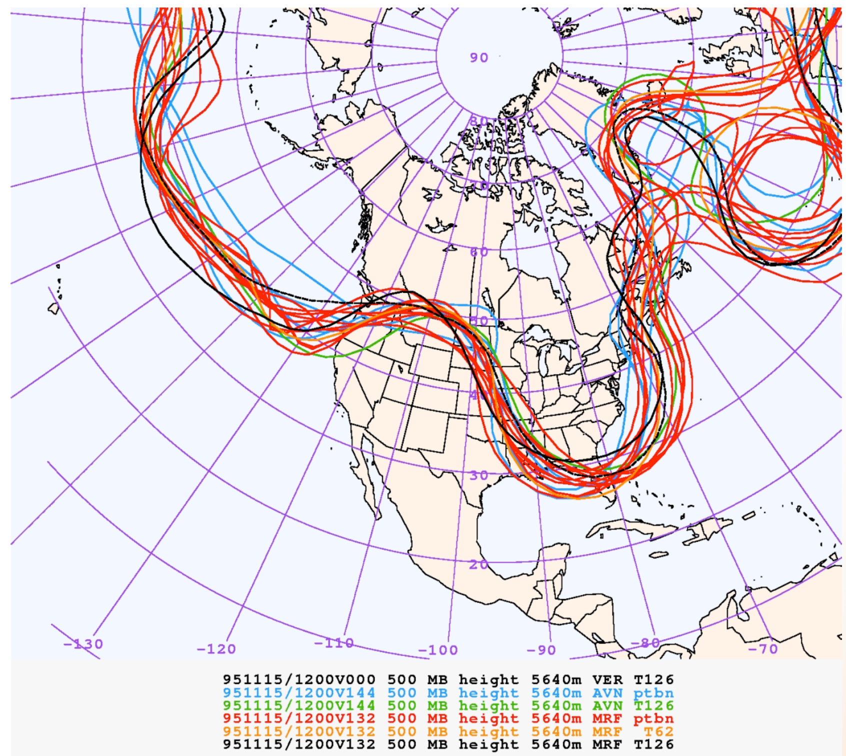



**The bred vectors are the growing atmospheric perturbations: "errors of the day"**

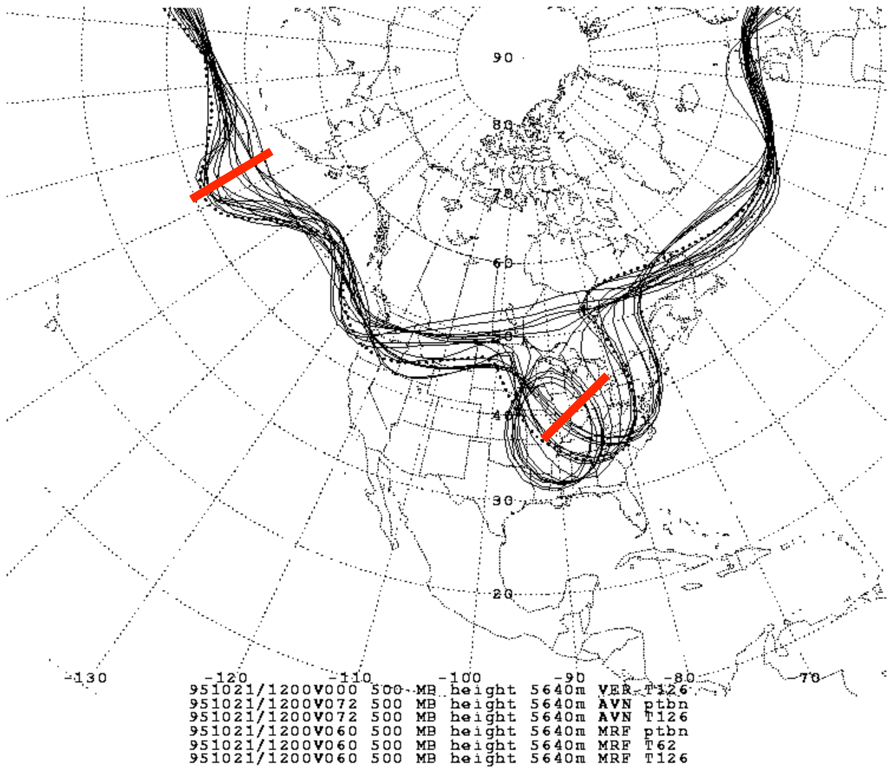
# The errors of the day are instabilities of the background flow. At the same verification time, the forecast uncertainties have *the same shape*

4 days and 6 days ensemble forecasts verifying on 15 Nov 1995



# Strong instabilities of the background tend to have simple shapes (perturbations lie in a low-dimensional subspace of bred vectors)



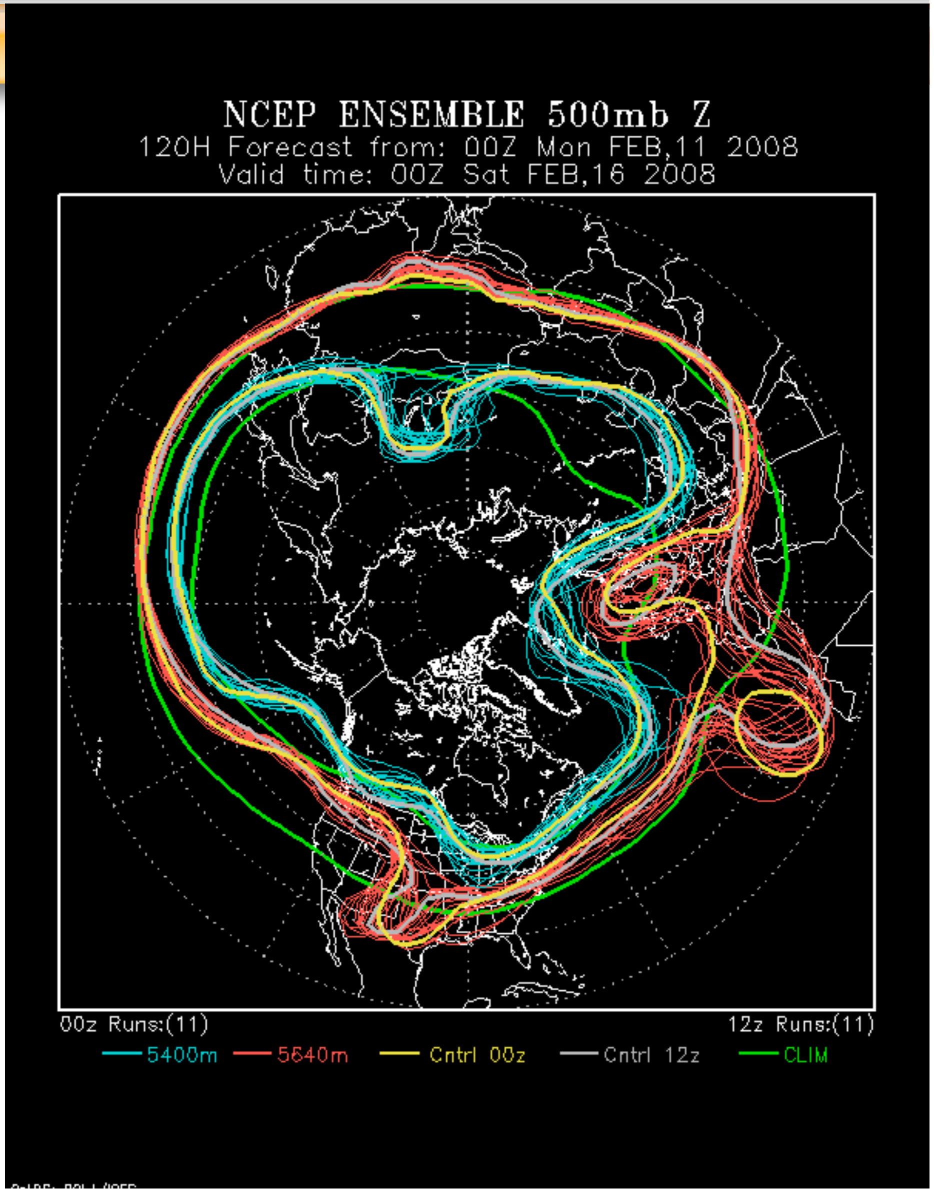

2.5 day forecast verifying on 95/10/21.

Note that the bred vectors (difference between the forecasts) lie on a 1-D space

This simplicity (**local low-dimensionality**, Patil et al. 2000) inspired the Local Ensemble Transform Kalman Filter (Ott et al. 2004, Hunt et al., 2007)

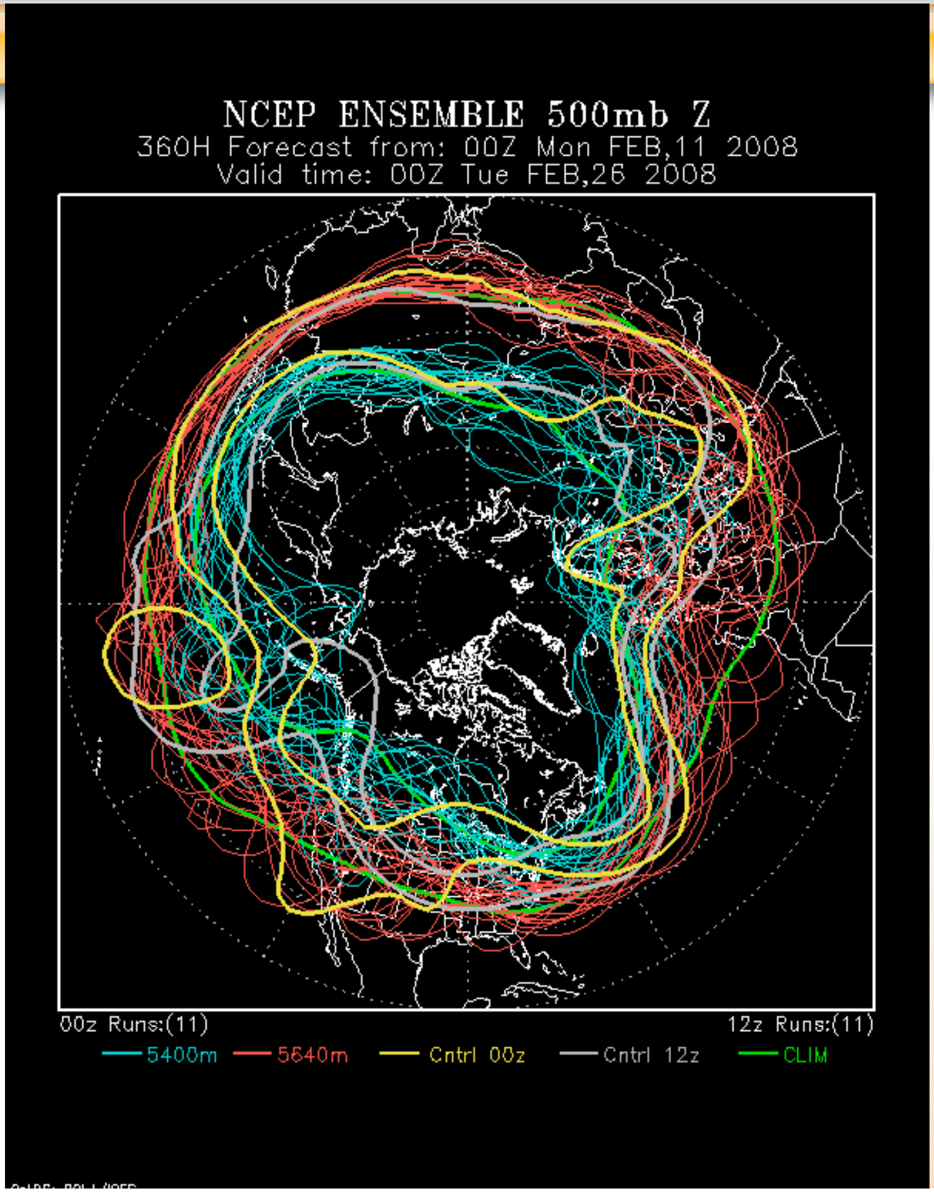
# 5-day forecast "spaghetti" plot

- The ensemble is able to separate the areas that are predictable from the ones that are chaotic.
- Even the chaotic ones have local low-dimensionality
- This is what makes possible to do Ensemble Kalman Filter with 50 (not a million!) ensemble members with good results



# 15-day forecast "spaghetti" plot: Chaos!

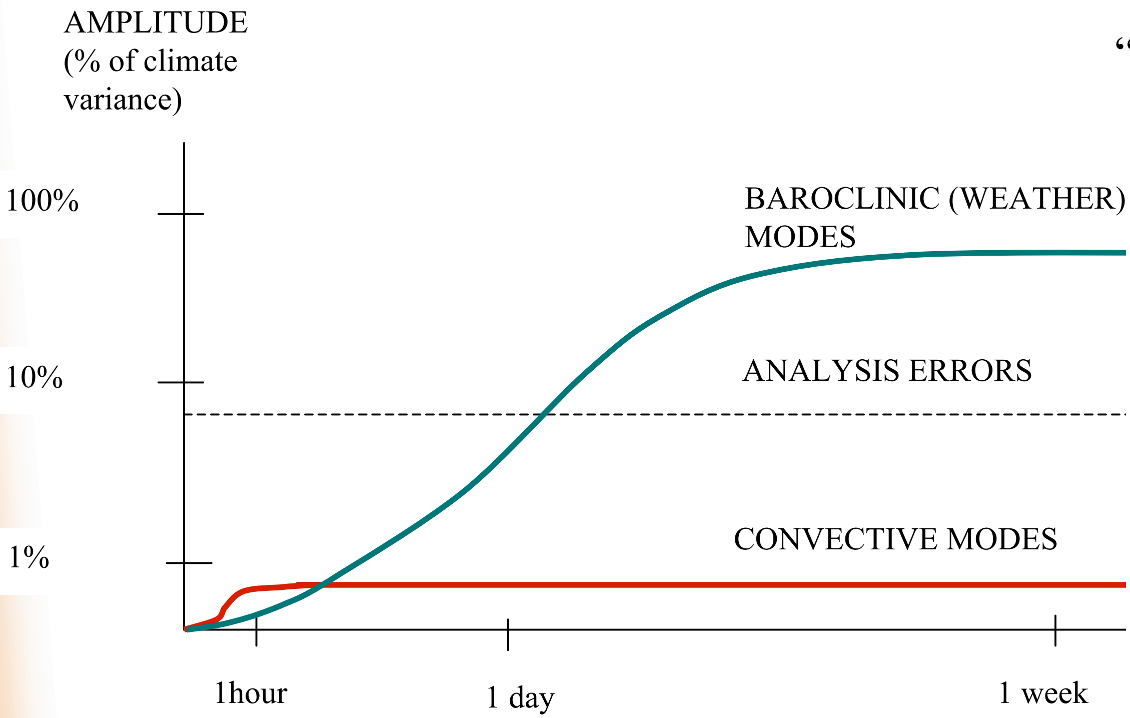
After 15 days, Lorenz' chaos has won!
No predictability left in the 15-day forecast (except in East Asia)



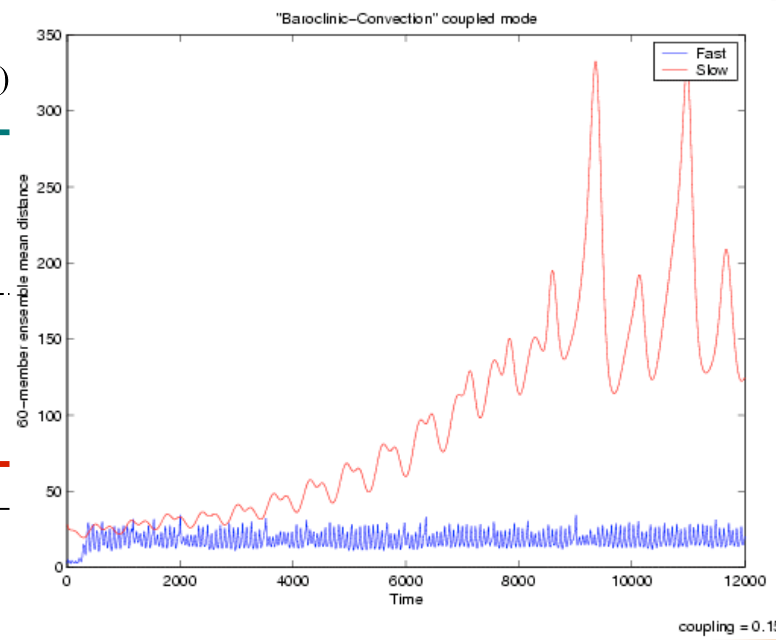
# In the rest of this talk, we will study chaos in coupled fast-slow systems

- The atmosphere has fast (e.g., convective clouds, 20 min) and slow instabilities (e.g., baroclinic or weather instabilities 3-7 days)
- The coupled ocean-atmosphere system has even slower instabilities (El Niño-Southern Oscillation, 3-7 years)
- In order to predict these phenomena, we need to isolate fast and slow instabilities
- If we can predict ENSO, we can predict climate anomalies a year or more in advance

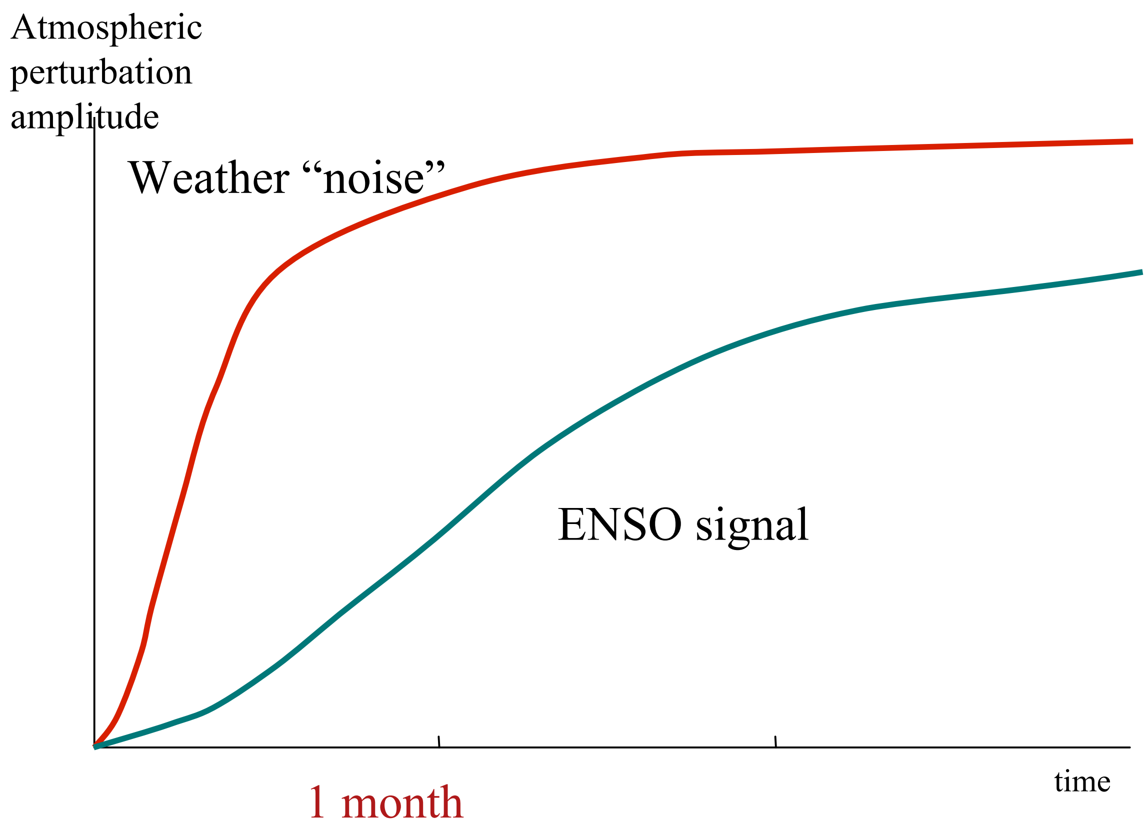
# In the atmosphere there are many instabilities, e.g., fast (convective clouds) and slow (baroclinic)
# Nonlinear breeding saturates convective noise




“weather + convection” coupled model



# Coupled ocean-atmosphere modes (El Niño-Southern Oscillation) The "weather noise" has large amplitude! Must use the fact that the coupled ocean modes are slower…




Need a long rescaling interval, like 2 weeks or one month

# Breeding in a coupled system

- Breeding: finite-amplitude, finite-time instabilities of the system (~Lyapunov vectors)
- In a coupled system there are fast and slow modes,
- A **linear** approach (like Singular or Lyapunov Vectors) will only capture **fast** modes.
- Can we do breeding of the **slow** modes?

# We coupled slow and a fast Lorenz (1963) 3-variable models (Peña and Kalnay, 2004)

Fast equations

$$\frac{dx_1}{dt} = \sigma(y_1 - x_1) - C_1(Sx_2 + O)$$

$$\frac{dy_1}{dt} = rx_1 - y_1 - x_1 z_1 + C_1(Sy_2 + O)$$

$$\frac{dz_1}{dt} = x_1 y_1 - bz_1 + C_1(Sz_2)$$

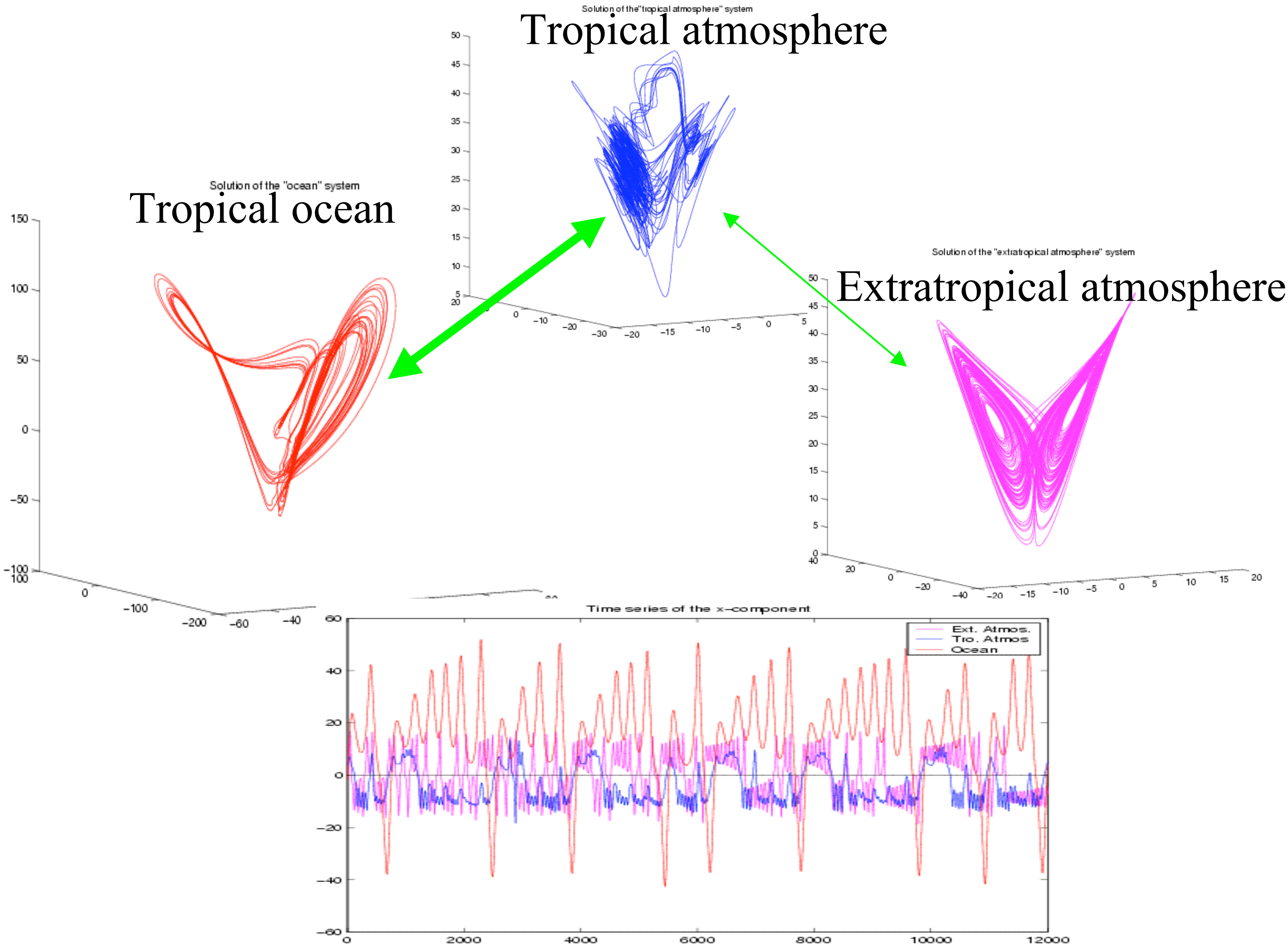
Slow equations

$$\frac{1}{\tau}\frac{dx_2}{dt} = \sigma(y_2 - x_2) - C_2(x_1 + O)$$

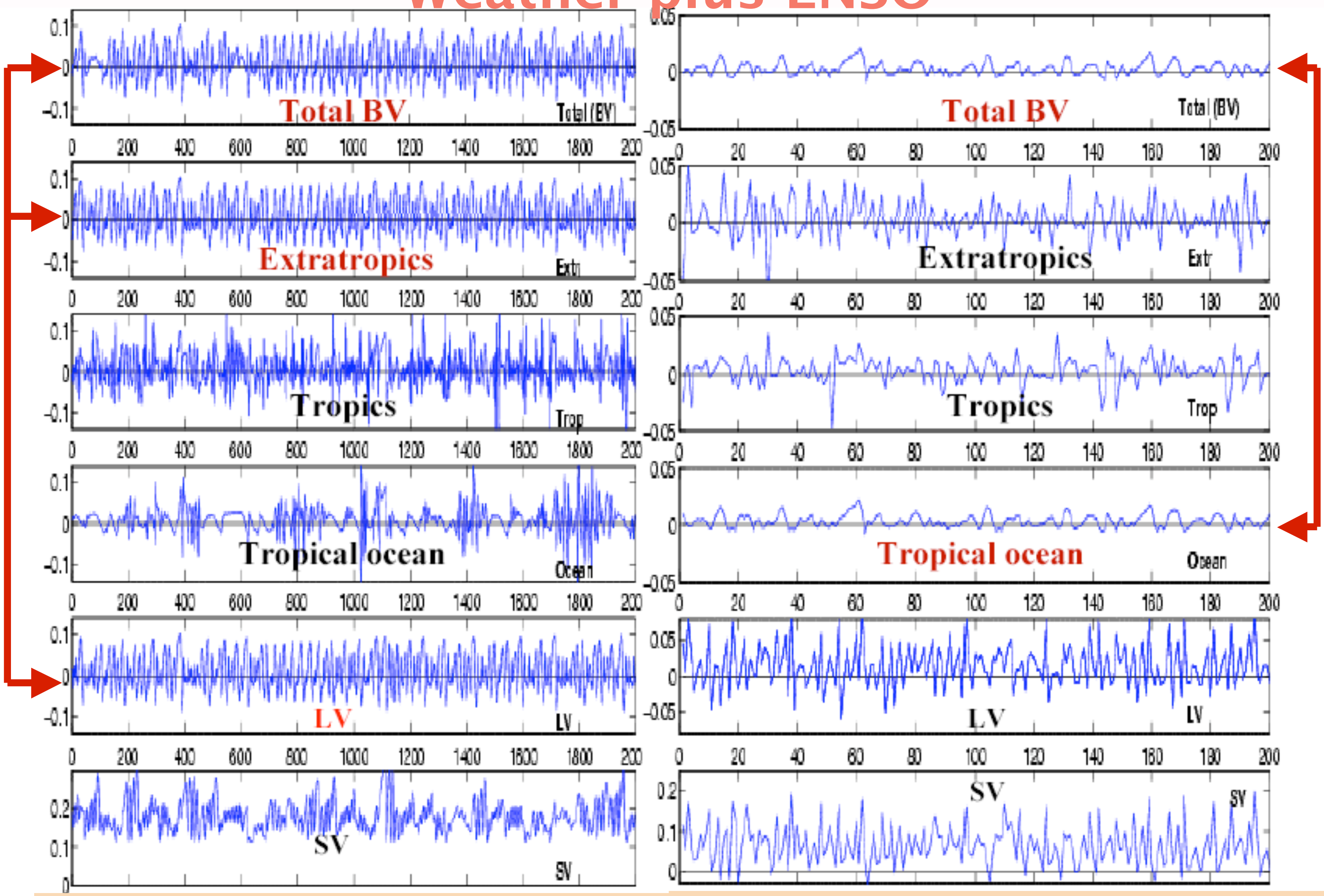
$$\frac{1}{\tau}\frac{dy_2}{dt} = rx_2 - y_2 - Sx_2 z_2 + C_2(y_1 + O)$$

$$\frac{1}{\tau}\frac{dz_2}{dt} = Sx_2 y_2 - bz_2 + C_2(z_1)$$

"Tropical-extratropical" (triply-coupled) system: the ENSO tropical atmosphere is weakly coupled to a fast "extratropical atmosphere" with weather noise


Solution of the"tropical atmosphere" system
Tropical atmosphere
Solution of the "ocean" system
Tropical ocean
Solution of the "extratropical atmosphere" system
Extratropical atmosphere
Time series of the x-component
Ext. Atmos.
Tro. Atmos
Ocean

# Breeding in a coupled Lorenz model: "Weather plus ENSO"




Short rescaling interval (5 steps) and small amplitude: fast modes

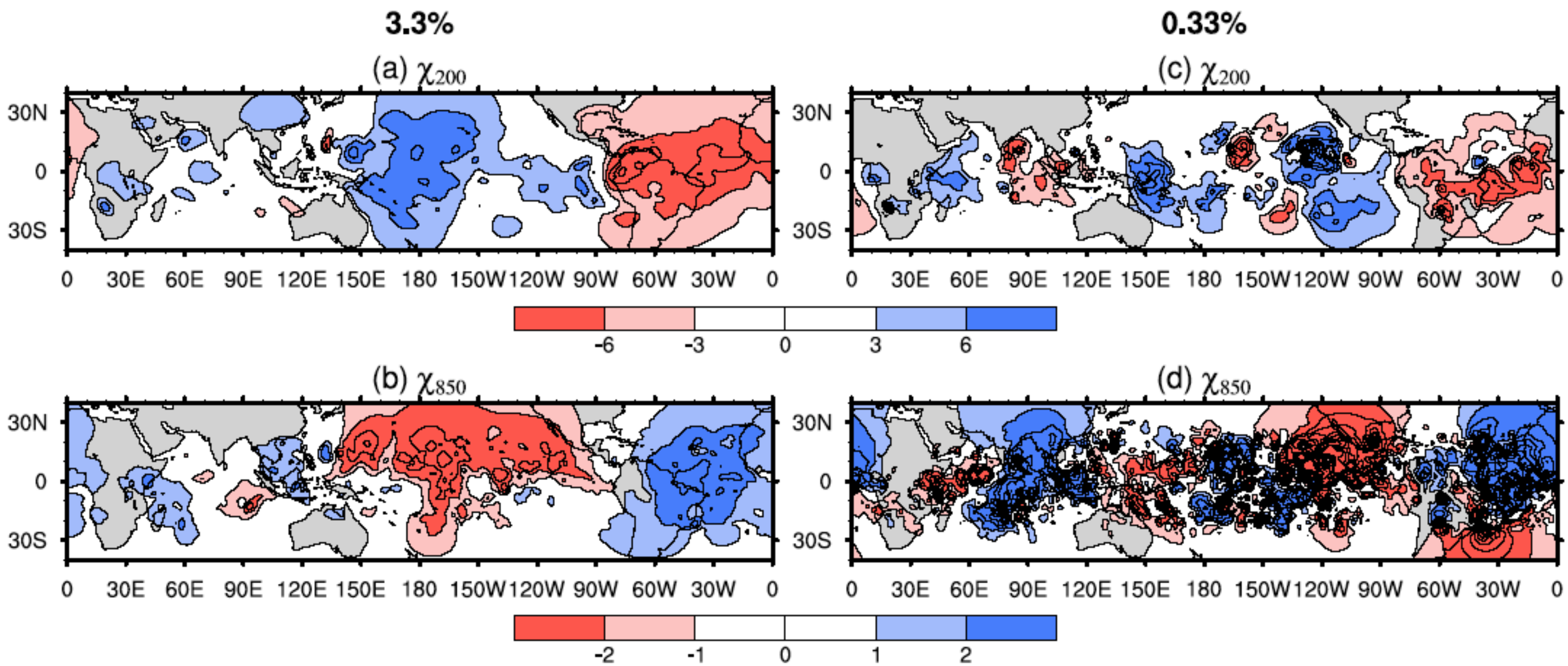
Long rescaling interval (50 steps) and large amplitude: ENSO modes

The linear approaches (LV, SV) cannot capture the slow ENSO signal

# Examples of breeding in a coupled ocean-atmosphere system with coupled instabilities

- In coupled fast/slow models, we can do breeding to isolate the slow modes
- We have to choose a slow variable and a long interval for the rescaling
- This identifies coupled instabilities. Examples
  - Madden-Julian Bred Vectors (Chikamoto et
  - NASA operational system with real observations (Yang et al 2007, MWR)
  - Ocean instabilities and their physical mechanisms (Hoffman et al, 2009, GRL)
  - Mars instabilities (Greybush et al, in preparation)

# Chikamoto et al (2007, GRL): They found the Madden-Julian instabilities BV by choosing an appropriate rescaling amplitude (only within the tropics)

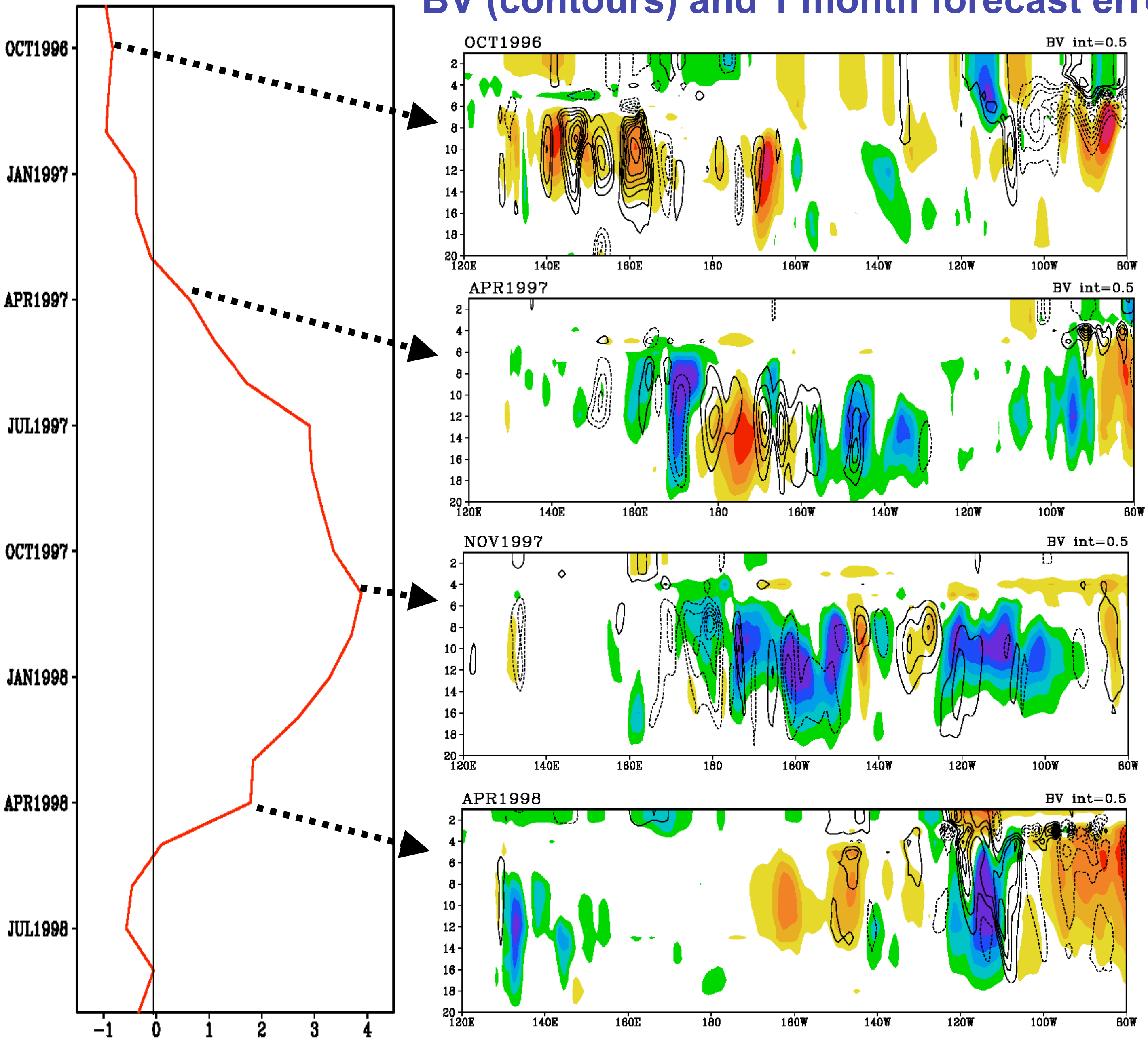


# Finding the shape of the errors in El Niño forecasts to improve data assimilation

- **Bred vectors**:
  - Differences between the control forecast and perturbed runs:
  - **Should show the shape of growing errors**
- **Advantages**
  - Low computational cost (two runs)
  - Capture coupled instabilities
  - Improve data assimilation

# Niño3 index

# Yang (2005): Vertical cross-section at Equator for BV (contours) and 1 month forecast error (color)



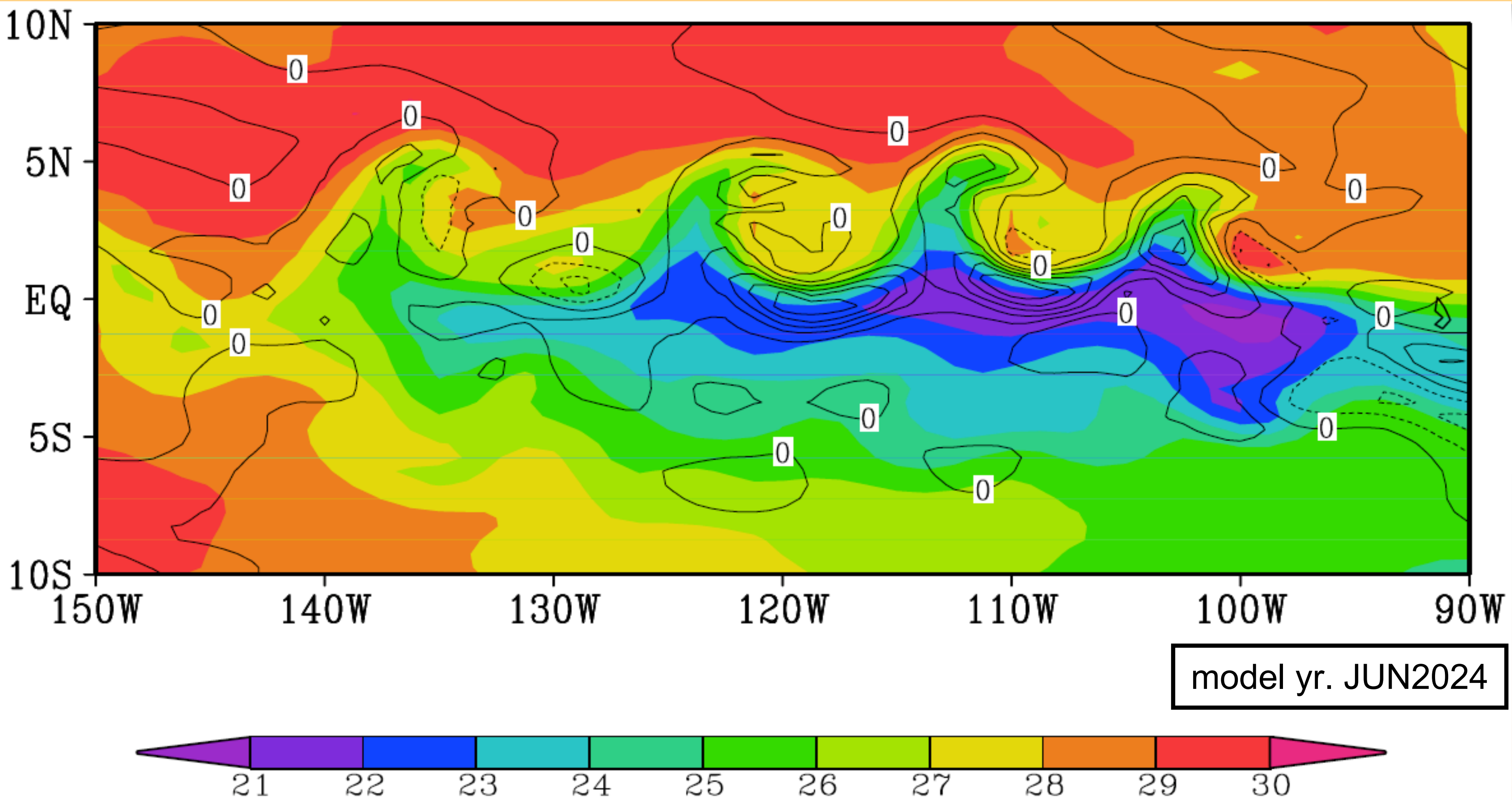

Before 97’ El Niño, error is located in W. Pacific and near coast region

During development, error shifts to lower levels of C. Pacific.

At mature stage, error shifts further east and it is smallest near the coast.

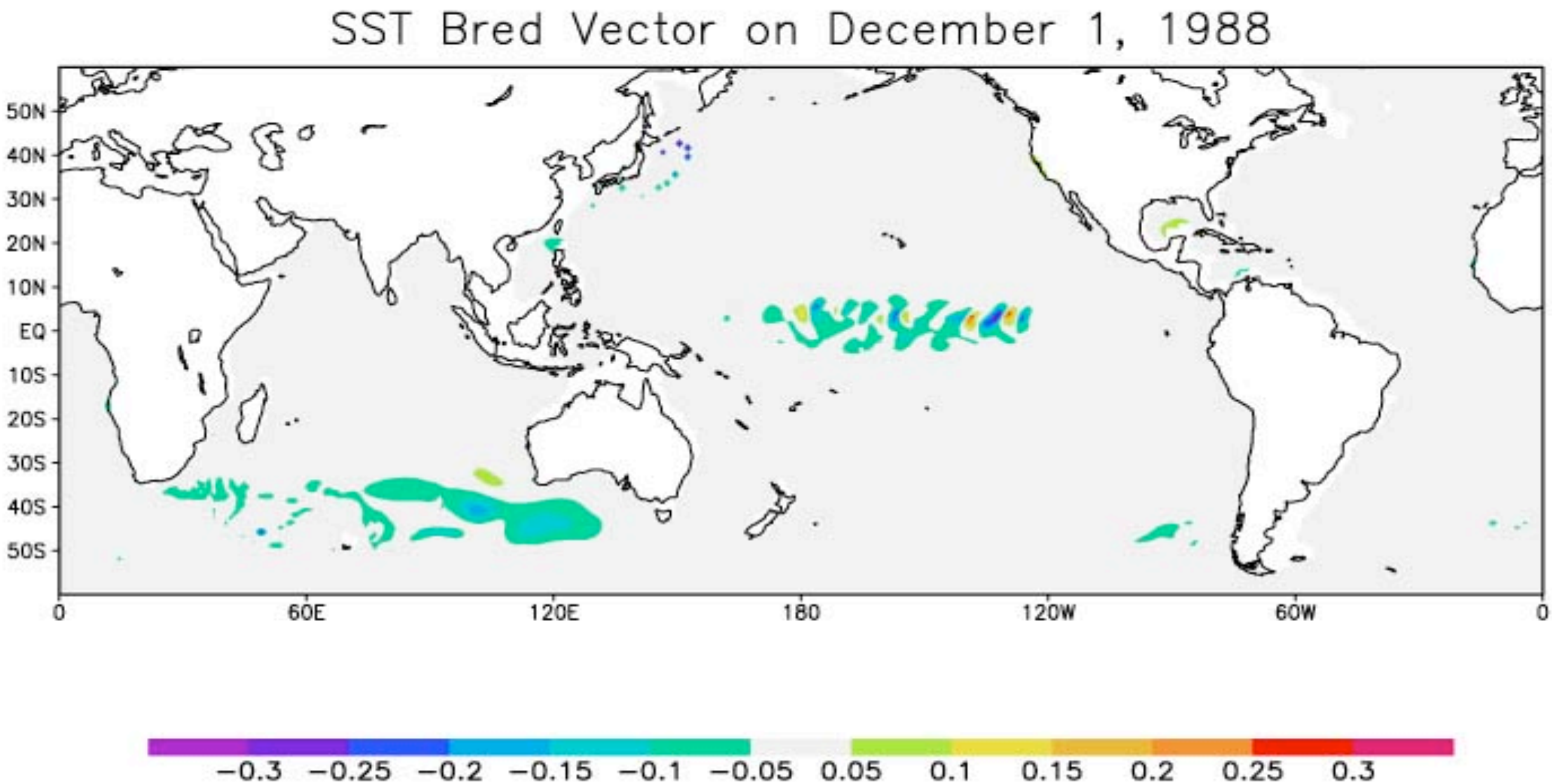
After the event, error is located mostly in E. Pacific.

# Yang et al., 2006: Bred Vectors (contours) overlay Tropical Instability waves (SST): making them grow and break!

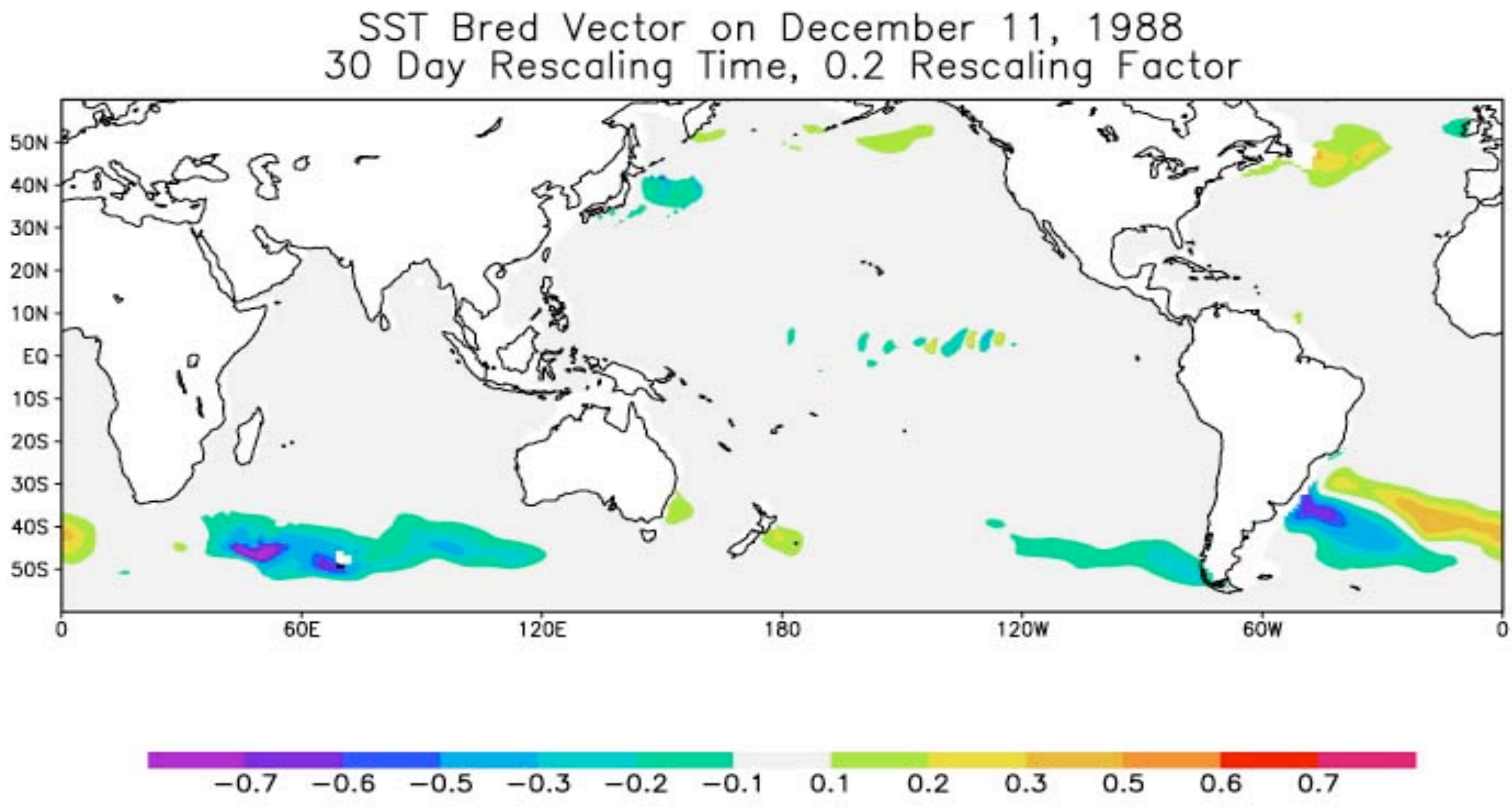


# Hoffman et al (2008): finding ocean instabilities with breeding time-scale 10-days captures tropical instabilities

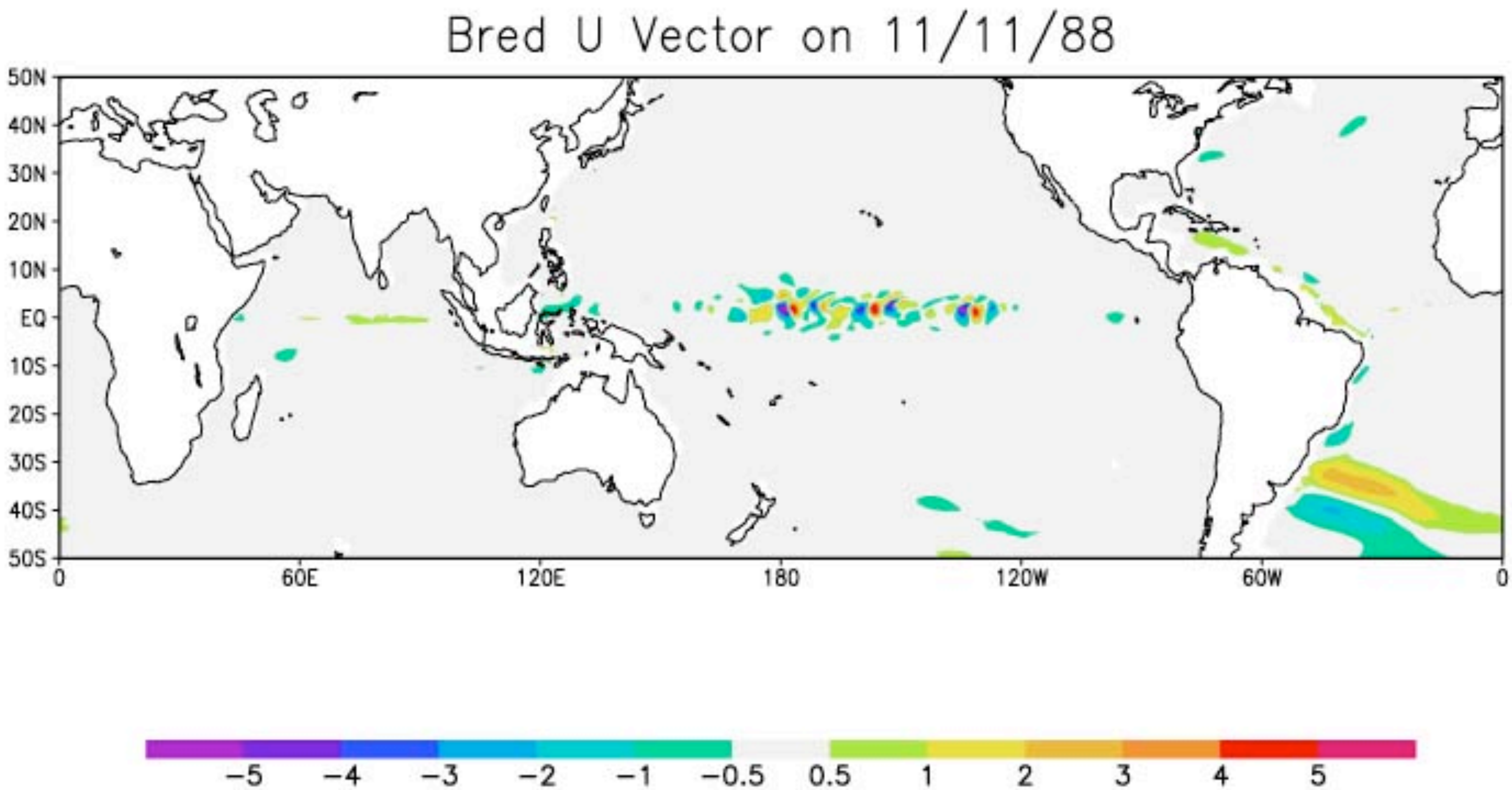
Breeding time scale: 10 days



# When the rescaling time scale is 30 days, extratropical instabilities dominate



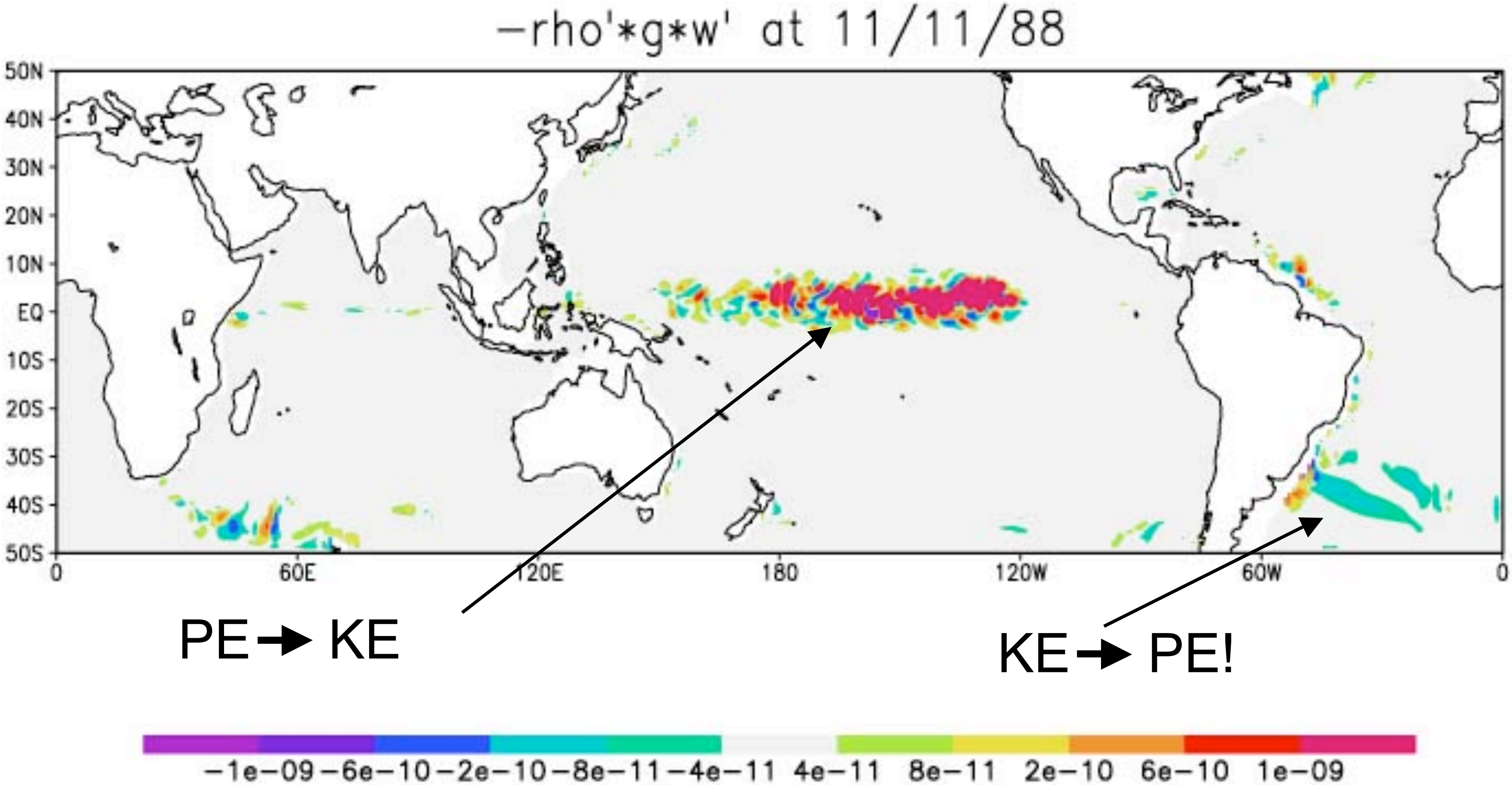
## Here we have both tropical and "South Atlantic Convergence Zone" instabilities. Can we determine the dynamic origin of the instabilities?



**The Bred Vector Kinetic Energy equation can be computed exactly because both control solution and perturbed solution satisfy the full equations!**

$$\frac{\partial KE_{bv}}{\partial t} = horizontal\ \ fluxes - \rho_b g w_b + \ldots$$

Conversion from potential to kinetic energy

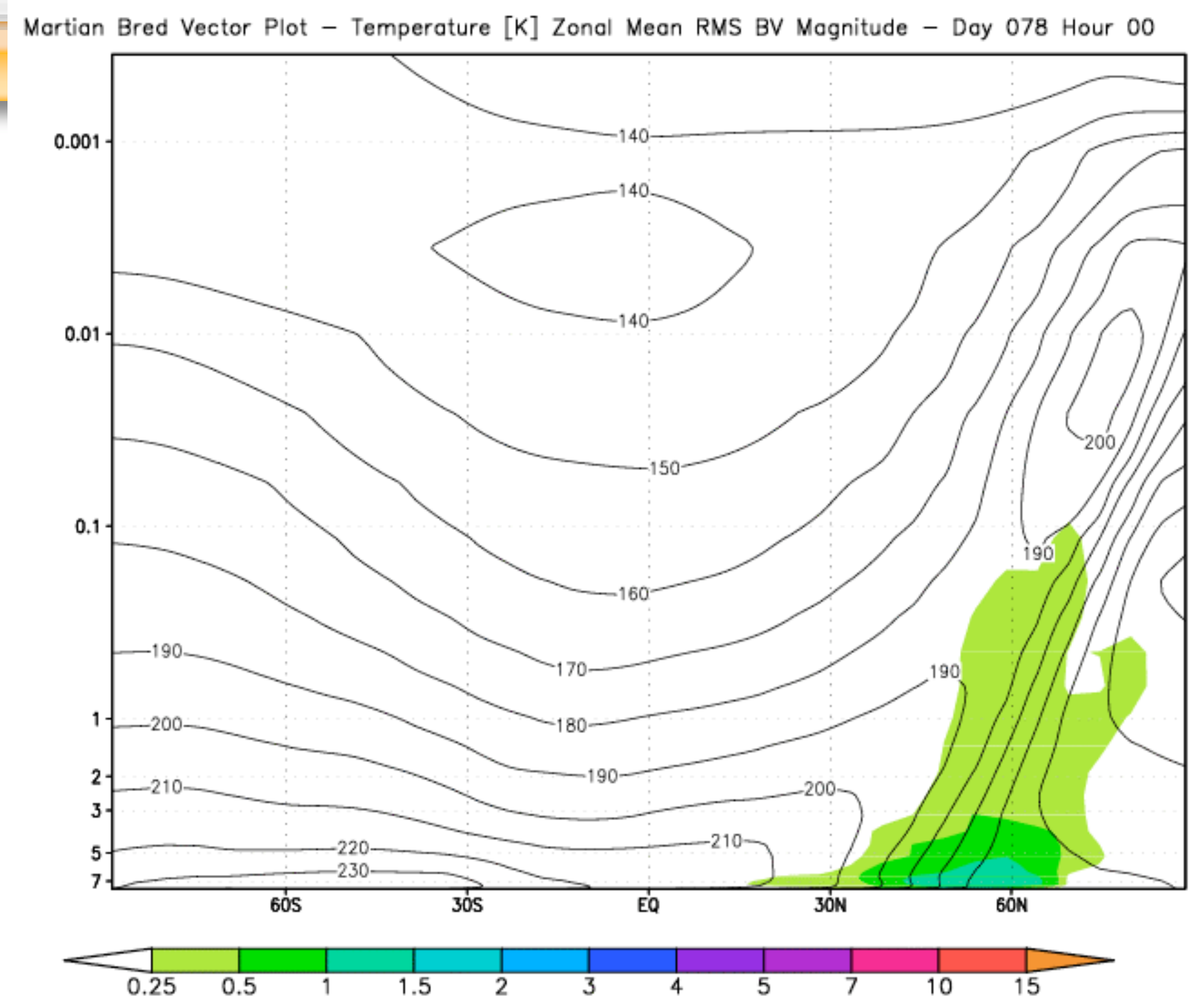



PE → KE

KE → PE!

# Bred vectors in Mars: annual cycle

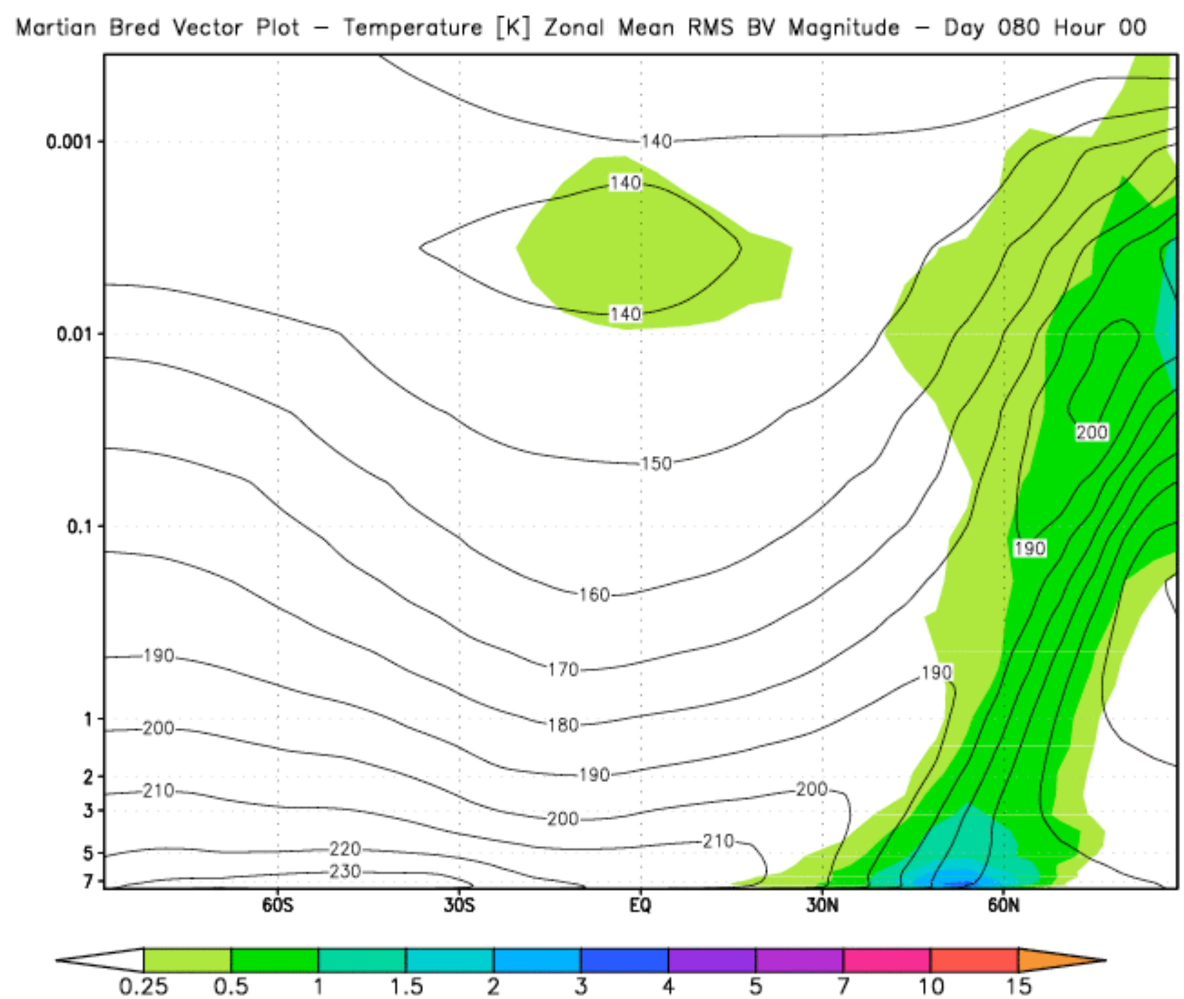
Steve Greybush




winter

Day 078 (Ls= 302, Boreal Winter): BV activity near surface temperature front begins to flare up.

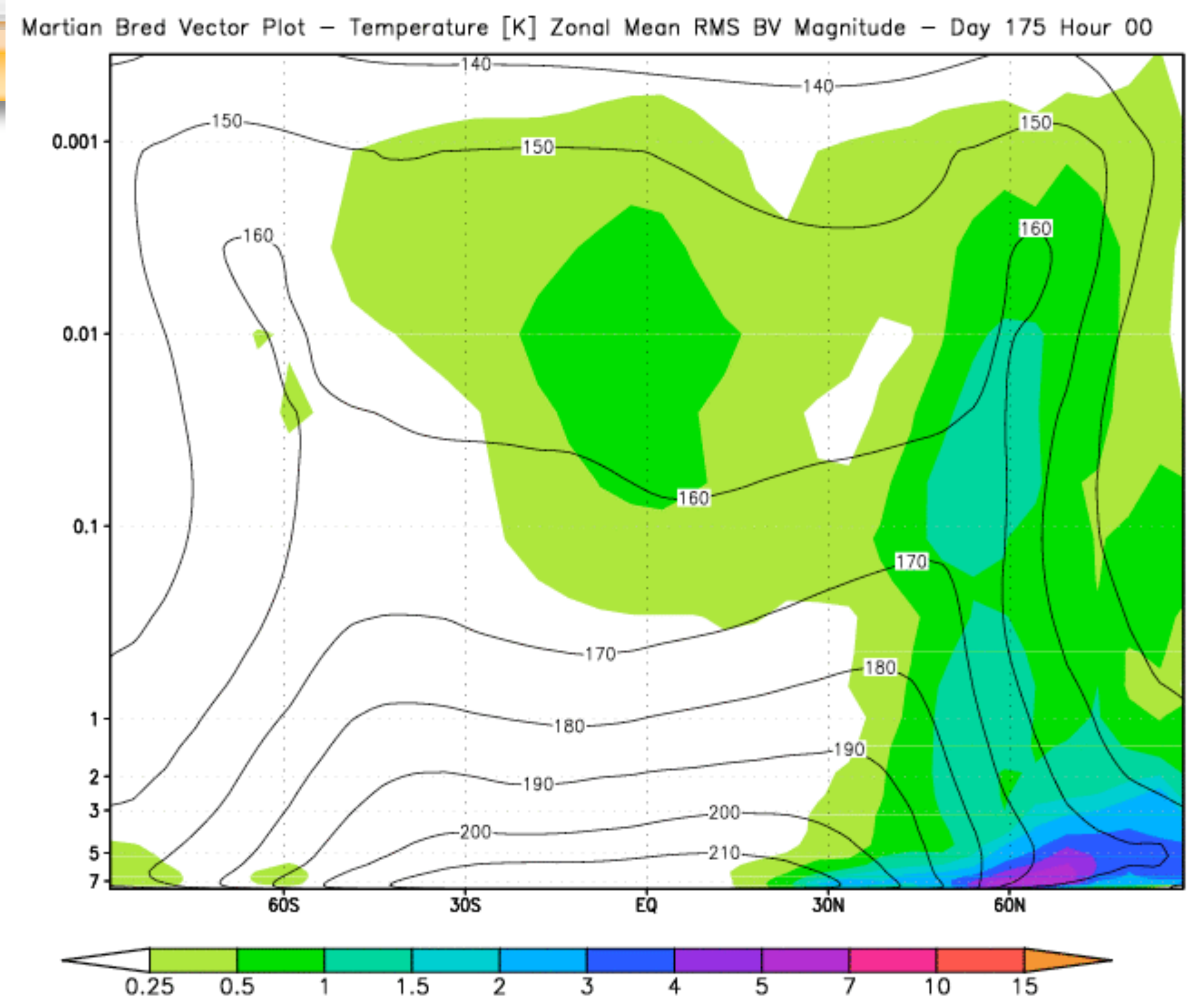
# MGCM-LETKF-TES Martian Atmosphere Reanalysis Project




2 days later

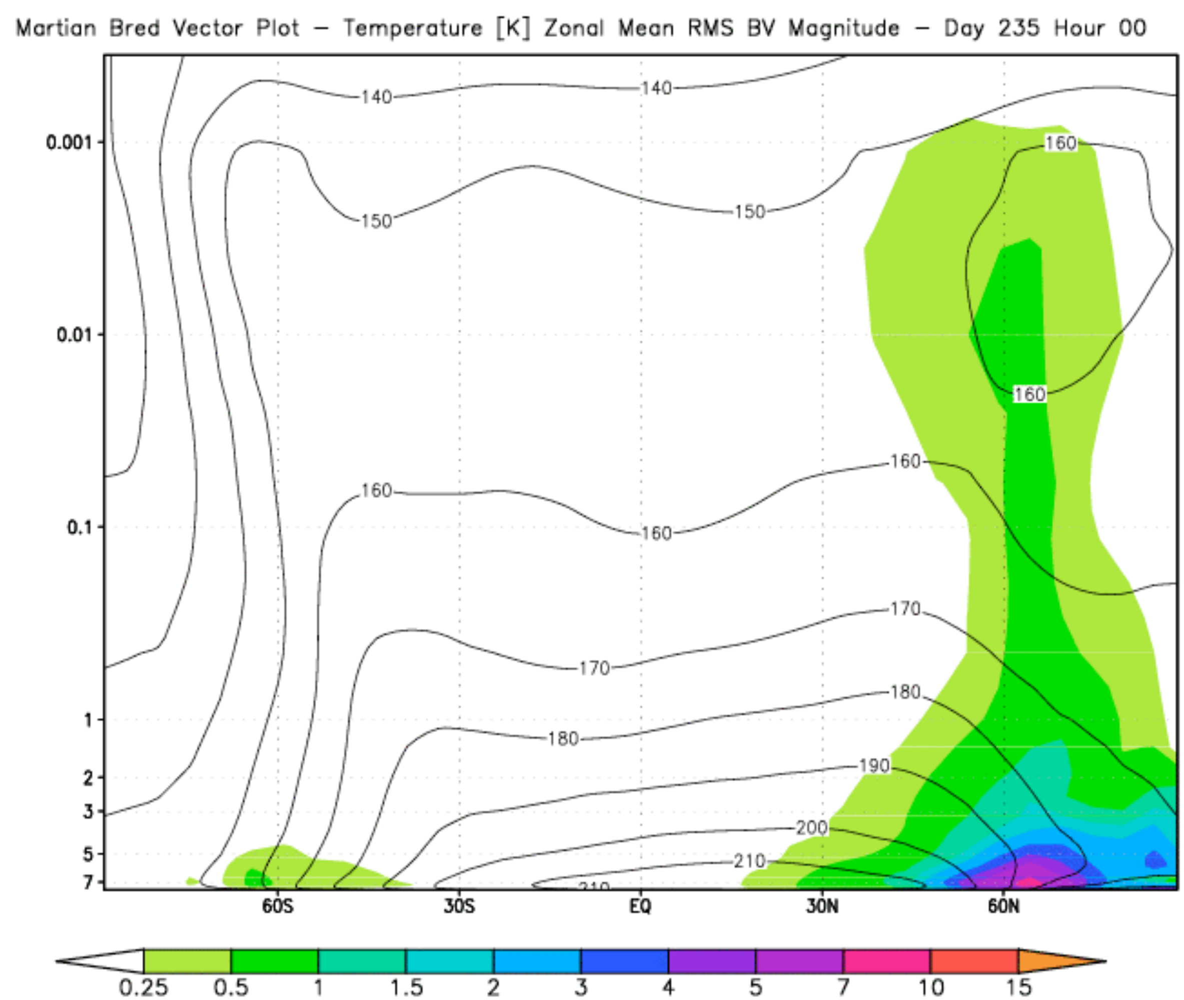
Day 080 (Ls= 304, Boreal Winter): Just two days later, BV now extends vertically along the length of the front. Connection to the upper level tropics begins.

# MGCM-LETKF-TES Martian Atmosphere Reanalysis Project




equinox

Day 175 (Ls= 358, near boreal vernal equinox): Typical winter BV activity along temperature front with upper level tropical connection. First hint of southern hemisphere activity.



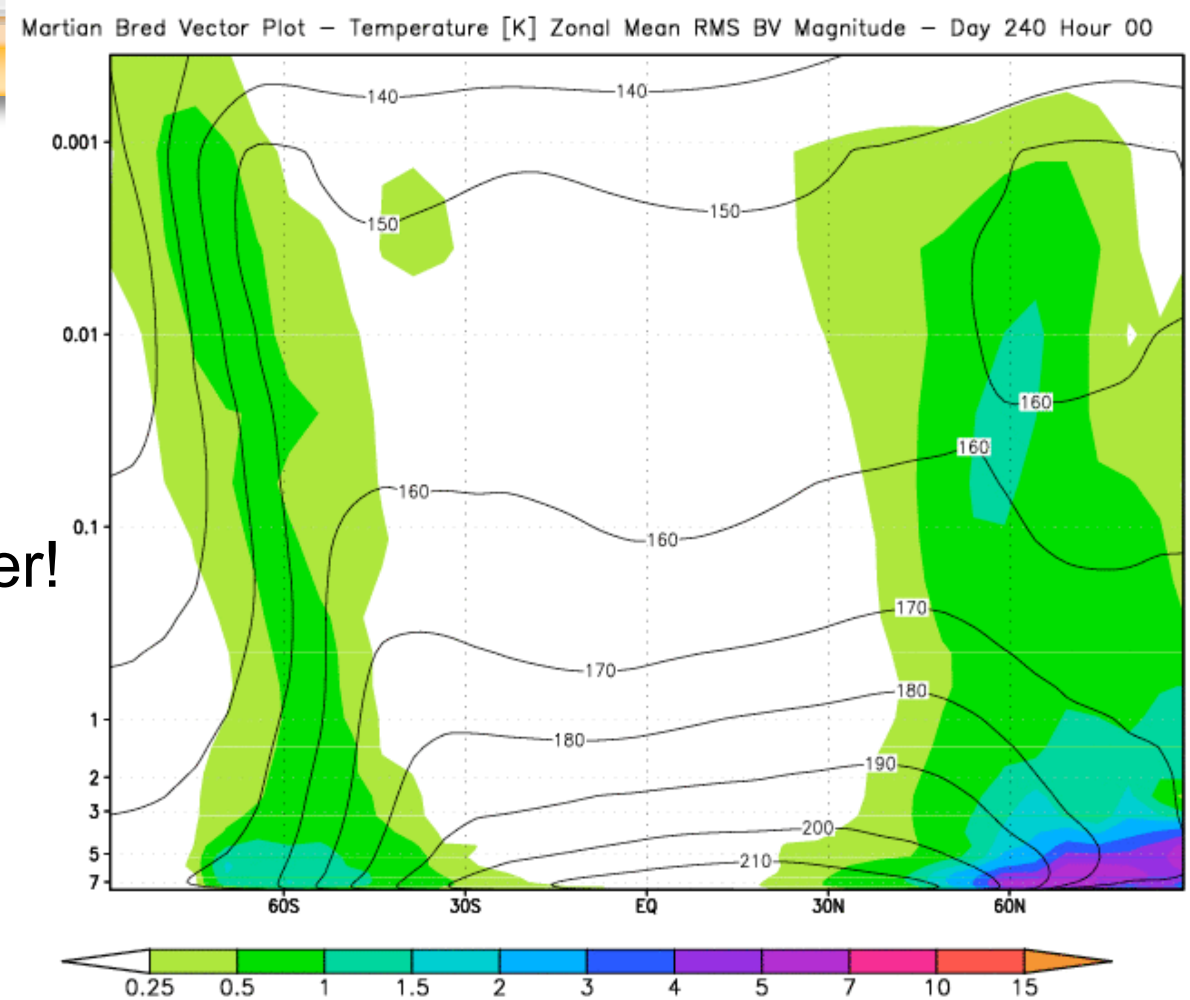

spring

Day 235 (Ls= 22, early boreal spring): Winter BV activity has begun to weaken, as the tropical connection has disappeared.

MGCM-LETKF-TES Martian Atmosphere Reanalysis Project

fall

5 days later!

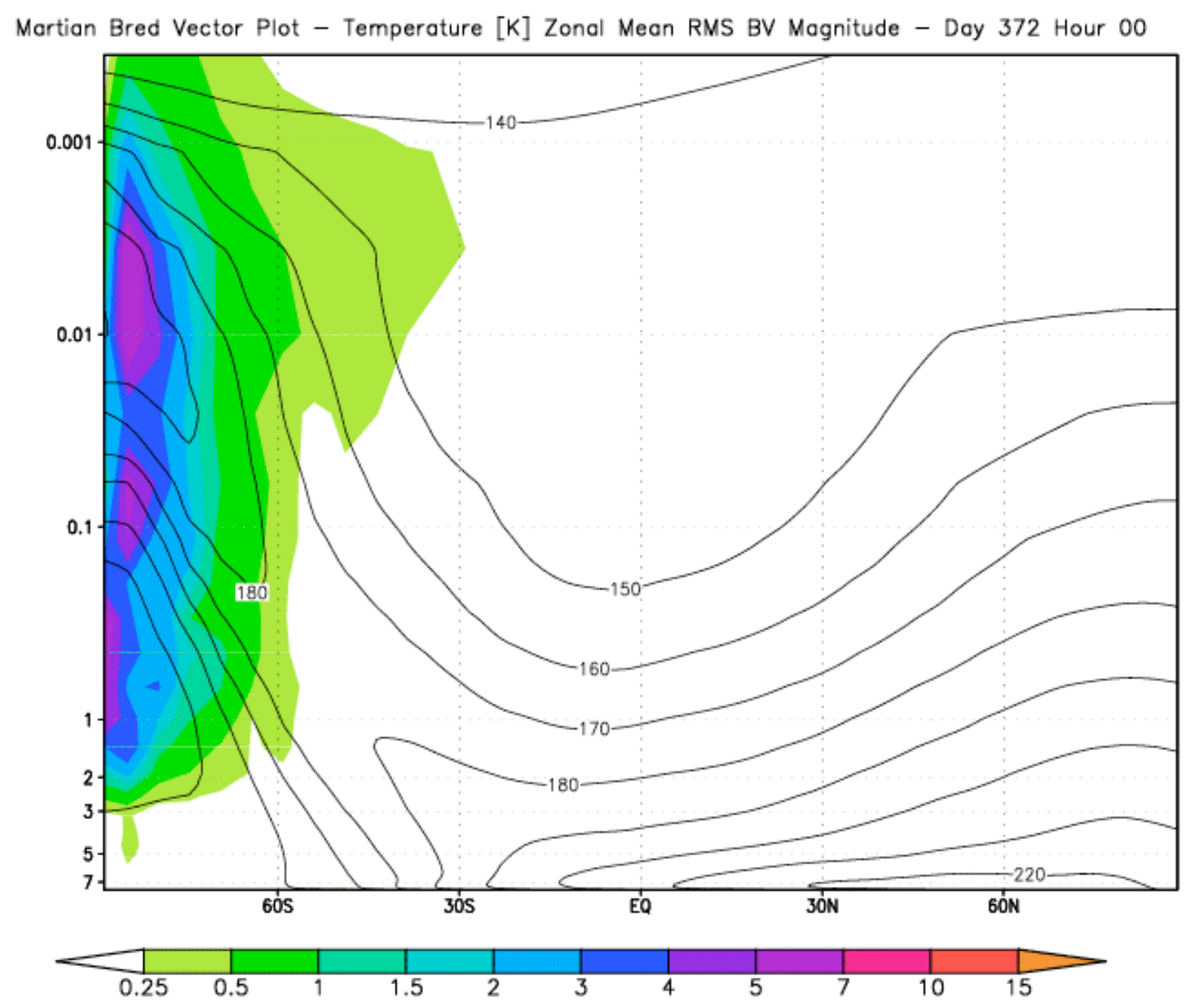



spring

Day 240 (Ls= 230, early boreal spring): Southern hemisphere activity has now grown rapidly along austral front.

MGCM-LETKF-TES Martian Atmosphere Reanalysis Project

winter




Day 372 (Ls= 90, austral winter solstice): Southern hemisphere active in polar regions; northern hemisphere activity has subsided.

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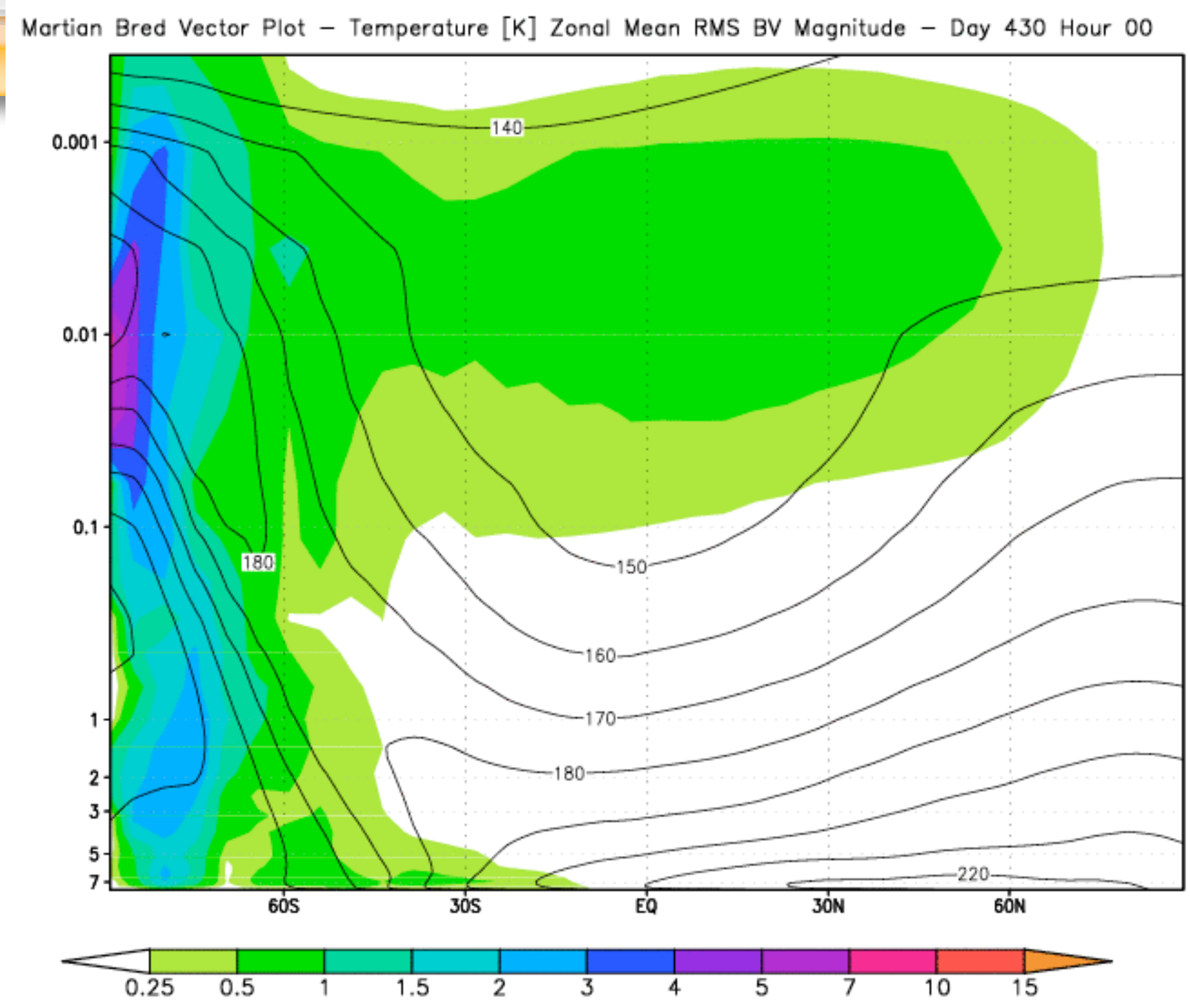
winter




Day 430 (Ls= 116. austral mid-winter): Southern hemisphere BV activity now assumes full spatial extent.

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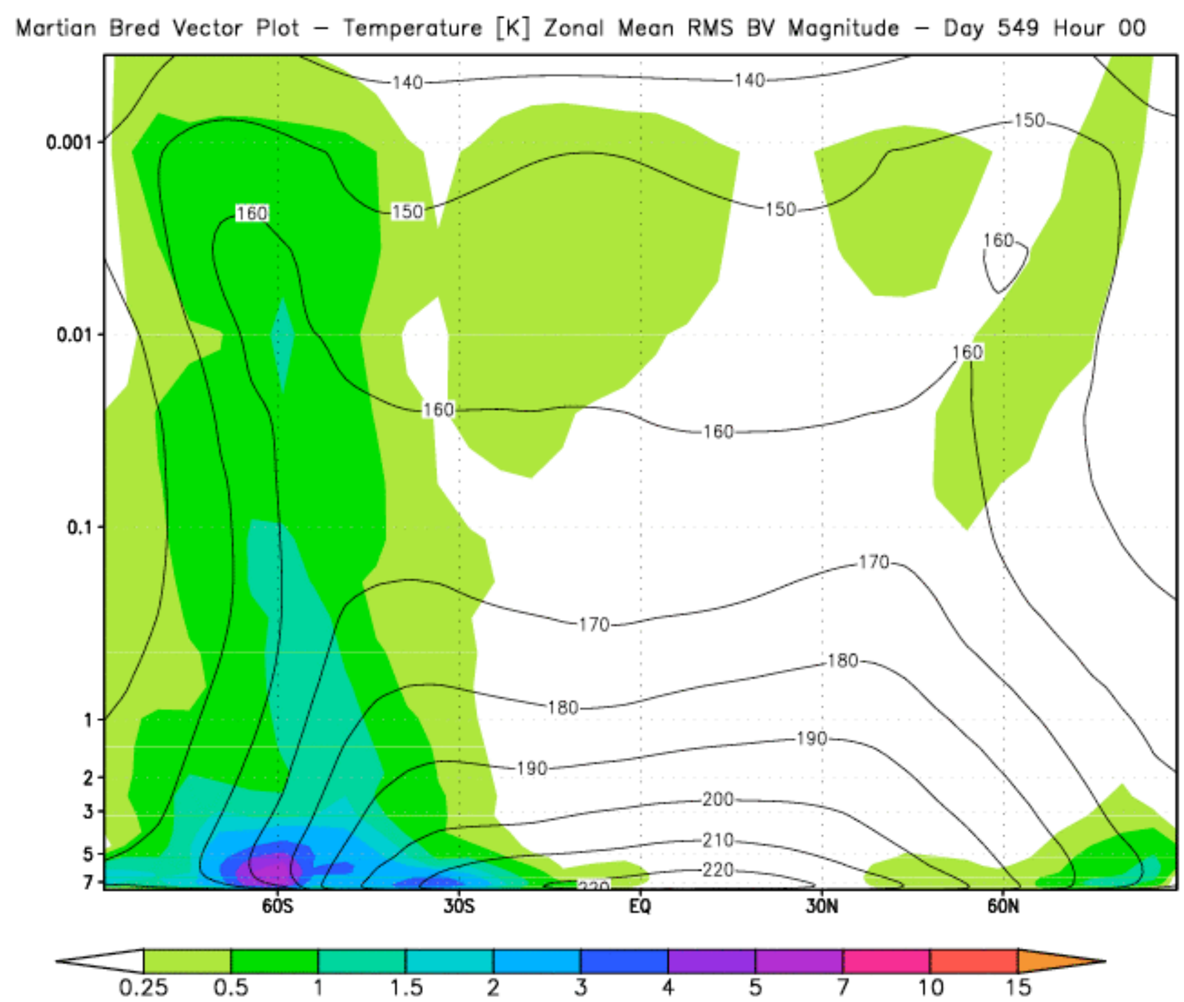


equinox

Day 549 (near boreal autumn equinox): Signs BV of activity in the northern hemisphere have resumed.

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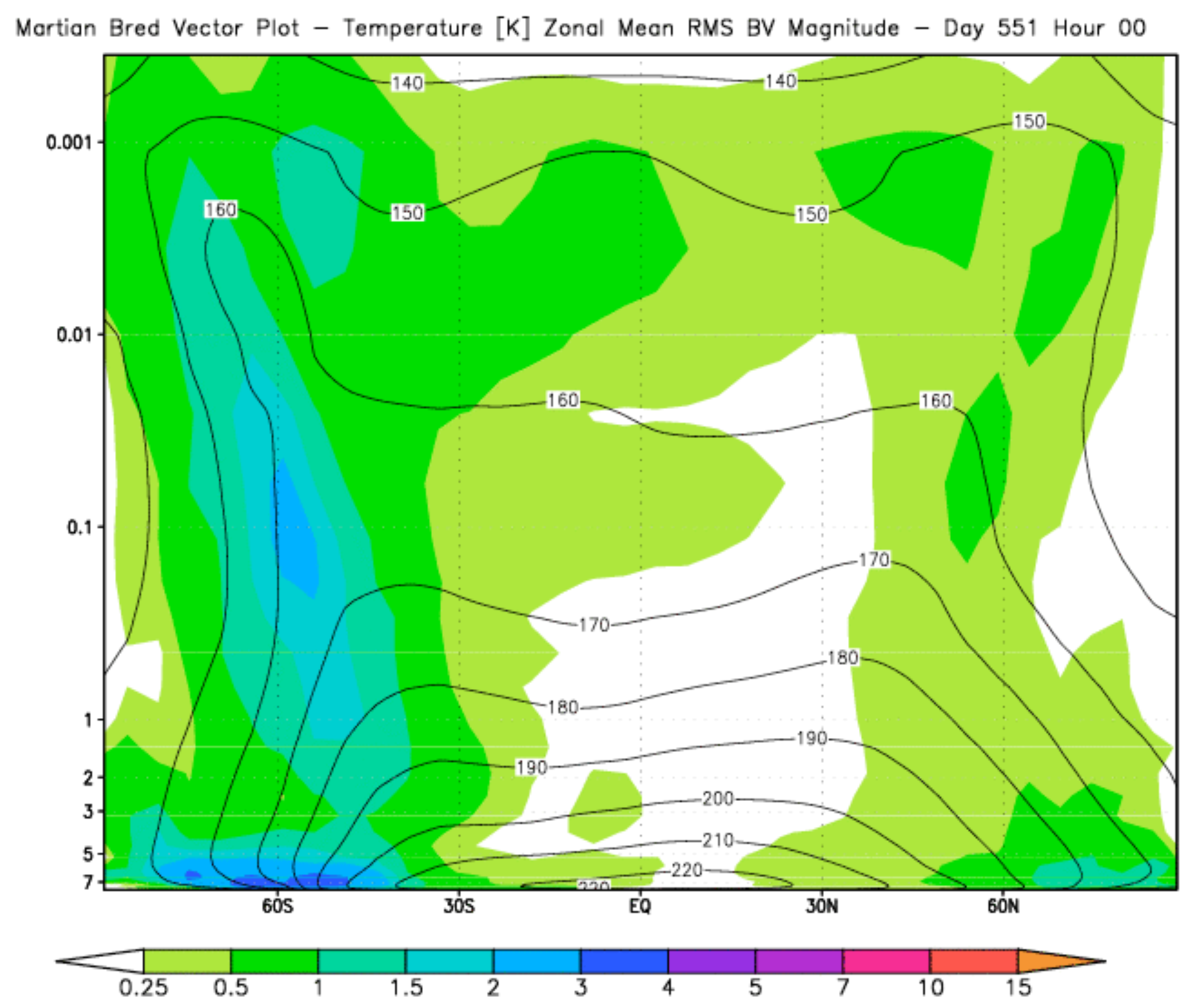


Day 551 (Ls= 180, boreal autumn equinox): Activity in northern hemisphere has extended vertically.

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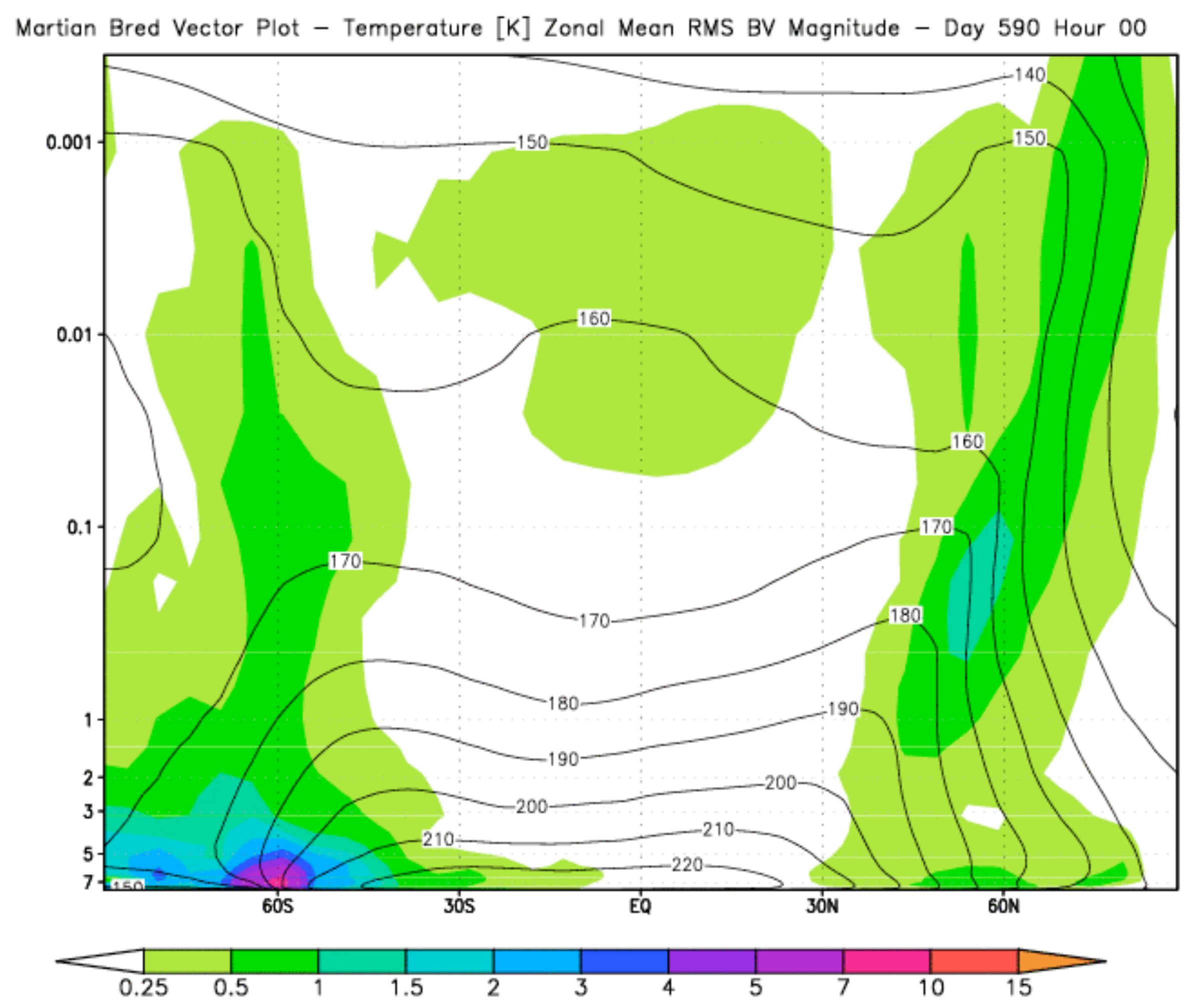


Day 590 (mid boreal autumn): Activity in both hemispheres, but most intense along southern polar front.

fall

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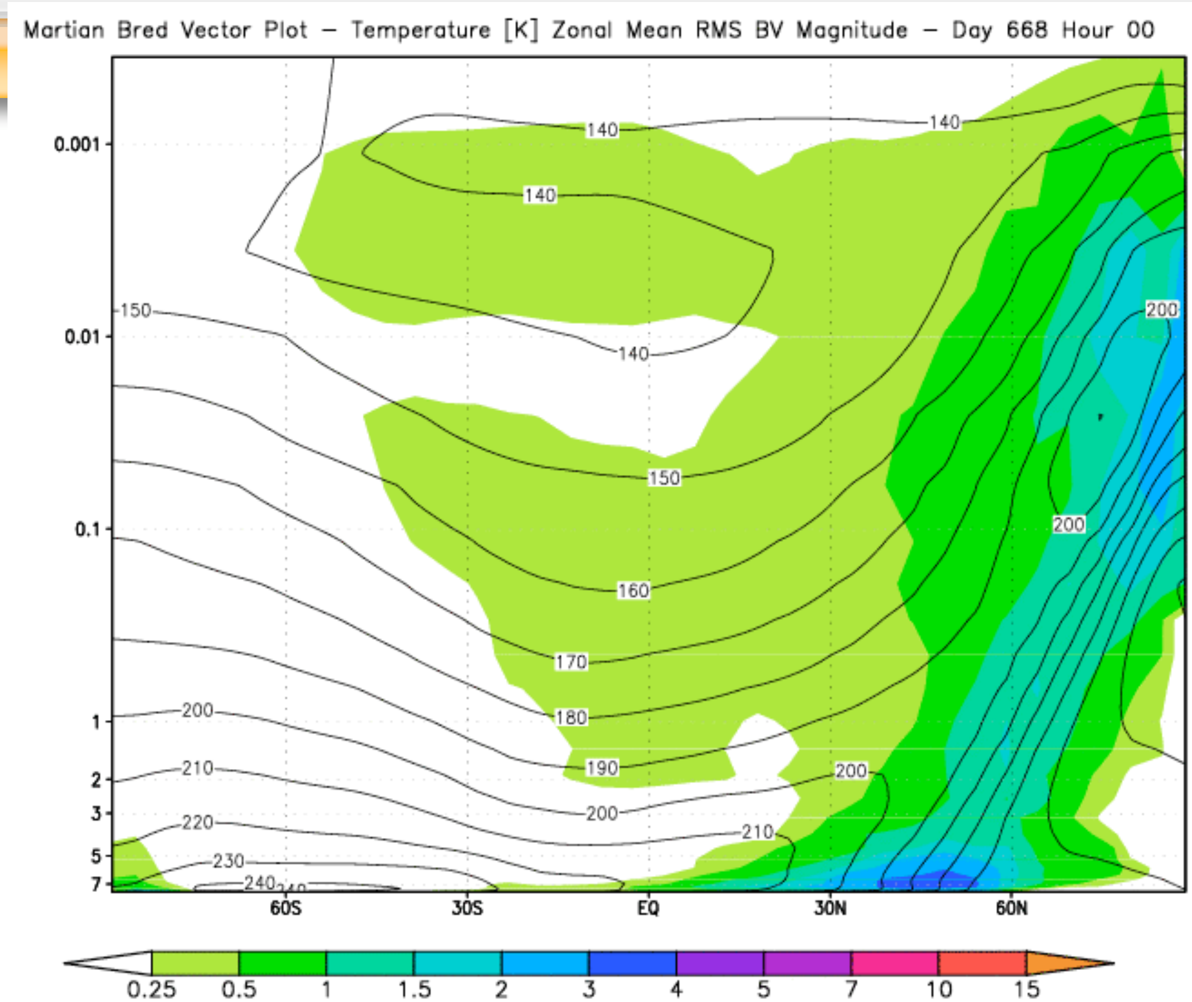


winter

Day 668 (Ls= 252, prior to boreal winter solstice): The seasons have returned full circle, with southern hemisphere activity fading and northern winter dominant.

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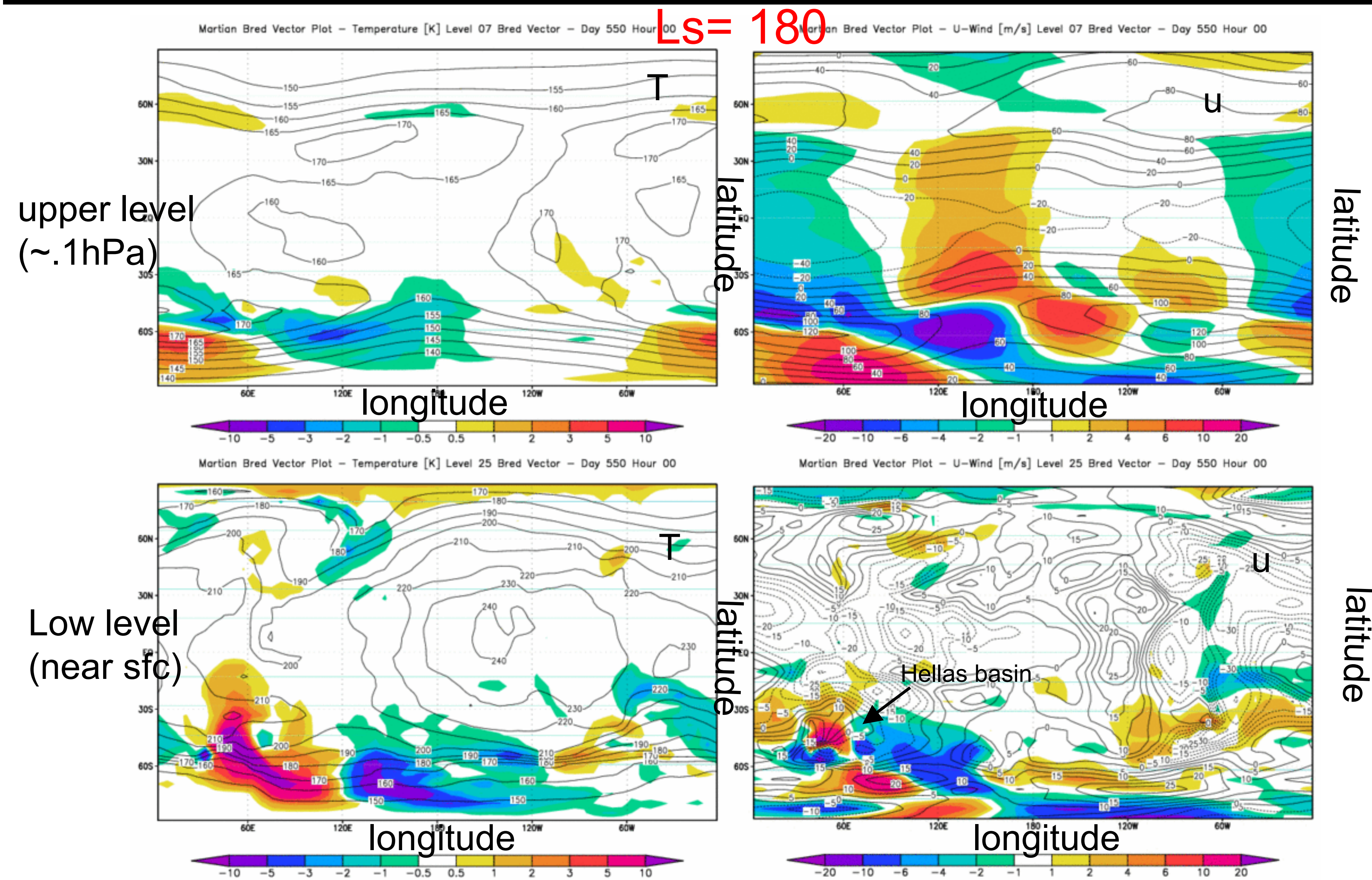

# Energy Equations

- Begin from the equations of motion for the MGCM (momentum equation in sigma coordinates).
- Control run and perturbed run both satisfy these equations exactly.
- Derive kinetic energy equation for bred vectors (difference between control and perturbed runs).
- These equations should allow us to diagnose precisely the physical origin of the instabilities: the KE of the bred vectors can change due to: Baroclinic or barotropic instabilities, pressure work, heating, or friction.

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# BV Kinetic Energy Equation

$$\frac{\partial K_b}{\partial t} = -\left[\boldsymbol{v} \bullet \nabla K_b + \dot{\sigma}\frac{\partial K_b}{\partial \sigma}\right] - \left[\nabla \bullet (\boldsymbol{v}_b \Phi_b) + \frac{\partial \dot{\sigma}_b \Phi_b}{\partial \sigma}\right] - \left[\dot{\sigma}_b \alpha_b p_{sb}\right] - \left[\boldsymbol{v}_b \bullet \left((\boldsymbol{v}_b \bullet \nabla)\boldsymbol{v}_c + \dot{\sigma}_b \frac{\partial \boldsymbol{v}_c}{\partial \sigma}\right)\right]$$

$$-\frac{\Phi_b}{p_{sb}}\left(\frac{\partial p_{sb}}{\partial t} + \boldsymbol{v}_b \bullet \nabla p_{sb}\right) + \boldsymbol{v}_b \bullet \left(-\sigma\alpha\nabla p_s + \sigma_c \alpha_c \nabla p_{sc}\right)$$

Term 1: Transport of BV KE by the total flow

Term 2: Pressure Work (ageostrophic)

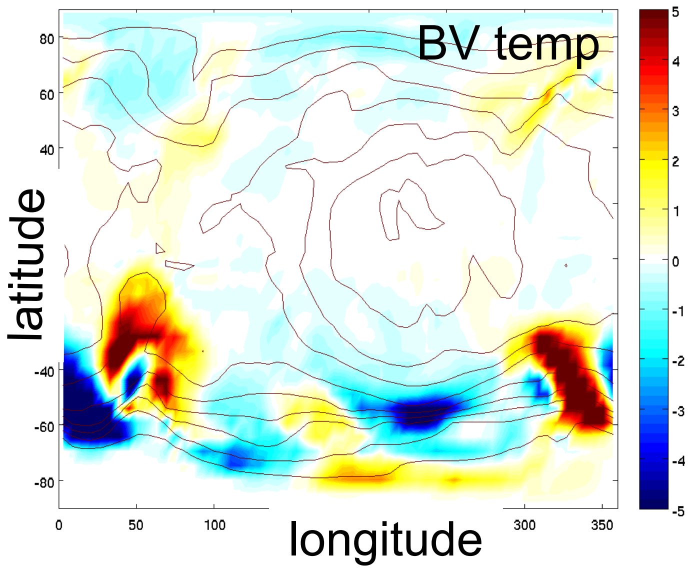
Term 3: Baroclinic Conversion Term

Term 4: Barotropic Conversion Term

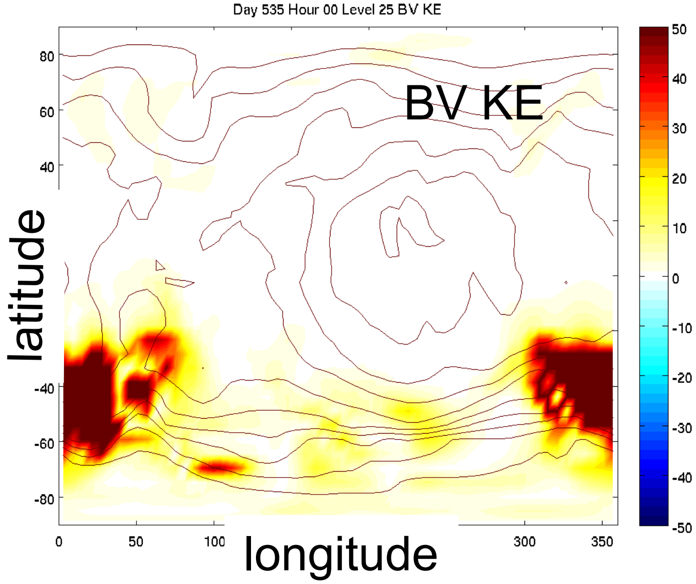
Term 5: Coordinate Transform Term

# MGCM-LETKF-TES Martian Atmosphere Reanalysis Project

Day 535 Hour 00 Level 25 Temperature BV [K]

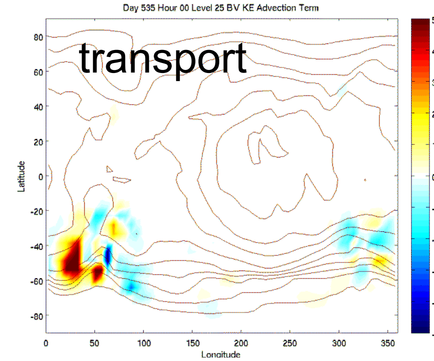



Day 535 Hour 00 Level 25 BV KE

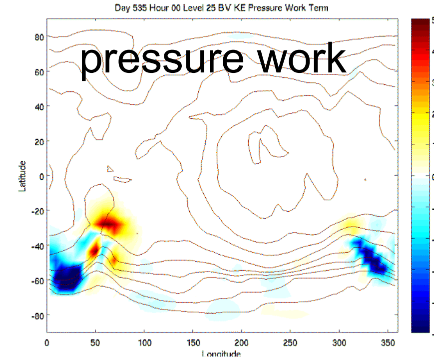



* LOW LEVEL, PRELIMINARY RESULTS *

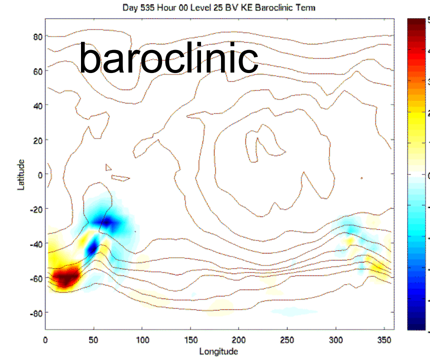
Day 535 Hour 00 Level 25 BV KE Advection Term



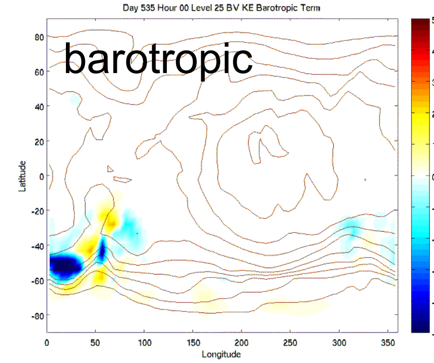

Day 535 Hour 00 Level 25 BV KE Pressure Work Term



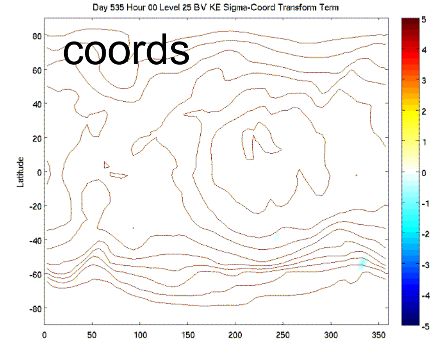

Day 535 Hour 00 Level 25 BV KE Baroclinic Term



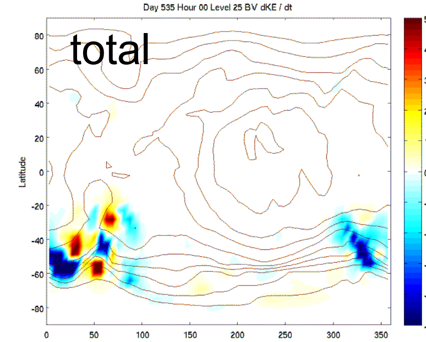

Day 535 Hour 00 Level 25 BV KE Barotropic Term




Day 535 Hour 00 Level 25 BV KE Sigma-Coord Transform Term




Day 535 Hour 00 Level 25 BV dKE / dt



# Summary: We can fight chaos and extend predictability by understanding error growth

- Chaos is not random: it is generated by physical instabilities
- Breeding is a simple and powerful method to find the growth and shape of the instabilities
- These instabilities also dominate the forecast errors: we can use their shape to improve data assimilation.
- Ensemble Kalman Filter is the ultimate method to explore and "beat chaos" through data assimilation.
- In the "chaotic" Lorenz model the growth of bred vectors predicts regime changes and how long they will last.
- Nonlinear methods, like Breeding and EnKF, can take advantage of the saturation of fast weather noise and isolate slower instabilities.
- Bred Vectors predict well the evolution of coupled forecast errors, and help explain the physical origin of ocean instabilities
- Ensembles of BV improve the seasonal and interannual forecast skill, especially during the "spring barrier"
- Mars!

REFERENCES: www.weatherchaos.umd.edu
www.atmos.umd.edu/~ekalnay