Chaos in a Generalized Lorenz System

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Abstract

A three-component dynamic system describing a quantum cavity electrodynamic device with a pumping, nonlinear dissipation and additional external perturbation is studied. Various dynamical regimes are investigated in terms of divergent trajectories approaches and fractal statistics. It has been shown that time dissipative structures can be formed in such system, with variation of pumping and nonlinear dissipation rates. Transitions to chaotic regime and the corresponding chaotic attractor are studied in detail. It has been shown that transitions between regular and chaotic regimes can be realized by scenarios of Ruelle-Takens, Feigenbaum and due to chaotic layer appearance.

Introduction

Objective

The main objective is to consider a character of transition to chaos in a three-component dynamic system describing a quantum cavity electrodynamic device.

Main tasks

1. To make a model of a quantum cavity electrodynamic device with pumping, nonlinear dissipation and additional external perturbation.
2. To investigate an influence of nonlinear dependence of an order parameter relaxation time vs. its value on dynamical regimes changing in the model without external perturbation.
3. To set possible scenarios of transition to chaotic dynamic in the model with nonlinear dissipation and external perturbation.
4. To determine type of realized attractors using divergent trajectories approaches and fractal statistics.

The Model

The equations governing the Lorenz oscillator

$$\frac{dX}{dt} = \sigma(Y - X), \quad \frac{dY}{dt} = X(r - Z) - Y, \quad \frac{dZ}{dt} = XY - \beta Z \tag{1}$$

The Lorenz-Haken model for Fabry-Perot cavity

$$\frac{d\eta}{dt} = -\eta + h, \quad \sigma\frac{dh}{dt} = -h + \eta S, \quad \varepsilon\frac{dS}{dt} = (r - S) - \eta h \tag{2}$$

with renormalized variables

$$t' = \sigma t,\ \eta = X/\sqrt{\beta},\ h = Y/\sqrt{\beta},\ S = r - Z,\ \varepsilon = \sigma/\beta$$

Dispersion of relaxation time (cavity loss) $\tau(\eta)$:

$$\frac{1}{\tau(\eta)} = 1 + \frac{\kappa}{1+\eta^2} \tag{3}$$

An additional external perturbations are provided by external potential in a form of the fold catastrophe

$$V_e = A\eta + \frac{C}{3}\eta^3 \tag{4}$$

A generalized Lorenz system has the form

$$\frac{d\eta}{dt} = -\eta + h + f_\tau + f_e, \quad \sigma\frac{dh}{dt} = -h + \eta S, \quad \varepsilon\frac{dS}{dt} = (r - S) - \eta h \tag{5}$$

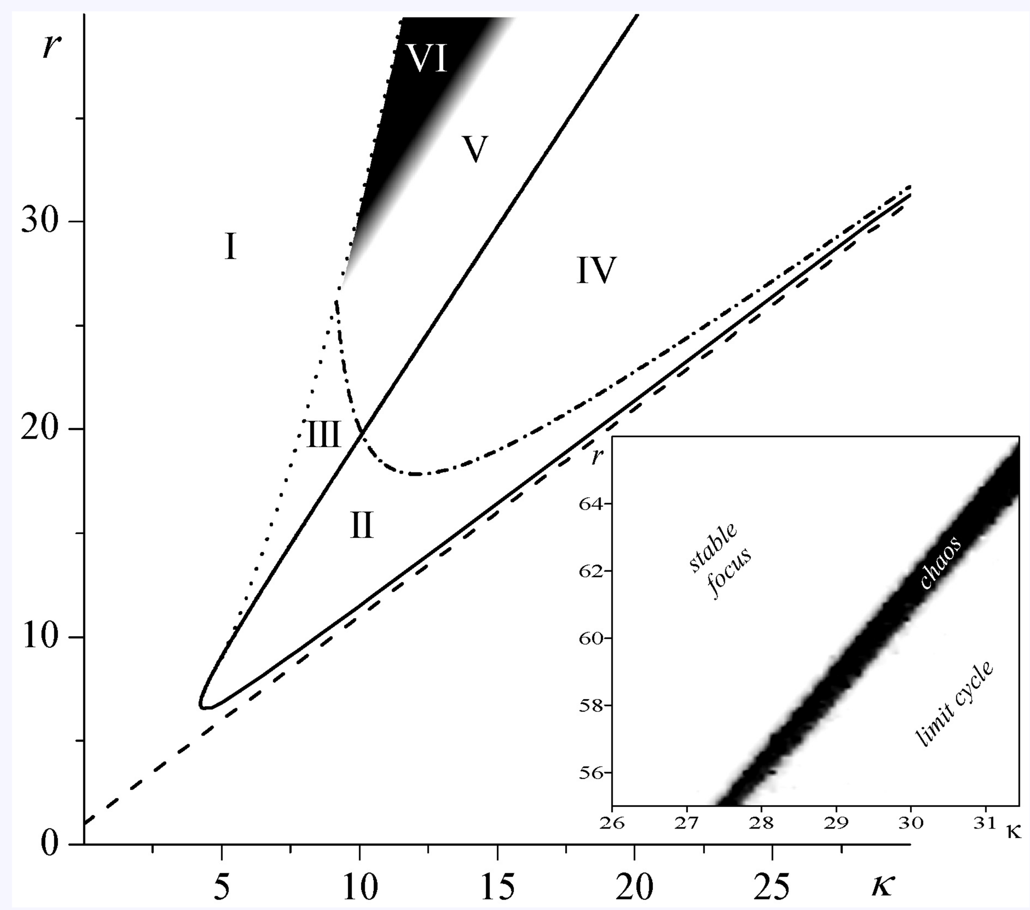
where

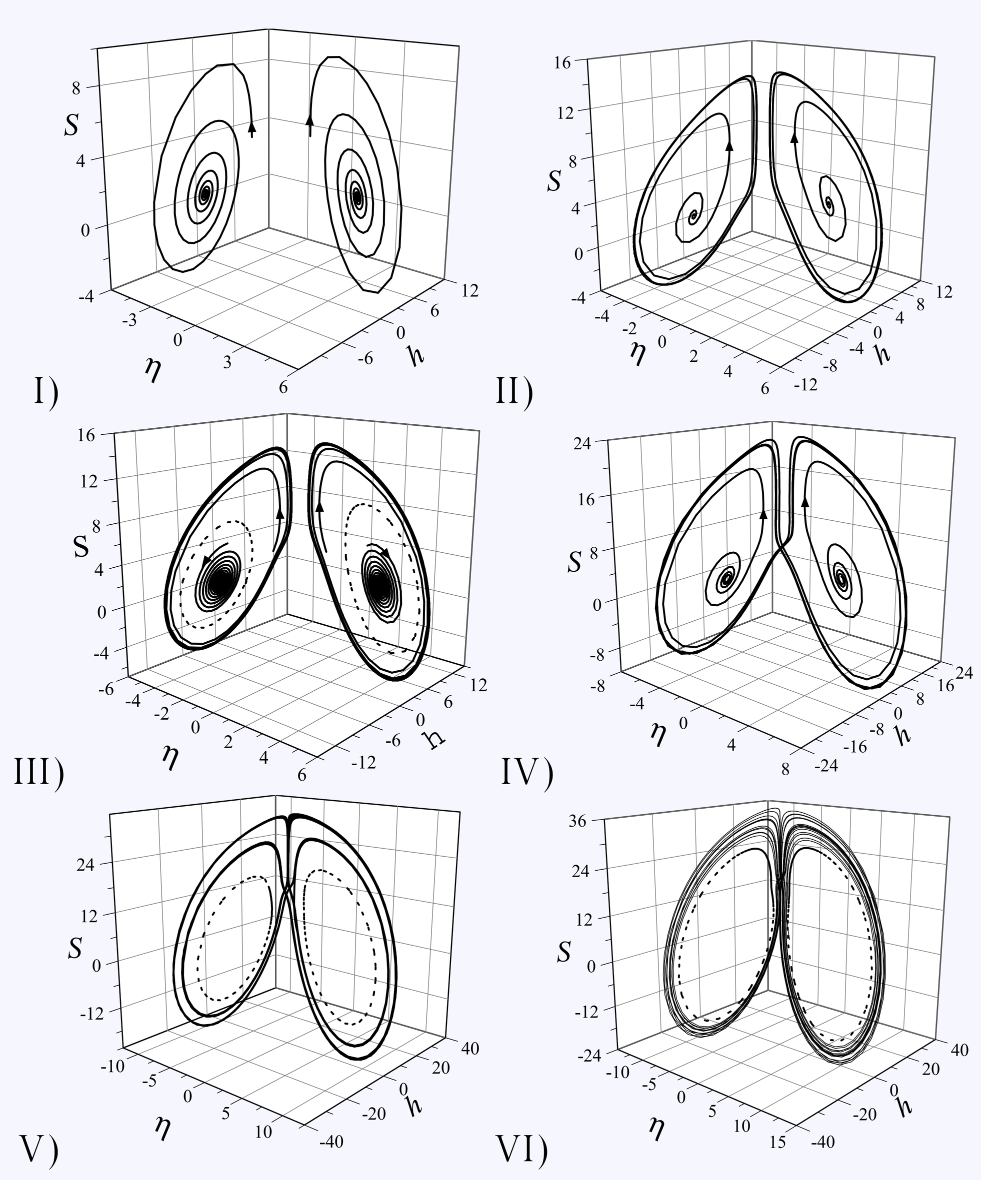
$$f_\tau \equiv -\kappa\left(1+\eta^2\right)^{-1}, \quad f_e \equiv -A - C\eta^2$$

Dynamical regimes reconstruction

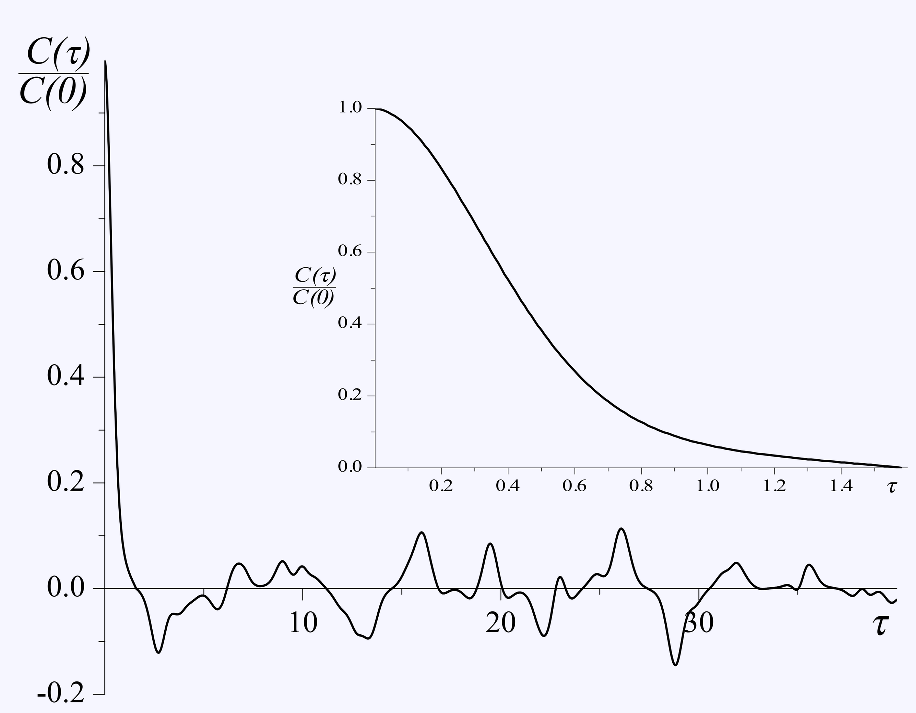
No external perturbations

Stability diagram of the system (5) at $A = C = 0$ and $\sigma = \varepsilon = 1$ (insertion corresponds to $\sigma/\varepsilon = 0.5$)

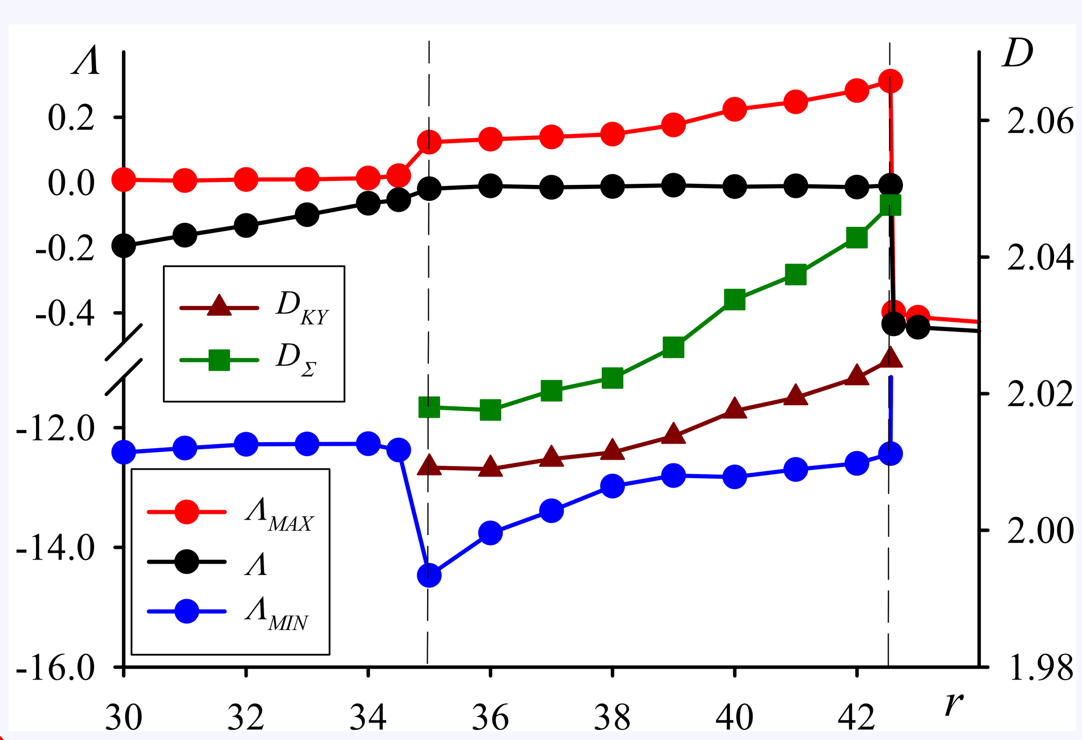




Autocorrelation function

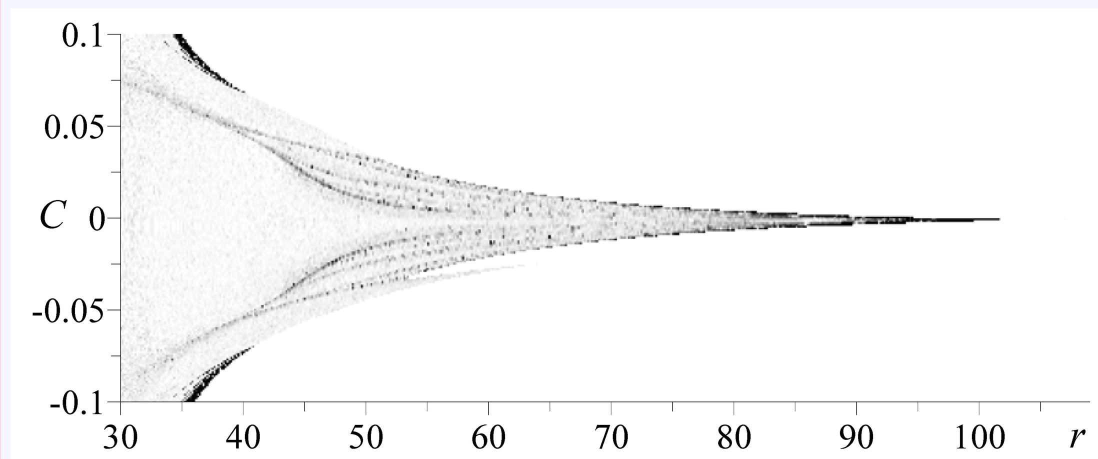


Fractal dimensions of strange attractor

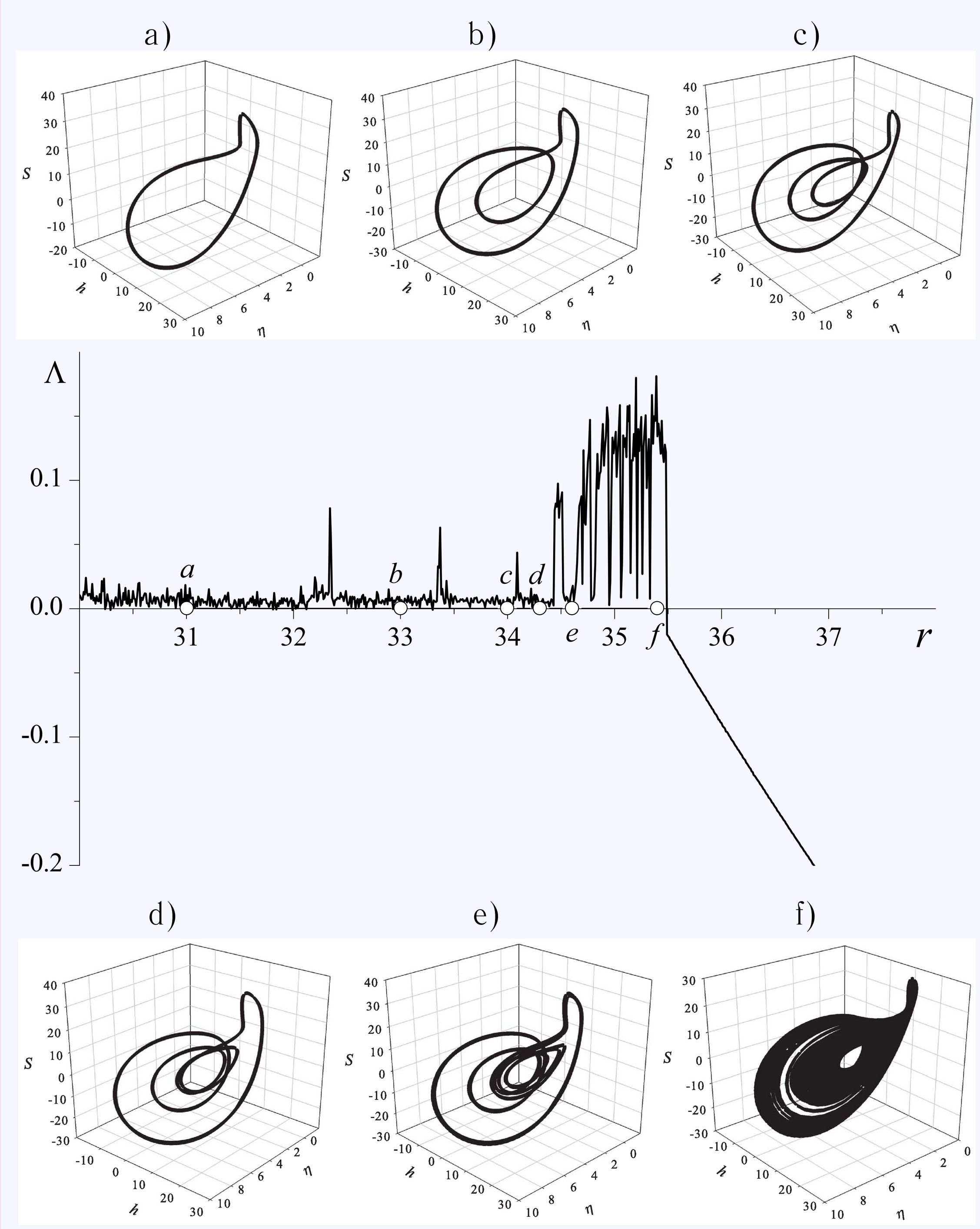


Regimes of chaos transition

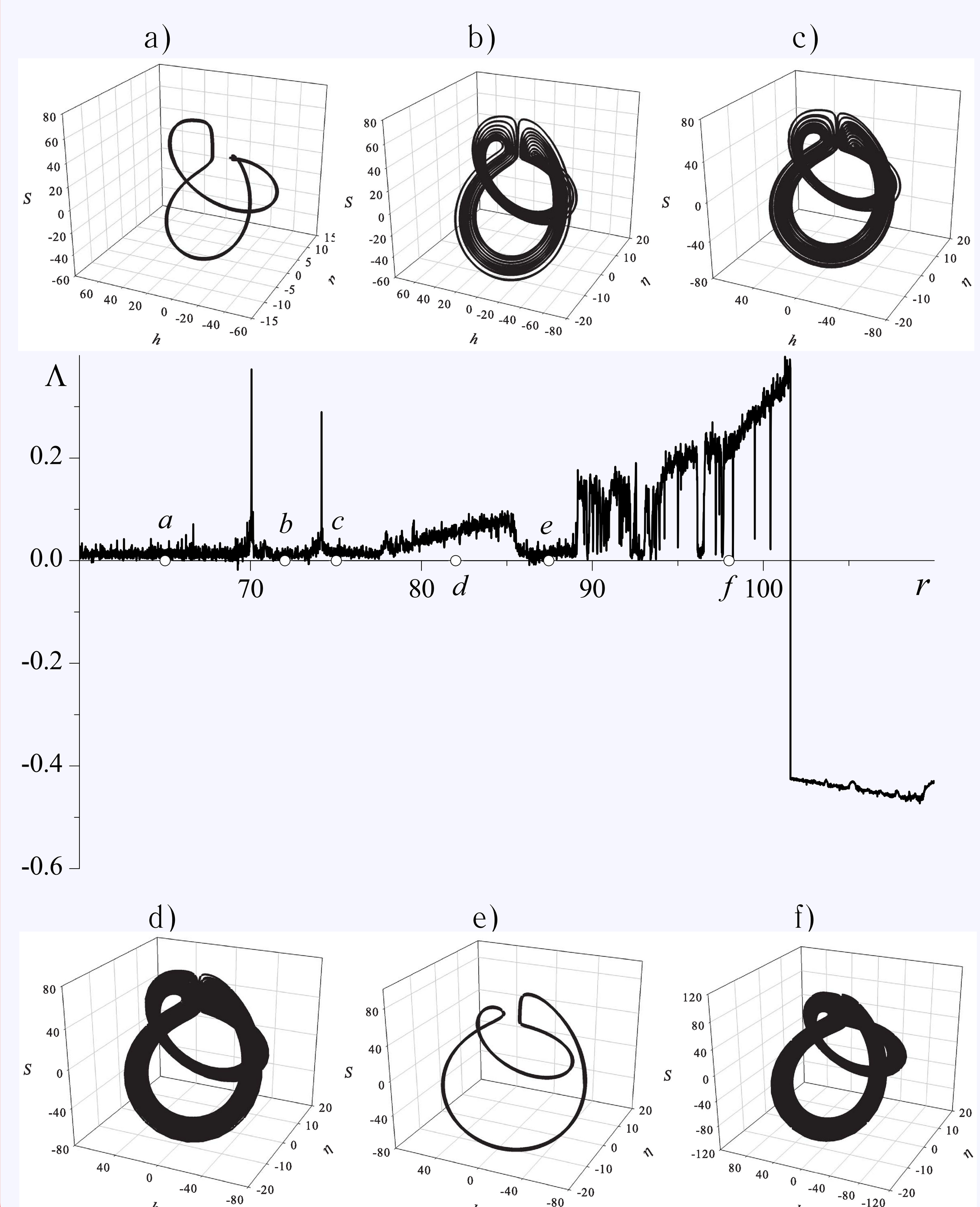
The Lyapunov map for the system (5) at $A = 0.1$, $\kappa = 25$ and $\sigma = \varepsilon = 1$



The Feigenbaum scenario at $A = 0.1$, $C = 0.1$



The Ruelle-Takens scenario at $A = 0.1$, $C = 0.0$



Publications

1 Chaos, Solitons & Fractals, **41**, 2595, (2009).

2 Functional Materials, **15(4)**, 496, (2008).

3 Journ. of Phys. Stud. (under review).