1. Motivation
Starting from the Navier-Stokes Equation, the analysis of turbulence is a hard numerical and mathematical task. Using a Dynamical Systems approach, e.g., a Coupled Map Lattice (CML), it is possible to understand basic features of turbulence. Our special interest lies in the understanding of the scaling of lifetimes of turbulent puffs with the system parameters.

2. The Model
Our CML is given by the evolution equation
\[ x(n+1) = \alpha x(n) f(x(n-1), \ldots), \]
with periodic boundary conditions.

3. Trajectories
The initial condition is given by a finite perturbation at the first site. By iterating the system, this perturbations travels through the lattice and eventually decays after a certain lifetime, which can be interpreted as a turbulent puff. The control parameters are the height \( h \), the coupling strength \( \alpha \) and the shift \( \delta \) (will be fixed to \( \delta=0.1 \) throughout).

4. Lifetimes
The lifetime for a fixed set of parameters can be computed by following trajectories of many initial conditions and calculating the exponential decay rate from the chaotic region. For the uncoupled lattice, the tent map \( f(x) \) has an average lifetime
\[ \tau_0 = \frac{1}{\lambda_1} = \frac{1}{\log(1/\lambda_1)} = \frac{1}{\log(2)} \approx 2. \]
In contrast, for super-long transients the average lifetime (cf. [5]) of the system should scale like
\[ \tau \sim \exp(\lambda_1 G_{\infty}), \quad \chi \ll 0, \lambda \approx 2. \]

5. Lyapunov vectors
The susceptibility to perturbations of a trajectory of the CML can be inferred from the linearized dynamics \( M \). The growth of perturbations is governed by its singular values, i.e., it is dominated by the largest eigenvalue of \( M^* M \). We wonder in what respect the corresponding eigenvectors of \( (M^* M) \) and \( (M M^*) \) characterize the most unstable directions of the forward and backwards dynamics, respectively.

References:

For zero coupling, the lifetime of the system follows the analytic solution Eq. (1).

In the coupled case, the average lifetime grows to significantly larger times, following the scaling law Eq. (2).

The average lifetime \( \tau \) depends sensitively on the coupling strength \( \alpha \). Close to \( \alpha=0.7 \) we observe very long average lifetimes of up to \( \tau=200000 \).

We summarize the \( h \) and \( \alpha \) dependence of the lifetime in the false color plot to the left. The complex shape of the isolines mimics the complex parameter dependence of lifetimes, which is commonly observed in models of turbulent pipe flow [2].

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