colloquium: Max-Planck-Institut für Physik komplexer Systeme, Dresden, Germany, 12 November 2012

esicqw12: International Workshop on Entanglement Spectra in Complex Quantum Wavefunctions

Entanglement, Topology and Geometry in the Fractional Quantum Hall Fluid

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- The previously-overlooked collective degree of freedom in the FQHE is a dynamical internal geometry associated with incompressibility
- This geometry can be found in the Laughlin state (and other model states), but was hidden for the last 30 years because of misinterpretation of the meaning of the "Laughlin wavefunction"
- Leads to a quantitive description of FQHE incompressibility explaining earlier results of Girvin, MacDolnald and Platzman (1985)
- A new (2D) "guiding-center spin" characterizes FQH incompressibility

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Revolutions in quantum condensed matter theory

- 1957: BCS theory: superconductivity is easy to understand in the second-quantized formalism, with the concept of **broken symmetry.** Feynman diagram methods become widespread. Single-particle "wavefunction"-based approaches are abandoned except for band-structure calculations (and localization).

- 1983: The newly-discovered (and completely unexpected) fractional quantum Hall effect proves completely intractable to second-quantized diagrammatic methods. Laughlin discovers his **wavefunction**. The new concept of **topological order** is a key to understanding it.

- Despite the 30 years since its discovery, some aspects of the FQHE (the origin of its incompressibility) are not well-understood theoretically.

- Numerical methods(exact diagonalization of finite-size system on a computer) have proved to be the **only** source of reliable results, analytic methods are missing.

- There is an interesting parallel with the problem of quark confinement, where numerical lattice-gauge theory methods confirm the existence of confinement, where analytic methods have not managed to do this.

The problem can perhaps be traced to the fact that **"out of the blue"** Bob Laughlin arrived with a very complete solution that he had come across (**and recognized!)** while looking at a 3-particle system to study Landau-level mixing by the short-distance singularity of the Coulomb interaction.

- It soon became clear that Laughlin's wavefunction was the **essential description** of the simplest FQHE states, and the basis for generalizations to the others.

- It works! But **WHY** ? In retrospect, the "explanations" to date seem to have just been plausible-sounding rationalizations of the fact that it does work!

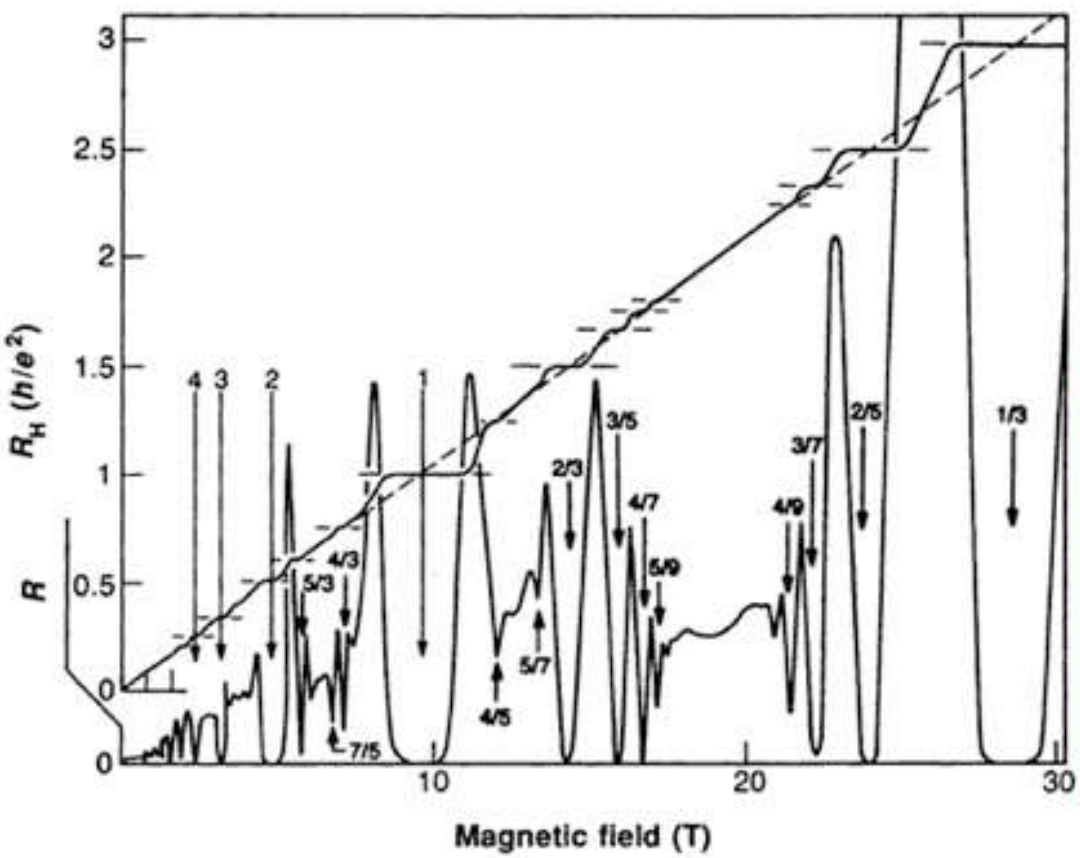
significance of Landau levels?

- Laughlin conceived of his state as a "lowest Landau level wavefunction" (despite the fact that it also described a FQHE in the second Landau level).
- More recently, it has be found to occur in numerical studies of lattice systems ("fractional Chern insulators") which are very different from Landau levels. These are currently the subject on intense study (e.g. at this esicqw12 meeting)

- I will argue on Wednesday 09:00 for a reinterpretation of the Laughlin state as quantum geometry,

- Fractional quantum Hall effect in 2D electron gas in high magnetic field (filled Landau levels)

$$\Psi_{1/3} = \prod_{i<j} (z_i - z_j)^3 \prod_i e^{-\frac{1}{2} z_i^* z_i}$$



$\nu = \frac{1}{3}$

- Laughlin (1983) found the wavefunction that correctly describes the 1/3 FQHE , and got Nobel prize,

- It is known that it works, (tested by finite-size numerical diagonalization) but **WHY** it works has never really been satisfactorily explained!

The usual Schrödinger presentation:

$$\langle \boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_N | \Psi_L \rangle = \Psi_L(\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_N)$$

$$\boxed{= \prod_{i<j} (z_i - z_j)^q \prod_i e^{-\frac{1}{2} z_i^* z_i}}$$

(unnormalized)

dimensionless complex coordinate

$$z_i = \frac{(x_i + i y_i)}{\sqrt{2}\ell_B}$$

"magnetic length" $\ell_B = \left(\frac{\hbar}{|eB|}\right)^{1/2}$

- when q=1, this is the uncorrelated Slater determinant state of the filled lowest Landau level. For $q > 1$, it is a highly correlated many-body state

annihilates lowest-Landau-level states

$$a_i = \frac{1}{2} z_i + \partial_{z_i^*}$$

$$a_i^\dagger = \frac{1}{2} z_i^* - \partial_{z_i}$$

Landau-level ladder operators

$$[a_i, a_j^\dagger] = \delta_{ij}$$

an elegant wavefunction!

$$\Psi_L = \prod_{i<j} (z_i - z_j)^q \prod_i e^{-\frac{1}{2} z_i^* z_i}$$

holomorphic factor Gaussian factor

- Two other useful forms:

$$\Psi_L = \langle N | \Psi_q^\dagger(z_1) \ldots \Psi_q^\dagger(z_N) | 0 \rangle$$

correlator of a chiral cft (conformal block)

$$\Psi_L = J_{\lambda_0}^{\left(\frac{-2}{q-1}\right)}(z_1, \ldots, z_N) \prod_i e^{-\frac{1}{2} z_i^* z_i}$$
$$= J_{\lambda_0}^{\left(\frac{-2}{q-2}\right)}(z_1, \ldots, z_N) \prod_{i<j}^{i} (z_i - z_j) \prod_i e^{-\frac{1}{2} z_i^* z_i}$$

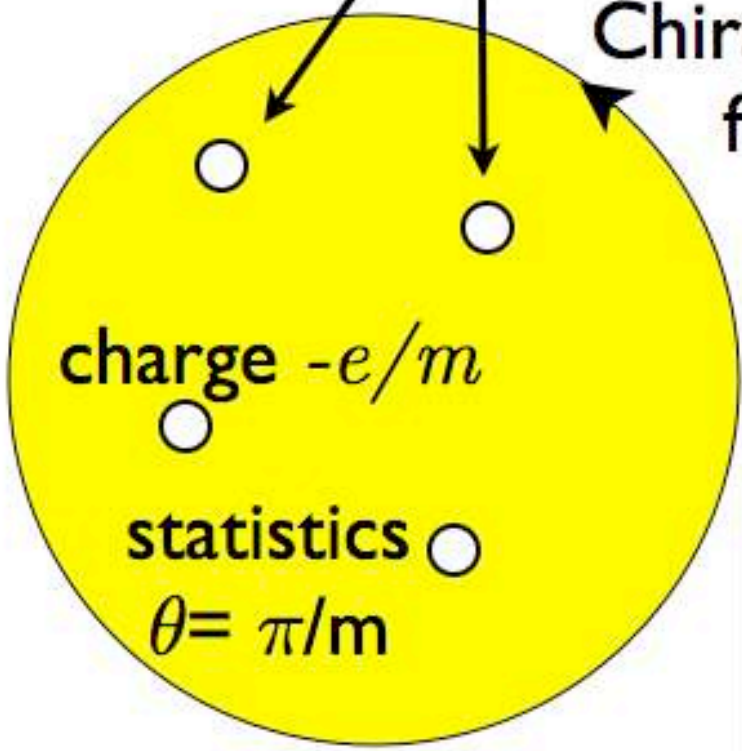
Jack polynomials with negative rational Jack parameter

$$\Psi_L = \prod_{i<j} (z_i - z_j)^q \prod_i e^{-\frac{1}{2} z_i^* z_i}$$

- Despite the apparent simplicity, analytic tools for calculating properties of this state (except for $q=1$) have still not been developed.....
- Its properties are still only known from finite-size numerical calculations

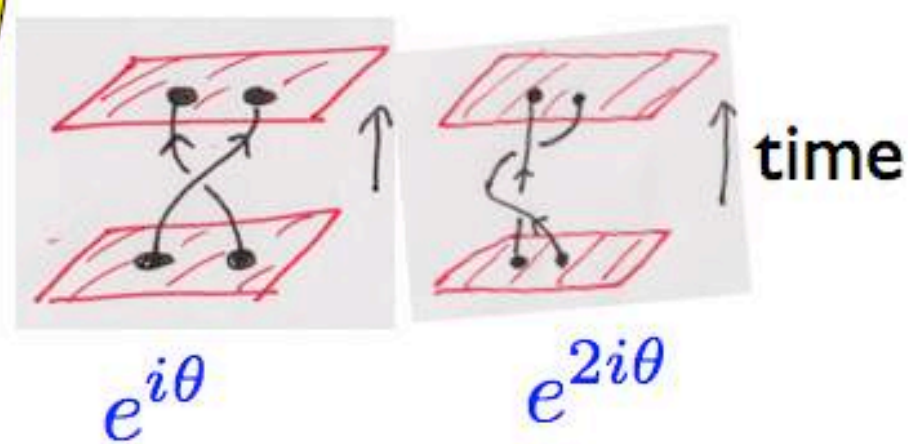
fractional-charge, fractional statistics vortices

$$\Psi = \prod_{i,\alpha}(z_i - w_\alpha)\prod_{i<j}(z_i - z_j)^m \prod_i e^{-\frac{1}{2}z_i^* z_j}$$



Chiral edge states at edge of finite droplet of fluid (Halperin, Wen)

e.g., m=3



- Physicists seem to have been "mesmerized" by the mathematical beauty of the LLL wavefunctions.....

- The Vandermonde determinant provides a natural "precursor" to the Laughlin state

$$\Psi = \prod_{i<j}(z_i - z_j)\prod_i e^{-\frac{1}{2}z_i^* z_i}$$

Filled N-particle LLL droplet

Laughlin wavefunction

Laughlin's "little" modification!

$$\Psi_L = \prod_{i<j}(z_i - z_j)^q \prod_i e^{-\frac{1}{2}z_i^* z_i}$$

NOBEL PRIZE!

- In retrospect, Laughlin's solution seemed so evidently correct and complete that it appears to have **frozen** the development of any understanding of the origin of **FQHE incompressibility** for almost 30 years.

- The Laughlin wavefunction for the strongest (1/3) FQHE state works, but why?

- In the attempt to "explain" **why** the Laughlin wavefunction works, theorists have been reduced to making nice-sounding pronouncements like

"The holomorphic wavefunction cleverly puts its zeroes on top of the other particles to lower the Coulomb energy"

Various attempts to explain FQHE incompressibility:

- Ginzburg-Landau superfluidity with a Higgs-like effect due to a Chern-Simons term
- Filling of "effective Landau levels" by "composite fermions"
- "Hamiltonian theory" of composite fermions
- non-commutative Chern-Simons theory with diffeomorphism invariance

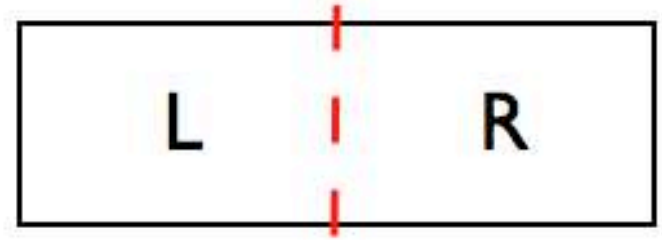
- There is no doubt that **Chern-Simons** theories as Topological Quantum Field theories capture the essential **topological** features of FQHE liquids, but TQFT provides NO information about **energy gaps** and **incompressibility**: because .....

TQFT has an effective Hamiltonian: H=0!

- for numerical calculations, other geometries (sphere, torus, cylinder) are useful.
- Physical edges exhibit 1+1d gapless chiral Luttinger liquid edge states reflecting topological order in the gapped incompressible bulk of the fluid.
- On the torus, (or other Riemann surfaces with higher genus) there is a topological degeneracy of the gapped ground state (related to topological field theories).
- An interesting observation I made a few years ago with my student H. Li, opened up the study of the **entanglement spectrum** for characterizing topological order.

Bipartite entanglement of (pure) quantum states

$$|\Psi\rangle = \sum_{\alpha=1}^{N_L} \sum_{\beta=1}^{N_R} W_{\alpha\beta} |\phi_\alpha^L\rangle \otimes |\phi_\beta^R\rangle$$



$$\langle \phi_\alpha^L | \phi_{\alpha'}^L \rangle \langle \phi_\beta^R | \phi_{\beta'}^R \rangle = \delta_{\alpha\alpha'} \delta_{\beta\beta'}$$

- Spatial decomposition of wavefunction into two parts
- Singular value (Schmidt)decomposition of W:

$$W_{\alpha\beta} = \sum_{\lambda=1}^{N_0} e^{-\frac{1}{2}\xi_\lambda} u_{\alpha\lambda}^L u_{\beta\lambda}^R \qquad \sum_\alpha (u_{\alpha\lambda}^L)^* u_{\alpha\lambda'} = \sum_\beta (u_{\beta\lambda}^R)^* u_{\beta\lambda'} = \delta_{\lambda\lambda'}$$

$N_0 \leq N_L, N_R$

Schmidt eigenvalues
(real positive)

$$\langle \Psi | \Psi \rangle = \sum_\lambda e^{-\frac{1}{2}\xi_\lambda}$$

Von Neuman (bipartite) entanglement entropy:

$$\boxed{S_{VN} = -\sum_{\lambda} p_\lambda \ln p_\lambda} \qquad \sum_{\lambda} p_\lambda = 1$$

$$p_\lambda = \frac{e^{-\xi_\lambda}}{Z} \qquad Z = \sum_{\lambda} e^{-\xi_\lambda}$$

Renyi entanglement entropy:

$$H(\beta) = \frac{1}{1-\beta} \ln \left(\sum_{\lambda} p_\lambda^\beta \right)$$

- reduces to S_{VN} when $\beta \to 1$
- typically evaluated with replicas, $\beta = n$

- In terms of the thermodynamics of a system with energy levels equivalent to the entanglement spectrum, the Renyi "entropy" is a rescaled free energy:

focusses on β = 1 as special

$$H(\beta) = \frac{\beta(F(1) - F(\beta))}{1 - \beta}$$

No 3rd law! ugly?

- In terms of the entanglement spectrum, a more transparent quantity is just the "true" entropy

$$S(\beta) = -\sum_{\lambda} p_\lambda(\beta) \ln p_\lambda(\beta)$$

3rd law!

$$P_\lambda(\beta) = \frac{e^{-\beta\xi_\lambda}}{Z(\beta)}$$

$$S_{VN} \equiv S(1)$$

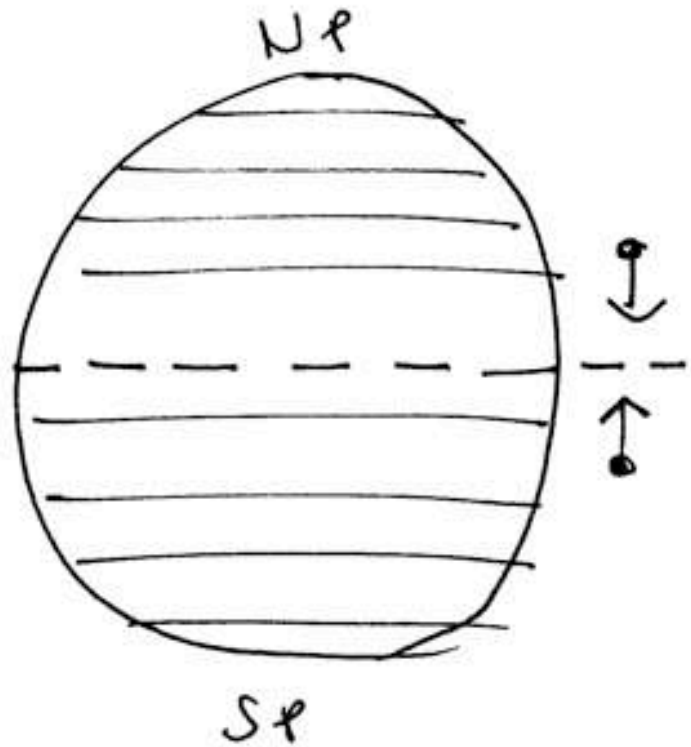
$$Z(\beta) = \sum_{\lambda} e^{-\beta\xi_\lambda}$$

- Rather than just representing entanglement by a single number S_{VN} , it is useful to examine the full "entanglement spectrum" ξ_λ
- In particular, examine its "low-energy" structure (or examine $S(\beta)$ for large β (low "temperature" limit)

result: gapped systems with <u>topological order</u> appear to always have a gapless entanglement spectrum

FQHE states in spherical geometry

- Schmidt decomposition of Fock space into N and S hemispheres.
- Classify states by Lz and N_e in northern hemisphere, relative to dominant configuration.

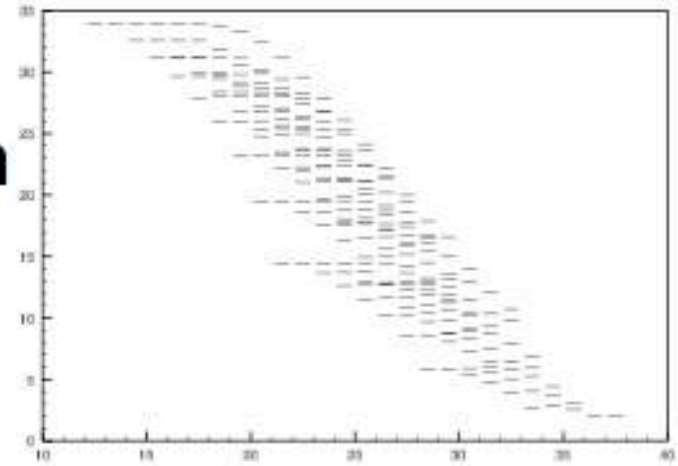


FQHE states have L=0

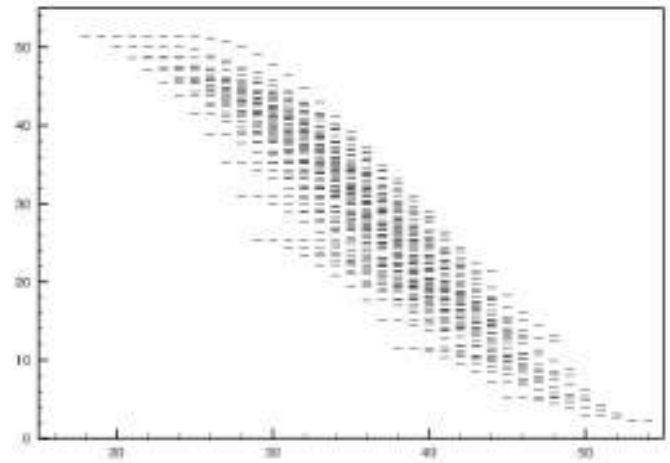
Represent bipartite Schmidt decomposition like an excitation spectrum (with Hui Li)

$$|\Psi\rangle = \sum_{\alpha} e^{-\beta_\alpha/2} |\Psi_{N\alpha}\rangle \otimes |\Psi_{S\alpha}\rangle$$

- like CFT of edge states.
- A lot more information than single number (entropy)
- many zero Schmidt eigenvalues (infinite "pseudoenergies) $e^{-\beta_\alpha} = 0$



(a) $N = 10$, $N_\phi = 27$

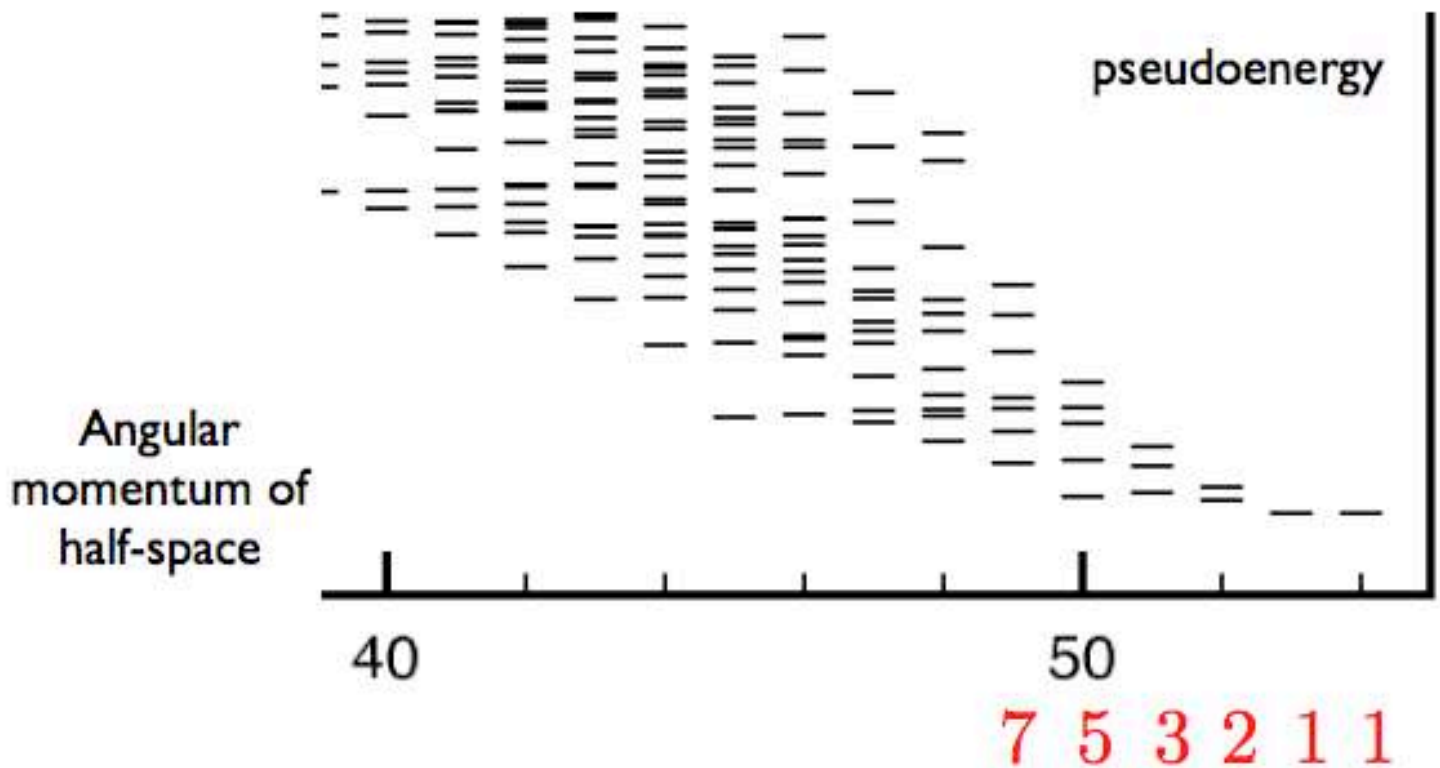


(b) $N = 12$, $N_\phi = 33$

FIG. 1: Entanglement spectrum for the 1/3-filling Laughlin states, for $N = 10, m = 3, N_\phi = 27$ and $N = 12, m = 3, N_\phi = 33$. Only sectors of $N_A = N_B = N/2$ are shown.

(due to"squeezing" (dominance) property of Laughlin wavefunction)

(from the Jack polynomial property)



gapless cft even though bulk is gapped!

- Spectrum allows characters of cft (up to a finite-size truncation of Virasoro level) to be "read off".
- Spectrum gives (a) conformal anomaly c (b) quantum dimensions of each sector, etc.

Interpolation between 1/3 Laughlin state and "true" ground state of Coulomb interaction.

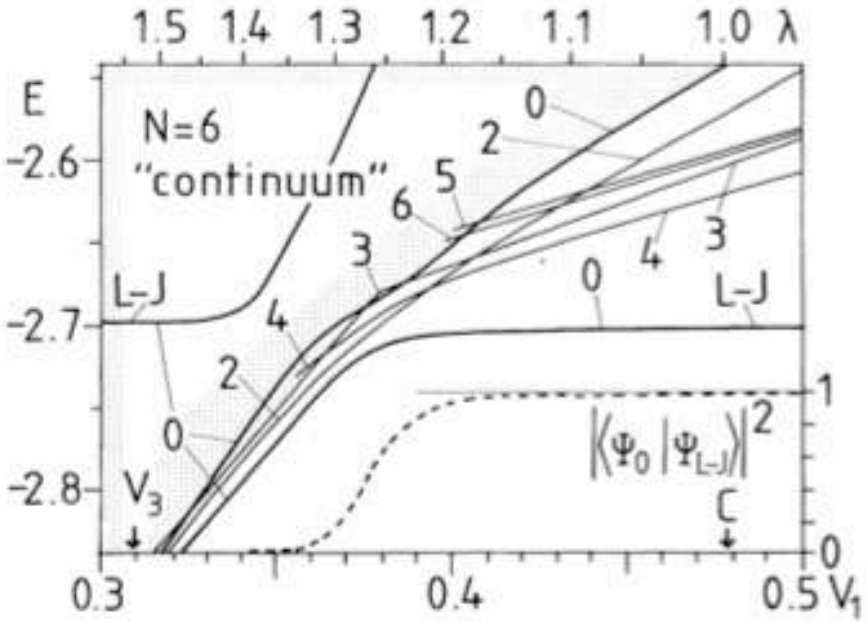


FIG. 6. Low-lying states at $N=6$, $2S=15$ ($\nu=\frac{1}{3}$) as the "hard-core" pseudopotential component V_1 is varied. The other V_m take their Coulomb values. V_3 and the Coulomb value (C) of V_1 are marked. Angular momentum quantum numbers L are indicated. Also shown is the projection of the LJ state on the ground state. In the gapless regime ($\lambda > 1.25$), the LJ state reappears as the *hightest* $L=0$ level.

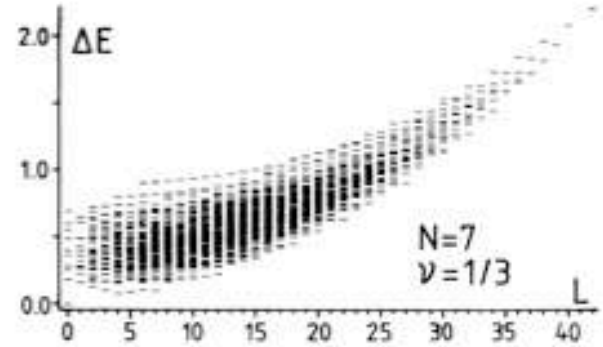


FIG. 1. The spectrum of 1656 multiplets (50 388 states) of the $N=7$ electron, $2S=18$ flux quanta system with Coulomb interactions, grouped by total angular momentum L. Energies (in units of $e^2/4\pi\epsilon l$) are shown relative to the incompressible ($\nu=\frac{1}{3}$) isotropic ($L=0$) ground state.

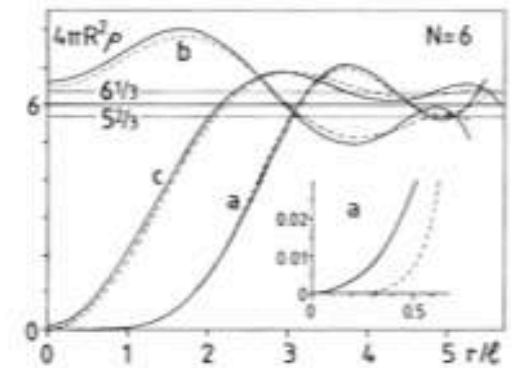
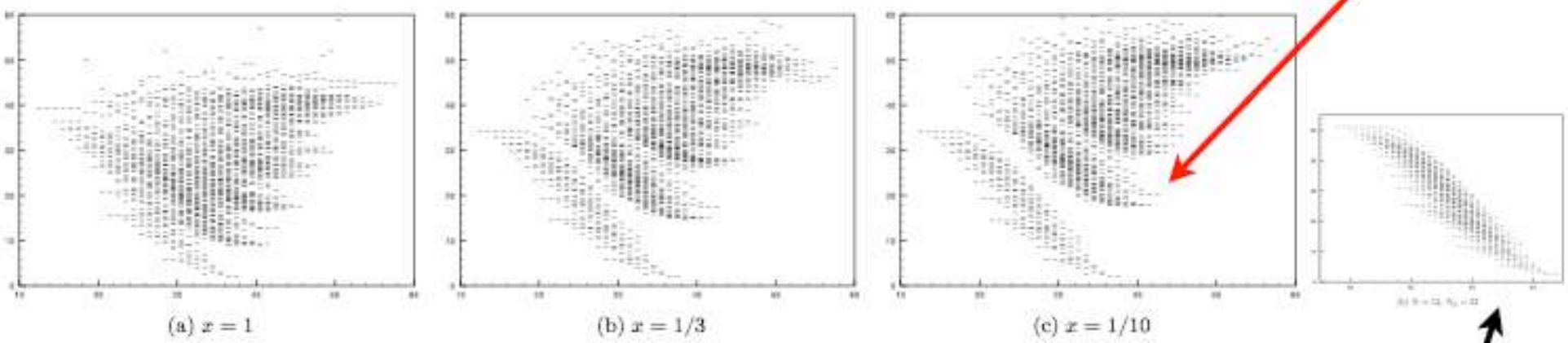


FIG. 3. (a) Ground-state pair correlation function for $\nu=\frac{1}{3}$, $N=6$. (b),(c) Density profiles of localized quasiparticle and quasihole defects. The condensate density ρ satisfies $4\pi R^2\rho=6$, $5\frac{2}{3}$, and $6\frac{1}{3}$, respectively. Filled curves, Coulomb interaction; broken curves, model Laughlin-Jastrow wave functions.

Look at difference between Laughlin state,entanglement spectrum and state that interpolates to Coulomb ground state.

dressing of ground state by one quasiparticle - quasihole pair



(a) $x = 1$ (b) $x = 1/3$ (c) $x = 1/10$

FIG. 2: Entanglement spectrum for the ground state, for a system of $N = 10$ electrons in the lowest Landau level on a sphere enclosing $N_\phi = 27$ flux quanta, of the Hamiltonian in Eq. (12) for various values of x.

x=0 is pure Laughlin

$$H = xH_c + (1 - x)V_1$$

Can we identify topological order in "physical as opposed to model wavefunctions from low-energy entanglement spectra?

Similar calculations for Moore-Read state

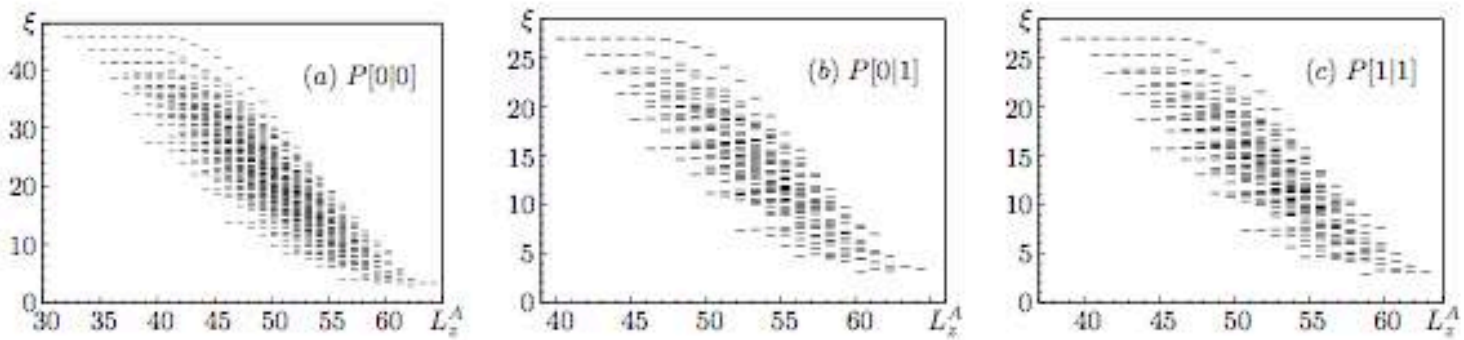


FIG. 1: The complete entanglement spectra of the $N_e = 16$ and $N_{orb} = 30$ Moore-Read state (only the relative values of ξ and L_z^A are meaningful).

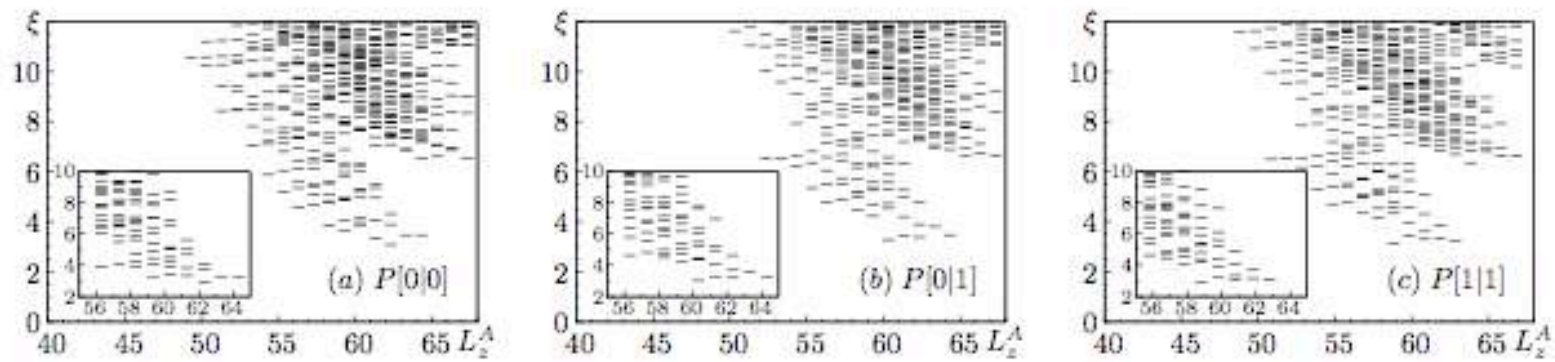


FIG. 2: The low-lying entanglement spectra of the $N_e = 16$ and $N_{orb} = 30$ ground state of the Coulomb interaction projected into the second Landau level (there are levels beyond the regions shown here, but they are not of interest to us). The insets show the low-lying parts of the spectra of the Moore-Read state, for comparison [see Figure (1)]. Note that the structure of the low-lying spectrum is essentially identical to that of the ideal Moore-Read state.

Gap to non-cft entanglement levels states of "generic" states appear to remain finite in the thermodynamic limit.

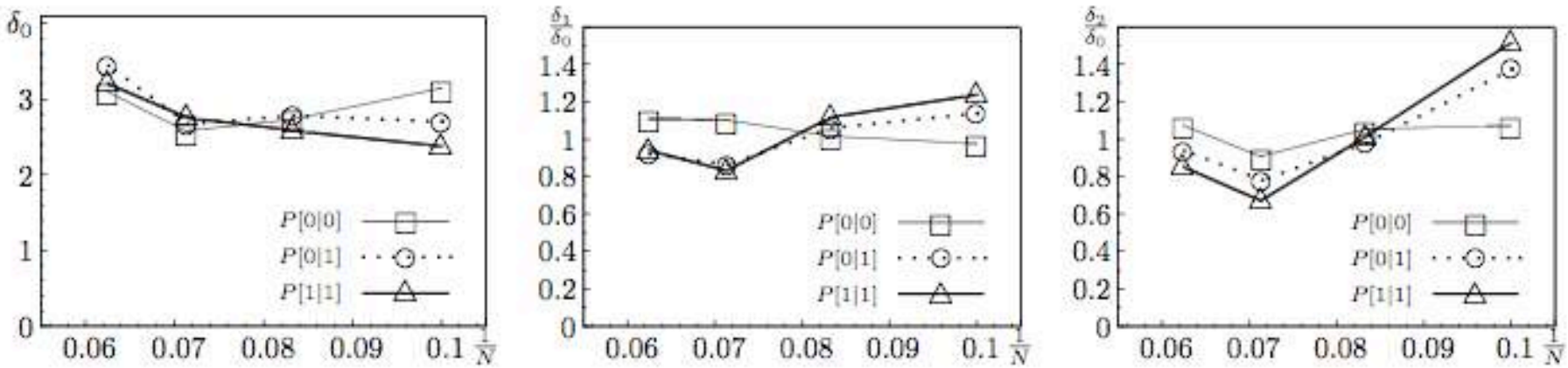
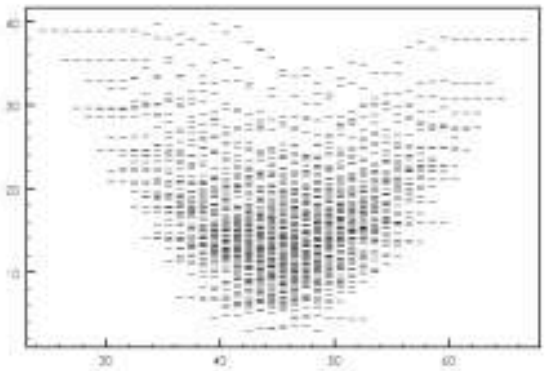
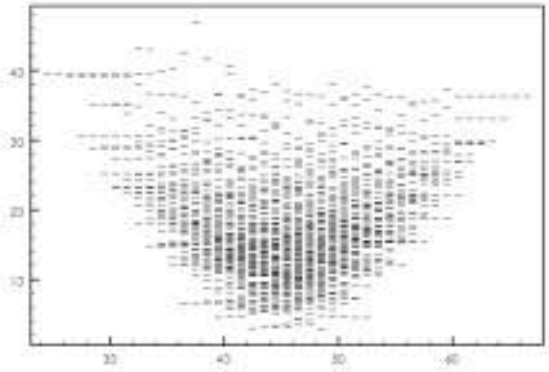
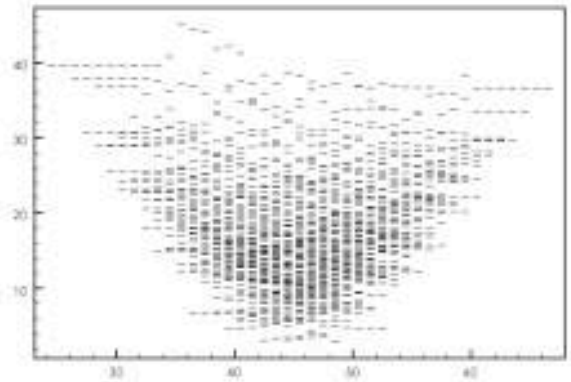
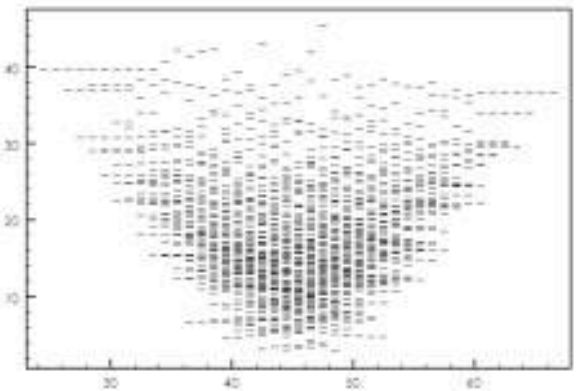


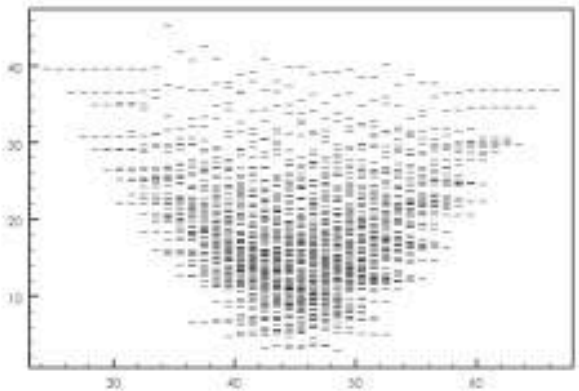
FIG. 3: Entanglement gap as a function of $1/N$. δ_0 is the gap at $\Delta L = 0$, i.e., the distance from the single CFT level at $\Delta L = 0$ to the bottom of the generic (non-CFT) levels at $\Delta L = 0$. At $\Delta L = 1, 2$, the gap $\delta_{1,2}$ is defined as the distance from the average of the CFT levels to the bottom of the generic levels. See Table I for the details of various partitionings.

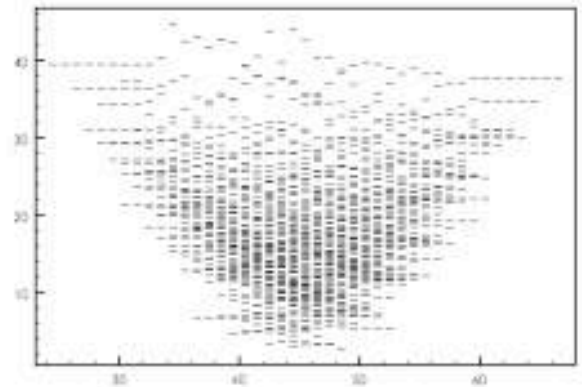


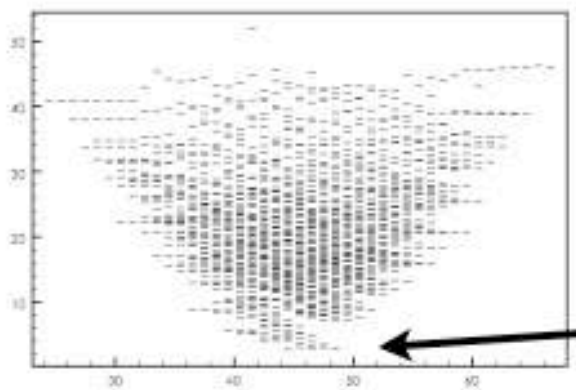












Driving the 2nd LL MR state to the gapless phase by varying V_1

(relative to pure Coulomb)

low-lying entanglement spectrum matches that of pure MR state

"Area law"

- The entanglement entropy of a gapped system is believed to have the form

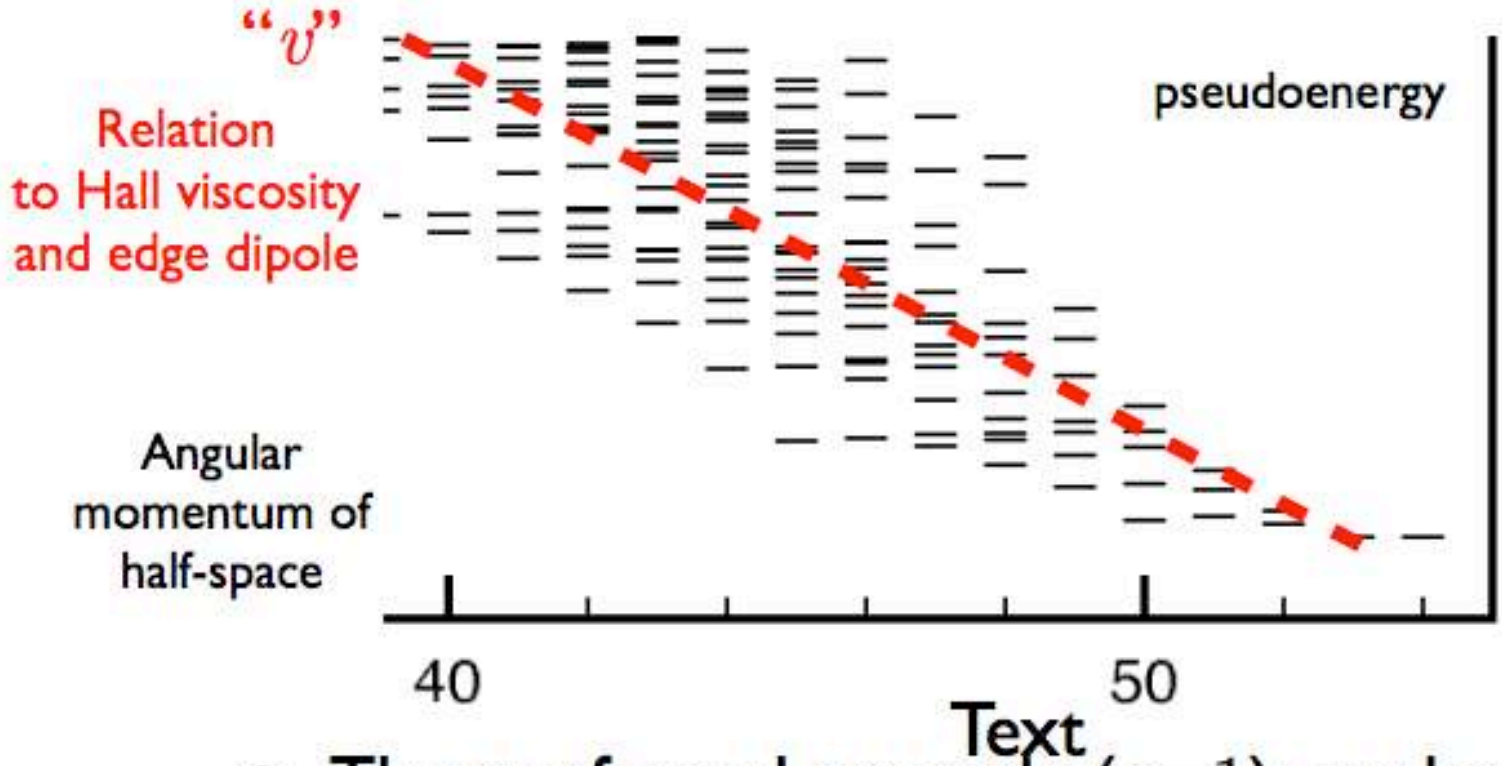
$$\lim_{A\to\infty} S = \alpha A + O(1)$$

non-universal, geometrical

(d-1)-dimensional area of surface dividing a d-dimensional region in two

universal, topological (if present)

non-universality of the area law is likely the case for Von-Neumann and Renyi "entropy", but for the low-``temperature" "true" entropy, there **is** structure



gapless cft even though bulk is gapped!

- The conformal anomaly (c=1) can be read from the characters (level count).

- If this was a cft, the entropy would vanish at T=0 (3rd law) and be

$$s = cL\frac{\pi^2}{6}\frac{k_B^2 T}{\hbar v}$$

$$ds^2 = dx^2 - v^2 dt^2$$

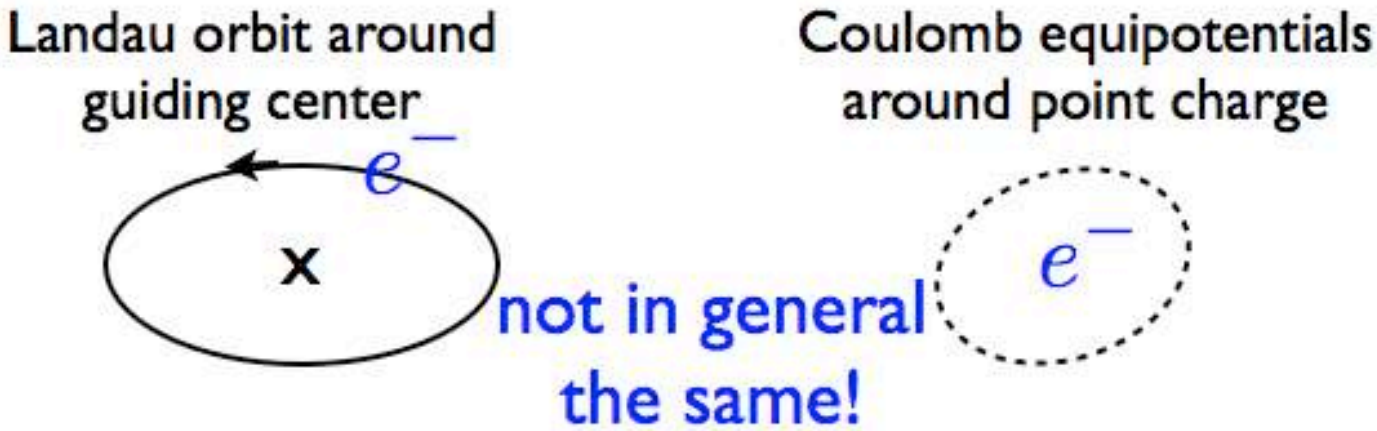
(expect a Euclidean 2D spatial metric to replace the space-time metric)

Reexamine FQHE geometry

- what is "x + iy" in the Laughlin state?
- its not just mathematics, or the Euclidean metric of flat space-time, it derives from the cyclotron effective-mass tensor, so the shape of a Landau orbit around the origin is

$$x^2 + y^2 = \text{constant}$$

- it changes if the magnetic field is "tilted" relative to the normal, or the underlying lattice is strained.

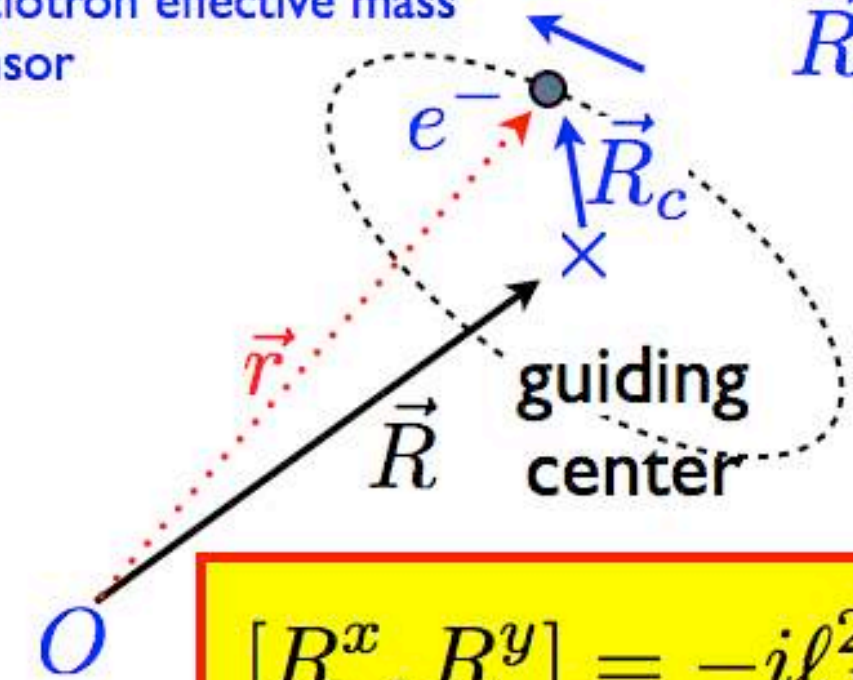


- The FQHE is controlled by the short-distance part of the Coulomb interaction, and by the shape of the equipotentials around a point change, **not** in general the same as the shape of the Landau orbits!

- The physics of the FQHE resides in the **guiding-centers** of the Landau orbits, not the orbits themselves.

- Non-commutative geometry of Landau-orbit guiding centers

shape of orbit around guiding center is fixed by the cyclotron effective mass tensor



$\vec{r}$ displacement of electron from origin

$\vec{R}$ displacement of guiding center from origin

$\vec{R}_c$ displacement of electron relative to guiding center of Landau orbit

$$\vec{\boldsymbol{r}} = \vec{\boldsymbol{R}} + \vec{\boldsymbol{R}}_c$$

$$[r^x, r^y] = 0$$

classical geometry

$$[R^x, R^y] = -i\ell_B^2$$

quantum geometry

$$[R_c^x, R_c^y] = +i\ell_B^2$$

Landau orbit (harmonic oscillator)

guiding centers commute with Landau radii $[R^a, R_c^b] = 0 \quad (a, b \in \{x, y\})$

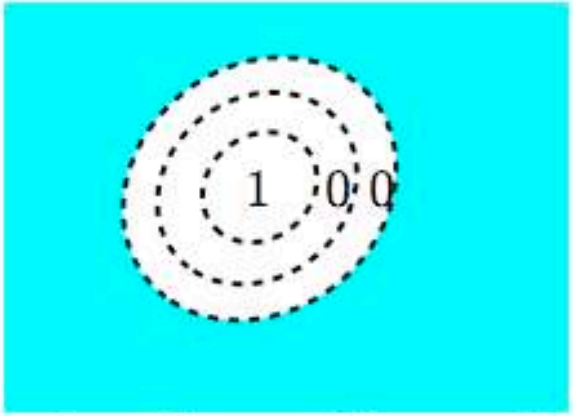
- A basis of states within a Landau level can be defined by the guiding-center operator

$$L = \frac{1}{2\ell_B^2} g_{ab} R^a R^b |\psi_m\rangle = (m + \tfrac{1}{2})|\psi_m\rangle$$

arbitrary positive-definite unimodular metric
(det g = 1)

one choice
of g

another choice
of g



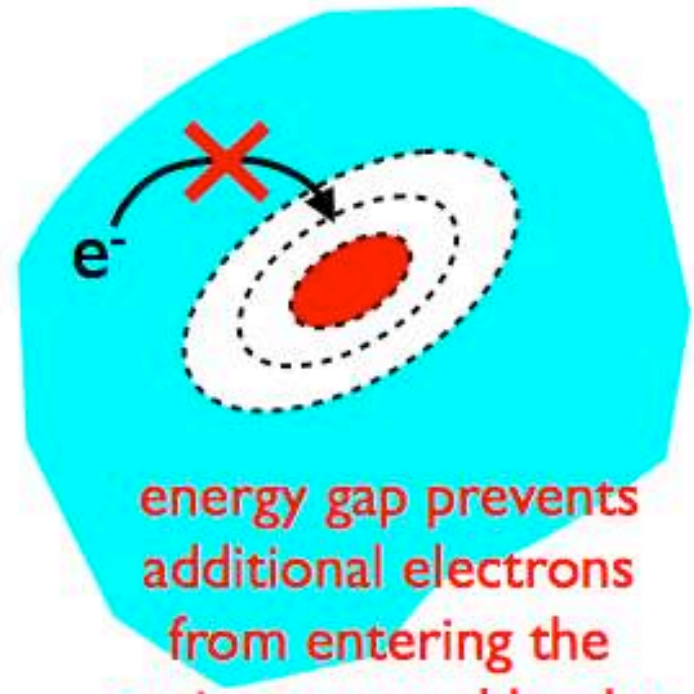
- essential physics of Laughlin state: occupation pattern $10\ 0$ .....
- if central orbital is occupied, next two are always empty

$$H(g) = \frac{V_1}{N_\Phi} \sum_{\boldsymbol{q}} L_1(q_g^2 \ell_B^2) e^{-\frac{1}{2} q_g^2 \ell_B^2} \sum_{i<j} e^{i\boldsymbol{q}\cdot(\boldsymbol{R}_i - \boldsymbol{R}_j)}$$

- "Pseudopotential" model for which Laughlin is exact ground state depends explicitly on metric: $q_g^2 \equiv g^{ab} q_a q_b$

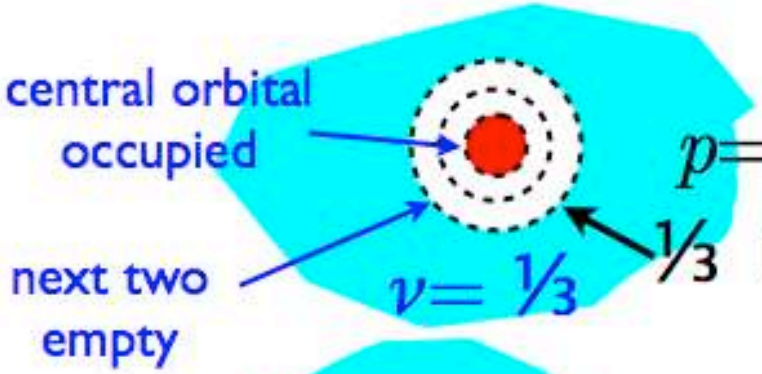
- Origin of FQHE incompressibility is analogous to origin of **Mott-Hubbard gap** in lattice systems.
- There is an energy gap for putting an **extra particle** in a quantized region that is **already occupied**

- **On the lattice** the "quantized region" is an atomic orbital with a fixed shape
- **In the FQHE** only the **area** of the "quantized region" is fixed. The **shape** must adjust to minimize the correlation energy.



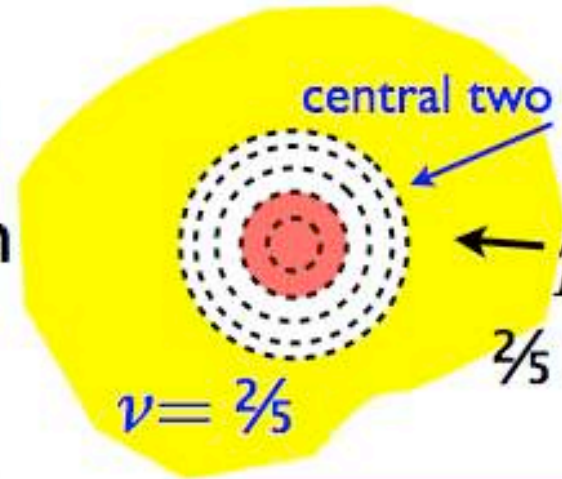
energy gap prevents additional electrons from entering the region covered by the composite boson

- elementary unit of the FQHE fluid with $\nu = p/q$ is a "**composite boson**" of p electrons that **exclude** other electrons from a region with q London (h/e) flux quanta



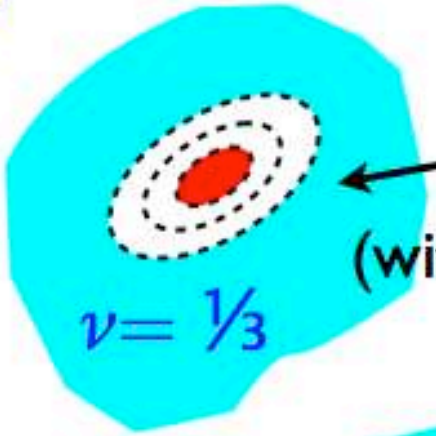
$p=1,\ q=3$

⅓ Laughlin



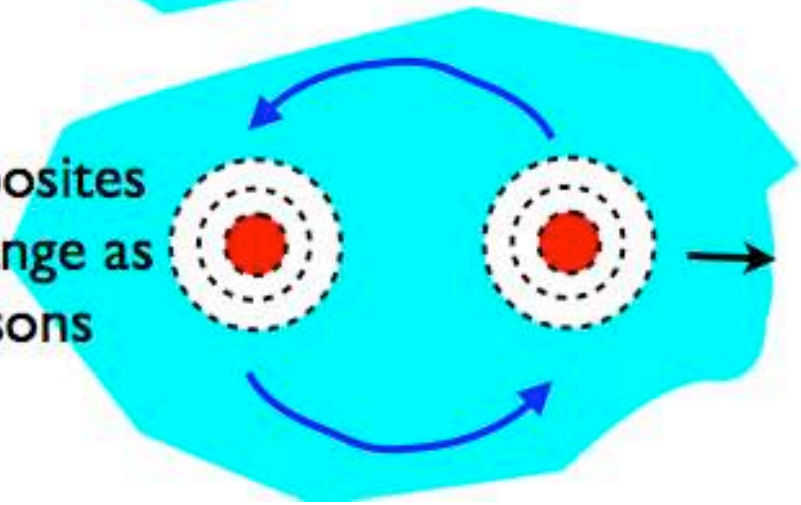
$p=2,\ q=5$

⅖ Hierarchy/Jain



⅓ Laughlin (with different shape)

the rule formerly known as "**odd-denominator**", (but Moore-Read has p=2, q=4)



Statistical selection rule

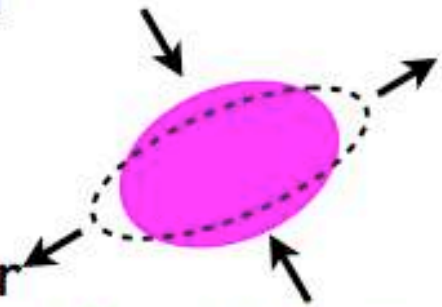
$$(-1)^p \times (-1)^{pq} = +1$$

exchange of p fermions

Berry phase (exchange of "exclusion zones")

composite is a boson

- The metric (shape of the composite boson) has a preferred shape that minimizes the correlation energy, but fluctuates around that shape
- The zero-point fluctuations of the metric are seen as the O(q^4) behavior of the "guiding-center structure factor" (Girvin et al, (GMP), 1985)



$\delta E \propto (\text{distortion})^2$

- The metric has a companion "guiding center spin" that is topologically quantized in incompressible states.

configuration of "elementary droplet" (composite boson)

subtract total L (=L_{ref}) of reference configuration (uniform occupation p/q)

⅓ Laughlin

total L

$L=$	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$	
	1	0	0	$\frac{1}{2}$
	⅓	⅓	⅓	$\frac{3}{2}$

$$s = \left(\frac{1}{2} - \frac{3}{2}\right) = -1$$

- dissipationless Hall viscosity (relates traceless stress tensor to gradient of drift velocity):

$$\sigma^a_c = \eta^{ab}_{cd}\partial_b v^c$$

$$v^a = \epsilon^{ab}(E_b/B)$$

$$\eta^{ab}_{cd} = B\epsilon_{ae}\epsilon_{bf}\Gamma_A^{ebfd}$$

$$\Gamma_A^{abcd} = \Gamma^{bacd} = -\Gamma^{cdab} = \frac{1}{2}\left(\epsilon^{ac}\gamma_H^{bd} + \epsilon^{ad}\gamma_H^{bc} + \epsilon^{bd}\gamma_H^{ac}\right)$$

fractional charge quantum

topologically-quantized guiding-center spin

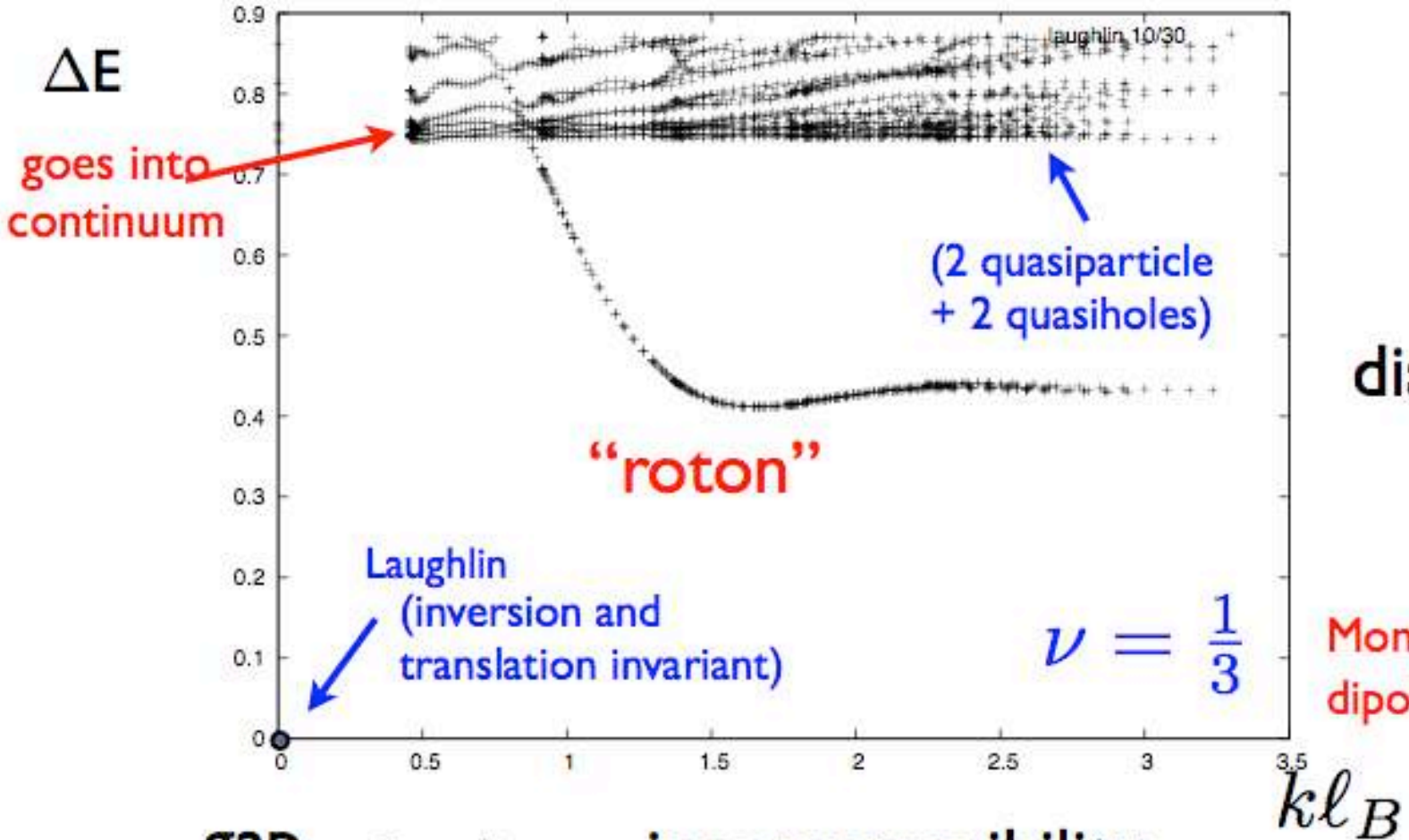
$$\gamma_H^{ab}(\boldsymbol{x}) = \frac{e^* s}{4\pi} g^{ab}(\boldsymbol{x})$$

rank-2 symmetric "Hall viscosity"

(local) unimodular inverse metric

$$e^* = \frac{e}{q}$$

unfortunately, long-wavelength limit of "graviton" collective mode is hidden in "two-roton continuum"



numerical finite-size diagonalization

Momentum $\propto$ dipole moment

gap $\longleftrightarrow$ incompressibility

Gap for tangential electric polarization (no dielectric screening)

single-mode approx. $E(\boldsymbol{q})s(\boldsymbol{q}) \leq \frac{1}{2}G^{abcd}q_a q_b q_c q_d \ell_B^2.$

- Coupling to geometry: Wen and Zee (1992), also Fröhlich and Stueder

a (2D) "spin"

$$\mathcal{L} = \frac{\hbar K}{4\pi}\epsilon^{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda + \frac{1}{2\pi}\left(qeA_\nu + \hbar s\Omega_\mu\right)\epsilon^{\mu\nu\lambda}\partial_\nu a_\lambda$$

Chern-Simons

electromagnetic gauge field

curvature gauge field (**spin connection**)

This was introduced to model the coupling to the **static extrinsic** curvature when electrons move on a curved surface

curl of spin connection = Gaussian Curvature

now use this to model coupling to **dynamic intrinsic** geometry

- Geometric action

(after Chern-Simons fields are integrated over)

electromagnetic gauge potentials

spin connection of metric

$$S = \int d^3x \mathcal{L}_0 - \mathcal{H}_0$$

$$\mathcal{L}_0 = \frac{1}{4\pi pq\hbar}\epsilon^{\mu\nu\lambda}\left(peA_\mu - s\Omega^g_\mu\right)\partial_\nu\left(peA_\lambda - \hbar s\Omega^g_\lambda\right)$$

(reduces to electromagnetic Chern-Simons action when s = 0 (integer QHE))

$$\mathcal{H}_0 = J^0 U(J^0 g) \qquad J^0 = \frac{1}{2\pi pq\hbar}\left(peB - \hbar s J^0_g\right)$$

energy function

Gaussian curvature

$$J^\mu_g = \epsilon^{\mu\nu\lambda}\partial_\nu\Omega^g_\lambda$$

correlation energy density

composite-boson density

Geometric distortion energy

correlation energy density

$$\mathcal{H}_0 = (\det G)^{1/2} U(G) = J^0 U(J^0 g)$$

geometric chemical potential (of composite bosons)

$$\boxed{\mu_g = U(G) + G_{ab} \frac{\partial U}{\partial G_{ab}}}$$

shear-stress tensor (traceless)

$$\boxed{\sigma^a_b = 2G_{bc} \frac{\partial U}{\partial G_{ac}} - \delta^a_b G_{cd} \frac{\partial U}{\partial G_{cd}}}$$

$$\boxed{\sigma^a_a = 0}$$

$$\sigma^a_c(x)\epsilon^{bc} = \sigma^{bc}(x)\epsilon^{ac}$$

$$\sigma^a_c(x)g^{bc}(x) = \sigma^{bc}(x)g^{ac}(x)$$

both expressions are symmetric in $a \leftrightarrow b$

Stress tensor is traceless because the gapped quantum incompressible fluid does not transmit pressure

(unlike incompressible limit of classical incompressible fluid, which has speed of sound $v_s \to \infty$

Euler equation

- action is minimized by Hall viscosity condition

composite boson current

$$J^0 \sigma^a_b(G) = \eta^{ac}_{bd}(G) \nabla^g_c J^d$$

Traceless stress-tensor

covariant spatial gradient of $J^a = J^0 v^a$

fluid flow-velocity

Hall viscosity

$$\eta^{ac}_{bd}(G) = \frac{1}{2}\hbar s \epsilon_{be}\epsilon_{df} J^0 \Gamma^{aecf}_H(g)$$

$$\Gamma^{abcd}_H(g) = \frac{1}{2}\left(\epsilon^{ac}g^{bd} + \epsilon^{ad}g^{bc} + \epsilon^{bc}g^{ad} + \epsilon^{bd}g^{ac}\right)$$

$$\eta^{ac}_{bd} = -\eta^{ca}_{db}$$

dissipationless

$$\eta^{ab}_{ac} = \eta^{ba}_{ca} = 0$$ incompressible

- composite boson current

Gaussian curvature density and current

$$J^0 = \frac{1}{2\pi pq\hbar}\left(\epsilon^{ab} peB - \hbar s J_g^0\right)$$

$$J^a = \frac{1}{2\pi pq\hbar}\left(\epsilon^{ab}(peE_b - \partial_b \mu_g) - \hbar s J_g^a\right)$$

responds to gradient of geometric chemical potential as well as electric field

$$peE_a J^a \neq 0$$

Energy flow from electromagnetic field to FQH fluid

$$pe(J^0 E_a + \epsilon_{ab} J^a B) \neq 0$$

tangential momentum flow from electromagnetic field to FQH fluid

- Action gives gapped spin-2 (graviton-like) collective mode that coincides at long wavelengths with the "single-mode approximation" of Girvin-MacDonald and Platzman.

- charge fluctuations relative to the background charge density fixed by the magnetic flux are given by the Gaussian curvature

$$J_g^0 = -\frac{1}{2}\partial_a\partial_b g^{ab} + \frac{1}{8} g_{ac}\epsilon_{bd}\epsilon^{ef}\left(\partial_e g^{ab}\right)\left(\partial_f g^{cd}\right)$$

$$\delta J_e^0 = \frac{e^* s}{2\pi} J_g^0$$

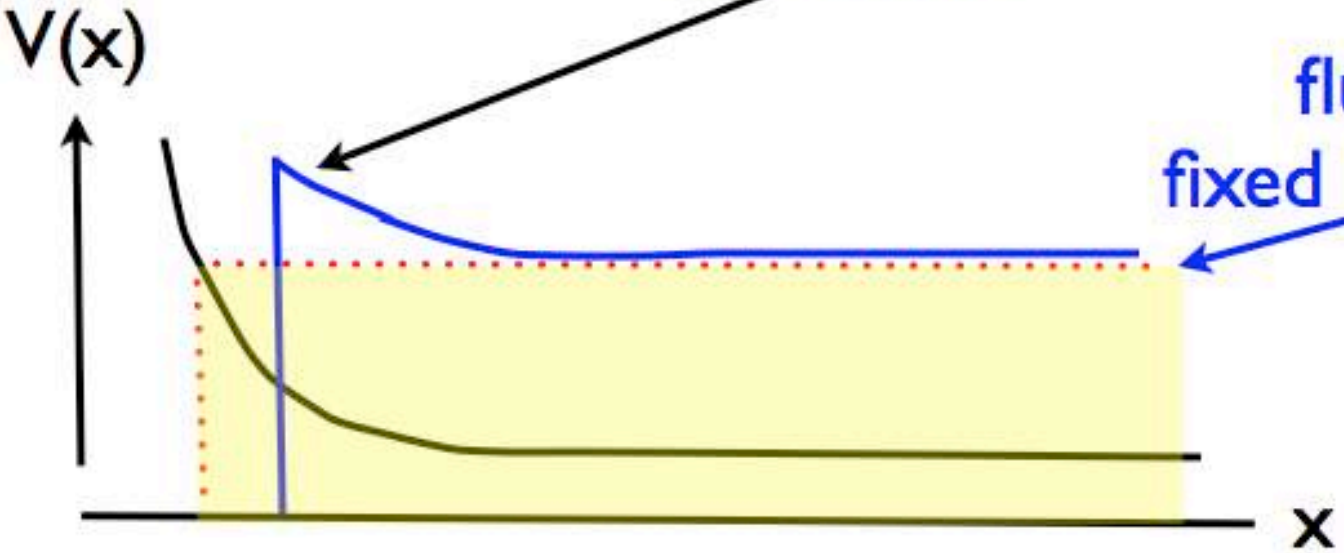
second derivative of metric

zero-point fluctuations of gaussian curvature give quantitatively correct $O(q^4)$ structure factor

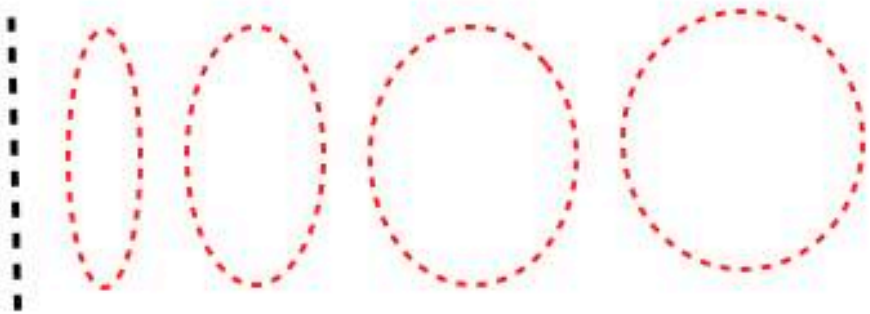
- near edges:

fluid is compressed at edges by creating Gaussian curvature

$$\delta J_e^0 = \frac{e^* s}{2\pi} J_g^0$$



fluid density fixed by flux density



$$g = \begin{pmatrix} \alpha(x) & 0 \\ 0 & \frac{1}{\alpha(x)} \end{pmatrix}$$

$$J_g^0 = -\frac{1}{2}\frac{d^2}{dx^2}\frac{1}{\alpha(x)}$$

For larger s, fluid becomes more compressible (less distortion needed for a given density change)

SUMMARY

- New collective geometric degree of freedom leads to a description of the origin of incompressibility in FQHE in a continuum "geometric field theory"
- many new relations: guiding-center spin characterizes coupling to Gaussian curvature of intrinsic metric, stress in fluid, guiding-center structure-factors, etc.

http://wwwphy.princeton.edu/~haldane

Can be also be accessed through Princeton University Physics Dept home page
(look for Research:condensed matter theory)

also see arXiv (search for author=haldane)