Due on Monday, February 15th.

If pictures are needed/relevant, please provide them with your solutions.

— — — — — — — — — — — — — — ⋆ — — — — — — — — — — — — —

1. **[10 pts.]** A charged sphere has radius a_0 and volume charge density

$$\rho = \begin{cases} \rho_0 (r/a_0)^2 & \text{when} \quad r \leq a_0 \\ 0 \quad \text{when} \quad r > a_0 \end{cases}$$

   where r is the distance from the center of the sphere. What is the total charge on the sphere?

   Note: The volume charge density, also known just as the charge density, is the charge per unit volume.

   Hint: it might help to consider the charge on a shell of infinitesimal width, and then integrate.

2. **Electric Dipole.**

   In this problem, we ignore the z-direction. All points are on the xy plane.

   A postiive charge q is placed at position $(0, +d/2)$, and a negative charge $-q$ is placed at postion $(0, -d/2)$. This is an *electric dipole.*

   (a) **[2 pts.]** For this electric dipole, what is the magnitude of the dipole moment? What is the direction of the dipole moment?

   (b) **[5 pts.]** Calculate the electric field at the point (x_0,0) on the x-axis, by calculating the fields due to each charge and adding them. This would be vector addition. Make sure to show pictures if relevant, indicating directions and angles.

   (c) **[5 pts.]** Calculate the electric potential $V(x, y)$ at an arbitrary point on the xy plane, by scalar addition of the contributions from the two charges.

   (d) **[5 pts.]** Use your expression for $V(x, y)$ to calculate the electric field at the point (x, y), using the relationship between $\mathbf{E}$ and V.

   Check that this yields the correct answer at the point (x_0,0), where you have already calculated the electric field by vector addition.

   Note: the z-direction does not affect the answer, so you can neglect the z-derivative in the gradient.

3. **Line (thin rod) of continuous charge.**

In this problem also, we ignore the z-direction. All points are on the xy plane.

Consider a uniformly charged rod of length L, lying along the y-axis. The endpoints are at $(0,-L/2)$ and $(0,+L/2)$ and the center is at the origin. The linear charge density (charge per unit length) is λ.

(a) [**1 pt.**] What is the total charge on the rod?

(b) [**2 pts.**] Consider the point P on the x-axis, with coordinates $(x_0, 0)$. We want to calculate the electric field at this point. By symmetry, which direction should the field point towards?

(c) [**7 pts.**] Take an infinitesimal element of the rod, between point $(0,\ y)$ and point $(0,\ y+dy)$. What is the charge within this element? What is the electric field contribution at point P due to this infinitesimal charge element? Evaluate separately the x- and y-components, dE_x and dE_y.

(d) [**6 pts.**] Using the expressions for dE_x and dE_y. calculate the total electric field components at P by adding up contributions from all infinitesimal elements. This means integrating from $y = -L/2$ to $y = +L/2$.

It might help to know that

$$\int \frac{du}{(u^2+a^2)^{3/2}} = \frac{u}{a^2(u^2+a^2)^{1/2}}$$

(e) [**3 pts.**] Simplify the expression for the case $x_0 \gg L$. Is the expression similar to the expression for a point charge? Explain why you could have expected this.

(f) [**4 pts.**] Now imagine that the rod is infinitely long, i.e., $L \to \infty$. By taking the limit of the expression derived for finite L, calculate the electric field in this limit.

Comment: The electric field for an infinite line of charge (if you did this correctly) is inversely proportional to x_0, not to the square of x_0. The decrease of electric field with increasing distance is *slower* than compared to the electric field due to a single charge. Please think about whether this makes sense physically.