Due on Monday, March 29th.

If pictures are needed/relevant, please provide them with your solutions.

The questions marked **[SELF]** are for yourself and need not be submitted.

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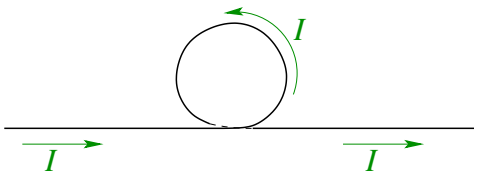
1. **[6 pts]** A very long wire carrying current I is straight except for a circular loop of radius R. (An approximate drawing is shown.) Find the magnetic field at the center of the loop.



2. Steady electric current I is flowing through a square-shaped loop of wire; let's call the square $ABCD$ with each letter denoting a corner of the square. Each side of the square has length L. We wish to calculate the magnetic field generated at the center of the square, which we denote as point P.

   Let's first concentrate on one of the sides, say AB. Choose an infinitesimal element of this side, of length $|d\mathbf{l}|$. Define $\mathbf{l}$ as its displacement from the center of AB. The line joining this element to the point P makes an angle θ with the line joining the point P to the center of the side AB.

   (a) **[2 pts]** Sketch a top view of the square wire and an element $d\mathbf{l}$, showing clearly the angle θ, the distance l, and the direction of the current. Take the current to flow counter-clockwise.

   (b) **[6 pts]** What is the distance between the element and the point P? Write two expressions for this distance: one in terms of L and l, and one in terms of L and θ.

   (c) **[8 pts]** Using the Biot-Savart law, write down the magnitude of the magnetic field $d\mathbf{B}$ created at P due to the chosen element. Express the magnitude in terms of L and l. (There should be no θ in your expression.)

   (d) **[3 pts]** What is the direction of $d\mathbf{B}$?

(e) **[8 pts]** Find the magnetic field at P due to the current in the side AB, by integrating your expression for $d\mathbf{B}$ over the displacement l. What are the limits of integration? It might help to know that

$$\int \frac{du}{(u^2+a^2)^{3/2}} = \frac{u}{a^2(u^2+a^2)^{1/2}}$$

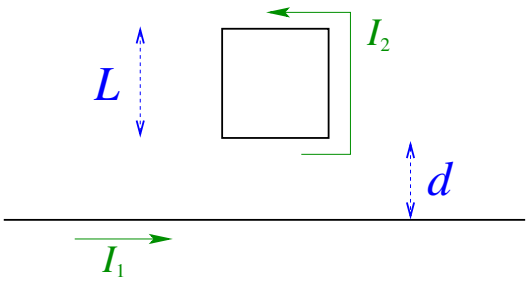
(f) **[SELF]** Express l as a function of θ, and use this equation to find dl as a function of θ and $d\theta$.

(g) **[SELF]** Express the magnitude of $d\mathbf{B}$ in terms of L, θ, and $d\theta$.

(h) **[SELF]** Find again the magnetic field at P due to the current in the side AB, by integrating over the angle θ. What are the limits of integration?

(i) **[3 pts]** You have found the magnetic field at P due to the current through one side of the square-shaped wire. What is the total magnetic field at P due to the complete current-carrying square wire?

3. A square-shaped wire loop (each side of length L) carrying steady current I_2, is placed near an infinitely long straight wire carrying steady current I_1, with the nearest side being at distance d away. (See figure.)



(a) **[5 pts]** Each side of the square experiences a force due to the magnetic field produced by the long wire. What are the directions of the force on each side? (Either draw the square with arrows showing the forces, or state clearly in words.)

(b) **[9 pts]** Calculate the total force acting on the square loop due to the current in the long wire. Hint: the forces on two of the sides will cancel by symmetry and thus don't need to be calculated.