Due on Monday, April 26th.

If pictures are needed/relevant, please provide them with your solutions.

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1. [**4 pts**] Consider a capacitor consisting of two parallel metallic plates, each of area A. We showed in an earlier problem set that, when the plates carry opposite charges $+Q$ and $-Q$, the electric field in the region between the plates has magnitude $E = Q/(A\epsilon_0)$.

   Consider now the situation that the capacitor is *discharging*, i.e., the magnitude of the charge on each plate is decreasing, as $Q(t) = Q_0 e^{-t/T_0}$. Find the displacement current density in the region between the two plates.

2. An electromagnetic field is described by the scalar potential V and vector potential $\mathbf{A}$, given by

$$V = \frac{2B_0}{\mu_0 \epsilon_0} t \,; \quad \mathbf{A} = B_0 \left[x\hat{i} + (y \sin \omega t - 3z)\hat{k} \right]$$

   where B_0 and ω are positive constants.

   (a) [**4 pts**] Find the electric and magnetic fields in this system.

   (b) [**4 pts**] Show explicitly that Maxwell's third equation is satisfied. (Maxwell's 3rd eq. is Faraday's law; involves the curl of **E**.)

   (c) [**SELF**] If **E** and **B** are derived from potentials, then the 3rd equation is expected to be satisfied automatically. Why? Which of Maxwell's other equations is satisfied automatically?

   (d) [**4 pts**] Find the displacement current density $\mathbf{J}_D$ and the current density **J**.

   (e) [**3 pts**] Show whether or not the continuity equation is satisfied.

3. (a) [**3 pts**] Maxwell's first equation in differential form is an expression for the divergence of the electric field. Rewrite this equation in terms of the scalar and vector potentials, V and **A**.

   (b) [**3 pts**] Write down the Coulomb gauge condition for the vector potential. Use this condition to express Maxwell's first equation as an equation for the scalar potential V.

4. A square loop with sides of length L lies in the x-y plane in a region in which the magnetic field points in the z-direction and changes over time as

$$\mathbf{B}(t) = B_0 e^{-5t/t_0} \hat{k}$$

   (a) **[4 pts]** Find the magnitude of the EMF induced in the wire, and sketch a plot of the EMF as a function of time.

   (b) **[2 pts]** You are looking from 'above' onto the x-y plane, so that the positive z-axis points out of the paper toward you. What is the direction of the induced current in the loop? (Counterclockwise or clockwise?) You will only get credits for your answer if you justify it clearly using Lenz's law.

5. A thin glass rod of length L lies along the y axis with one end at the origin and the other end at $(0, L, 0)$. The rod carries a uniformly distributed postive charge Q.

   (a) **[6 pts]** Consider the point $(0, y_0, 0)$, also on the y axis, with $y_0 > L$. Calculate the electric field generated at this point. You will have to first consider an infinitesimal element of the rod, and then integrate over appropriate limits.

   (b) **[2+1 pts]** Consider the limit $y_0 \gg L$ and approximate your result for this limit. Explain why you could have expected this result for the electric field far from the rod, by comparing with the field due to a point charge.

   (c) **[10 pts]** Consider the point $(x_0, 0, 0)$ on the x axis. Calculate the electric field generated at this point.

   In this case you will have to consider both components of the field created by the infinitesimal element, and integrate separately. You might need the integrals

$$\int \frac{du}{(u^2 + a^2)^{3/2}} = \frac{u}{a^2\sqrt{u^2 + a^2}}\,; \qquad \int \frac{u\,du}{(u^2 + a^2)^{3/2}} = -\frac{1}{\sqrt{u^2 + a^2}}\,.$$