(2) * Two systems of units in use! (at least)

Example using Coulomb law:

~~SI / MKSI~~

$$\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^2} \left(\widehat{\vec{r}_1 - \vec{r}_2}\right) = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{r}$$

CGS / Gaussian

$$\vec{F} = \frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^2} \left(\widehat{\vec{r}_1 - \vec{r}_2}\right) = \frac{q_1 q_2}{r^2} \hat{r}$$

* This module: use SI units, but look up or learn the CGS form for some important equations

* Aim of this class: MAXWELL'S EQUATIONS

→ unify electricity & magnetism, predict electromagnetic waves (light, X-rays, γ-rays, radio-waves, microwaves).

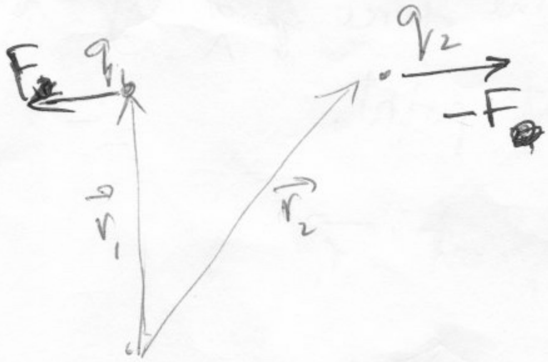
$$\vec{\nabla} \cdot \vec{E} = \frac{1}{\epsilon_0} \rho$$ (Coulomb's law / Gauss's law)

$$\vec{\nabla} \cdot \vec{B} = 0$$ (absence of magnetic monopoles)

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$ (electromagnetic induction / Faraday's law)

$$\vec{\nabla} \times \vec{B} = \frac{1}{\epsilon_0 c^2} \vec{J} + \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t}$$

Coulomb's law



~~Charge~~ Force felt by charge q_1

$$\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^2} \widehat{(\vec{r}_1 - \vec{r}_2)}$$

* $\wedge$ indicates unit vector in direction of $\vec{r}_1 - \vec{r}_2$

* ϵ_0 = permittivity of free space

$\simeq 8.85 \times 10^{-12}$ Coulomb^2/Newton-m^2

* Question: does the origin of the coordinate system matter?

* $q_1(\vec{r}_1)$, $q_2(\vec{r}_2)$

$Q(\vec{r})$

$q_3(\vec{r}_3)$ ...

Force on charge Q at position $\vec{r}$ due charges $q_1, q_2, q_3 \ldots$ at $\vec{r}_1, \vec{r}_2, \ldots$

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \ldots$$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{q_1 Q}{|\vec{r}-\vec{r}_1|^2}\widehat{(\vec{r}-\vec{r}_1)} + \frac{q_2 Q}{|\vec{r}-\vec{r}_2|^2}\widehat{(\vec{r}_0-\vec{r}_2)} + \ldots\right]$$

$$= Q\vec{E} \longleftarrow$$

④

Electric Field

Def^n $\vec{E}$ at point $\vec{r}$ is the force experienced by a unit charge at that point.

* Due to single charge q_1 at $\vec{r}_1$:

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{q_1 \cdot 1}{|\vec{r}-\vec{r}_1|^2}\left(\widehat{\vec{r}-\vec{r}_1}\right)$$

Due to collection of charges, $q_1, q_2, \ldots$ at $\vec{r}_1, \vec{r}_2, \ldots$:

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{|\vec{r}-\vec{r}_1|^2}\left(\widehat{\vec{r}-\vec{r}_1}\right) + \frac{q_2}{|\vec{r}-\vec{r}_2|^2}\left(\widehat{\vec{r}-\vec{r}_2}\right) + \ldots\right)$$

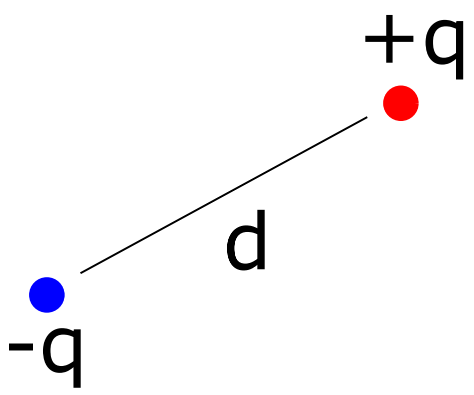
$$= \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{|\vec{r}-\vec{r}_i|^2}\left(\widehat{\vec{r}-\vec{r}_i}\right)$$

$$= \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{\mathscr{r}^2}\hat{\mathscr{r}} \quad \text{(Griffiths' notation)}$$

* "Principle of superposition" $\longrightarrow$

$\vec{E}$-field due to collection of charges is the sum of $\vec{E}$-fields due to each individual charge.

Electric dipole



Vector with magnitude $p = qd$

and direction from negative to positive charge

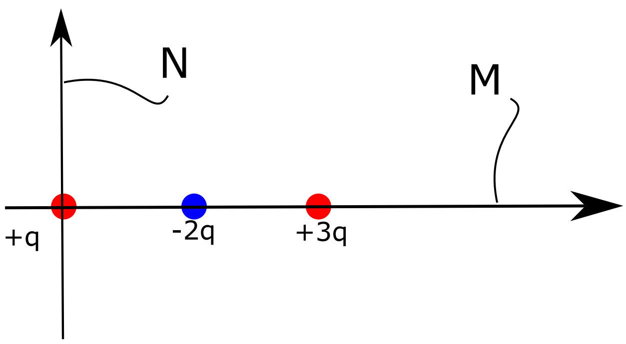
Ideally: each is a point charge, and d is very small.

In practice, charges could be extended.

Then you would calculate the direction

vector from one center of charge to another.

Electric field due to collection of charges



Electric field at point M?

Electric field at point N?.

Electric field at point M? Calculate Field due to EACH charge at M. Lucky they all have only x-components. Use vector addition.

Electric field at point N? Calculate field due to EACH charge at N. Not so lucky now. Use vector addition.

⑥ LECTURE 2 [redacted]

[redacted]

Electric Potential

With any static electric field is associated a SCALAR FIELD,

$$\vec{E}(\vec{r}) = -\vec{\nabla} V(\vec{r}) = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right)$$

Will sometimes use $\hat{x}, \hat{y}, \hat{z}$ instead of $\hat{i}, \hat{j}, \hat{k}$

Remarkable! a scalar ~~function~~ field (one number per position) gives all the information of a vector ~~function~~ field (3 numbers per position).

$$* \quad dV(\vec{r}) = -\vec{E}\cdot d\vec{r}$$

$$V(\vec{r}) = V(\vec{r}_0) - \int_{\vec{r}_0}^{\vec{r}} \vec{E}\cdot d\vec{r}$$

Work done to bring a unit charge from $\vec{r}_0$ to $\vec{r}$.

"line integral"

"Differentiation is opposite of integration – 3D version"

$V(\vec{r}_0)$ is constant of integration

Note that a reference point $(\vec{r}_0)$ has to be specified. The path between $\vec{r}_0$ and $\vec{r}$ in the integral doesn't matter.

Usually $\vec{r}_0$ taken to be at infinity, far from charges, so $V(\vec{r}_0) = 0$

* Potential due to single charge (q_1 at $\vec{r}_1$)

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{q}{|\vec{r}-\vec{r}_1|}$$

no unit vector! Why?

* EXERCISE: Show, $V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{q_1}{|\vec{r}-\vec{r}_1|}$ leads to $\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q_1}{|\vec{r}-\vec{r}_1|^2}(\widehat{\vec{r}-\vec{r}_1})$

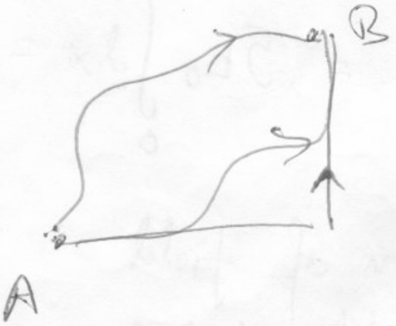
* Principle of superposition

q_1 at $\vec{r}_1$, q_2 at $\vec{r}_2$, ...

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1}{|\vec{r}-\vec{r}_1|} + \frac{q_2}{|\vec{r}-\vec{r}_2|} + \cdots\cdots\right)$$ a scalar sum

$\longrightarrow$ Do line of charge (p. 7a-7b) first

* Potential difference between two points:



$$V(\vec{r}_B) - V(\vec{r}_A) = -\int_{\vec{r}_A}^{\vec{r}_B} \vec{E}\cdot d\vec{r}$$

= the work done in carrying a unit charge from A to B.

Work done in carrying charge q is $W = q\Delta V$

* Relation between el. potential and $\vec{E}$-field is similar to relation between potential energy & force.

* Difference of electric potentials = voltage difference between two points

* Unit of potential: Joule/Coulomb = Volt

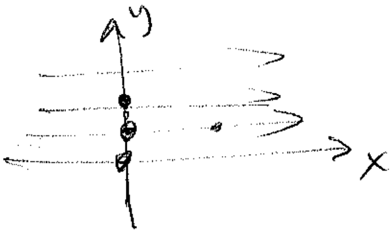
Unit of $\vec{E}$-field: N/Coulomb = Volt/m

7'

* Example

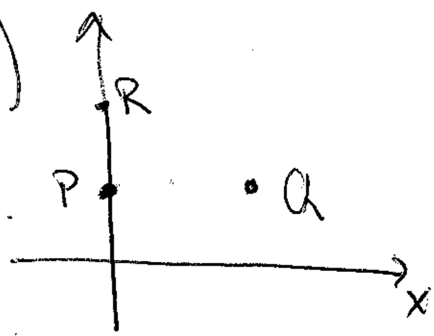
EASY CASE : CONSTANT $\vec{E}$-FIELD

$$\vec{E} = 5E_0\,\hat{i}$$



$P(0, d_0)$, $Q(l, d_0)$

$R(0, 2d_0)$



$$V_{QP} = V_Q - V_P = -\int_{\vec{r}_P}^{\vec{r}_Q} \vec{E}\cdot d\vec{r}$$

$$= -\int_{x=0}^{x=l} \left(5E_0\,\hat{i}\right) dx\,\hat{i} = 5E_0 \int_0^l dx = 5E_0\, l$$

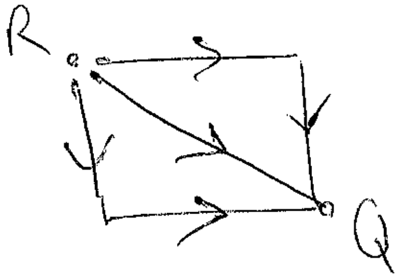
Voltage difference in direction of field = field $\times$ distance

Exercise

① Show $V_{PR} = 0$

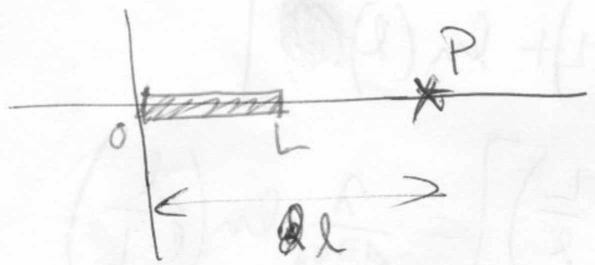
Voltage difference $\perp$ to field $= 0$

② Calculate $V_{RQ} = V_Q - V_R$



Can choose any path!
should get same result.

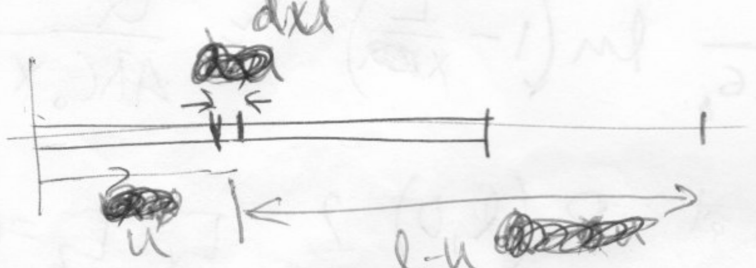
Example: line of charge:



Linear charge density λ

Total charge on line/rod: $Q = \lambda L$

Electric Potential at point $P(l, 0)$?



Take element of length du

Could be $(u, u+du)$

or $\left(u - \frac{du}{2}, u + \frac{du}{2}\right)$,

doesn't matter.

Can think of element as point charge λdu

Potential at P due to this element:

$$dV = \frac{1}{4\pi\epsilon_0} \frac{\lambda\, du}{l - u}$$

Total potential?

$$V = \int dV = \frac{\lambda}{4\pi\epsilon_0} \int_0^L \frac{du}{l - u}$$

Integrate from $u=0$ to $u=L$

"Adding" over all infinitesimal elements

(7b)

$$V(l,0) = \frac{\lambda}{4\pi\epsilon_0}\left[-\ln(l-u)\right]_{u=0}^{u=L}$$

$$= \frac{\lambda}{4\pi\epsilon_0}\left[-\ln(l-L)+\ln(l)\right]$$

$$= \frac{\lambda}{4\pi\epsilon_0}\left[-\ln\left(1-\frac{L}{l}\right)\right] = \frac{\lambda}{4\pi\epsilon_0}\ln\left(\frac{l}{l-L}\right)$$

* Exercise : What if $l >> L$?

$$V(x,0) = -\frac{\lambda}{4\pi\epsilon_0}\ln\left(1-\frac{L}{x}\right) \simeq \frac{Q}{4\pi\epsilon_0 x}$$

* Electric field at $P(l,0)$? $E_y = E_z = 0$

Due to the element : $dE_x = \frac{1}{4\pi\epsilon_0}\frac{\lambda\, du}{(l-u)^2}$

$$E_x = \frac{\lambda}{4\pi\epsilon_0}\int_0^L \frac{dx}{(l-u)^2} = \frac{\lambda}{4\pi\epsilon_0}\left[\frac{+1}{(l-u)}\right]_0^L$$

$$= \frac{\lambda}{4\pi\epsilon_0}\left(\frac{1}{l-L}-\frac{1}{l}\right)$$