Due on Monday March 29th.

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1. Consider a set of inertial frames which are aligned at the commonly defined zero time, but might be moving in different relative directions.

   (a) [**1+2 pts**] Write down the Lorentz boost matrices for relative velocity v_1 between frames in the x-direction, and for relative velocity v_2 in the y-direction.

   (b) [**5 pts**] Find out by matrix multiplication the result of successive application of these two transformations. Consider both possible orderings: applying the x-direction boost first and then the y-direction boost, and vice versa.

   This will look cumbersome, so I suggest that you use $c = 1$ units and the notation $\gamma_1 = \gamma(v_1)$, $\gamma_2 = \gamma(v_2)$. Another hint: do you really need to multiply 4×4 matrices?

   (c) [**3 pts**] Do the two Lorentz boost matrices commute? Compare with Lorentz boosts in the same direction.

   (d) [**1+2 pts**] Consider any of the matrices resulting from successive application of the two boosts. Is the matrix symmetric?

   Remembering from Problem Set 5 that Lorentz boost transformations in arbitrary directions are symmetric matrices, interpret this observation.

2. The kinetic energy of motion of a particle is the relativistic total energy minus the rest energy.

   (a) [**2 pts**] A particle has rest mass M and speed v. If $v \ll c$, then show that the kinetic energy due to motion is approximated by the well-known non-relativistic expression for the kinetic energy.

   (b) [**3 pts**] Derive the leading correction to the non-relativistic expression.

   (c) [**3 pts**] What is a particle's speed if its kinetic energy T is equal to its rest energy? What is a particle's speed if its kinetic energy is n times its rest energy?

   (d) [**3 pts**] Express the momentum p of the particle as a function of the mass M and the kinetic energy of motion, which we will call T. Your expression should contain M and T, not v or γ_v.
   Hint: use the relation between E and p that does not involve v or γ_v.

3. Relative velocities, longitudinal & transverse.

   (a) **[2 pts]** A rocket traveling at speed $\frac{1}{2}c$ with respect to frame Σ shoots forward bullets at speed $\frac{3}{4}c$ relative to the rocket. What is the speed of the bullets relative to Σ?

   (b) **[5 pts]** Now imagine that the bullets are fired *perpendicular* to the direction of rocket motion, as perceived from the rocket. What is the speed of the bullets relative to Σ?

   (c) **[4+2 pts]** An X-ray beam is sent out from the rocket, perpendicular to the direction of rocket motion. Relative to Σ, what are the components of the X-ray beam velocity parallel and perpendicular to the direction of rocket motion?

   Calculate from your velocity components the speed of the X-ray beam relative to Σ. Explain why the speed relative to Σ could have been expected.

4. A stick of proper length L moves past you at speed v. There is a time interval between the front end passing you (first event) and the back end passing you (second event). We are interested in this time interval and in the length of the stick, as seen from various frames.

   (a) **[1 pt.]** Draw snapshots of the two events as seen from your frame, marking with an arrow what is moving (you or the stick).

   (b) **[4 pts.]** Find the length of the stick as seen from your frame, and the time between the two events, as seen from your frame.

   (c) **[1 pts]** Draw snapshots of the two events as seen from the frame of the stick, marking with an arrow what is moving, as seen from this frame.

   (d) **[4 pts.]** Find the length of the stick and the time between the two events, as seen from the frame of the stick.

   (e) **[2 pts.]** In which frame is the time interval the *proper* time interval: your frame or the stick's frame? Explain why.