Due on Monday, April 26th

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1. As measured in frame Σ, the coordinates of events A and B are respectively (ct_A, x_A) and (ct_B, x_B). The interval between the events are spacelike.

   There is then a frame Σ' in which the two events are simultaneous.

   (a) [**4 pts**] Plot the two events on a spacetime diagram, taking the Σ frame to have ct and x axes perpendicular to each other. Plot also the axes of the Σ' frame, so that it is clear that A and B are simultaneous in this frame. You only get credits if this last feature is evident in your drawing.

   (b) [**4 pts**] Using the spacetime diagram, calculate the speed v of the Σ' frame relative to the Σ frame, in terms of t_A, t_B, c, x_A, x_B.

   (c) [**3 pts**] Now calculate the same expression for v using the Lorentz transformations.

2. [**4 pts**] The 4-velocity and 4-acceleration of a particle are given by

$$V \;=\; (\gamma_v c,\; \gamma_v \vec{v})\,, \qquad A \;=\; \left(\frac{\gamma_v^4}{c} v\dot{v},\; \frac{\gamma_v^4}{c^2} v\dot{v}\vec{v} + \gamma_v^2 \vec{a}\right)\,,$$

   where $\vec{v}$ is the 3-velocity, $v = \sqrt{\vec{v}\cdot\vec{v}}$ is the speed, and $\dot{v} = \dfrac{dv}{dt}$ is NOT the 3-acceleration $\vec{a}$. Show that the 4-velocity and 4-acceleration are 'orthogonal' to each other, i.e., have zero inner product.

   From problem set 1, we know $v\dot{v} = \vec{v}\cdot\vec{a}$.

3. Four-momenta.

   (a) [**3 pts**] Show that the four-momentum of a particle with nonzero mass is always timelike.

   (b) [**3 pts**] Write down the four-momentum of a photon moving in the y direction. The frequency of the photon is denoted by f.

   (c) [**4 pts**] Find out whether the 4-momentum of a photon is timelike, spacelike, or lightlike/null.

4. Four-velocities.

   (a) [**4 pts**] Write down the four-velocity of a particle with nonzero mass m moving in the y direction with speed v. Show that this four-vector is timelike.

   (b) [**5 pts**] Find the four-velocity of a particle with nonzero mass m and velocity $\vec{u} = (c/2, c/2, c/2)$. Show that this four-vector is timelike.

   Is this particle massless?

   (c) [**2 pts**] What is the four-velocity of the particle with mass m in its own rest frame, i.e., in the frame where it is at rest?

   (d) [**2 pts**] Try writing down the four-velocity of a photon moving in the y direction. Is there a problem?

5. Collision/decay problems.

   (a) [**6 pts**] A π-meson (a.k.a. pion) of mass M at rest disintegrates into a muon of mass m_μ and a neutrino of effectively zero mass. Show that the kinetic energy of motion of the muon is

$$T = \frac{(M - m_\mu)^2}{2M} c^2$$

   As usual: first draw the situations before and after the reaction/decay, write down equations for energy and momentum conservation in terms of the most useful variables, and then manipulate these equations as required.

   You should decide early whether it is helpful to work with speed and γ, or whether to avoid these and work with momentum instead.

   You probably want to use $c = 1$ units for this problem.

   (b) [**6 pts**] A neutral pion of rest mass m, while moving in the positive x direction, decays into two photons. One of the photons is emitted in the same direction as the original pion, and the other in the opposite direction. If the first photon has three times the energy of the second, find the pion's original speed.